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An experimental study of the effects of pulsating and steady internal fluid flow on an elastic tube subjected to external vibration

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Abstract

The results of an experimental study on both pulsating and steady Newtonian fluid flow in an initially stretched rubber tube subjected to external vibration are reported. A circulating loop system was designed to maintain constant hydrostatic pressure throughout the tests so that the influence of external excitation on the fluid flow could be properly distinguished. The effects of fluid flow velocity and initial stretch rates on the dynamic response and damping of the tube conveying fluid were examined, and it was observed that damping ratios increase with increasing flow velocities, and generally decrease with increasing initial stretch rates for the tube conveying fluid. It was also noted that dynamic responses increase with increasing initial stretch rates, and decrease with increasing flow velocities. The effect of external vibration on fluid flow rates is small in a tube with a thickness-to-radius ratio $(D_{out} - D_{in})/D_{in} = 0.617$. Fluid pressures vary, in terms of frequency and amplitude, with external vibration as well as Womersley number.

1. Introduction

Fluid flow in elastic tubes subject to external vibration is encountered in a variety of engineering applications, including above-ground pipelines exposed to wind gusts [1] and spanning sub-sea

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pipelines excited by cross-flow currents [2]. It is well known that tubes carrying fluid at a critical flow velocity or excited at the critical frequency can develop large and catastrophic vibration. However, potential interaction between a pulsating flow in a tube and external excitation may vary these conditions. It is thus highly desirable to understand the underlying characteristics and elastic/dynamic behaviour of the system so that suitable strategies for avoiding destructive vibration can be developed.

As demonstrated by Païdoussis and Li [3], although extensive theoretical studies have been performed, experiments on the dynamic behaviour of flexible tubes conveying fluid are not numerous. Gregory and Païdoussis [4] first investigated experimentally the effect of flow velocity on natural frequency and mode shape for a cantilevered rubber tube conveying water and air. Such tubes lose stability by flutter [5–7] while a fixed–fixed rubber tube conveying fluid loses stability by divergence [8]. Naguleswaran and Williams [9] conducted an experiment with a clamped–clamped neoprene tube conveying water and investigated the effect of pressurization on the lowest natural frequency of the system. Jendrzejczyk and Chen [8] conducted a careful set of experiments for polyethylene and acrylic tubes conveying water with various support conditions. Recently, Zhang et al. [10,11] conducted experiments on simply supported small-diameter tubes conveying water, examined mode coupling, and found that vibration nodes shifted downstream as the flow velocity increases, and that natural frequencies increase with increasing initial stretch rates. However, all these investigations have been confined to tubes conveying steady fluid flow to establish: (1) the critical flow velocity at which large tube displacement develops, (2) the dynamic response of the systems without initial axial tensions at a sub-critical flow velocity, and (3) the influence of initial axial tension, flow velocity and fluid pressure on natural frequency.

A parametric resonance analysis of a silicon rubber tube conveying a pulsating flow of water was conducted by Païdoussis and Issid [12], although this system was only subjected to internal pulsating excitation. It was found that the tube, which was unstable by the mean (steady) flow, could be re-stabilized by parametric excitation—adding a pulsating flow component at certain frequencies and amplitudes. Based on this stabilizing mechanism, a vibration suppression method for a hanging silicon rubber tube was presented [13]. These two experimental studies were for tubes conveying fluid in the absence of initial tensions and external vibration.

The present paper reports an experimental study of steady and pulsating fluid flow in an elastic tube subjected to an external harmonic excitation. It describes the influence of external excitation on fluid flow and the effect of initial stretch rates and flow velocities on the dynamic behaviour of the system. The rubber tube initial stretch rates are in the range 1.42–16.5%, and steady and pulsating fluid flow in the tube are in the Reynolds number range, $Re = 0-30\,000$, and a Womersley number range $0 \leqslant Wo \leqslant 10$.

2. Experimental apparatus and procedures

The experimental apparatus comprised two main parts: the hydraulic piping and excitation system, and the instrumentation. A schematic of the closed flow loop is shown in Fig. 1. A speed-adjustable motorized cam was used to produce reproducible pulsating flow in the test section (the distribution curve at the outlet for a typical cycle is shown in Fig. 2), and the mean flow rate was adjusted by a manual/automatic control valve. Using a pump to feed water from the sink reservoir

