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Letter to the Editor

Transmissibility of strain produced in PVDF actuator to elastic beam

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1. Introduction

The use of the piezoelectric materials as sensors and actuators in active vibration control of highly flexible structures is the research topic since decades. One such material is the piezopolymer—polyvinylidene fluoride (PVDF). Fuller et al. [1], and Sunar and Rao [2] gave an exhaustive literature survey on vibration sensing and vibration control of flexible structures using such piezoelectric materials. It is observed that the use of the PVDF film as vibration sensor and as actuator for flexible structures is well recognized. However, it is important to know at the design stage as to how much fraction of the strain produced in the piezoelectric materials when used as an actuator due to high voltage input analog signal is getting transferred to the structure. This will obviously depend upon the physical and material properties of both film and structure as well as the effectiveness of film bonding on the structure by the adhesive. Since the film is very flexible compared to the structures, the strain transferred to the structure is expected to be small. In fact, Fuller et al. [1] gave a material-geometric constant for the piezoelectric actuator based on the assumption that the actuator is perfectly bonded to the surface of the structure, which can be used to estimate such strain transmissibility factor (STF). Fuller et al. [1] have further demonstrated the use of this constant in the analytical prediction by comparing the measured responses of a beam obtained experimentally by Clark et al. [3] using the piezoelectric ceramic actuator (PZA). In the paper, the STF has been estimated based on the experimentally observed modal response of the cantilever beam when the PVDF films have been used as the actuator through a finite element (FE) simulation of experiments. The estimated STF has also been compared with the theory given in Ref. [1].

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2. Experiments using PVDF films as an actuator

The schematic of the laboratory test setup of a cantilever beam is shown in Fig. 1. The physical dimensions and the material properties are listed in Table 1. Initially, the modal tests were carried out on the cantilever beam by a small non-contacting shaker using step sine excitation [4]. The frequencies—10.625 and 67.25 Hz have been identified as the first two bending modes of the beam and 3.85% damping measured at 10.625 Hz. The high damping seen is due to the bonding of the PVDF films on the beam; otherwise, for the beam alone, the experimentally observed damping at first mode was 0.213%.

For the present study, the readily available PVDF films from M/s. Kynar Film [5] have been used. The physical dimensions and constructional details of different PVDF films used in the experiment are shown in Fig. 2. The material properties and physical dimensions of a typical PVDF film are also listed in Table 1 for the easy comparison with the beam properties. To test the effectiveness of the different PVDF films as actuator, a couple of experiments was conducted according to the schematic diagram shown in Fig. 1.

The PVDF films have been bonded on the surface of the beam using cyanoacrylate adhesive. This is a kind of adhesive that is recommended by Kynar Piezo Film technical manual [5]. It has been bonded near the fixed end of the cantilever as shown in Fig. 1. One and two films together have been used. The use of two films together (mounted on either side of the beam) was expected to produce an excitation double compared to that of one film. It is because if one film contracts, the other will expand in case the input voltage supply is reversed and hence in turn high excitation level.

The beam was excited at its first mode by the PVDF films as an actuator using sinusoidal analog voltage input signal to the films. The maximum voltage level was ±100 V. The test was conducted with one PVDF-1 film as an actuator and two PVDF-2 films simultaneously acting as an actuator. The response spectrum of the beam using PVDF-1 film is shown in Fig. 3. The

