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A combined finite element–stiffness equation transfer method for steady state vibration response analysis of structures

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Abstract

An extended finite element transfer matrix method, in combination with stiffness equation transfer, is applied to dynamic response analysis of the structures under periodic excitations. In the present method, the transfer of state vectors from left to right in a combined finite element-transfer matrix (FE-TM) method is changed into the transfer of general stiffness equations of every section from left to right. This method has the advantages of reducing the order of standard transfer equation systems, and minimizing the propagation of round-off errors occurring in recursive multiplication of transfer and point matrices. Furthermore, the drawback that in the ordinary FE-TM method, the number of degrees of freedom on the left boundary be the same on the right boundary, is now avoided. A FESET program based on this method using microcomputers is developed. Finally, numerical examples are presented to demonstrate the accuracy as well as the potential of the proposed method for steady state vibration response analysis of structures.

1. Introduction

The most powerful and most widely used numerical method in structural analysis is the finite element (FE) method. The disadvantage of FE method, however, is that in the case of complex structures, it is necessary to use a large number of nodes, resulting in very large matrices which require large computers for their management and regulation. In order to reduce the size of the matrices, some substructure techniques have been proposed which consist of keeping the important degrees of freedom and suppressing the less important ones. Which degrees of freedom in the substructure are to be retained depends on judgment and on the physical system. However, this approach may lead to considerable inaccuracy if wrong degrees of freedom are suppressed.

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The combined finite element-transfer matrix (FE-TM) method was proposed for the first time by Dokanish [1] for free plate vibration problems. Since the publication of Dokanish's paper, several authors have proposed refinements and extensions of this method [2–10]. This method has the advantage of reducing stiffness matrix size to much smaller than that obtained with the FE method and has been successfully applied to various linear and non-linear structural problems, such as static structural analysis, natural frequencies of structure, transient and steady state vibration response of structure, and non-linear dynamic response of structure. However, it is pointed out that, in the standard FE-TM method, recursive multiplication of the transfer and point matrices are main sources of round-off errors. Particularly, in calculating high resonant frequencies or the response of a long structure, the numerical instability would be appeared and it leads to an unwanted solution. The technique of exchanging state vectors and the riccati transformation of state vectors have been, respectively, used in Refs. [5–8] for solving this problem. In addition, the derivation of the transfer matrix from the dynamic stiffness $[G]_i$ for strip i requires the inversion of sub-matrix $[G_{12}]_i$ [1–3,5–9]. In a strict sense, the inversion is possible only if $[G_{12}]_i$ is a square matrix. But, $[G_{12}]_i$ is a square matrix only if there are equal numbers of nodes on the right boundary and on the left boundary, Therefore, most of the previous formulations of the combined FE-TM method are only applicable to the models which have the same number of nodes on all the substructure boundaries. Degen et al. have proposed a new FE-TM method based on a mixed finite element formulation which alleviates the restriction on the finite element model [4]. Bhutani and Loewy proposed a procedure for deriving a transfer matrix by adding the zero elements to the state vectors which allows different number of nodes on the right and on the left boundary [10]. However, in these methods, recursive multiplication of the transfer and the point matrices are still necessary. The numerical instabilities that are inherent to mixed methods in general, and to transfer methods in particular, must be circumvented. Even though various techniques for treating these problems have been presented [5–8], researches on this problem are as yet insufficient.

The purpose of this paper is to present an extended finite element–stiffness equation transfer method (FE–SET) to overcome simultaneously both these two disadvantages in the ordinary FE-TM method. In the present method, because the transfer of state vectors from left to right in the FE-TM method is transformed into a transfer of general stiffness equations in every section from left to right, the inverse matrix of sub-matrix $[G_{12}]_i$ of the FE-TM method becomes the inverse matrix of sub-matrix $[G_{11}]_i$ of the present method. It is well known that $[G_{11}]_i$ is always a square matrix whether the structures are rectangular or not. Since the numerical solution of a two-point boundary value problem in the FE-TM method is converted into the numerical solution of an initial value problem in the present method, the propagation of round-off errors occurring in recursive multiplication of the transfer and point matrices is avoided. The present method is applicable to different kinds of structural analysis in the same manner as the ordinary FE-TM method. For simplicity, in the present paper, we only discuss the application of this method in steady state vibration response analysis of structures under periodic excitations. Extension to analyze other problems of structures is straightforward and will be presented in subsequent publications.