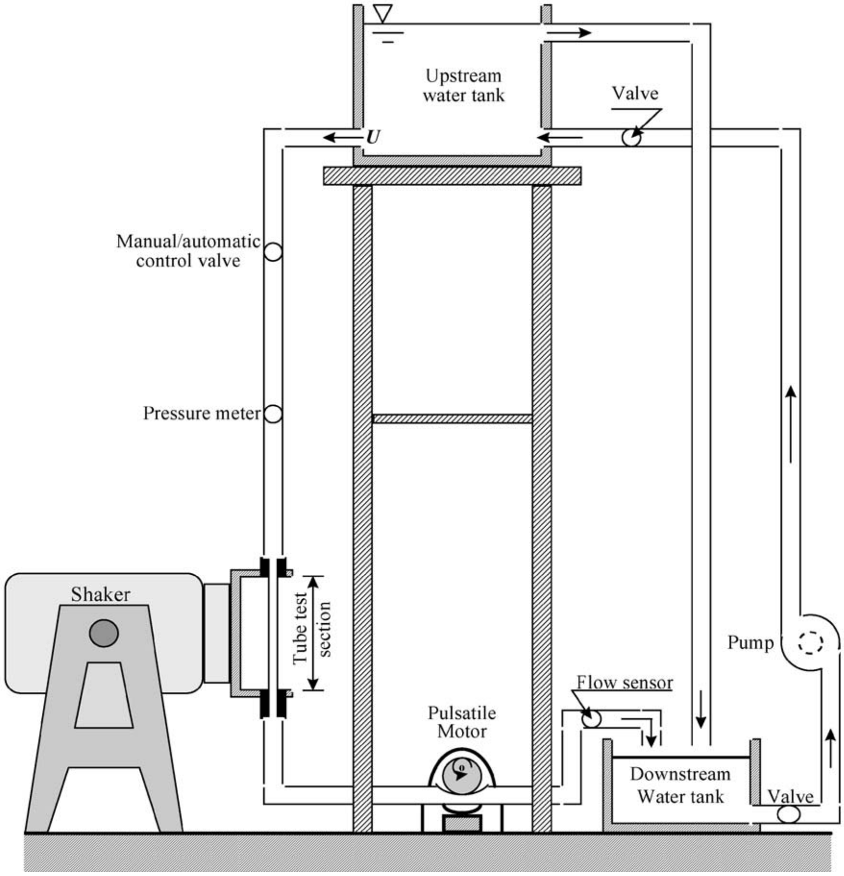

Fig. 1. Schematic of the closed flow loop experimental facility.

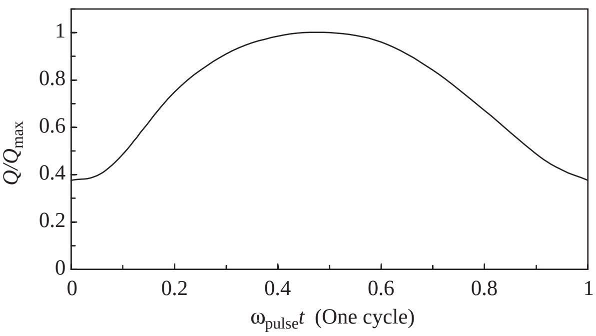

Fig. 2. Pulsating flow in the tube for one cycle.

back to the overhead supply reservoir with overflow tube caused a constant hydrostatic pressure to be imposed throughout the tests.

The test section consisted of a circular rubber tube. Various tubes with natural lengths (L_0) in the range 0.311–0.357 m were stretched to a fixed setting length of 0.362 m (thereby realising stretch rates in the range 1.42–16.5%), and then mounted between upstream and downstream rigid thin-walled pipes, which are themselves mounted on a rigid platform. The resonance frequencies of the rigid platform were well in excess of the frequency range of interest (i.e., the natural frequencies of the fluid-conveying tubes under test). The upstream and downstream rigid pipes were connected to two flexible hoses leading to the upstream supply and downstream sink tanks respectively, with the aim of not introducing noise into the test section from another part of the piping system. As the rubber tube was easily deformed, it had to be carefully clamped at both ends: in order to avoid distorting the force signal and introducing errors in measurement, a symmetric clamping condition was imposed. The complete assembly was symmetrically clamped onto a shaker driven by an amplified sine-wave signal from a 5 MHz function generator.

The shaker excitation was measured by an accelerometer mounted on the shaker platform. A laser velocity transducer with an accuracy of $\pm 0.3\%$ was used to measure the vibration response of the tube. The laser was aligned to focus on a small piece of retro reflective tape attached to the surface of the tube. Two dynamic pressure transducers measured fluid pressure within the test tube at both ends. Flow rates were measured by a flow rate sensor with an accuracy of $\pm 2\%$. In order to avoid both the dynamic response of the tube and wave propagation through the tube wall during the squeezing of the tube affecting the pulsating flow signals, the flow rate sensor was located far from the test section. Signals from the accelerometer, pressure transducers and laser velocity transducer were examined using the oscilloscope and phase meter. These signals were then passed to a real-time fast Fourier transform (FFT) analyzer. Subsequently, all data was then transferred to a PC for further processing.

The physical and geometric properties of the test section were measured and are listed as follows: $L = 0.362\,\mathrm{m}$, $D_{in} = 6.0 \times 10^{-3}\,\mathrm{m}$, $D_{out} = 9.7 \times 10^{-3}\,\mathrm{m}$, $\rho_f = 1000.0\,\mathrm{kg\,m^{-3}}$, $\rho_t = 1128.56\,\mathrm{kg\,m^{-3}}$, $\nu = 0.5$, $E = 2.0924 \times 10^6\,\mathrm{N\,m^{-2}}$, $\mu = 1.307 \times 10^{-3}\,\mathrm{N\,s\,m^{-2}}$.