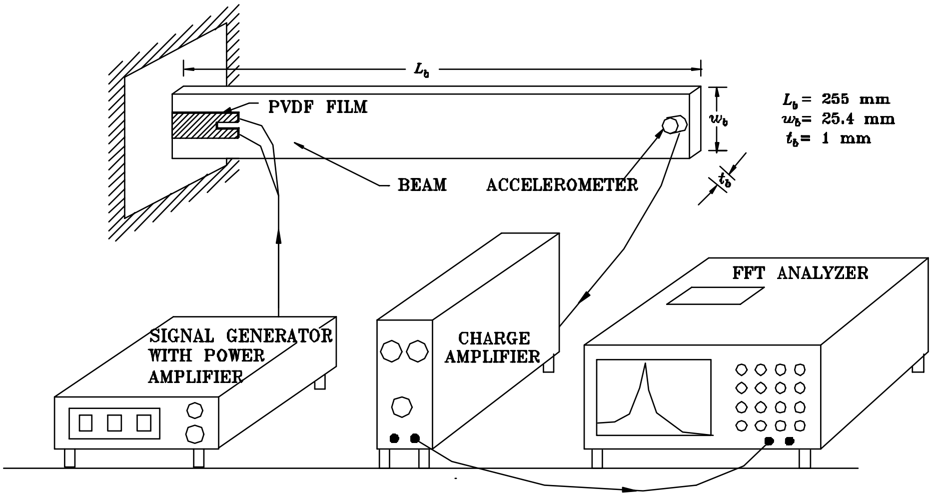

Fig. 1. Laboratory test setup of the cantilever beam.

Table 1
Properties of the beam and the PVDF film

Parameters	Cantilever beam	Piezoelectric film
Material	Stainless steel	PVDF
Young's modulus (E) (kg/m^2)	2.0×10^{10}	2.0×10^{8}
Density (ρ) (kg/m^3)	7800.0	1780.0
Poisson ratio (μ)	0.30	0.30
Thickness (t) (µm)	1000.0	28.0
Width (w) (mm)	25.4	15.0
Length (L) (mm)	255.0	35.0
Piezo-strain coefficient (d_{31}), (m/m)/(V/m)	—	23×10^{-12}

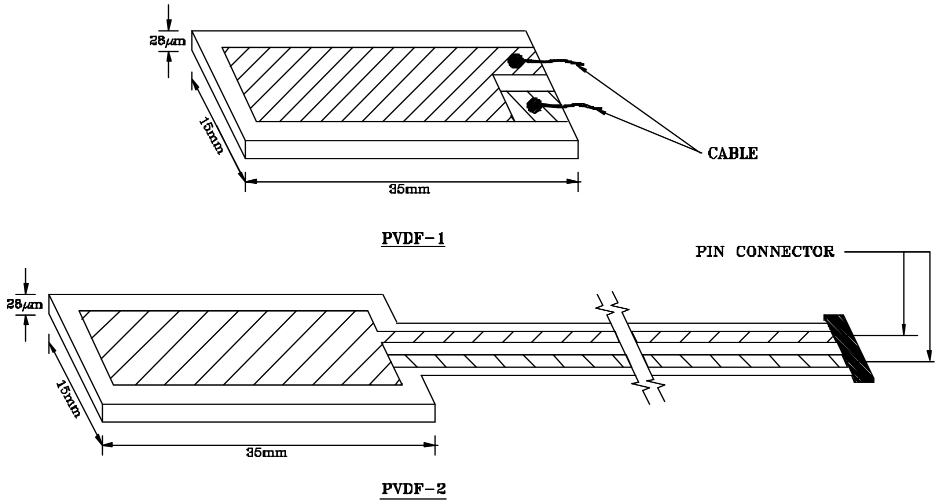

Fig. 2. Different types of PVDF films used in the experiment.

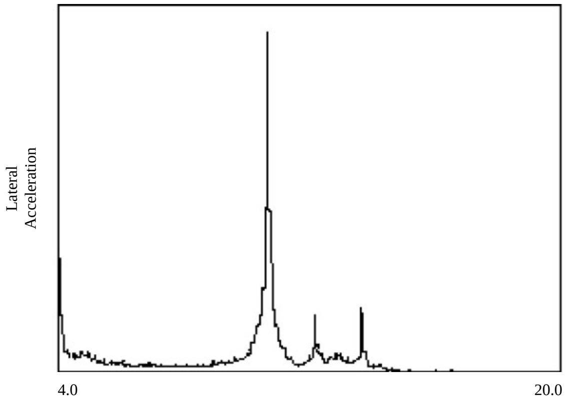

Fig. 3. Frequency response of beam due to PVDF actuator.

