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Sensitivity analysis applied to a dynamic railroad model

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Abstract

An analytical method of analyzing sensitivity is presented. It is shown that in a special case, when the dynamical problem is described by differential equations (of any order) with constant coefficients, first and second order semilogarithmic (semirelative) sensitivity functions can be determined analytically. The method is applied to the practical problem of railway track vibration, with the intention of using it for the identification of railway track model parameters in the future. The railway track model is an infinite beam resting on multi-parameter viscoelastic subsoil.

1. Introduction

The sensitivity of linear dynamic systems has been the subject of many research papers [1–3]. The range of possible uses of the sensitivity analysis is wide and includes such problems as: the approximation of the solutions in the neighbourhood of a known solution, gradient method optimization (including the identification problem) for specified objective functions and the analysis of (measurement) error sensitivity [4]. Generally, the sensitivity problem in practical dynamical cases is so complex that the only viable way of handling it is through computer numerical analysis.

It will be shown here that in a special case, when the dynamical problem is described by differential equations (of any order) with constant coefficients, first and second order sensitivity functions can be determined in an analytical form. The only difficulty is the solution of the characteristic equation. Since in the proposed method only characteristic equation root values are needed, the equation can always be solved numerically (e.g., using the *Mathematica* software).

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The method is applied to the practical problem of railway track vibration, with the intention of using the sensitivity function for the identification of railway track model parameters in the future. In the identification procedure, the particularly interesting parameters are the ones which are unkown and may have an influence on the capacity of the structure. Such design parameters were chosen from a set of all the parameters. The influence of velocity, though the latter was not a design parameter, was also analyzed. This parameter particularly affects the value of railroad displacement and it is easy to control during the identification test. If too many parameters are identified, the error of their identified values can be large. Therefore, the dynamic system identification method requires that the model of the system should not be too complicated.

An infinitely long prismatic beam resting on multiparameter viscoelastic subsoil and loaded with a set of moving forces was assumed as the optimal railway track model. Since the engine moved with a constant velocity, only the stationary problem was considered in the identification procedure. The assumption of stationarity allows a set of moving forces to be substituted for the complicated engine model (a set of sprung and unsprung masses) [5].

The system vibration problem was solved by the Fourier transformation method. The displacement function was obtained in an integral form. Using the residua theorem (the complex function theory) the integral solution was transformed to a closed form. Then by applying theorems concerning the calculation of implicit function derivatives an analytical form of the semilogarithmic (semirelative) sensitivity function was obtained.

The semilogarithmic sensitivity function of quantity w with respect to b was defined by the expression $\partial w/\partial \ln b = b \cdot \partial w/\partial b$ (see also Eqs. (12) and (13) and Refs. [2,5]). Owing to the use of such functions it became possible to study the system's sensitivity to parameter variation for different dimensions and values of the parameters. The used semilogarithmic function indicates what absolute increments of the displacement function will be for the same relative increments of the design parameters. The sensitivity analysis method was tested for all the design parameters and the results for three of them are presented graphically. The *Mathematica* software was used to represent the results as three-dimensional graphs in order to facilitate sensitivity assessment.

2. Problem formulation

An infinite beam resting on a five-parameter elastic subsoil (Fig. 1) and subjected to load $p(x,t) = \sum_j P_j \delta(x - vt - u_j)$ moving with constant velocity v was assumed as the track structure model. This system in a stationary system of co-ordinates x, z is described by the following

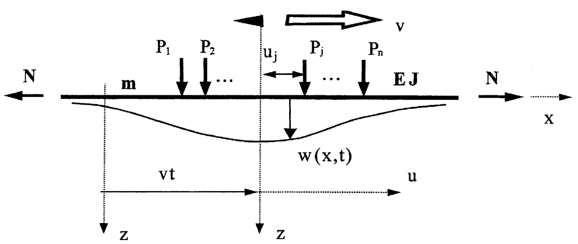

Fig. 1. Model of track structure with load.

differential equation:

$$EJ\frac{\partial^4 w}{\partial x^4} - N\frac{\partial^2 w}{\partial x^2} + m\frac{\partial^2 w}{\partial t^2} = p(x,t) - (k_0\ w + c_0\ \dot{w}) + k_2\frac{\partial^2 w}{\partial x^2} + c_2\frac{\partial^2 \dot{w}}{\partial x^2}, \tag{1}$$

with boundary conditions $w \to 0$ and $\partial w/\partial x \to 0$ for $|x| \to \infty$. In Eq. (1), w denotes the displacement function; EJ the stiffness of a rail; N the axial force (in the rail); m the mass per unit length of the railway track; $p(x,t)$ the railway track's load; and k_0, k_2, c_0, c_2 are the stiffness and viscosity of the subsoil model.

After transformation to movable co-ordinate system $u = x - vt, \quad z$, Eq. (1) becomes an ordinary differential equation describing a stationary tracking wave

$$EJ\ \ w^{IV} + C_2\ \ w^{III} - K_2\ w^{II} - C_0\ \ w^{I} + K_0\ \ w - \sum_j P_j\ \delta(u - u_j) = 0, \tag{2}$$

where:

$$K_0 = k_0, \quad C_0 = vc_0, \quad C_2 = vc_2, \quad K_2 = k_2 + N - v^2 m, \quad (\ )' = \frac{\partial}{\partial\, u}. \tag{3}$$

By performing the Fourier transformation defined by this formula

$$\bar{w}(\lambda) = \int_{-\infty}^{\infty} w(u)\mathrm{e}^{\mathrm{i}\lambda u}\mathrm{d}u = \bar{w}_c(\lambda) + i\bar{w}_s(\lambda) \tag{4}$$

on Eq. (2), determining function $\bar{w}(\lambda)$ and performing the inverse transformation we get

$$w(u) = \frac{1}{2\pi}\sum_j \int_{-\infty}^{\infty} \frac{P_j\ \ \mathrm{e}^{-\mathrm{i}\lambda(u-u_j)}}{\left(\lambda^4 EJ + i\lambda^3 C_2 + \lambda^2 K_2 + i\lambda C_0 + K_0\right)}\,\mathrm{d}\lambda = \sum_j P_j\ w_j(u). \tag{5}$$

