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Design of an active suspension to suppress the horizontal vibrations of a spray boom

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Abstract

Spray-boom vibrations are one of the main causes of a non-homogeneous distribution of agro-chemicals. Yawing and jolting motions of the boom are most critical. A horizontal active suspension, reducing yawing and jolting is designed. The use of special and consequently expensive equipment hampers the breakthrough of active suspensions. Therefore the proposed active suspension is built from standard hydraulic equipment, suited for mobile applications. Coulomb friction and asymmetric behaviour introduces considerable non-linearities, complicating the derivation of a linear model and controller design. It is explained how to identify a model, approximating in the best-possible way the general linear behaviour of the system. By the H^∞ methodology and an iterative procedure, introduced in this paper, non-linearities are taken into account in the controller design without increasing the controller dimensions. The resulting controller is stable and shows good performance.

1. Introduction

Application of chemical plant protection means and fertiliser is one of the most important field operations. Pesticides are dissolved in a carrier liquid and sprayed by means of an agricultural spray boom. Fertilisers are distributed over the field as small particles by a spreader or in liquid form by a spray boom. This paper concentrates on agricultural spray booms. Due to non-proper application of agro-chemicals, the effectiveness of these products is reduced. Overdoses are applied to compensate for these inefficiencies. Reduction of the spray-boom motions turns out to be one of the key factors in lowering the amount of pesticides used. Spray-boom vibrations are

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induced by soil unevenness transmitted through the tires, via the tractor to the boom. The tractor acts like a filter: at the eigenfrequencies of the tractor, vibrations are amplified, from a certain frequency, vibrations are attenuated. Unfortunately, the eigenfrequencies of the tractor are close to the eigenfrequencies of the spray boom [1–4]. Numerous studies and field measurements show variations in spray deposition pattern between 0% and 800% (100% is ideal) [5–9]. The most important boom vibrations affecting the spray distribution pattern are rolling, yawing and jolting. Boom roll is the rotation of the boom around a horizontal axis along the driving direction of the tractor. Yawing is a rotational motion around a vertical axis and jolting is a translational motion in the driving direction. This paper concentrates on horizontal boom movements: yawing and jolting. Experimental and theoretical research have shown that yawing and jolting of the boom are more critical with respect to an uneven spray pattern than rolling boom motions. It has been calculated [5] that as long as the distance between the spray nozzles and the soil or crop does not exceed 50% of the desired distance, variation in spray distribution pattern with rolling will remain between 85% and 140% of the desired distribution. In the horizontal direction however, even small vibrations at the boom tips with an amplitude around 30 cm can cause overdoses of 3 times the desired dose [5]. An investigation into the performance of horizontal spray-boom suspensions, showed that active systems are much more efficient than their passive counterparts [10]. A burden, hampering the breakthrough of active suspensions is their considerable installation cost. The objective of this paper is to design an active horizontal suspension (reducing yawing and jolting) by using 'off-the-shelf' components, i.e., standard and cheap hydraulic equipment, designed for mobile machinery. The main purpose is to show how to deal with non-linear behaviour, occurring typically when instead of servo-hydraulics, standard equipment for mobile machinery is used. First the set-up for which the active suspension is designed, is discussed. A next section describes the linear modelling procedure of the system. For controller design, the H^∞ methodology is employed. It is explained how model deviations are taken into account into the design, guaranteeing robustness of the controller. Finally the performance of the active suspension is evaluated.

2. Description of the set-up

Fig. 1 shows a plan view of the boom with active suspension. The boom is a Delvano Junior with a width of 12 m and weight of approximately 70 kg. It consists of a central frame, which may be considered as rigid. The flexibility of the boom is located in the side arms. The boom is mounted via the active suspension on a shaker. The shaker consists of a platform, having a translational and rotational degree of freedom. Two excitation actuators excite it and reproduce yawing and jolting motions, transmitted from the tractor to the boom. Resonance frequencies of the platform show up in the frequency band of interest. No attempt has been made to stiffen the platform. In practice, tractor dynamics must be taken into account, which can be represented by the vibration modes of the platform. The suspension aims to isolate the boom from the tractor. Therefore, the boom is given a jolting degree of freedom through the axles on which the sledge slides (through shaft sliding collars) and a revolute joint allowing a yawing degree of freedom. The actuators of the active suspension, originating from the company Büter, are single rod, having a stroke of 297 mm and a piston and rod diameter of, respectively, 40 and 35 mm. These are cheap