Table 2
Effectiveness of different PVDF films as actuator

PVDF film		Max. voltage supplied (V)	Max. displacement at free end (y) (μm) (peak)	STF (f)	
Type	Quantity			Estimated	Theoretical
PVDF-1	1	100	152	8.138×10^{-4}	8.625×10^{-4}
PVDF-2	2	100	330	8.138×10^{-4}	8.620×10^{-4}

maximum steady state response at the free end of the beam observed to this excitation by different PVDF films used is listed in Table 2. As can be seen from Table 2, the response observed in case of PVDF-1 is less than half compared to PVDF-2 for the same input voltage level though they are dimensionally equal (see Fig. 2). The riveted joints at the connectors could be reducing the effective length of film actuator in the case of PVDF-1 and hence reducing the beam response.

3. Estimation of STF

The differential equation of motion suggested in Ref. [1] for the beam-piezoelectric actuator system can be used to simulate the experiments. However, considering the difficulties in the boundary condition simulation in the differential equation of motion, the FE method [6] is used which is the most appropriate for such practical problems, and can handle complex geometric and different boundary conditions for structure. Two noded Euler–Bernoulli beam elements have been used to model the cantilever beam of the experimental setup. Since the beam was mechanical clamped at one end, it may have zero displacement (i.e., pin support) but may allow some rotation at the clamp end. Hence, a rotational boundary stiffness (k_θ) was adjusted in the FE model using gradient-based model updating method [7,8] so that the computed natural frequencies match well with the experimental natural frequencies. Apart from the model updating, the masses of the accelerometer and cables connected to the PVDF films, though they are very small, were also included in the model to avoid a small difference between the experiment and the FE simulation. A rotational stiffness (k_θ) of 35 N m/rad has been realized to simulate the boundary stiffness of the beam in the setup and the computed frequencies are 10.58 and 67.65 Hz, which are close to the experimental values. A stiffness-proportional damping matrix has also been included in the updated FE model using the measured modal damping. Hence the updated FE model shown in Fig. 4 has now been an accurate representation of the experimental setup of the cantilever beam, and it has been used to estimate the STF.

It is generally assumed that the strain in finite length piezoelectric actuator due to the analog sinusoidal voltage input produces a uniform moment over the length of the film to the beam [1]. The induced moment distribution $M(x,t)$ is derived as below.

The input analog signal at high voltage would cause the strain in the PVDF film actuator. This strain, $\varepsilon_a(t)$, is given by [5]

$$\varepsilon_a(x,t)=\frac{n_f d_{31} v(t)}{t_a}, \tag{1}$$

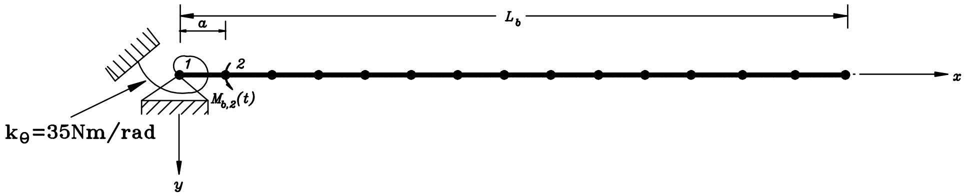

Fig. 4. FE model of beam used in the study.

where $v(t)$ is the input voltage of analog signal to the PVDF Film. d_{31} is the property of the PVDF film which relates the strain produced in one direction (length) to the voltage applied in three directions (thickness). n_f and t_a are the number of films used in experiment and thickness of the film, respectively. The strain induced in the actuator would not be totally transferred to the beam due to wide difference in material and geometrical properties of the PVDF actuator and the beam, and the effectiveness of bonding of actuators to the beam surfaces. Thus, the strain in the beam, $\varepsilon_b(x,t)$, due to strain in the PVDF film actuator can be written as

$$\varepsilon_b(x,t) = \varepsilon_a(t)f, \tag{2}$$

where f = STF, the strain transferred to the beam to the total strain in the actuator. The strain in the beam produces bending action. The bending moment distribution $M(x,t)$ over the actuator length induced due to this bending action can be written as

$$M(x,t) = \frac{E_b \varepsilon_a(t) I_b}{t_b/2} f = \frac{2 n_f E_b I_b d_{31} v(t)}{t_a t_b} f, \tag{3}$$