Being different from common methods of computing steady state vibration response of structures, the derivation of analytical procedures for the present method requires neither time integration nor modal superposition. In the response analysis by modal superposition, one must first solve the free, undamped vibration problem to obtain the solution of the eigenvalues and eigenvectors of the finite element assemblage. The distributed loading must then be expanded into the series, each term of which has the form of an eigenvector. The present paper demonstrates a method for solving vibration problems of the structure which does not require a knowledge of the free vibration eigenvectors, the problems are solved only as an initial value problem in a straightforward manner. Hence, the computational efficiency as well as accuracy is greatly increased. The FESET program based on this method using a microcomputer is developed. Some numerical examples of steady state dynamic problems are also given and their results compared with those obtained with the ordinary finite element method.

2. A combined finite element-stiffness equation transfer method (FE–SET)

Without losing generality, we consider the plate shown in Fig. 1. It is divided into n strips and each strip is subdivided into finite elements. The vertical sides dividing or bordering the strips are called sections. It is apparent that the right of section i is the left of strip i.

Let $\{U\}_i^L$, $\{N\}_i^L$ and $\{U\}_i^R$, $\{N\}_i^R$ be the left and right steady forced vibration displacement and force vectors of section i.

Similarly as in generalized riccati transformation of state vectors [11], we may assume that the generalized stiffness equations which relate the force vectors to the displacement vectors on the left of section i are given by

$$\{N\}_i^L = [T]_i\{U\}_i^L + \{E\}_i \qquad (i \geqslant 2). \tag{1}$$

2.1. Transfer at section i

The displacements are continuous across section i, so that we obtain

$$\{U\}_i^R = \{U\}_i^L. \tag{2}$$

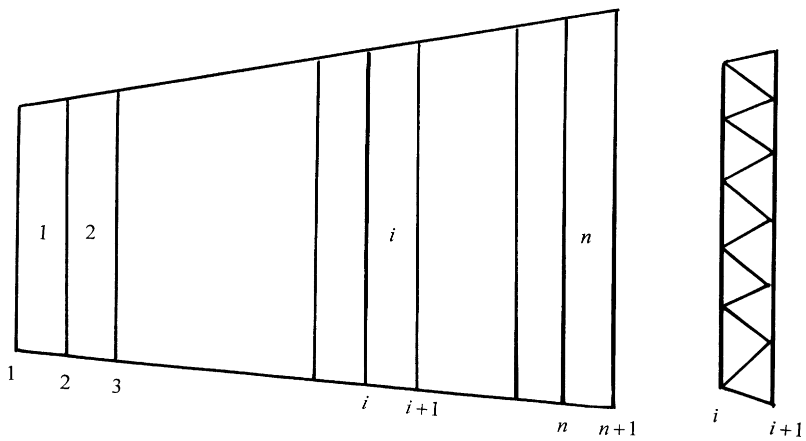

Fig. 1. Subdivision of structure into strips and finite elements.

Without losing generality, we suppose that there is no concentrated external load acting on section i (concentrated external load acting on section i may be treated as generalized external force on the left of strip i). Due to the continuity of force at section i, we obtain

$$\{N\}_i^R = -\{N\}_i^L. \tag{3}$$

Substituting Eqs. (2) and (3) into Eq. (1), we obtain

$$\{N\}_i^R = -[T]_i\{U\}_i^R - \{E\}_i. \tag{4}$$

Eq. (4) describes the relation between the forced vibration internal force vectors and the displacement vectors on the right of section i.

2.2. Transfer in strip i

The discrete finite element equations of motion for a substructure i take the form [9]:

$$[M]_i\{\ddot{U}\}_i + [C]_i\{\dot{U}\}_i + [K]_i\{U\}_i = \{N\}_i + \{Q\}_i, \tag{5}$$

where matrices $[M]_i$, $[C]_i$ and $[K]_i$ represent the mass, damping and stiffness properties of strip i, respectively. $\{U\}_i$, $\{N\}_i$ and $\{Q\}_i$ are respectively the nodal displacement vector, internal force vector and the generalized external force vector on the left and the right of strip i. When a periodic force is acted on the structure, it may be represented by a set of harmonic forces through Fourier transformation. Without losing generality, we consider one component of harmonic forces, $\{Q\}_i$, $\{U\}_i$ and $\{N\}_i$ may be represented by

$$\begin{aligned}
\{Q\}_i &= \{Q_s\}_i \sin\omega t + \{Q_c\}_i \cos\omega t,\\
\{U\}_i &= \{U_s\}_i \sin\omega t + \{U_c\}_i \cos\omega t,\\
\{N\}_i &= \{N_s\}_i \sin\omega t + \{N_c\}_i \cos\omega t,
\end{aligned} \tag{6}$$

where ω is the frequency of harmonic exciting forces.