3. Results and discussion

Throughout the experiments, the coherence function, $\gamma^2(\Omega) = H_1(\Omega)/H_2(\Omega)$, was always greater than zero but less than or almost equal to unity. During the tests it was important to examine the resonance frequency of the combined structure (rigid platform together with the test section) before taking measurements on the test section. The first resonance frequency of the structure is around 100 Hz. As the lowest several resonance frequencies are of practical interest, and the first four or five resonance frequencies of the test tube section are lower than 100 Hz (see Fig. 3), it was therefore concluded that the experimental set-up was well designed. The magnitudes of several frequency response functions (FRF) and the corresponding coherence functions were measured at the midpoint of the tube.

Comparing Figs. 3(a–d), it can be seen that the effect of the presence of flowing fluid is to reduce natural frequencies, as expected. Similarly, it can also be seen from Fig. 3 that the larger the stretch rates the larger the natural frequencies. The coherence function shown in Fig. 4 demonstrates that for low excitation frequency the correlation between excitation and measurement is not good. But it can be seen from Figs. 3(c, d) and 4 that $\gamma^2(\Omega)$ is very close to unity around the resonance frequencies of the test section. However, when the excitation frequency is smaller than 10 Hz, the coherence function is less than unity. This is probably due to

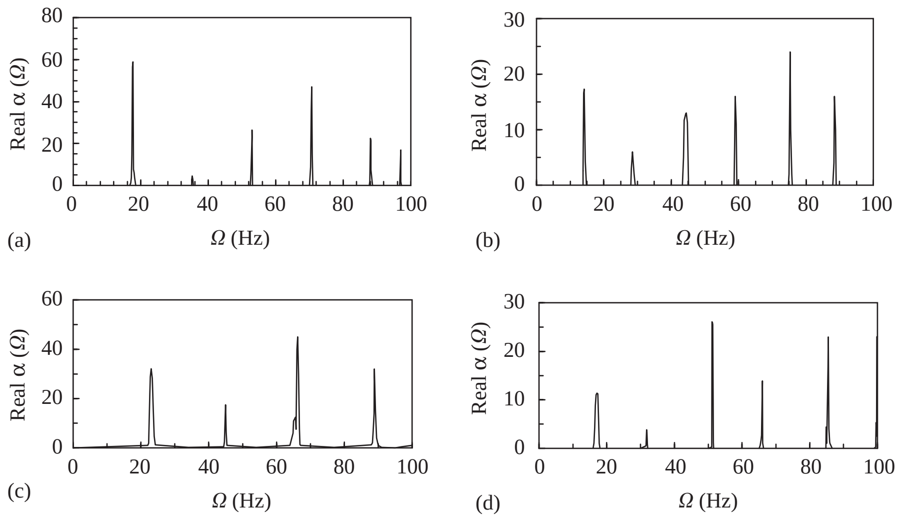

Fig. 3. Frequency response function versus excitation frequency at stretch rates (a,b) $\lambda = 1.415\%$ and (c,d) $\lambda = 11\%$; (a,c) the empty tube, (b,d) the tube conveying fluid at a flow rate $Q = 50.1 \times 10^{-6}\ \mathrm{m^3\ s^{-1}}$ and $Wo = 0$.

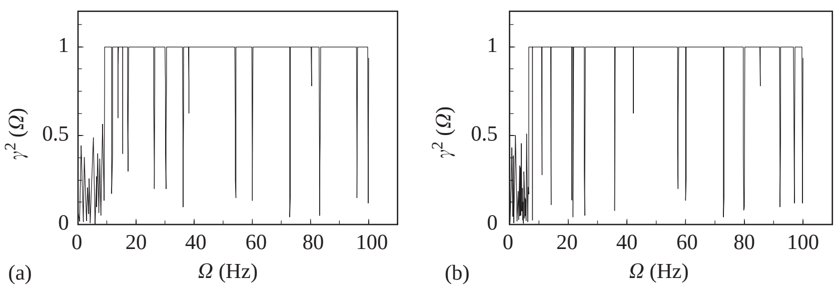

Fig. 4. Coherence function versus excitation frequency at $\lambda = 11\%$; (a) the empty tube, (b) the tube conveying fluid with $Q = 50.1 \times 10^{-6}\ \mathrm{m^3\ s^{-1}}$ and $Wo = 0$.

one, or more, of the following three factors: (1) extraneous noise is present in the FRF measurements; (2) the system relating excitation force $\mathbf{f}(t)$ and dynamic response $\mathbf{u}(t)$ is not linear; (3) the measured response $\tilde{\mathbf{u}}(t)$ is due to other external inputs besides $\mathbf{f}(t)$.

For the tube with a larger stretch rate, when fluid is flowing, $\gamma^2(\Omega)$ is very close to unity around the resonance frequency. But the correlation is not good for a low excitation frequency: due to the flexibility of the rubber tube it is difficult to measure its in-plane vibration around the first natural frequency when the stretch rate is low. Hopf-bifurcation easily appeared when the tube with a low stretch rate is excited around the first natural frequency. It was observed that the relationship between $\mathbf{f}(t)$ and $\mathbf{u}(t)$ was non-linear when $\Omega < 10$ Hz. This may be a main contributing reason why $\gamma^2(\Omega) < 1$. Figs. 3 and 4 show that the experimental set-up has been demonstrated to yield useful results for the first few vibration modes, whose frequencies are larger than 10 Hz and less than 100 Hz.