Using the residua theorem, integral solution (5) was transformed to a closed form. Then the displacement function $w(u)$ and its derivatives are expressed by the formulae [5]

$$w_j^{(n)}(u) = \begin{cases} \dfrac{\partial^n w_j}{\partial u^n} = \mathrm{i}\left(\dfrac{(-\mathrm{i}\lambda_1)^n \mathrm{e}^{-\mathrm{i}\lambda_1(u-u_j)}}{\psi'(\lambda_1)} + \dfrac{(-\mathrm{i}\lambda_2)^n \mathrm{e}^{-\mathrm{i}\lambda_2(u-u_j)}}{\psi'(\lambda_2)}\right) & \text{for } \ u - u_j < 0, \\ \dfrac{\partial^n w_j}{\partial u^n} = -\mathrm{i}\left(\dfrac{(-\mathrm{i}\lambda_3)^n \mathrm{e}^{-\mathrm{i}\lambda_3(u-u_j)}}{\psi'(\lambda_3)} + \dfrac{(-\mathrm{i}\lambda_4)^n \mathrm{e}^{-\mathrm{i}\lambda_4(u-u_j)}}{\psi'(\lambda_4)}\right) & \text{for } \ u - u_j \geqslant 0, \end{cases} \tag{6}$$

where: $\psi(\lambda) = \lambda^4 EJ + \mathrm{i}\lambda^3 C_2 + \lambda^2 K_2 + \mathrm{i}\lambda C_0 + K_0$, $\psi'(\lambda) = \partial\psi/\partial\lambda$, and $\lambda_1, \ \lambda_2, \ \lambda_3, \ \lambda_4$ are complex roots of equation $\psi(\lambda) = 0$ fulfilling conditions $\mathrm{Im}\ \lambda_{1,2} > 0$, $\mathrm{Im}\ \lambda_{3,4} < 0$. Taking into account the fact that roots $\lambda_1, \ \lambda_2, \ \lambda_3, \ \lambda_4$ of equation $\psi(\lambda) = 0$ are functions of the design parameters $\mathbf{b} = [b_1, \ b_2, \ b_3, \ b_4, \ b_5, \ b_6, \ b_7, \ b_8] = [k_0, \ k_2, \ c_0, \ c_2, \ N, \ m, \ EJ, \ v]$, for the deflection and its derivatives $w_j^{(n)} = \partial^n w_j/\partial u^n$ the first and second order sensitivity functions are determined which are expressed by the following formulae:

$$\frac{\partial w_j^{(n)}}{\partial b_m} = \mathrm{i}\left(\sum_{k=1}^{2}(-\mathrm{i}\lambda)^n e^{-\mathrm{i}\lambda(u-u_j)}\left[\frac{\partial\psi(\lambda)}{\partial\lambda}\right]^{-1}\right.$$
$$\left. \times\left[\frac{\partial\lambda}{\partial b_m}\frac{n}{\lambda} - \mathrm{i}\frac{\partial\lambda}{\partial b_m}(u-u_j) - \left(\frac{\partial\psi(\lambda)}{\partial\lambda}\right)^{-1}\left(\frac{\partial^2\psi}{\partial\lambda^2}\frac{\partial\lambda}{\partial b_m} + \frac{\partial^2\psi}{\partial\lambda\partial b_m}\right)\right]\ \Bigg|_{\lambda=\lambda_k}\right) \quad \text{for } \ u - u_j < 0, \tag{7}$$

$$\frac{\partial w_j^{(n)}}{\partial b_m} = -\mathrm{i}\left(\sum_{k=3}^{4}(-\mathrm{i}\lambda)^n e^{-\mathrm{i}\lambda(u-u_j)}\left[\frac{\partial\psi(\lambda)}{\partial\lambda}\right]^{-1} \times\left[\frac{\partial\lambda}{\partial b_m}\frac{n}{\lambda}-\mathrm{i}\frac{\partial\lambda}{\partial b_m}(u-u_j)-\left(\frac{\partial\psi(\lambda)}{\partial\lambda}\right)^{-1}\left(\frac{\partial^2\psi}{\partial\lambda^2}\frac{\partial\lambda}{\partial b_m}+\frac{\partial^2\psi}{\partial\lambda\partial b_m}\right)\right]\Bigg|_{\lambda=\lambda_k}\right) \quad \text{for } u-u_j\geqslant 0,$$

and

$$\begin{aligned}\frac{\partial^2 w_j^{(n)}}{\partial b_l\partial b_m} = \mathrm{i}\Bigg\{\sum_{k=1}^{2}&(-\mathrm{i}\lambda)^n \mathrm{e}^{-\mathrm{i}\lambda(u-u_j)}\left(\frac{\partial\psi(\lambda)}{\partial\lambda}\right)^{-3}\lambda^{-2}\\ &\times\Bigg[\left(\frac{\partial\psi(\lambda)}{\partial\lambda}\right)^2\left(\left((n-1)n-2in(u-u_j)\lambda-(u-u_j)^2\lambda^2\right)\left(\frac{\partial\lambda}{\partial b_l}\right)\left(\frac{\partial\lambda}{\partial b_m}\right)+\lambda(n-\mathrm{i}(u-u_j)\lambda)\frac{\partial^2\lambda}{\partial b_l\partial b_m}\right)\\ &+2\lambda^2\left(\frac{\partial^2\psi(\lambda)}{\partial\lambda\,\partial b_l}+\frac{\partial\lambda}{\partial b_l}\frac{\partial^2\psi(\lambda)}{\partial\lambda^2}\right)\left(\frac{\partial^2\psi(\lambda)}{\partial\lambda\,\partial b_m}+\frac{\partial\lambda}{\partial b_m}\frac{\partial^2\psi(\lambda)}{\partial\lambda^2}\right)-\frac{\partial\psi(\lambda)}{\partial\lambda}\lambda\Bigg(\lambda\left(\frac{\partial^3\psi(\lambda)}{\partial\lambda\,\partial b_l\partial b_m}+\frac{\partial^2\lambda}{\partial b_l\partial b_m}\frac{\partial^2\psi(\lambda)}{\partial\lambda^2}\right)\\ &+\frac{\partial\lambda}{\partial b_l}\left((n-\mathrm{i}(u-u_j)\lambda)\frac{\partial^2\psi(\lambda)}{\partial\lambda\,\partial b_l}+\lambda\frac{\partial^3\psi(\lambda)}{\partial\lambda^2\,\partial b_l}\right)+\frac{\partial\lambda}{\partial b_m}\left((n-\mathrm{i}(u-u_j)\lambda)\frac{\partial^2\psi(\lambda)}{\partial\lambda\,\partial b_m}+\lambda\frac{\partial^3\psi(\lambda)}{\partial\lambda^2\,\partial b_m}\right)\\ &+\left(\frac{\partial\lambda}{\partial b_l}\right)\left(\frac{\partial\lambda}{\partial b_m}\right)\left(2\left(n-\mathrm{i}(u-u_j)\lambda\right)\frac{\partial^2\psi(\lambda)}{\partial\lambda^2}+\lambda\frac{\partial^3\psi(\lambda)}{\partial\lambda^3}\right)\Bigg)\Bigg]\Bigg|_{\lambda=\lambda_k}\Bigg\} \quad \text{for } u-u_j<0, \end{aligned} \tag{8a}$$