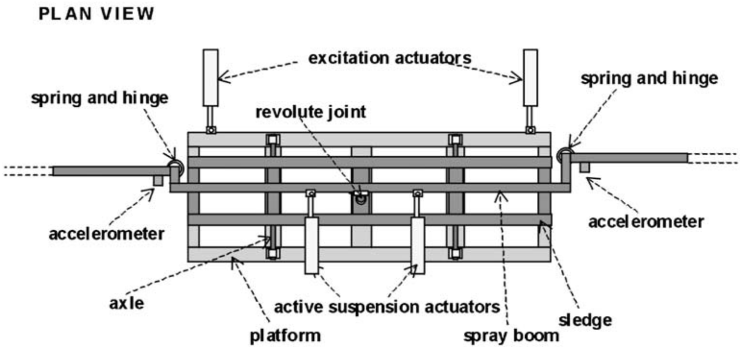

Fig. 1. Plan view of the boom with active suspension mounted on an excitation platform.

standard cylinders, often encountered in mobile applications. Contrary to servo-actuators, they are not equipped with special designed low-friction seals and do not incorporate extra conducts coping with leakage, introduced to loosen the seals for lowering the friction. The actuators are driven by proportional pressure control valves (Bosch 0 532 006 028), specially designed for mobile applications. Instead of requiring 24 V supply voltage, as is the case for industrial valves, the ones used, need only 12 V. The valves can operate at pressures up to 130 bar and can handle flow rates of 150 l/min. The response time of the valve is guaranteed to be 40 ms. It is the time between the moment the valve is 10% open and the moment when the valve is for the first time 90% open, if a step input is applied, which switches the valve from closed to entirely open state. Depending on the damping, this corresponds to a bandwidth between 7 and 10 Hz. A PWM current amplifier operates the valve. By the PWM signal a small dither is put on the valve, reducing the Coulomb friction and improving the response time. A gear pump, delivering a flow rate of 50 l/min supplies the hydraulics. In order to position the boom perpendicular to the driving direction of the tractor and to avoid drift, always present in hydraulics, the position of the piston rods of the actuators is fed back through a proportional controller. LVDT position sensors (Monitran Ltd., MTN/IEUR150-10) measure the displacement of the piston rods. The proportional controller is tuned by hand such that if the same signal is send to the actuators, both perform a motion in phase. Despite its simplicity, the selected P-controller showed satisfactory results, as a precise position control of the cylinder in itself is not a prime requirement, but only the avoidance of drift. An additional advantage of the position control loop is that it linearizes the non-linear behaviour of the hydraulic devices, simplifying the control problem of the active suspension. Horizontal boom vibrations are measured in the interested frequency band by two LVDT-type accelerometers (HBM B12/200), which can capture acceleration signals to 200 m/s^2 with a frequency content between 0 and 200 Hz. Accelerometers were selected because they are cheap and the only devices that can measure absolute boom motions (i.e., with respect to the soil) with sufficient accuracy. Precisely absolute boom vibrations influence the spray deposition pattern. Alternatively, a laser could be used. However, lasers are expensive, need a fixed reference in the field and fail to work under harsh field conditions. The place of the accelerometers determines whether the controller is an active compensator or an active isolator. In case of an active compensator, tractor vibrations are allowed to travel to the boom, causing boom vibrations. Sensors on the boom capture the vibrations and the controller calculates a suitable control action, which is sent to the actuator to compensate these vibrations. As the sensors for an active compensator are placed on the boom, the control will be non-collocated, which can result in so-called right half plane zeros [11]. Right half plane zeros make controller design difficult and have a dramatic effect on the performance of the final system [12,13]. Therefore, a vibration isolator has been created. To create an active isolator, the accelerometers, used for the boom stabilization, are fixed on the arms, just after the hinges. When the friction in the shaft sliding collars and the revolute joint is low, the only transmission path for relevant horizontal tractor vibrations are the compensation actuators and the central frame. By keeping this central frame in stand still, the tractor vibrations are isolated from the flexible frame parts of the boom. If the controller makes the measured vibrations zero, the side arms will not deform or perform rigid body motions. Although the control is still non-collocated, no right half plane zeros will occur, as there is no time delay between the actuator movements and the registered accelerations. Actuator motions are directly transmitted to the accelerometers because the central frame is rigid, the supports of the actuators are stiff and the actuators are position controlled.