Fig. 5 shows the spectrum of the tube in the absence of and containing flowing fluid when the stretch rate $\lambda = 11\%$. As demonstrated earlier (see Fig. 4), the correlation results for excitation

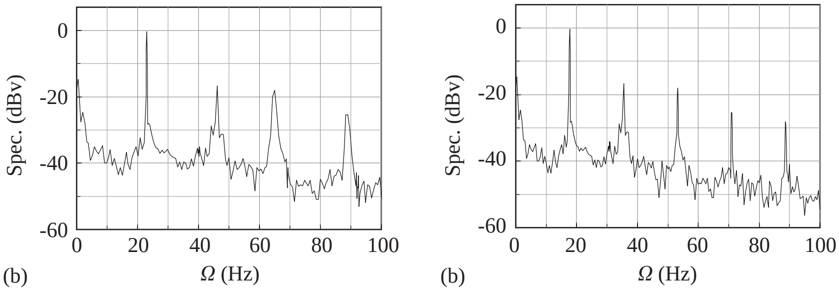

Fig. 5. Spectrum for the tube at $\lambda = 11\%$; (a) the empty tube, (b) the tube conveying fluid with $Q = 50.1 \times 10^{-6}\ \mathrm{m^3\,s^{-1}}$ and $Wo = 0$.

Table 1
Damping ratios for the tube conveying steady fluid flow at various initial stretch rates and flow velocities

Stretch rate λ	Flow velocity U (m/s)						Damping ratio η_d (%)						η_d[a] (%)
	U_1	U_2	U_3	U_4	U_5	U_6	η_{d_1}	η_{d_2}	η_{d_3}	η_{d_4}	η_{d_5}	η_{d_6}	
5.0%	0.0	0.78	2.91	4.88	5.57	6.66	3.83	3.87	3.12	4.39	5.02	5.51	4.01
9.0%	0.0	1.54	3.07	4.34	5.42	6.43	3.07	2.97	3.12	3.25	3.62	3.88	3.12
12.75%	0.0	0.93	3.34	4.28	5.55	—	2.45	2.52	2.56	2.76	2.91	—	2.21
14.75%	0.0	—	—	—	—	—	2.26	—	—	—	—	—	2.14
15.75%	0.0	—	—	—	—	—	2.24	—	—	—	—	—	2.51
16.5%	—	—	—	—	—	—	—	—	—	—	—	—	2.34

[a] The damping ratio for the tube in the absence of fluid.

frequencies less than 10 Hz are far from unity. At these very low excitation frequencies it can also be seen from Fig. 5 that the spectrum results are not reasonable.

The damping ratios for the tube conveying steady fluid flow under various initial stretch rates were evaluated using the so-called half-power points [14], viz.

$$\eta_\mathrm{d} = (\Omega_2^2 - \Omega_1^2)/2\Omega_n^2, \tag{1}$$

(see Nomenclature for definition of standard symbols). They are shown in Table 1 and Fig. 6 where it can be seen that, generally, they increase with increasing flow velocities except in two cases (viz. $\lambda = 5\%$ and $U = 2.91\ \mathrm{m\,s^{-1}}$; $\lambda = 9\%$ and $U = 1.54\ \mathrm{m\,s^{-1}}$). Damping ratios decrease with increasing initial stretch rates for the tube conveying fluid. However, when the tube is without fluid, this trend exists only up to $\lambda = 14.75\%$. This is consistent with the fact that initial tensions within the tube wall affect the vibration of a pre-stressed flexible system causing stiffening of the tube and an increase in natural frequencies [10]. Provided that the tube wall thickness reduces to some extent as the tube is stretched, the effect of stretch rate on η_d is not considerable.

In order to appreciate the influence of excitation amplitude, vibration mode and undisturbed flow rate (i.e., the flow rate for the system without any external excitation, $a_m = 0$), an experiment was performed for two undisturbed flow rates and the first three vibration modes by varying the

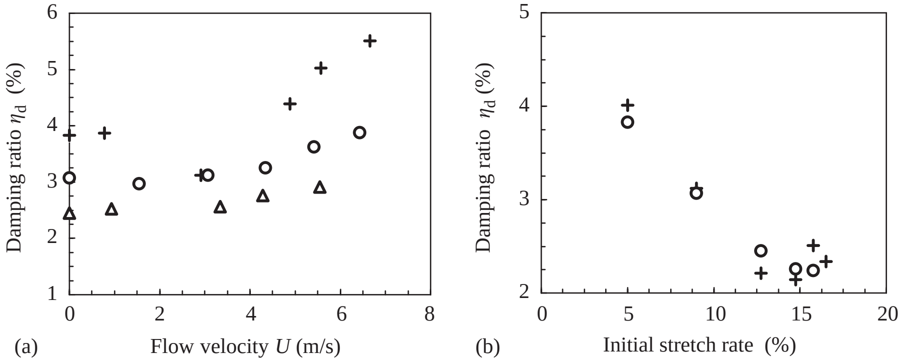

Fig. 6. (a) Variation of damping ratios with flow velocities, $\lambda = 5.0\%$ (+ +), $\lambda = 9.0\%$ (○ ○), $\lambda = 12.75\%$ (Δ Δ); (b) variation of damping ratios with initial stretch rates, in the absence of fluid (+ +), tube containing quiescent fluid (○ ○).