$$\begin{aligned}\frac{\partial^2 w_j^{(n)}}{\partial b_l\partial b_m} = -\mathrm{i}\Bigg\{\sum_{k=3}^{4}&(-i\lambda)^n \mathrm{e}^{-\mathrm{i}\lambda(u-u_j)}\left(\frac{\partial\psi(\lambda)}{\partial\lambda}\right)^{-3}\lambda^{-2}\\ &\times\Bigg[\left(\frac{\partial\psi(\lambda)}{\partial\lambda}\right)^2\left(\left((n-1)n-2\mathrm{i}n(u-u_j)\lambda-(u-u_j)^2\lambda^2\right)\left(\frac{\partial\lambda}{\partial b_l}\right)\left(\frac{\partial\lambda}{\partial b_m}\right)+\lambda(n-\mathrm{i}(u-u_j)\lambda)\frac{\partial^2\lambda}{\partial b_l\partial b_m}\right)\\ &+2\lambda^2\left(\frac{\partial^2\psi(\lambda)}{\partial\lambda\partial b_l}+\frac{\partial\lambda}{\partial b_l}\frac{\partial^2\psi(\lambda)}{\partial\lambda^2}\right)\left(\frac{\partial^2\psi(\lambda)}{\partial\lambda\partial b_m}+\frac{\partial\lambda}{\partial b_m}\frac{\partial^2\psi(\lambda)}{\partial\lambda^2}\right)-\frac{\partial\psi(\lambda)}{\partial\lambda}\lambda\Bigg(\lambda\left(\frac{\partial^3\psi(\lambda)}{\partial\lambda\partial b_l\partial b_m}+\frac{\partial^2\lambda}{\partial b_l\partial b_m}\frac{\partial^2\psi(\lambda)}{\partial\lambda^2}\right)\\ &+\frac{\partial\lambda}{\partial b_l}\left((n-\mathrm{i}(u-u_j)\lambda)\frac{\partial^2\psi(\lambda)}{\partial\lambda\partial b_l}+\lambda\frac{\partial^3\psi(\lambda)}{\partial\lambda^2\partial b_l}\right)+\frac{\partial\lambda}{\partial b_m}\left((n-\mathrm{i}(u-u_j)\lambda)\frac{\partial^2\psi(\lambda)}{\partial\lambda\partial b_m}+\lambda\frac{\partial^3\psi(\lambda)}{\partial\lambda^2\partial b_m}\right)\\ &+\left(\frac{\partial\lambda}{\partial b_l}\right)\left(\frac{\partial\lambda}{\partial b_m}\right)\left(2\left(n-\mathrm{i}(u-u_j)\lambda\right)\frac{\partial^2\psi(\lambda)}{\partial\lambda^2}+\lambda\frac{\partial^3\psi(\lambda)}{\partial\lambda^3}\right)\Bigg)\Bigg]\Bigg|_{\lambda=\lambda_k}\Bigg\} \quad \text{for } u-u_j\geqslant 0, \end{aligned} \tag{8b}$$

where derivatives $\partial\lambda/\partial b_m$, $\partial^2\lambda/\partial b_m^2$ are derivatives of implicit function $\lambda(k_0,\ k_2,\ c_0,\ c_2,\ N,\ m,\ EJ,\ v)$ defined by this equation

$$\begin{aligned}\psi(b_1,b_2,\ldots;\lambda) &= \psi(k_0,k_2,c_0,c_2,N,m,EJ,v;\lambda)\\ &= \lambda^4 EJ+\mathrm{i}\lambda^3 vc_2+\lambda^2(k_2+N-v^2 m)+\mathrm{i}\lambda vc_0+k_0=0.\end{aligned} \tag{9}$$

The values of the derivatives at points $\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4$ can be calculated from the following formulas for the implicit function's first and second derivative:

$$\left.\frac{\partial\lambda}{\partial b_l}\right|_{\lambda=\lambda_k} = -\left.\left(\frac{\partial\psi}{\partial b_l}\right)\left(\frac{\partial\psi}{\partial\lambda}\right)^{-1}\right|_{\lambda=\lambda_k}, \tag{10}$$

$$\left.\frac{\partial^2\lambda}{\partial b_l\,\partial b_m}\right|_{\lambda=\lambda_k} = -\left.\left(\frac{\partial^2\psi}{\partial b_l\,\partial b_m}\left(\frac{\partial\psi}{\partial\lambda}\right)^2 - \frac{\partial^2\psi}{\partial\lambda\partial b_l}\frac{\partial\psi}{\partial b_l}\frac{\partial\psi}{\partial\lambda} - \frac{\partial^2\psi}{\partial\lambda\partial b_m}\frac{\partial\psi}{\partial b_m}\frac{\partial\psi}{\partial\lambda} + \frac{\partial^2\psi}{\partial\lambda^2}\frac{\partial\psi}{\partial b_l}\frac{\partial\psi}{\partial b_m}\right)\left(\frac{\partial\psi}{\partial\lambda}\right)^{-3}\right|_{\lambda=\lambda_k}. \tag{11}$$

The semilogarithmic first order sensitivity function of function $w^{(n)} = \partial^n w/\partial u^n$ is defined by this formula [1–3]

$$\frac{\partial w^{(n)}}{\partial \ln b_l} = \frac{\partial w^{(n)}}{\partial b_l/b_l} = b_l\frac{\partial w^{(n)}}{\partial b_l} = b_l s_l^{(n)}. \tag{12}$$

The semilogarithmic second order sensitivity function of function $w^{(n)} = \partial^n w/\partial u^n$ has this form [1–3]

$$\frac{\partial^2 w^{(n)}}{\partial\ln b_l\partial\ln b_m} = \frac{\partial^2 w^{(n)}}{\left(\partial b_l/b_l\right)\left(\partial b_m/b_m\right)} = b_l b_m\frac{\partial^2 w^{(n)}}{\partial b_l\partial b_m} = b_l b_m s_{l,m}^{(n)}. \tag{13}$$

In formula (12), $s_l^{(n)} = \partial w^{(n)}/\partial b_l$ is a first-order sensitivity function and in formula (13), $s_{l,m}^{(n)} = \partial^2 w^{(n)}/\partial b_l\partial b_m$ is a second order sensitivity function.