3. Black box modelling

From Fig. 1, it is clear that the system has two inputs (two hydraulic cylinders) and two outputs (two accelerometers) such that MIMO black box identification techniques are required to determine a model. In Refs. [14,15], it is shown that by constant unitary transformations, the system $\mathbf{G}(s)$ can be diagonalized:

$$\mathbf{G}(s) = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \sigma_t(s) & 0 \\ 0 & \sigma_r(s) \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}, \tag{1}$$

in which $\sigma_t(s)$ and $\sigma_r(s)$ describe the translational and rotational behaviour of the spray boom respectively. By this, MIMO identification reduces to two SISO separate identifications. As the behaviour of the boom is dominantly linear, linear black box identification techniques are employed. In this paper the non-linear least-squares estimator (NLSE) [16], a frequency domain method, identifies a model. It minimizes the following cost function:

$$\theta = \arg \min_{\hat{\theta}} \left(\sum_{k=1}^{N} |(\sigma(\mathrm{j}\omega_k) - P(\mathrm{j}\omega_k, \hat{\theta}))|^2 \right), \tag{2}$$

where $\sigma(\mathrm{j}\omega_k)$ is either σ_t or σ_r evaluated on the imaginary axis at frequency point ω_k. $P(\mathrm{j}\omega_k, \hat{\theta})$ is the model of the system dependent on parameter vector $\hat{\theta}$, making Eq. (2) minimal at θ. N denotes the total number of frequency lines. The NLSE is a frequency domain method and allows direct identification of continuous time models, which is an advantage for incorporating a priori physical information. This information is often in the form of differential equations and can immediately, without any conversion rule, be inserted in the model structure. Furthermore in controller design, elaborated in one of the next sections, algorithms formulated in continuous time, minimize an H^∞ criterion. H^∞ optimizations in the Z-domain are often solved by transforming the criterion back to the Laplace domain and after optimization, transforming the continuous time controller to the discrete time. There is no need to perform all these transformations when working directly in the Laplace domain. In practice $\sigma(j\omega_k)$ in Eq. (2) is unknown and a good estimate of $\sigma(j\omega_k)$ is required. This guess is obtained by calculating the frequency response function (FRF):

$$FRF(j\omega) = \frac{Y(j\omega)}{X(j\omega)}, \tag{3}$$

where $Y(j\omega)$ and $U(j\omega)$ are the Fourier transform of the output and input respectively. In Refs. [1,17] it is shown that important non-linearities are present in the system. They originate mainly from the hydraulics, which contains important Coulomb friction and asymmetrical behaviour. It is proved in Refs. [18,19] that the measured FRF of a non-linear system can be split into the following contributions:

$$FRF(j\omega_k) = FRF_0(j\omega_k) + FRF_B(j\omega_k) + FRF_S(j\omega_k) + N(j\omega_k), \tag{4}$$