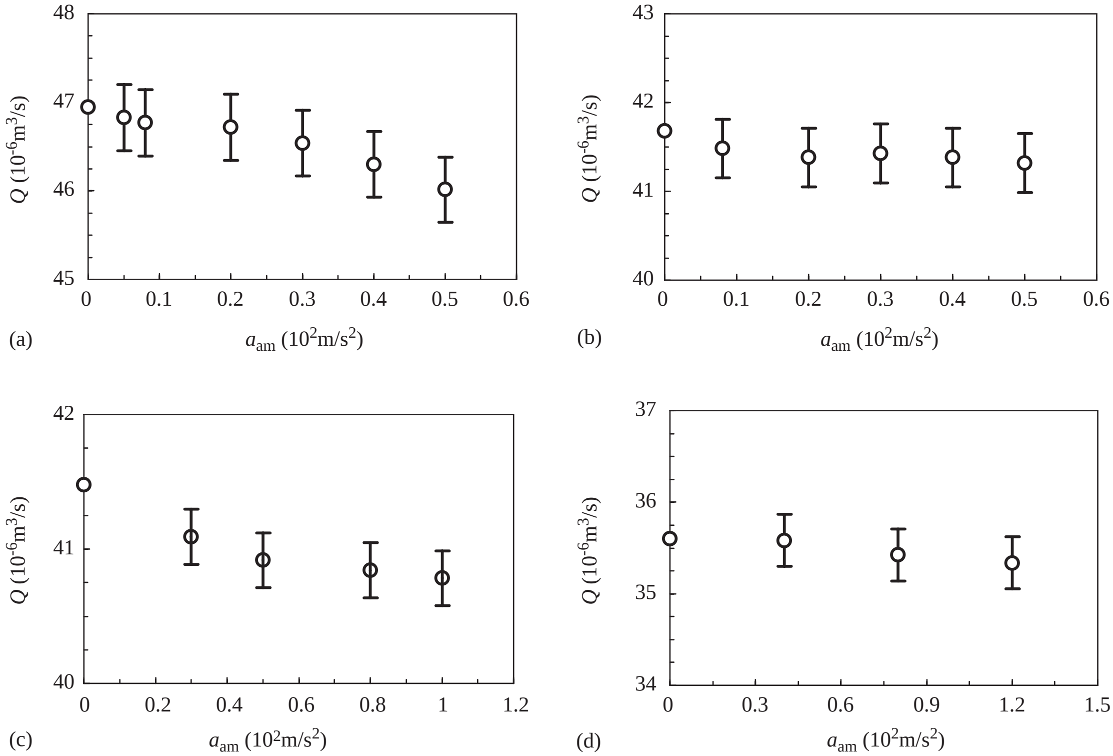

Fig. 7. Effect of excitation amplitude on fluid flow rate for the initially stretched tube conveying fluid with $\lambda = 11\%$ and $\omega_{pulse} = 0$ (steady flow); (a) $\Omega = 17.49$ Hz, initial flow rate $Q = 47 \times 10^{-6}$ m^3 s^{-1}; (b) $\Omega = 17.49$ Hz, initial flow rate $Q = 41.7 \times 10^{-6}$ m^3 s^{-1}; (c) $\Omega = 35.2$Hz, initial flow rate $Q = 41.5 \times 10^{-6}$ m^3 s^{-1}; (d) $\Omega = 55.62$ Hz, initial flow rate $Q = 35.6 \times 10^{-6}$ m^3 s^{-1}.

excitation amplitude in the range $0-120\,\mathrm{m\,s^{-2}}$. Fig. 7(a) shows the effect of harmonic excitation amplitude on the fluid flow in the tube when the excitation frequency $\Omega = 17.49\,\mathrm{Hz}$, the stretch rate $\lambda = 11\%$ and the initial undisturbed flow rate $Q = 4.7 \times 10^{-5}\,\mathrm{m^3\,s^{-1}}$. It can be seen that the effect of external vibration is to reduce the flow rate. This is probably due to the larger excitation amplitudes inducing larger friction coefficients; however, this flow reduction is small. It can also be seen that external vibration causes fluid flow fluctuation, which is also small (around 1.6%). For a smaller undisturbed initial flow rate a similar trend holds, as shown in Fig. 7(b), but the flow reduction is generally smaller. Figs. 7(b)–(d) show the effect of excitation amplitude on fluid flow in the tube at the first three resonance frequencies. It can be seen that the influence of excitation amplitude and frequency on the flow is small. This may be due to the relatively high thickness of the tube wall (viz. $(D_{out} - D_{in})/D_{in} = 0.617$) and the Newtonian nature of the fluid. The former factor leads to the dominance of beam vibration modes and causes a reduction in the flow rate as a result of an increase in friction. The latter factor means that the fluid is neither shear-thinning (which would enhance the flow), nor shear-thickening (which would reduce the flow).