3. Results

Numerical calculations were performed to illustrate the proposed method of sensitivity analysis. The results are presented for three design parameters $k_0, k_2,\ m$ (de facto, for more parameters since the semilogarithmic sensitivity function with respect to parameter N is the same as the semilogarithmic sensitivity function with respect to parameter k_2). The choice of the three design parameters was not accidental: the parameters, chosen from a set of all of the parameters, were unknown and had an influence on the load capacity of the structure. Because of the paper's limited space, parameters $c_0,\ c_2$ were omitted. The following nominal parameter values $k_0 = 5\times10^6\,\mathrm{N/m^2}$; $k_2 = 5\times10^7\,\mathrm{N}$; $c_0 = 0.0\,\mathrm{N\,s/m^2}$; $c_2 = 0.0\,\mathrm{N\,s}$; $m = 1500\,\mathrm{kg/m}$; $N = 0.0\,\mathrm{N}$; $EJ = 1.26\times10^7\,\mathrm{Nm^2}$ were assumed. A system of four forces acting on the track at points with coordinates $u_{2,3} = \pm2.55$ m, $u_{1,4} = \pm5.40$ m was assumed as the vehicle model. The first and second order sensitivities were determined as a function of position u and velocity v, with the velocity in a range of 100–500 km/h and the argument in a range of –12 to 12 m. Three-dimensional graphs of the functions are shown in Figs. 2–11a. Graphs of the three-dimensional functions' cross-sections for $v = 100$ km/h (dashed line) and $v = 500$ km/h (full line) are shown in Figs. 2b–11b. The three-dimensional functions' cross-sections for $u_0 = 0$, $u_1 = 2.55$ m, $u_2 = 5.40$ m are shown in Figs. 2c–11c where the dot-and-dash line represents $u_0 = 0$, the solid line – $u_1 = 2.55$ m and the dashed line—$u_2 = 5.40$ m.

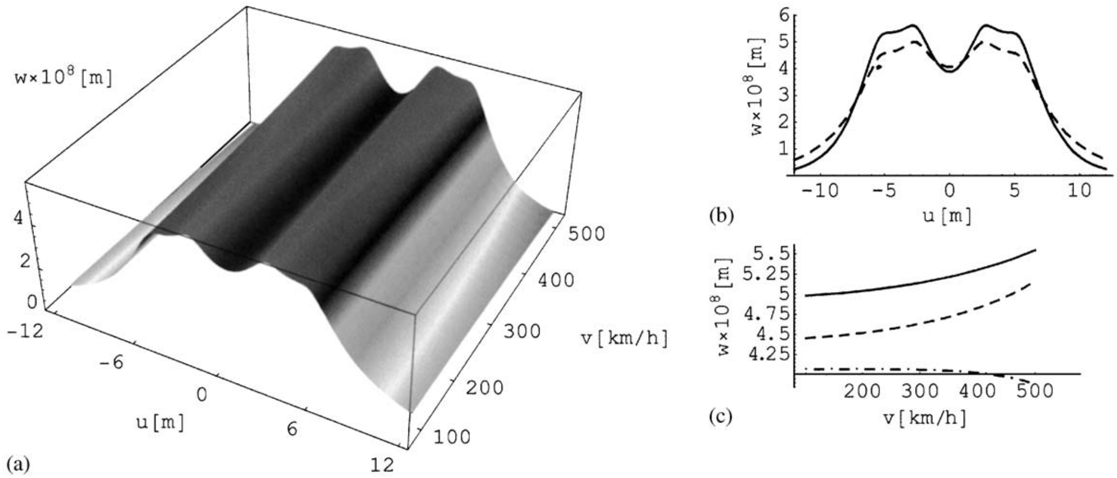

Fig. 2. Displacement function $w(u,v)$.

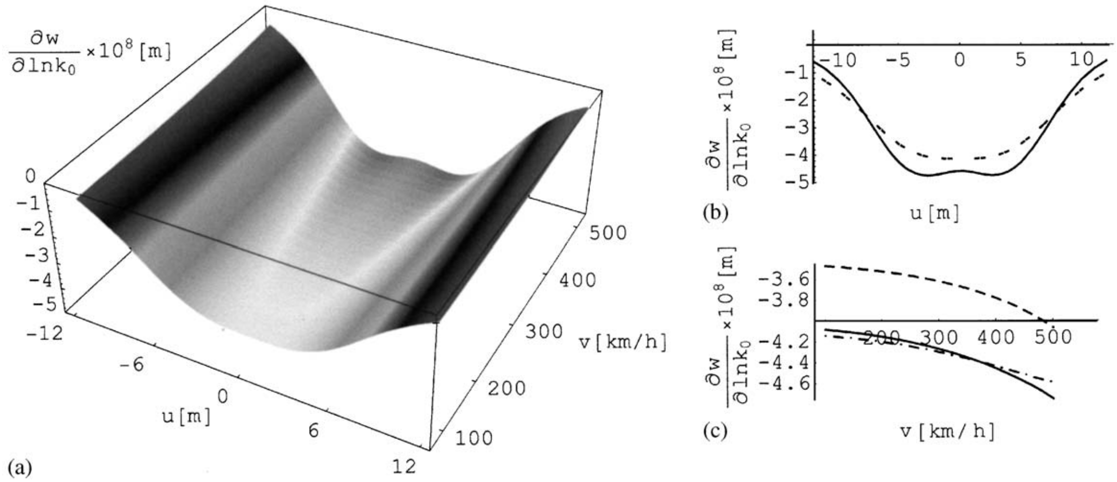

Fig. 3. First order sensitivity function for parameter k_0.

If the semilogarithmic sensitivity function values are positive, this means that the deflection increases with the value of the design parameter for which the sensitivity is tested. If they are negative, the deflection decreases as the parameter value increases.

A higher absolute value of the semilogarithmic sensitivity function corresponds to a greater change in deflection at the same relative change of the system parameters.

