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Authors' reply ☆

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We thank Mr. Wu very much for pointing out that a possible root $w = 0$ of the derived function $F(w)$ (or $G(w^2)$) was not taken into consideration in the delay-independent stability analysis in our previous paper (jsvi.1999.2282). This incompleteness makes it *possible* that some special points are not included in the delay-independent stable region, as in the two mentioned examples. Mr. Wu suggested in Ref. [1] changing the conditions "$F(w)$ ($G(w^2)$) has *no* real roots" to "$F(w)$ ($G(w^2)$) has *no nonzero* real roots". In our opinion, this may lead to a misunderstanding, since "negation of negation = affirmation". As seen below, the exact meaning that we want to express is "either $F(w)$ ($G(w^2)$) has no real roots or $w = 0$ is the only one if $F(w)$ ($G(w^2)$) has any real roots", or equivalently, "there is a non-negative integer k and a polynomial $\bar{F}(w)$ ($\bar{G}(w^2)$) that has no real roots such that $F(w) = w^k \bar{F}(w)$ ($G(w) = w^k \bar{G}(w)$)". We prefer to say "$F(w)$ ($G(w^2)$) cannot have real roots *other than zero*" for short.

To gain an insight into the case when $w = 0$ is a root of $F(w)$ ($G(w^2)$), let us first assume that $\tau_1 = \tau_2 = \tau$. Let

$$D(\lambda, \tau) = P(\lambda) + Q(\lambda)e^{-\lambda\tau}, \quad F(w) = |P(iw)|^2 - |Q(iw)|^2. \tag{1}$$

By following the notations of our paper, $D(iw, \tau) = 0$ gives the real and imaginary parts as follows:

$$r(w) \equiv P^R(w) + Q^R(w)\cos w\tau + Q^I(w)\sin w\tau = 0, \tag{2}$$

$$i(w) \equiv P^I(w) + Q^I(w)\cos w\tau - Q^R(w)\sin w\tau = 0 \tag{3}$$

and we have $r(0) = P^R(0) + Q^R(0)$ and $i(0) = 0$, because $P^I(w)$ and $Q^I(w)$ are odd functions in w. It follows that $D(0, \tau) = r(0) \neq 0$ for all $\tau \geqslant 0$ if $P(\lambda) + Q(\lambda)$ is Hurwitz stable. Then we have the following proposition.

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Proposition. *Assume that $D(\lambda, \tau)$ is the characteristic function of the delay system, then the system is delay-independent stable if and only if:* (1) *the polynomial $P(\lambda) + Q(\lambda)$ is Hurwitz stable and* (2) *the polynomial $F(w)$ cannot have real roots* other than zero.

Proof. On one hand, it is easy to know that any root w of $D(\mathrm{i}w, \tau) = 0$ must satisfy $F(w) = 0$, and conversely if $F(w)$ has a *nonzero* root w, then there exist some $\tau > 0$, determined by $r(w) = 0$ and $i(w) = 0$, such that $D(\mathrm{i}w, \tau) = 0$. When $w = 0$ is the unique real root of $F(w)$, on the other hand, $D(0, \tau) \neq 0$ for all $\tau \geqslant 0$ if $P(\lambda) + Q(\lambda)$ is Hurwitz stable as seen above. Thus, if $P(\lambda) + Q(\lambda)$ is Hurwitz stable, then $D(\mathrm{i}w, \tau) = 0$ has no real root w for all $\tau \geqslant 0$ if and only if either $F(w)$ has no real roots or $w = 0$ is the unique real root of $F(w)$. This confirms the proposition by using Theorem 2 of our paper. □

Accordingly, the stability condition concerning $G(w^2)$ should be changed to "$G(w^2)$ cannot have real roots *other than zero*" when the characteristic function is in the form

$$D(\lambda, \tau_1, \tau_2) = P(\lambda) + Q_1(\lambda)\mathrm{e}^{-\lambda\tau_1} + Q_2(\lambda)\mathrm{e}^{-\lambda\tau_2}. \tag{4}$$

This little change of the stability condition *may* result in some changes in the application of the presented method. In addition to the region determined by the Hurwitz condition when the delays disappear and by the condition "$F(w)$ (or $G(w^2)$) has no real roots", the delay-independent stable region should contain also the points determined by $F(0) = 0$ (or $G(0) = 0$) but $F(w)$ (or $G(w^2)$) has no other real roots, as well as by the Hurwitz stable condition.

In the application part (Section 4 of our paper), however, the main results are kept *unchanged* since the newly emerging case does not occur. In fact, we have

1. In Section 4.1, when $\tau_1 \neq \tau_2$, $G(0) = 0$ if and only if $r = 0$, namely $s_1 = -b$. For this case, however, it can be shown that $w^6 + pw^2 + q$ does *have* non-zero real roots. This means that the delay-independent stable region does not contain any point on the line $s_1 = -b$.

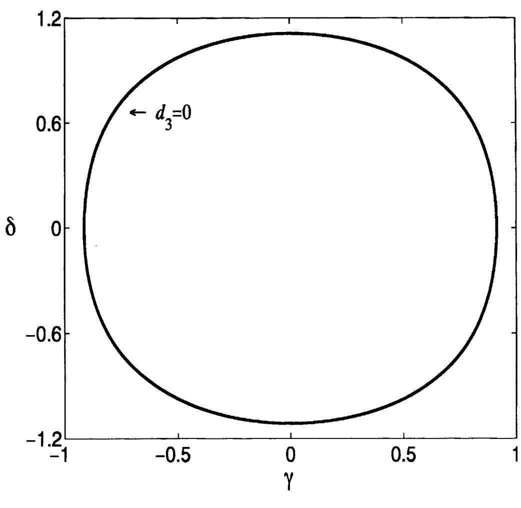

Fig. 1. The delay-independent stable region of active tendon structure in (γ, δ) plane when $\alpha = 2$, $\xi = 0.5$.

2. In Section 4.2, $F(0) = 0$ if and only if $b_3 = 0$, namely $\gamma = 1$. In order that $F(w)$ does not have non-zero real roots, one has $b_1 \geqslant 0$, $b_2 \geqslant 0$, or $b_1 < 0$, $b_1^2 - 4b_2 < 0$. This is *not* true for the case studies. So $\gamma = 1$ does not fall into the delay-independent stable regions.
3. In Section 4.3, $F(0) = 0$ if and only if $b_4 = 0$, namely $g_1 = 1$. One can also prove that for the case corresponding to Fig. 6, $F(w)$ *has* non-zero real roots. This means the delay-independent stable region is kept unchanged. When the system is equipped with sky-hook damper, we have $g_1 = 0$, so the delay-independent stable region remains unchanged, here too.

Finally, we regret to say, there is a typesetting error in the original paper: the coefficients a and b in Eq. (14) should be $-a$ and $-b$, respectively. And Fig. 5 in the original paper, where the axes γ and δ were interchanged in display of the plot, should be replaced by Fig. 1.

References

[1] S. Wu, Remarks on "Delay-independent stability of retarded dynamic systems of multiple degrees of freedom", Journal of Sound and Vibration 265 (2003) 693–694, this issue.