Figs. 8 and 9 show the effect of external harmonic excitation on gauge pressure when the stretch rate $\lambda = 5.8\%$, and the Womersley numbers $Wo = 6.42$ and 7.72, respectively. It should be pointed out that as the test section is mounted vertically and the fluid flows from top to bottom, the pressure at the downstream end of the tube is larger than at the upstream end. It can be seen that at $Wo = 6.42$ the pressure wave frequency for fluid flow in the tube subjected to an external excitation is larger than for the tube without excitation. When $Wo = 7.72$ the pressure wave frequency for fluid flow in the excited tube is smaller than for the tube without excitation. Therefore the direction in which the pressure wave frequency changes for fluid flow in the excited tube (compared with the pressure wave frequency for the system at $y_{am} = 0$) is associated with the Womersely number. For $Wo = 6.42$ the difference in the pressure wave amplitude within the excited tube compared to that in the unexcited tube is small, whereas for $Wo = 7.72$ the difference is significant.

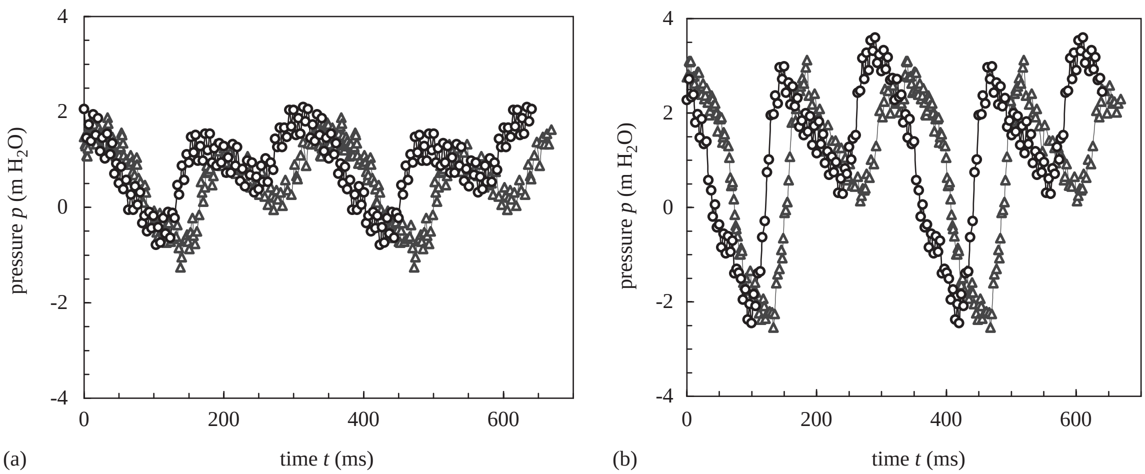

Fig. 8. Effect of external excitation on gauge pressure for the tube conveying fluid at $\lambda = 5.8\%$, $Re = 965$, $\Omega = 44.37\,\mathrm{Hz}$, $Wo = 6.42$, $y_{am} = 0.8\,\mathrm{mm}$ (○ ○), $y_{am} = 0$ (Δ Δ); (a) upstream, (b) downstream.

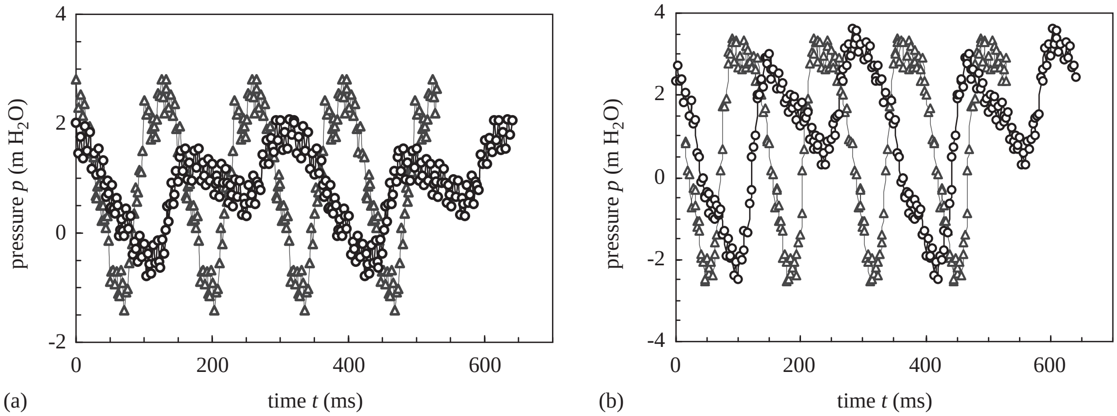

Fig. 9. Effect of external excitation on gauge pressure for the tube conveying fluid at $\lambda = 5.8\%$, $Re = 965$, $\Omega = 44.37\,\text{Hz}$, $Wo = 7.72$, $y_{am} = 0.8\text{mm}$ (○ ○), $y_{am} = 0$ (Δ Δ); (a) upstream, (b) downstream.