in which $FRF_0(j\omega_k)$ is the FRF of the underlying linear system, $FRF_B(j\omega_k)$ is the bias or systematic error due to non-linear distortions, $FRF_S(j\omega_k)$ the stochastic non-linear contributions and $N(j\omega_k)$ the noise on the measurements. The underlying linear system is the behaviour of the system at a certain operating point when the excitation signals are made arbitrarily small. For controller design, the objective is not directly to identify this underlying linear system but rather finding a linear model that predicts in the best possible way the non-linear system $\sigma(s)$. Because $FRF_S(j\omega_k)$ has the characteristics of noise, it cannot be taken into account in a linear parametric model. Therefore the best global linear approximation of the non-linear system is found by fitting a model on $FRF_0(j\omega_k) + FRF_B(j\omega_k)$. Contrary to $FRF_0(j\omega_k)$, $FRF_B(j\omega_k)$ is still non-linear as it is dependent on the amplitude of the input signal. The contribution of the noise $N(j\omega_k)$ on the FRF is reduced by applying periodic excitation signals and averaging over the measurement periods:

$$FRF_{EV}(k) = \frac{\frac{1}{m}\sum_{i=1}^{m} Y_i(k)}{\frac{1}{m}\sum_{i=1}^{m} X_i(k)}, \tag{5}$$

in which m is the total number of periods. The Fourier parameters $Y_i(k)$ and $U_i(k)$ are obtained by applying the FFT algorithm on the output respectively the input for each period of excitation. The FRF_{EV} computed in Eq. (5) is known as the errors in variables estimator and renders an unbiased estimate of the FRF [20]. It is the maximum likelihood estimator, if the input and output perturbations are complex normally distributed and some other conditions are fullfilled. $FRF_S(j\omega_k)$ depends on the phase of the input signal. Consequently, averaging over the measurement periods of one input signal cannot reduce $FRF_S(j\omega_k)$. Therefore, four different periodic signals, with different phase characteristics are applied to excite the system: a random sequence, a swept sine and two different multisines. Four different multisines with different phase characteristics could also have been selected. However the experience was that FRFs from multisines, with different phase, tend to be more close to each other than the FRF of a multisine and a sweptsine. The random sequence excitation is interesting to assess the noise during the measurements when the controller is operating. The effect of stochastic non-linear contributions is reduced by averaging over the $FRF_{EV}(j\omega_k)$ of the different excitation signals

$$FRF_{EVa}(j\omega_k) = \frac{1}{m_a}\sum_{i=1}^{m_a} (FRF_{EV_i}(j\omega_k)), \tag{6}$$

in which m_a is the number of excitation signals on which an FRF_{EV} is calculated and FRF_{EV_i} the FRF_{EV} of the ith excitation signal ($i = 1, \ldots, 4$).

Although 15 measurement periods have been collected for each excitation signal, the FRF_{EV_i} curves still look spiky, indicating that a lot of non-linearities are present. Averaging the FRF_{EV_i} curves according to Eq. (6), renders the much smoother FRF_{EVa} curve, which is depicted in Fig. 2. Before starting the identification of a parametric model on the FRF_{EVa} of the rotations and the translations, a certain model structure must be proposed. A priori physical information can be incorporated into the model structure. Owing to the position feedback, the applied voltages result in a certain displacement of the piston rods and of the central frame. Next to the central frame, the accelerations are measured, which are second derivatives of the positions. This information can immediately be inserted into the model structure by putting the last coefficients of the numerator polynomial zero. Because of the hydraulics and eigenfrequencies of the platform, it is almost impossible to postulate a certain model structure based on physical insight. Therefore several model orders have been tried out. A fifth and fourth order model with equal degree in numerator and denominator for respectively the translations and the rotations seemed to give the best trade-of between model accuracy and complexity. The identification results are shown in Fig. 2.