Substituting Eq. (6) into Eq. (5), we obtain

$$\begin{bmatrix} [K]-\omega^2[M] & -\omega[C] \\ \omega[C] & [K]-\omega^2[M] \end{bmatrix}_i \begin{Bmatrix} U_s \\ U_c \end{Bmatrix}_i = \begin{Bmatrix} N_s \\ N_c \end{Bmatrix}_i + \begin{Bmatrix} Q_s \\ Q_c \end{Bmatrix}_i. \tag{7}$$

For strip i, it includes the nodes on the right of section i and on the left of section $i+1$, so that we have

$$\begin{aligned}
&\{U_s\}_i = [\{U_s\}_i^R, \{U_s\}_{i+1}^L]^{\mathrm{T}} \quad \{U_c\}_i = [\{U_c\}_i^R, \{U_c\}_{i+1}^L]^{\mathrm{T}}\\
&\{N_s\}_i = [\{N_s\}_i^R, \{N_s\}_{i+1}^L]^{\mathrm{T}} \quad \{N_c\}_i = [\{N_c\}_i^R, \{N_c\}_{i+1}^L]^{\mathrm{T}},\\
&\{Q_s\}_i = [\{Q_s\}_i^R, \{Q_s\}_{i+1}^L]^{\mathrm{T}} \quad \{Q_c\}_i = [\{Q_c\}_i^R, \{Q_c\}_{i+1}^L]^{\mathrm{T}}.
\end{aligned} \tag{8}$$

Substituting Eq. (8) into Eq. (7), Eq. (7) is rearranged and repartitioned. We obtain

$$\begin{bmatrix} [G_{11}] & [G_{12}] \\ [G_{21}] & [G_{22}] \end{bmatrix}_i \begin{Bmatrix} \{U\}_i^R \\ \{U\}_{i+1}^L \end{Bmatrix} = \begin{Bmatrix} \{N\}_i^R \\ \{N\}_{i+1}^L \end{Bmatrix} + \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix}_i \tag{9}$$

in which

$$\begin{aligned} \{U\}_i^R &= [\{U_s\}_i^R, \{U_c\}_i^R]^{\mathrm{T}} \quad \{U\}_{i+1}^L = [\{U_s\}_{i+1}^L, \{U_c\}_{i+1}^L]^{\mathrm{T}}, \\ \{N\}_i^R &= [\{N_s\}_i^R, \{N_c\}_i^R]^{\mathrm{T}} \quad \{N\}_{i+1}^L = [\{N_s\}_{i+1}^L, \{N_c\}_{i+1}^L]^{\mathrm{T}}, \\ \{Q_1\}_i &= [\{Q_s\}_i^R, \{Q_c\}_i^R]^{\mathrm{T}} \quad \{Q_2\}_i = [\{Q_s\}_{i+1}^L, \{Q_c\}_{i+1}^L]^{\mathrm{T}}. \end{aligned} \tag{10}$$

By expanding Eq. (9), we obtain

$$[G_{11}]_i\{U\}_i^R + [G_{12}]_i\{U\}_{i+1}^L = \{N\}_i^R + \{Q_1\}_i, \tag{11}$$

$$[G_{21}]_i\{U\}_i^R + [G_{22}]_i\{U\}_{i+1}^L = \{N\}_{i+1}^L + \{Q_2\}_i. \tag{12}$$

Substituting Eq. (4) into Eq. (11), we obtain

$$\begin{aligned} \{U\}_i^R = & -([G_{11}] + [T])_i^{-1}[G_{12}]_i\{U\}_{i+1}^L \\ & + ([G_{11}] + [T])_i^{-1}(-\{E\} + \{Q_1\})_i. \end{aligned} \tag{13}$$

Substituting Eq. (13) into Eq. (12), we have

$$\{N\}_{i+1}^L = [T]_{i+1}\{U\}_{i+1}^L + \{E\}_{i+1}, \tag{14}$$

where

$$[T]_{i+1} = [G_{22}]_i - [G_{21}]_i([G_{11}] + [T])_i^{-1}[G_{12}]_i, \tag{15}$$

$$\{E\}_{i+1} = [G_{21}]_i([G_{11}] + [T])_i^{-1}(\{Q_1\} - \{E\})_i - \{Q_2\}_i. \tag{16}$$

Eq. (14) represents the relationships for the internal force vectors and the displacement vectors on the left of section $i+1$.