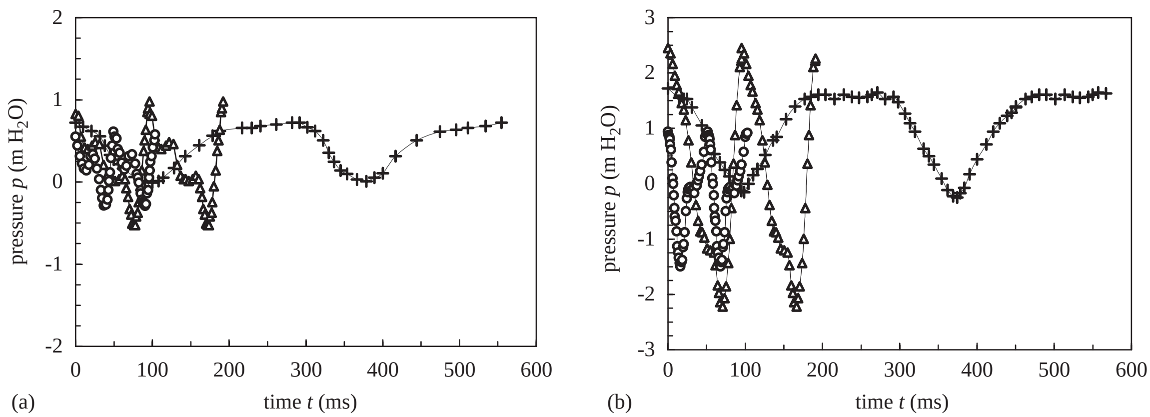

Fig. 10. Variation of gauge pressure with time for the tube conveying fluid subjected to external excitation at $\lambda = 11\%$, $Re = 965$, $\Omega = 16.88\,\text{Hz}$, $y_{am} = 0.304\,\text{mm}$, $Wo = 4.97$ (+ +), $Wo = 8.52$ (Δ Δ), $Wo = 9.99$ (○ ○); (a) upstream, (b) downstream.

Fig. 10 shows the influence of the Womersley number on gauge pressure waves in the tube subjected to external excitation with a stretch rate $\lambda = 11\%$. It can be seen that the largest amplitude of pressure wave appears at $Wo = 8.52$, and it is indicated that the largest positive or negative pressure that appears is associated with the Womersley number and the initial stretch rate.

The dynamic responses measured at the midpoint of the tube at the third vibration mode for several initial stretch rates are shown in Fig. 11. An increase in initial axial tension leads to an increase in vibration amplitude as a result of a smaller damping ratio.

The variation of dynamic responses with excitation frequencies for several flow velocities is shown in Fig. 12. It can be seen that increasing flow velocity leads to a decrease in vibration

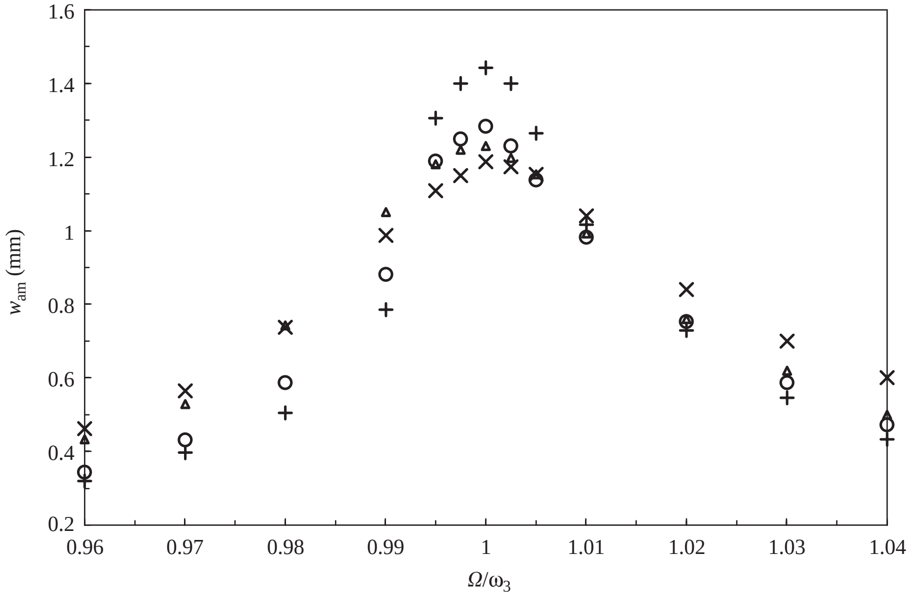

Fig. 11. Dynamic responses of the tube containing quiescent fluid for the third vibration mode at $\omega_{pulse} = 0$ and $y_{am} = 0.87$ mm,where ω_3 is the third natural frequency; $\lambda = 14.75\%$ (+ +), $\lambda = 12.75\%$ (○ ○), $\lambda = 9.0\%$ (Δ Δ), $\lambda = 5.0\%$ (× ×).

amplitude. This is generally attributable to the fact that the damping ratio increases with increasing flow velocity. At a smaller stretch rate, increasing flow velocity more significantly decreases the vibration amplitude.

4. Conclusions

In this paper, experiments on initially stretched rubber tubes conveying pulsating and steady water flow have been reported. The effect of fluid flow velocity and initial stretch rates on the dynamic response and damping of the tube conveying fluid has been assessed. The results show that:

- damping ratios generally increase with increasing flow velocities, and decrease with increasing initial stretch rates for the tube conveying fluid;
- dynamic responses increase with increasing initial stretch rates, decrease with increasing flow velocities and decrease more significantly with increasing flow velocities at a smaller stretch rate;
- the effect of external excitations is to reduce fluid flow rates in a tube with a thickness-to-radius ratio $(D_{out} - D_{in})/D_{in} = 0.617$, but this reduction is small;
- an external excitation causes small fluctuations of fluid flow rate and pressure in the tube;
- pressure wave frequency and amplitude vary with external excitation, and the directions of these variations are associated with the Womersley number.