The calculated sensitivity function values were also used to approximate the increment of the displacement function. The latter was expanded into a Taylor series in which the successive

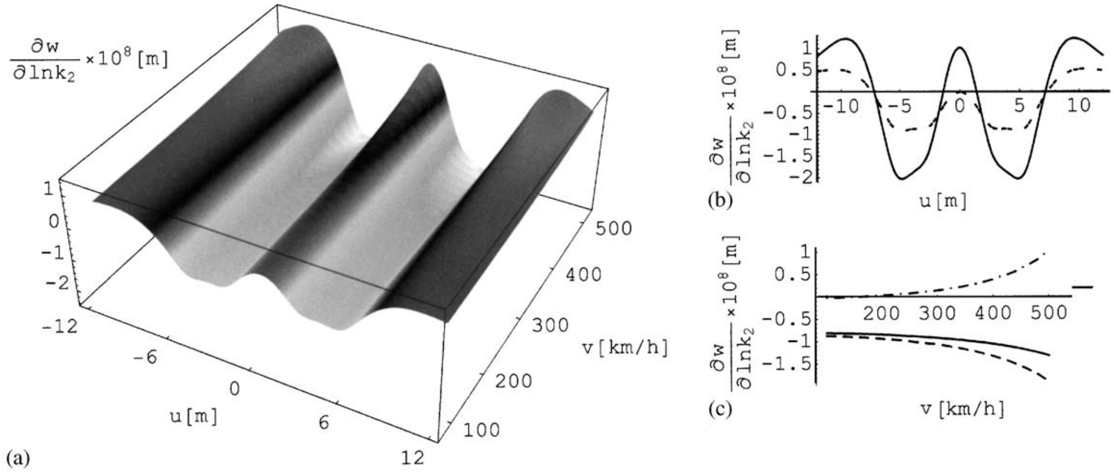

Fig. 4. First order sensitivity function for parameter k_2.

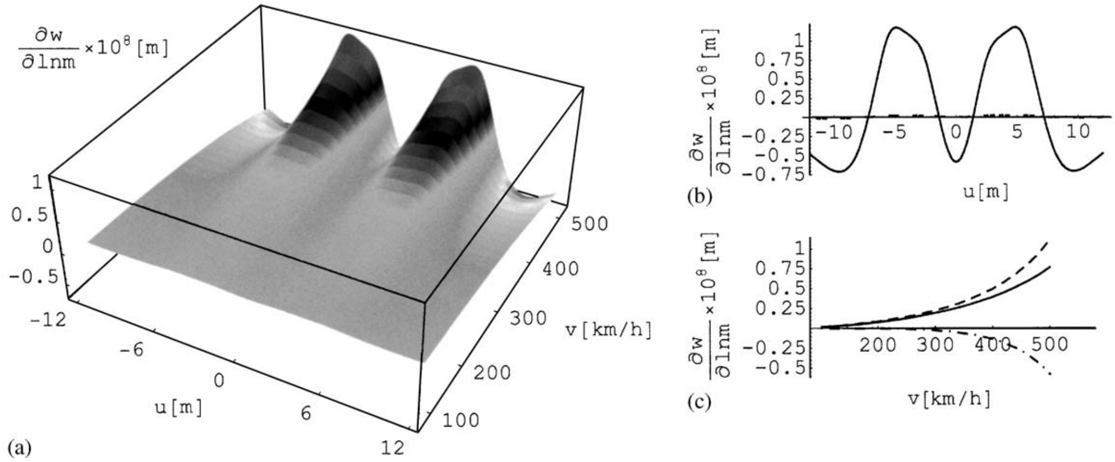

Fig. 5. First order sensitivity function for parameter m.

expansion terms are expressed by this well-known formula

$$d^n w = \sum_{n_1+n_2+\ldots+n_s=n} \frac{n!}{n_1!n_2!\ldots n_s!} \frac{\partial^n w}{\partial b_1^{n_1} \partial b_2^{n_2} \ldots \partial b_s^{n_s}} \Delta b_1^{n_1} \Delta b_2^{n_2} \ldots \Delta b_s^{n_s}, \tag{14}$$

where s stands for the dimension of the design variables vector.

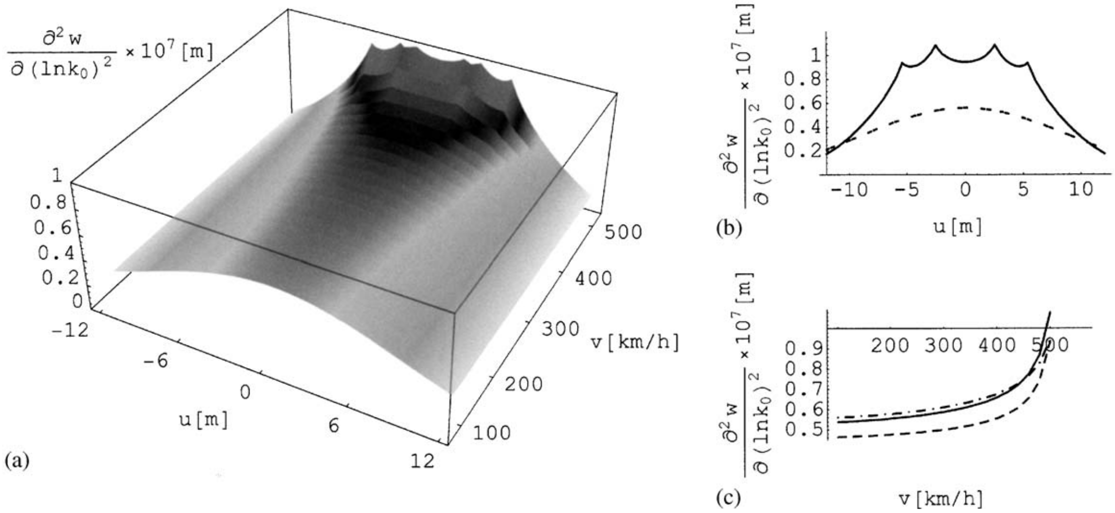

Fig. 6. Second order sensitivity function for parameter k_0.