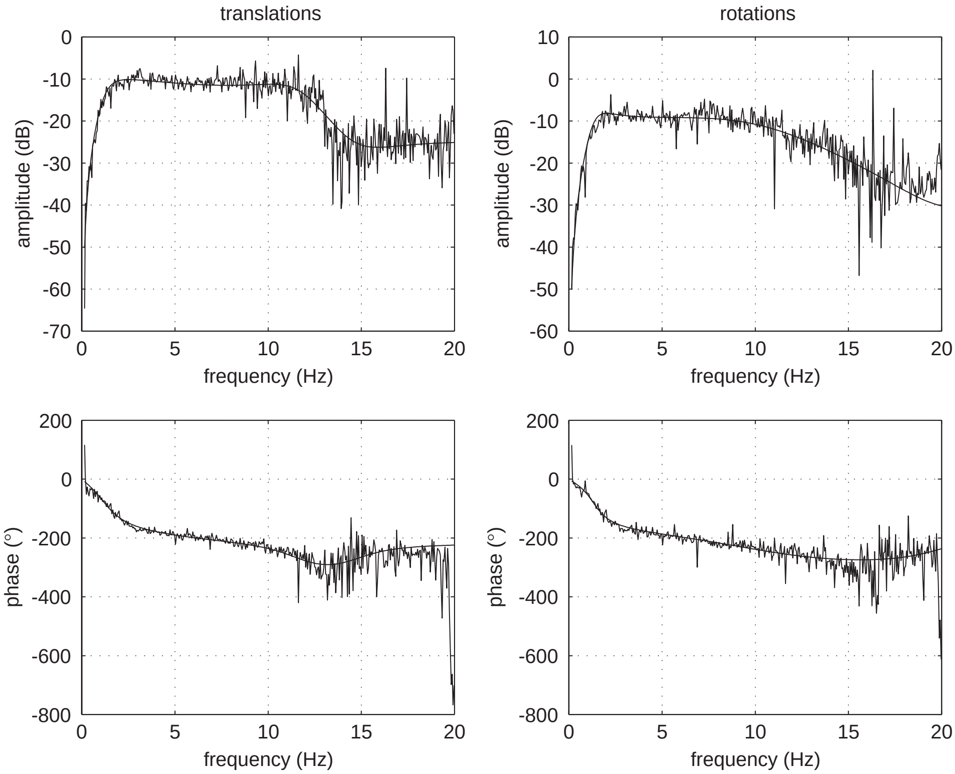

Fig. 2. FRF_{EVa} measurements and models for the translations and rotations.

4. Controller design

MIMO controller design reduces to two SISO designs because of Eq. (1). When the separate SISO controllers are stable and optimal, the resulting controller is also stable and optimal [21]. The H^∞ design methodology is selected to calculate a controller as apart from performance, robustness can be taken into account explicitly. This methodology is based on the small gain theorem formulated by Zames [22]. The small gain theorem is actually a stability criterion, but in H^∞ control theory, it is shown that a performance measure can be interpreted as a robustness measure. In this way, performance as well as robustness can be incorporated into the design objective function. The latter is formulated as a set of system transfer functions of which the H^∞-norm needs to be minimized. The H^∞-norm of a system is defined as the peak of the maximum singular value. In the case of the active horizontal suspension, the objective function is based on the scheme depicted in Fig. 3.

The objective of the active suspension is to reduce the influence of spray-boom vibrations on the spray distribution pattern. Spray-boom vibrations with a low-frequency content have the time to grow to large boom displacements whereas vibrations, having a high-frequency content cannot. Therefore, the suspension should act mainly in the low frequencies. On the other hand manoeuvres of the tractor such as accelerating and turning over the field should be followed. This implies that the suspension should act as a band stop filter. The manoeuvres and the vibrations transmitted from the tractor to the boom are considered as disturbances a_d [14]. The transfer function from a_d to y is the sensitivity function, which is defined by

$$S(s) = \frac{1}{[1 + k(s)P(s)]}, \tag{7}$$

in which $k(s)$ is the controller and $P(s)$ the model of the system.