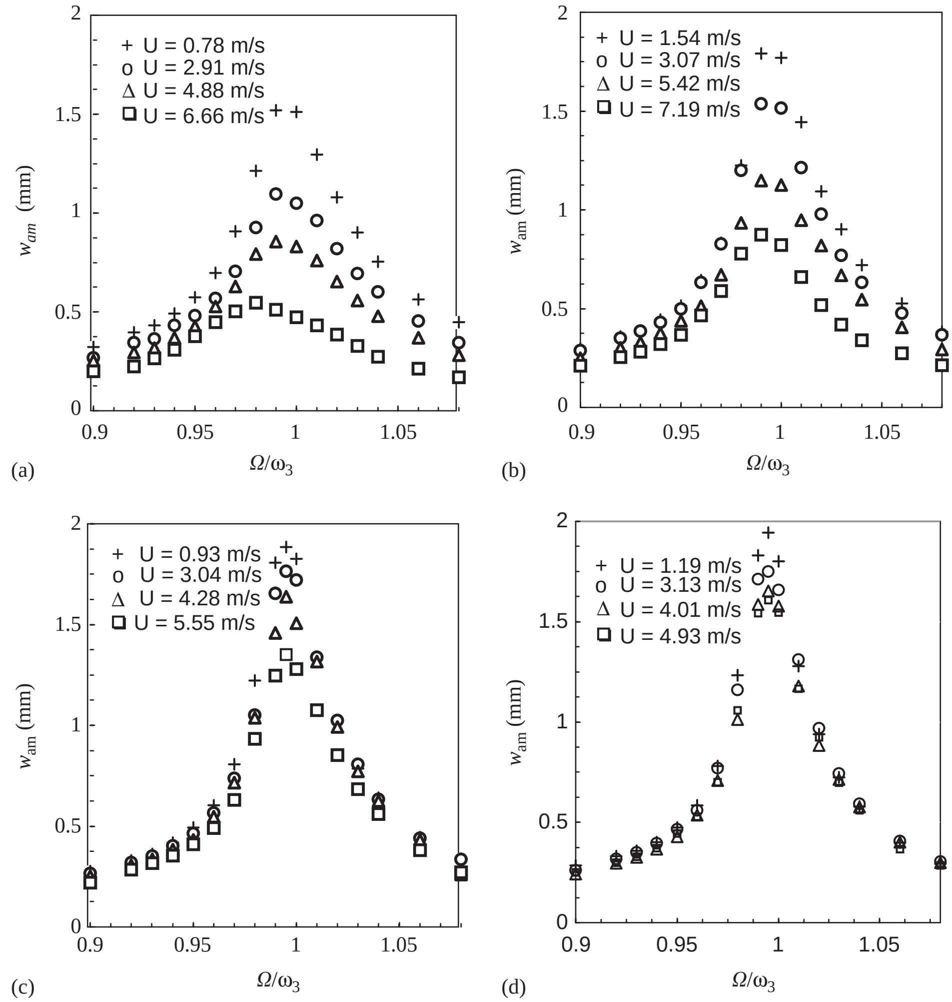

Fig. 12. Dynamic response of the tube conveying fluid for the third vibration mode and $\omega_{pulse} = 0$; (a) $\lambda = 5.0\%$, (b) $\lambda = 9.0\%$, (c) $\lambda = 12.75\%$, (d) $\lambda = 14.75\%$.

Appendix A. Nomenclature

a_{am}	harmonic excitation acceleration amplitude
D_{in}, D_{out}	internal and external diameters of the tube, respectively
E	Young's modulus of the tube
$\mathbf{f}(t)$	external excitation force
H_1, H_2	frequency response function estimators, respectively
L, L_0	tube setting and natural lengths, respectively
p	gauge pressure
Q	fluid flow rate

Re	Reynolds number, viz. $Re = \rho_f U D_{in}/\mu$
t	time
$\mathbf{u}(t)$	dynamic response of the tube
U	time mean cross-sectional average flow velocity
w_{am}	tube displacement amplitude
Wo	Womersley number, viz. $Wo = (D_{in}/2)(\rho_f \omega_{pulse}/\mu)^{1/2}$
y_{am}	excitation displacement amplitude
$\alpha(\Omega)$	receptance, viz. $\alpha(\Omega) = \mathbf{u}(t)/\mathbf{f}(t)$
γ^2	normalized coefficient of correlation between the measured force and response signals evaluated at each frequency
λ	initial stretch rate, viz. $\lambda = (L - L_0)/L_0$
ω	natural frequency
ω_{pulse}	pulsation frequency of the fluid
Ω	harmonic excitation frequency
Ω_1, Ω_2	frequencies corresponding to the flanks of the peak response for which the energy dissipated per cycle of oscillation is half that at resonance ($\Omega_2 > \Omega_1$)
Ω_n	frequency corresponding to the peak response
η_d	damping ratio
v	Poisson ratio
μ	coefficient of dynamic viscosity
ρ_f, ρ_t	fluid and tube densities, respectively

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