2.3. Transfer of entire structure

Supposing $[T]_2$ and $\{E\}_2$ are known, using Eqs. (15) and (16), $[T]$ and $\{E\}$ are transferred from the left of the second section to the right of the total structure. Hence we have

$$\{N\}_{n+1}^L = [T]_{n+1}\{U\}_{n+1}^L + \{E\}_{n+1}. \tag{17}$$

By considering boundary conditions, the known force or displacement variables on the right hand boundary of the total structure are substituted into Eq. (17) to determine the unknown force or displacement variables. After the force and displacement vectors on the right hand boundary of the total structure are solved, the force and displacement vectors at any section i are calculated by Eqs. (13) and (4).

It is noteworthy that the transfer matrix $[T]$ for the ordinary FE-TM method [9] is replaced by the transfer matrix $[T]_{n+1}$ in Eq. (17) for the FE–SET method. The dimension of the matrix $[T]_{n+1}$ is only half that of the matrix $[T]$. In the FE–SET method, The storage requirements would only about half of the FE-TM method. In addition, the transfer matrix $[T]_{n+1}$ is obtained by recursively using Eqs. (15) and (16), and not by recursive multiplication of transfer and point matrices, so the propagation of round-off errors occurring in recursive multiplication of transfer and point matrices is avoided.

2.4. The method of determining $[T]_2$ and $\{E\}_2$

In Eq. (1), let i be 2, we obtain

$$\{N\}_2^L = [T]_2\{U\}_2^L + \{E\}_2. \tag{18}$$

For strip 1, by expanding Eq. (9), we have

$$[G_{11}]_1\{U\}_1^R + [G_{12}]_1\{U\}_2^L = \{N\}_1^R + \{Q_1\}_1, \tag{19}$$

$$[G_{21}]_1\{U\}_1^R + [G_{22}]_1\{U\}_2^L = \{N\}_2^L + \{Q_2\}_1. \tag{20}$$

It is obvious that $\{U\}_1^R$ and $\{N\}_1^R$ may be determined by using the left hand boundary conditions of the total structure.

2.4.1. Displacement boundary condition

It is obvious that $\{U\}_1^R$ is known in a displacement boundary condition, hence by Eq. (20), we obtain

$$\{N\}_2^L = [G_{22}]_1\{U\}_2^L + [G_{21}]_1\{U\}_1^R - \{Q_2\}_1. \tag{21}$$

Comparing with Eq. (18), we have

$$[T]_2 = [G_{22}]_1, \tag{22}$$

$$\{E\}_2 = [G_{21}]_1\{U\}_1^R - \{Q_2\}_1. \tag{23}$$

2.4.2. Force boundary condition

It is obvious that $\{N\}_1^R$ is known in a force boundary condition, hence $\{U\}_1^R$ is obtained from Eq. (19).

$$\{U\}_1^R = -[G_{11}]_1^{-1}[G_{12}]_1\{U\}_2^L + [G_{11}]_1^{-1}(\{N\}_1^R + \{Q_1\}_1). \tag{24}$$

Substituting the $\{U\}_1^R$ into Eq. (20), we have

$$\{N\}_2^L = ([G_{22}]_1 - [G_{21}]_1[G_{11}]_1^{-1}[G_{12}]_1)\{U\}_2^L + [G_{21}]_1[G_{11}]_1^{-1}(\{N\}_1^R + \{Q_1\}_1) - \{Q_2\}_1. \tag{25}$$

Comparing with Eq. (18), we have

$$[T]_2 = [G_{22}]_1 - [G_{21}]_1[G_{11}]_1^{-1}[G_{12}]_1, \tag{26}$$

$$\{E\}_2 = [G_{21}]_1[G_{11}]_1^{-1}(\{N\}_1^R + \{Q_1\}_1) - \{Q_2\}_1. \tag{27}$$