By using standard hydraulic equipment, a lot of non-linearities are present which was explained in the previous section. They may not turn the controller unstable. These non-linearities are not incorporated in the model and can be considered as model deviations or uncertainties. As uncertainties have a considerable effect on the behaviour of the system, it is important to insert them explicitly into the design criterion. In this paper the uncertainties are considered multiplicative:

$$\sigma(s) = (1 + \Delta_M(s))P(s), \tag{8}$$

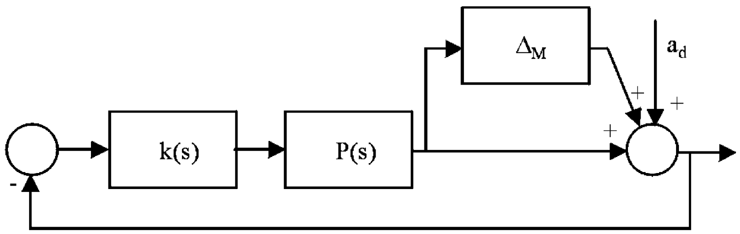

Fig. 3. Lay-out of the controller.

where $\Delta_M(s)$ is the multiplicative uncertainty. It can be calculated that the transfer function seen by $\Delta_M(s)$ is the complementary sensitivity function $T(s)$, defined by

$$T(s) = \frac{k(s)P(s)}{[1 + k(s)P(s)]}. \tag{9}$$

In Ref. [23], it is shown that by decreasing the H^∞-norm of $T(s)$, the robustness is increased.

Combining the performance and robustness requirements, results in the following objective function of which the H^∞-norm must be minimized:

$$\left\| \begin{array}{c} W(s)S(s) \\ \beta T(s) \end{array} \right\|_\infty, \tag{10}$$

which is known as the mixed sensitivity design. Function $W(s)$ shapes the sensitivity function to a band stop characteristic. Parameter β is a trade-off between performance and robustness. In classical design problems β is also a shape function, approximating the shape of the neglected dynamics and uncertainties. These are not known, because the model for the controller design is not obtained from a reduced elaborate physical model but by black box identification. Furthermore, all the dynamics inserted in the design weights increase the complexity of the controller. Therefore β is a scalar, determined from an iterative procedure. By replacing $\sigma(j\omega)$ in Eq. (8) by FRF_{EV_i}, four estimates of $\Delta_M(j\omega)$ can be calculated

$$\Delta_M(j\omega) = \frac{FEF_{EV_i}(j\omega)}{P(j\omega)} - 1. \tag{11}$$

From black box modelling techniques only variances, which are actually measures about uncertainties due to stochastics, i.e., measurement noise, can be deduced. Nothing can be said about bias errors due to under-modelling or errors due to approximation of a non-linear system by a linear model. Therefore Eq. (11) is used.

In Ref. [23] it is proven that the system is stable if and only if

$$\|\Delta_M(s)T(s)\|_\infty \leqslant 1. \tag{12}$$

Eq. (11) is satisfied if the following holds on the imaginary axis:

$$|\Delta_M(j\omega)| < \left| \frac{1}{T(j\omega)} \right|. \tag{13}$$

The transition from Eqs. (12) to (13) is justified by the fact that the singular value curve of a SISO system corresponds to the absolute value or modulus of the transfer function. Eq. (13) implies that the inverse of the absolute value of the complementary sensitivity function can serve as a robustness bound. This leads to the following iterative design scheme. First β gets an arbitrary value and a controller is calculated. Then it is checked whether one of the $|\Delta_M(j\omega)|$ curves crosses the robustness bound $\frac{1}{|T(j\omega)|}$. If so, more stress must be allocated on robustness in Eq. (10), which is performed by increasing β. In case that all the $|\Delta_M(j\omega)|$ curves are far below the robustness bound, a too conservative controller is found and the importance of the performance specification can be increased by decreasing β. The robustness check of the final step for jolting is depicted in Fig. 4. At some frequencies, the robustness bound is crossed. Attempting to put all multiplicative uncertainties below the robustness bound results in no performance. Therefore some spikes, crossing the robustness bound are discarded and care is taken that the general trend of the uncertainty curves is below the