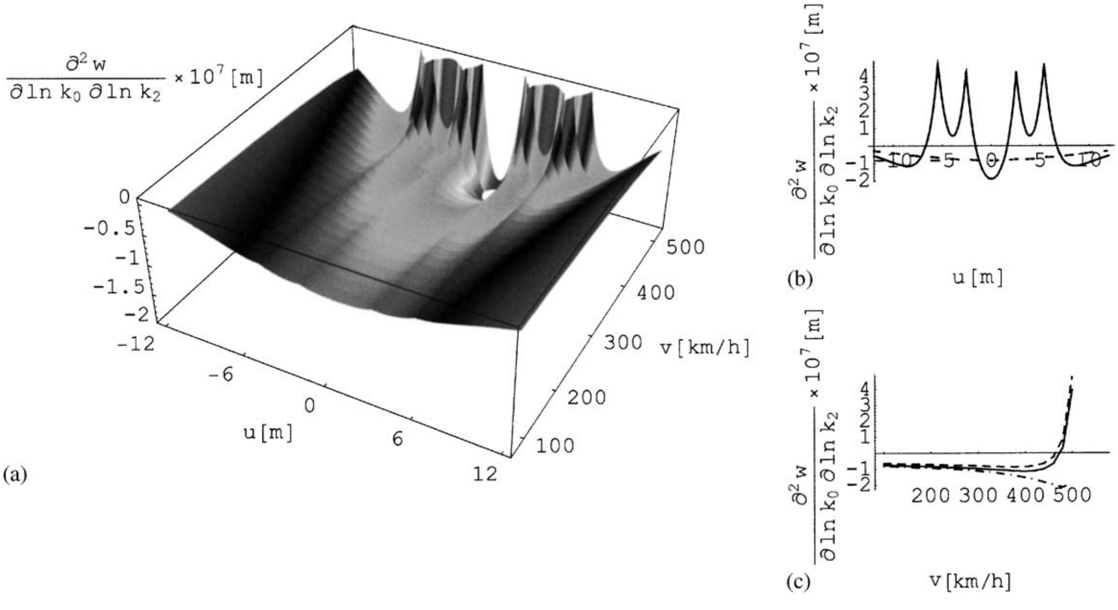

Fig. 7. Second order sensitivity function for parameters k_0 and k_2.

If the considerations are limited to the variability of only one parameter: b_i ($\Delta b_i \neq 0$, $\Delta b_k = 0$ for $k \neq i$), one gets

$$\Delta w = \frac{\partial w}{\partial b_i}\Delta b_i + \frac{1}{2}\frac{\partial^2 w}{\partial b_i^2}\Delta b_i \Delta b_i + \cdots. \tag{15}$$

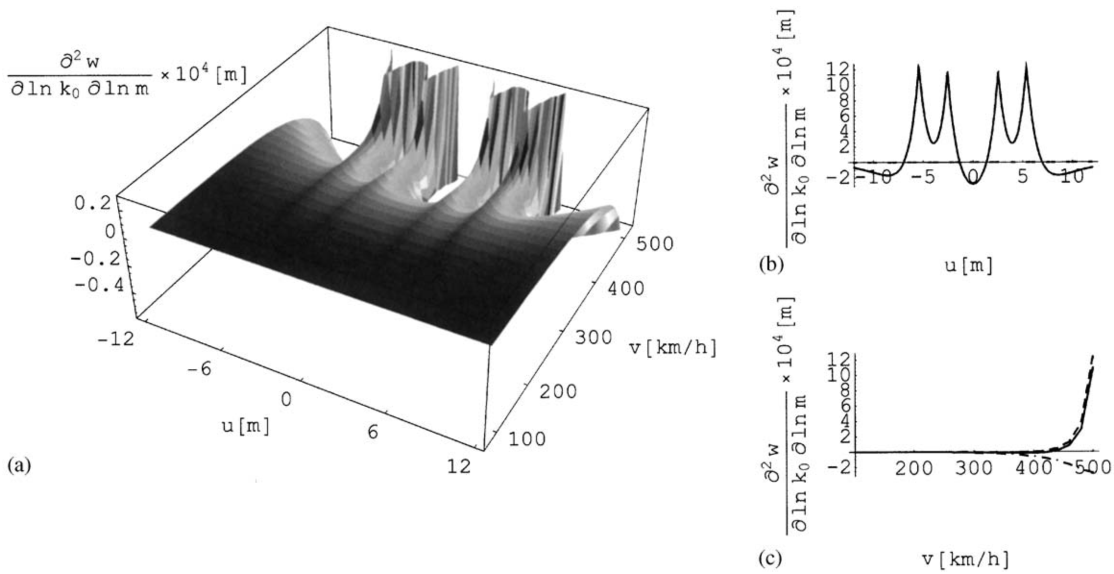

Fig. 8. Second order sensitivity function for parameters k_0 and m.

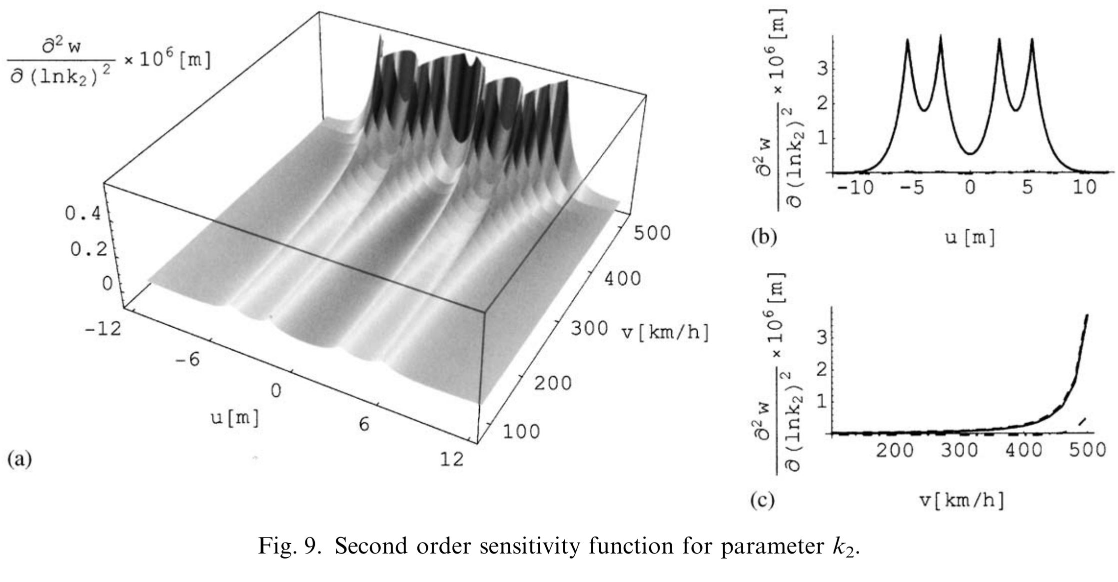

Fig. 9. Second order sensitivity function for parameter k_2.