2.4.3. Mixture boundary condition

In mixture boundary condition, we suppose $\{U\}_1^R = [\{U'\}_1^R, \{U''\}_1^R]^{\mathrm{T}}$ and the corresponding $\{N\}_1^R = [\{N'\}_1^R, \{N''\}_1^R]^{\mathrm{T}}$. If $\{U'\}_1^R$ is unknown and $\{U''\}_1^R$ is known, the corresponding $\{N'\}_1^R$ is known and $\{N''\}_1^R$ is unknown. For strip 1, Eq. (9) is rearranged and repartitioned, so we have

$$\begin{bmatrix} [H_{11}] & [H_{12}] & [H_{13}] \\ [H_{21}] & [H_{22}] & [H_{23}] \\ [H_{31}] & [H_{32}] & [H_{33}] \end{bmatrix} \begin{Bmatrix} \{U'\}_1^R \\ \{U''\}_1^R \\ \{U\}_2^L \end{Bmatrix} = \begin{Bmatrix} \{N'\}_1^R \\ \{N''\}_1^R \\ \{N\}_2^L \end{Bmatrix} + \begin{Bmatrix} \{Q_1'\}_1 \\ \{Q_1''\}_1 \\ \{Q_2\}_1 \end{Bmatrix}. \tag{28}$$

Expanding Eq. (28) and solving relations for $\{N\}_2^L$ and $\{U\}_2^L$, we obtain

$$[T]_2 = [H_{33}] - [H_{31}][H_{11}]^{-1}[H_{13}], \tag{29}$$

$$\{E\}_2 = [H_{31}][H_{11}]^{-1}\left(\{N'\}_1^R + \{Q_1'\}_1\right) + [H_{32}]\{U''\}_1^R - [H_{31}][H_{11}]^{-1}[H_{12}]\{U''\}_1^R - \{Q_2\}_1. \tag{30}$$

3. Numerical examples

In order to investigate the accuracy and the computational efficiency of our method, we developed a program FESET based on this method on a microcomputer. Many numerical examples can be given using the FE–SET approach. In this section, a vibrating Euler beam is first analyzed to obtain its natural frequencies and forced vibration displacements for checking purposes, and then the forced response of the trapeziform and circle plates is given to illustrate the validity of the proposed method.

A uniform rectangular beam with simple supports, as shown in Fig. 2, was modulled with eight elements. The beam has a modulus of elasticity $E = 1.5 \times 10^5$ MPa, and density $\rho = 8000\ \mathrm{Kg/m^3}$. The exciting force $F(t) = F_0 \sin \omega t (F_0 = 400\ \mathrm{N})$ is applied at the middle point of the beam. Using the proposed method, the response at the nodes of interest to a forced harmonic input versus forcing frequency can be easily obtained. In Table 1 the first three natural frequencies obtained by using the subspace iteration method [12] for a full eigenvalue problem are compared with the first two peak resonances calculated using the proposed FE–SET method. For the first and the third natural frequency there seems no difference of results between the two approaches. Because the second modal generalized force is zero in our example, the second peak resonance of the present method cannot be obtained. It is found that the accuracy of the peak resonance depends on the frequency step size. In order to save computational time and obtain accurate solution, one can first use a coarse step size to determine the frequency ranges of interest, and then repeat the

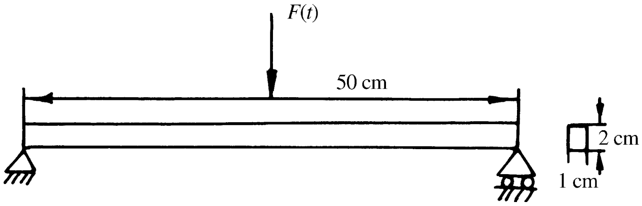

Fig. 2. A simply supported beam.