where E_b, t_b and I_b are Young's modulus of elasticity of beam material, thickness and the area moment of inertia of the beam, respectively. The moment $M(x,t)$ is derived based on the assumption that widths of the beam and the PVDF film are the same. But for the present case the widths are different; hence this needs to be accounted for in Eq. (3) for the moment:

$$M_c(x,t) = M(x,t)\frac{w_a}{w_b}, \tag{4}$$

where w_a and w_b are the widths of the PVDF film and the beam, respectively. It is difficult to directly include this uniformly distributed moment over the finite length of the actuator in the FE model. Hence an equivalent bending moment (see Fig. 4) acting at the mean position of the film i.e., $x = a$ (at node 2 for present case) was assumed, which is given by the following relation:

$$M_{b,2}(t)a = M_c(x,t)L_a. \tag{5}$$

The responses were computed using the equivalent bending moment. The Nemark-β method was used for numerical solution of the system dynamic equation. The time step used was 1/40th of the first natural period of the beam. A value of STF (f) equal to 8.138×10^{-4} is required to compute the 330 μm (peak) deflection same as observed experimentally for the case of the two PVDF-2 films acting together as an actuator. The experiment with one PVDF-1 film was also simulated using FE model. The steady state response is computed to be 155.80 μm (peak) when the STF,

$f = 8.138 \times 10^{-4}$ and the 90% (reduced effective length due to riveted connection at one end as shown in Fig. 2) of the PVDF-1 film length is considered. This computed response is also close to the experimental observation (see Table 2). Hence, the estimated STF, $f = 8.138 \times 10^{-4}$ could be a correct value for a combination of the PVDF films, beam and cyanoacrylate adhesive used in the experiment.

4. Active damping vibration control

The active vibration control of the beam of the test setup was also carried out using the PVDF actuator to further check the estimated STF. The schematic of the experiment is shown in Fig. 5. In the experiment, the disturbance in the cantilever beam of the test setup was generated by exciting at the first mode by the non-contacting shaker as shown in Fig. 5. The response measured by accelerometer was used to drive the PVDF actuator to attenuate the beam vibration actively. The velocity feedback (i.e., damping control) was used. The steady state vibration signal from the accelerometer was first converted into velocity signal. The voltage gain for the steady state velocity signal was initially adjusted to the maximum possible voltage of 100 V through control unit and the amplified signal was fed back to the PVDF actuator. The attenuation in the amplitude of the beam vibration was observed to be quite effective. This exercise was repeated for different levels of the external disturbance to the beam and their control using different PVDF films as actuators. The observations and results are summarized in Table 3.

An FE simulation of these experiments was also carried out using the STF, $f = 8.138 \times 10^{-4}$ for the PVDF actuator. It was observed that the computed responses were closely matching with the experimental observations, hence further confirming the reliability of the estimated STF.

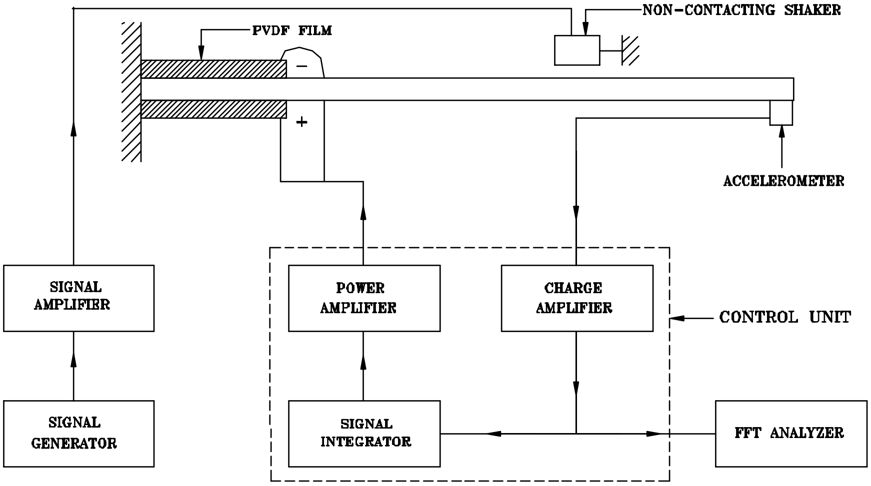

Fig. 5. Schematic of the test setup for active vibration control.