Respectively one

$$\Delta w_I = \frac{\partial w}{\partial b_i}\Delta b_i = \frac{\partial w}{\partial \ln b_i}\frac{\Delta b_i}{b_i} = S_i \frac{\Delta b_i}{b_i} \tag{16}$$

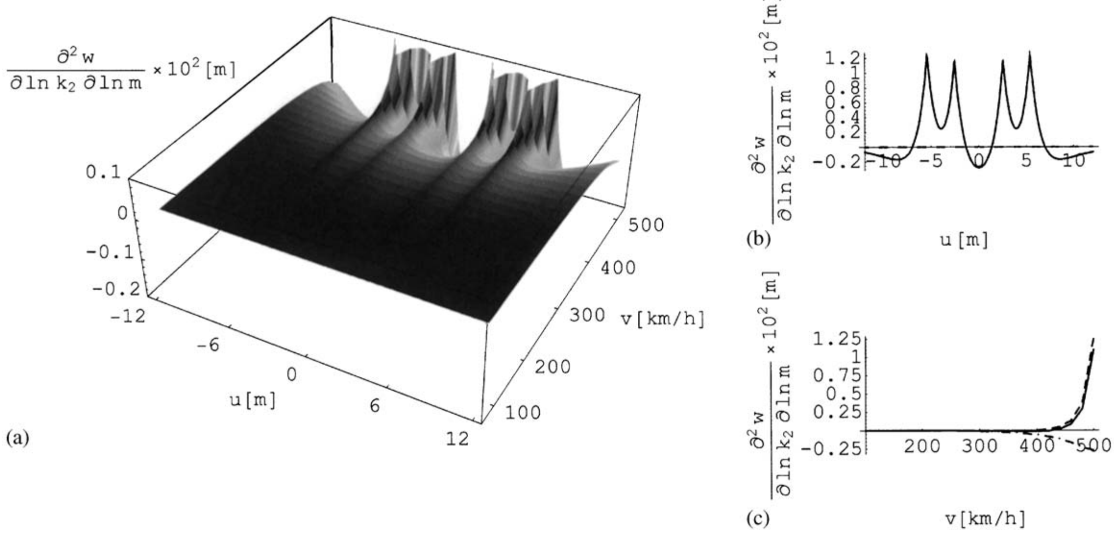

Fig. 10. Second order sensitivity function for parameters k_2 and m.

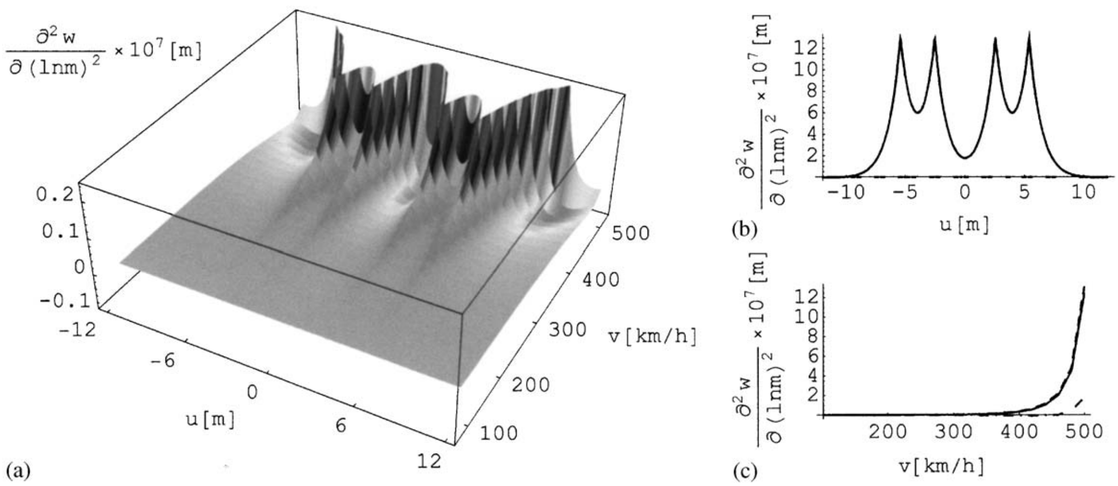

Fig. 11. Second order sensitivity function for parameter m.

and two series elements

$$\Delta w_{II} = \frac{\partial w}{\partial b_i}\Delta b_i + \frac{1}{2}\frac{\partial^2 w}{\partial b_i^2}(\Delta b_i)^2 = \frac{\partial w}{\partial \ln b_i}\frac{\Delta b_i}{b_i} + \frac{1}{2}\frac{\partial^2 w}{\partial (\ln b_i)^2}\left(\frac{\Delta b_i}{b_i}\right)^2 = S_i\frac{\Delta b_i}{b_i} + \frac{1}{2}S_{i,i}\left(\frac{\Delta b_i}{b_i}\right)^2 \quad (17)$$

were taken into account in the approximation. The values of functions Δw_I, Δw_{II} were compared with exact increment value $\Delta w = w(b_i + \Delta b_i) - w(b_i)$. Ten percent variability of design parameters in the neighbourhood of their nominal values was assumed. The results are presented as graphs in Figs. 12–14 (figures (a) — for $u_0 = 0\,\mathrm{m}$, figures (b) — $u_2 = 5.40\,\mathrm{m}$), where Δw the exact

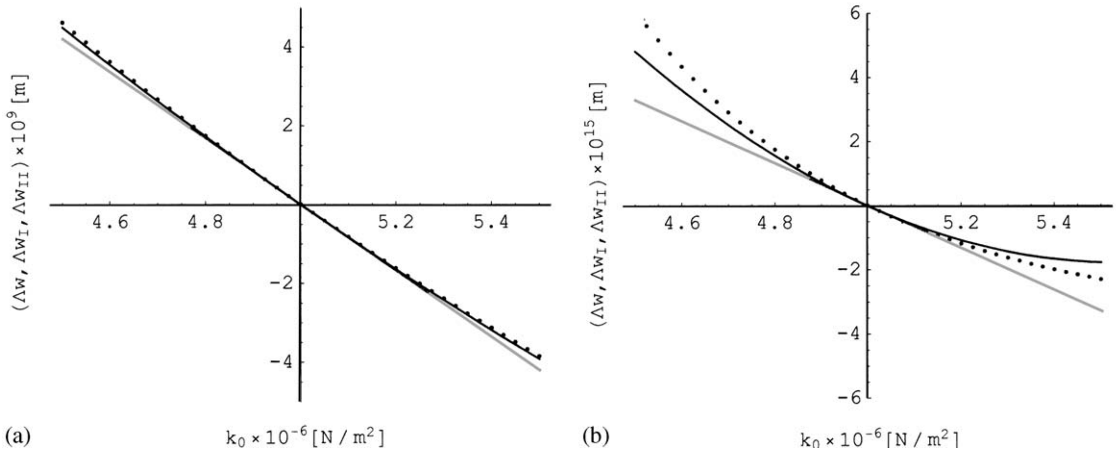

Fig. 12. Displacement increment functions Δw for parameter k_0, taking into account first order sensitivity function Δw_I and second order sensitivity function Δw_{II}.