Table 1
Natural frequencies of simply supported beam

Peak resonance using FE-SET (rad/s)	Solution of subspace iteration method (rad/s)
987–988	987.2
	3948.8
8884–8885	8884.9

Table 2
Vibration displacements at the nodal points of the simply supported beam (cm)

Number of node	1	2	3	4	5	6	7	8	9
Exact solution	0	2.79	5.16	6.75	7.30	6.75	5.16	2.79	0
FE Solution	0	2.79	5.14	6.70	7.25	6.70	5.14	2.79	0
FE-SET Solution	0	2.79	5.14	6.73	7.28	6.73	5.15	2.79	0

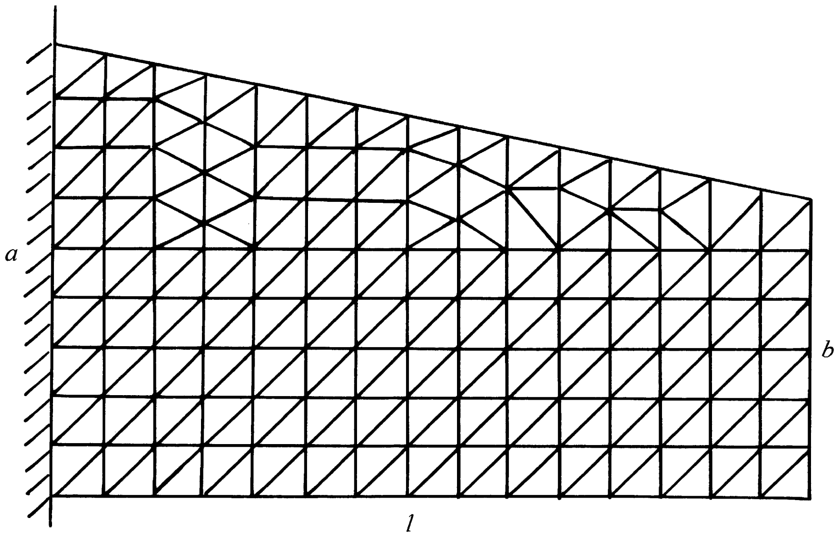

Fig. 3. Cantilever trapeziform plate model.

calculations with a smaller size near the desired response peaks to obtain more accurate results. For $\omega = 980\,\text{rad/s}$, the computed results of the vibration displacements are listed in Table 2. The solutions of the FE method using the modal superposition approach and the theoretical exact solutions are also listed in Table 2. In the FE method, the same elements as those used in the FE–SET method are employed. A comparison indicates that very little difference exists among the three results.

The second example is to obtain the forced response of a cantilever trapeziform plate under the harmonic uniform pressure as shown in Fig. 3, where the physical parameters of the plate are as follows: length $l = 150\,\text{cm}$, width $a = 90\,\text{cm}$, $b = 60\,\text{cm}$, thickness $t = 0.635\,\text{cm}$, a specific weight $\gamma = 78\,\text{KN/m}^3$, the Possion ratio $v = 0.3$, modules of elasticity $E = 2.0 \times 10^5\,\text{MPa}$, Rayleigh damping constant $\alpha = 0.1$, $\beta = 0.02$ and harmonic uniform pressure $q(t) = 800 \sin \omega t\,\text{KN/m}^2$. The plate is divided into 15 substructures which are divided into many triangular plate elements. The first three peak resonances calculated by the proposed FE–SET method and the first three natural frequencies obtained by the subspace iteration method [12] are listed in Table 3. From the above results, it can be seen that the computed results by the presented method are almost the same as those obtained by using the subspace iteration method. For $\omega = 837\,\text{rad/s}$, the vibration displacement at the middle point of the section b is listed in Table 4. The solutions of the finite element method using the modal superposition approach are also listed in Table 4. From the

Table 3
Natural frequencies of the cantilever plate

Peak resonance using FE-SET (rad/s)	Solution of subspace iteration method (rad/s)
16–17	16.5
74–75	74.5
94–95	94.4

Table 4
Vibration displacement at the middle point of the section b for the cantilever plate

Method by applying	FE	FE	FE	FE	FE-SET
Mode number	5	10	15	20	
Displacement (mm)	15.89	19.16	18.84	18.86	18.86
Computation time (s)	10	16	28	41	11