Table 3
Velocity feedback at the first mode of the beam

Film type	Without feedback			With feedback		
	Displ., y (μm) (peak)	Damping, ξ (%)	Static displ., $y_{st} = 2\xi y$ (μm) (peak)	Displ., y_f (μm) (peak)	Reduction $y - y_f$ (μm) (peak)	Damping achieved, $\xi_f = y_{st}/2y_f$ (%)
PVDF-1	186.00	3.85	14.32	33.76	152.24	21.21
	465.20	3.85	35.82	129.22	335.98	13.86
PVDF-2	497.92	3.85	38.34	168.62	329.30	11.37
	740.16	3.85	57.00	394.00	346.16	7.23

5. Comparison with theoretical STF

By comparing the expression for the moment distribution in Ref. [1] with Eq. (3), the theoretical STF, f can be written as

$$\frac{n_f f}{t_b/2} = K^f, \tag{6}$$

where the material-geometric constant

$$K^f = \frac{12E_bE_a(t_b/2)t_a(t_b + t_a)}{16E_b^2(t_b/2)^4 + E_bE_a(32(t_b/2)^3t_a + 24(t_b/2)^2t_a^2 + 8(t_b/2)t_a^3) + E_a^2t_a^4}$$

for one PVDF film actuator assuming the axial strain in the beam is negligible.

$$K^f = \frac{3E_a((t_b/2 + t_a)^2 - t_b^2)}{2E_a((t_b/2 + t_a)^3 - t_b^3) + E_b(t_b/2)^3}$$

for two films acting simultaneously as an actuator, where E_a is the modulus of elasticity for the PVDF film. The theoretically estimated values for the STF (f) are 8.625×10^{-4} and 8.620×10^{-4} for the PVDF-1 and PVDF-2, respectively. These values are listed in Table 2, and are slightly higher (nearly 5.6%) than the estimated values from the experiments. This indicates that the adhesive used for the bonding of the films on the beam surface causes small reduction in the strain transmissibility.

6. Conclusion

The actuation action of PVDF films and their use for active vibration control has been demonstrated through laboratory experiment on a cantilever beam. Based on the experiments and the FE simulation, the observation made is that only 0.08138% of the strain produced in the PVDF films is getting transferred to the beam when the film used as an actuator, and this is found to be in excellent agreement with the theory. The observation of a slightly lower estimated STF compared to the theoretical STF indicates the effect of the adhesive used for bonding the PVDF films on the beam surface. The effect of the different types of adhesive for bonding the films on the STF requires further study.

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References

[1] C.R. Fuller, S.J. Elliott, P.A. Nelson, Active Control of Vibration, Academic Press, London, 1996.

[2] M. Sunar, S.S. Rao, Recent advances in sensing and control of flexible structure via piezoelectric materials technology, Applied Mechanics Review 52 (1999) 1–16.

[3] R.L. Clark, C.R. Fuller, A.L. Wicks, Characterization of multiple piezoelectric actuators for structural excitation, Journal of the Acoustical Society of America 90 (1991) 346–357.

[4] D.J. Ewins, Modal Testing: Theory, Practice and Application, 2nd Edition, Research Studies Press, Hertfordshire, England, 2000.

[5] Kynar Piezo Film Technical Manual, Pennwalt Corporation, King of Prussia, PA, USA, 1983.

[6] O.C. Zienkiewicz, R.L. Taylor, The Finite Element Method: Vol. 1, Basic Formulations and Linear Problems, Fourth Edition, McGraw-Hill, New York, 1994.

[7] M.I. Friswell, J.E. Mottershead, Finite Element Model Updating in Structural Dynamics, Kluwer Academic Publishers, Dordrecht, 1995.

[8] J.K. Sinha, M.I. Friswell, Model updating: a tool for reliable modelling, design modification and diagnosis, The Shock and Vibration Digest 34 (2002) 27–35.