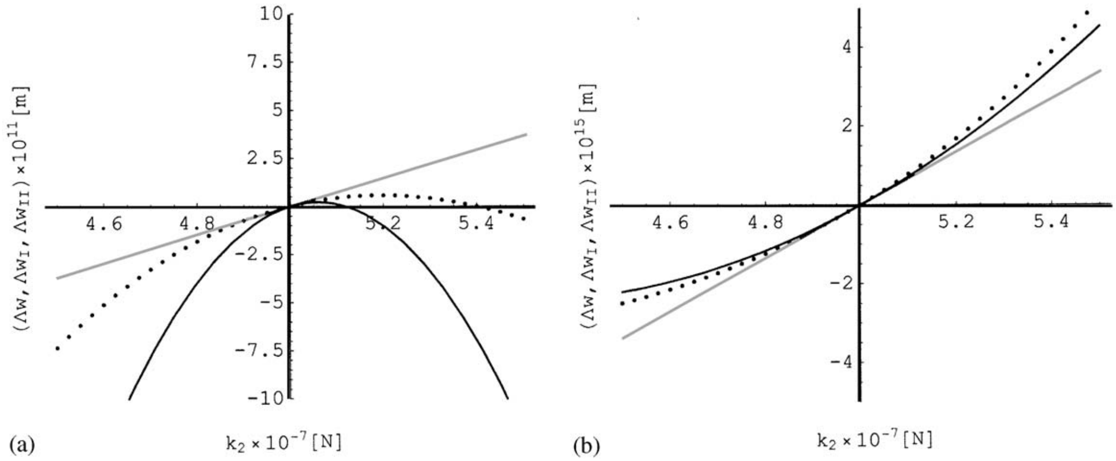

Fig. 13. Displacement increment functions Δw for parameter k_2, taking into account first order sensitivity function Δw_I and second order sensitivity function Δw_{II}.

displacement increment value (full line), Δw_I the increment exact to the first term in the Taylor series (grey line), and Δw_{II} the increment exact to two Taylor series terms (dotted line).

4. Conclusions

The authors' main intention was to develop an analytical method of determining the sensitivity function. The numerical example given illustrates the effectiveness of the method an it does not amount to a full analysis of the problem. Nevertheless, certain conclusions can be drawn even from this limited analysis.

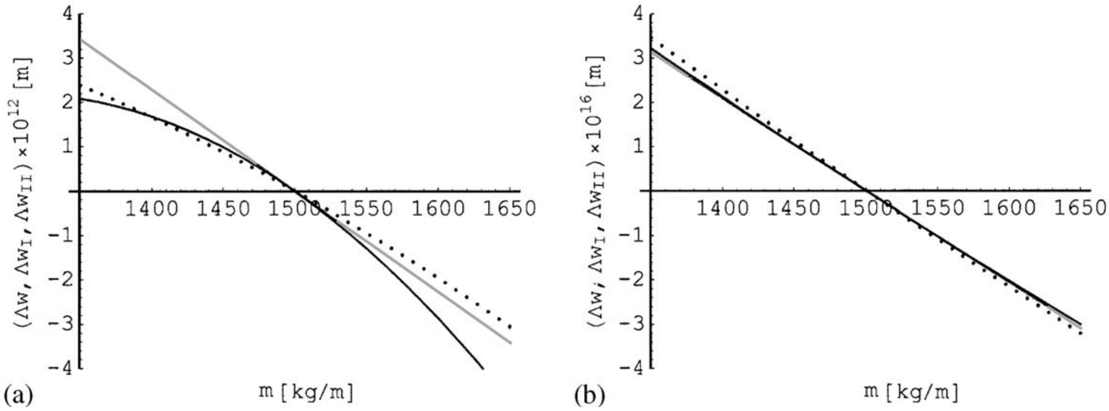

Fig. 14. Displacement increment functions Δw for parameter m, taking into account first order sensitivity function Δw_I and second order sensitivity function Δw_{II}.

An analysis of both the numerical results and the method of determining sensitivity functions shows that:

1. Sensitivity function values increase in the neighbourhood of force application points.
2. Sensitivity increases with velocity. This applies mainly to second order functions where already at a velocity of 450 km/h the values of the functions increase rapidly. Such a large increase was expected to occur only close to the critical velocity which for the assumed system parameters was 754 km/h ($v_{cr} = \sqrt{(\sqrt{4EJk_0} + k_2 + N)/m}$).
3. It follows from the obtained results that the first order sensitivity of deflection is the highest for parameter k_0 and the lowest for parameter m.
4. The second order sensitivity of deflection is the highest for parameters k_2, m and the lowest for parameters k_0.
5. The calculations have confirmed that the second order sensitivity function significantly improves the accuracy of displacement approximation.
6. One cannot compare first and second order sensitivity function values since the displacement difference value is affected not only by sensitivity function values but by the values of the weight coefficients (multipliers) which occur in formula (14).
7. When identifying a railway track's parameters, it is advisable to perform the identification test in high sensitivity areas. As the sensitivity increases, so does the accuracy of the method (at a constant measuring accuracy). Therefore the identification of mating mass m should be conducted at high vehicle speeds (Fig. 5). This requirement is not so stringent for parameter k_0, k_2 (Figs. 3 and 4) which can be identified by a quasi-static test.

The proposed method of testing track structure (considered to be an infinite beam resting on multiparameter elastic subsoil) sensitivity parameters seems to be a highly effective practical tool which allows one to isolate such a set of parameter values for which displacement function sensitivity values will be the highest (or the lowest).

The sensitivity analysis method's advantage is its universality due to the analytical form of the solution. The method is applicable to any problem described by differential equations with constant coefficients. Formulas derived for such problems will differ only slightly from the relations presented in this paper.

References

[1] J. Wicher, First and second order sensitivity derivatives of mechanical systems transfer matrix, Engineering Transactions. 35 (2) (1987) 229–240.
[2] P.M. Frank, Introduction to System Sensitivity, Academic Press, London, 1978.
[3] Z. Wójcicki, Sensitivity analysis of steady-state response of parametric systems, in: M. Papadrakakis, B.H.V. Topping (Eds.), Advances in Computational Mechanics, Civil-Comp Press, Edinburgh, 1994, pp. 215-223.
[4] P. Eykhoff, System Identification. Parameter and State Estimation, Wiley, London, 1974.
[5] L. Fryba, Vibration of Solids and Structures under Moving Loads, Academia, Prague, 1972.