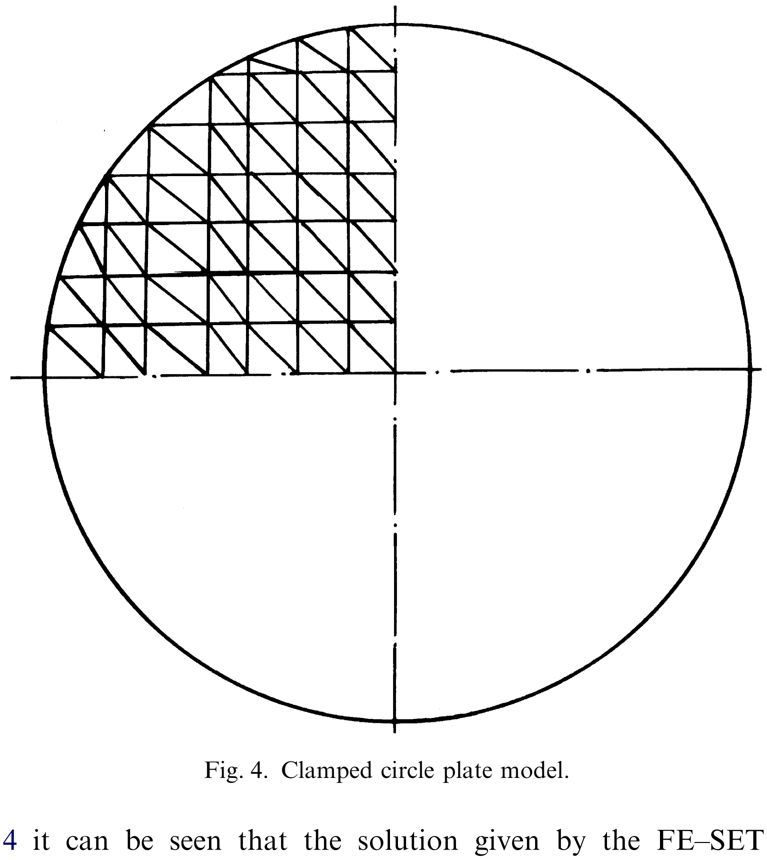

Fig. 4. Clamped circle plate model.

results in Table 4 it can be seen that the solution given by the FE–SET method coincide completely with that obtained from using the FE method (mode number = 20). A comparison of computation time shown in Table 4 indicates that a computation efficiency of the present method

Table 5
Vibration displacement at the central point of the circle plate

Method by applying	FE	FE	FE	FE	FE-SET
Mode number	5	10	15	20	
Displacement (mm)	33.18	32.01	31.04	31.03	31.03
Computation time (s)	5	8	14	30	7

is higher than that of the FE method. In this example, the number of nodes on the left boundary is 10, and that on the right boundary is 7. Most of the ordinary FE-TM method can only be applied to the chain-like structure which has equal number of degrees of freedom on the boundaries, so the ordinary FE-TM method [2–3,9] cannot be used in this case. The present method has potentially wider application than the ordinary FE-TM method. In comparison with the FE method, The FE program computes frequency response via a modal analysis whereas the present method allows a direct computation of frequency response. In addition, the size of the matrix in the FE method is much larger than that in the present method. The computational efficiency of our method is higher than that of the FE method.

In the third example, we analyzed a clamped circle plate under a uniform harmonic pressure as shown in Fig. 4. The physical parameters of the plate are as follows: radius $R = 1.4\,\mathrm{m}$, thickness $t = 1\,\mathrm{cm}$, modules of elasticity $E = 2.0 \times 10^5\,\mathrm{MPa}$, a specific weight $\gamma = 78\,\mathrm{KN/m^3}$, Rayleigh damping constant $\alpha = 0.001$, $\beta = 0.002$, the Possion ratio $v = 0.3$ and a uniform harmonic pressure $q(t) = 100 \sin 314t\,\mathrm{KN/m^2}$. It is shown in Fig. 4 that a quarter of the plate is divided into seven substructures that are divided into many triangular plate elements. Table 5 compares the harmonic vibration displacement at the central point of the plate resulting from the employment of both the FE method and the present method. Similar results as in Example 2 are obtained.

4. Conclusion

A combination of the finite element method and the stiffness equation transfer method for solving the steady state vibration response problems is proposed and illustrated by three examples. A FESET microcomputer program based on this method is developed. Some numerical examples presented in this paper show that the proposed method can be successfully applied to the steady state vibration response analysis of structures with random boundaries. In the present method, the transfer of state vectors from left to right in the ordinary FE-TM method is changed into the transfer of general stiffness equations of every section from left to right. This method has the advantages of reducing the order of standard transfer equation systems, and minimizing the propagation of round-off errors occurring in recursive multiplication of transfer and point matrices. It also has an additional advantage in that one does not need to calculate so many natural frequencies. Furthermore, the drawback that in the ordinary FE-TM method, the number of degrees of freedom on the left boundary be the same on the right boundary, is now avoided. Hence, the present method has potentially wider application than the ordinary FE-TM method.

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