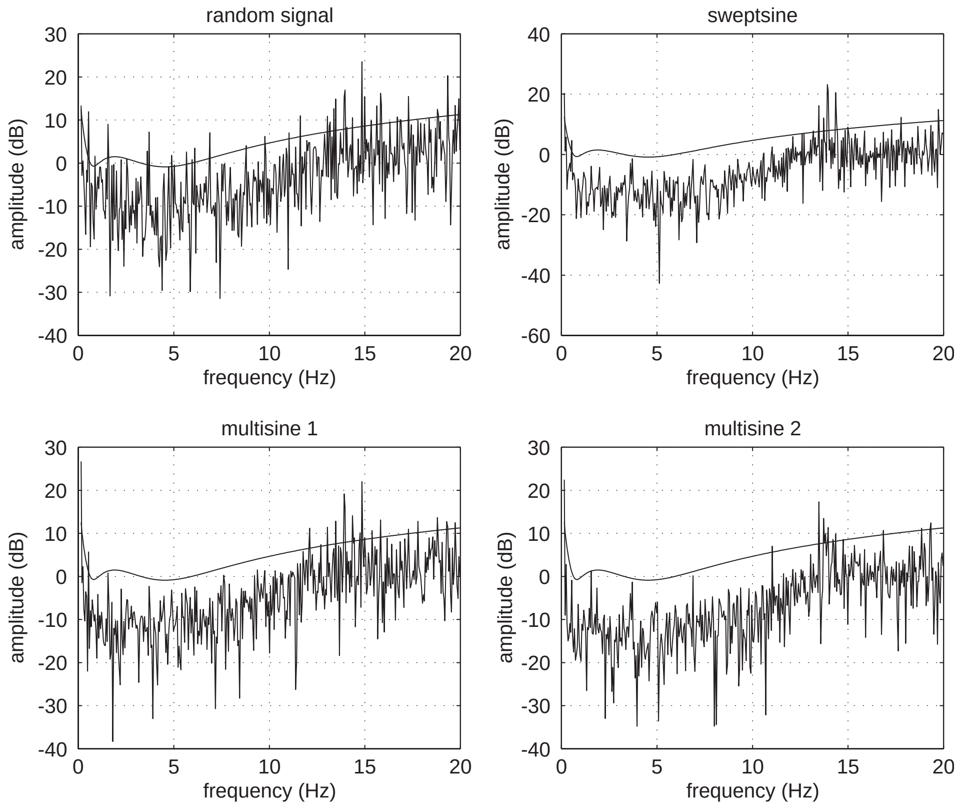

Fig. 4. Robustness check for the final controller design step for jolting.

bound. This can be justified by the fact that the crossings on the four plots occur rather randomly and are not located at particular frequencies. Consequently, the spikes crossing the robustness bound are more due to the excitation signal than due to the system and do not need to be taken into account.

5. Validation

The controller is implemented in the laboratory set-up and is stable, justifying the procedure of the previous section. Validation of the controller is performed by applying excitation signals to the actuators of the shaker. These signals are based on a standardized track [24] often used to assess and to compare the dynamic characteristics of agricultural machinery and off-road vehicles. Boom tip displacements, measured by a laser, with and without the controller are shown in Fig. 5. A reduction of boom tip displacements of almost three is achieved.

6. Conclusions

An active horizontal spray-boom suspension, reducing yawing and jolting, has been designed. The suspension uses cheap, standard hydraulic components, suited for mobile applications. Due

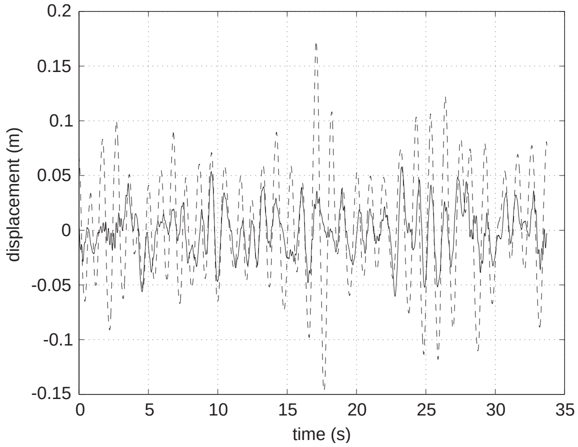

Fig. 5. Boom tip displacements with and without controller.

to Coulomb friction and asymmetric behaviour, a lot of non-linearities are present into the system. By applying different periodic excitation signals and using a frequency domain identification method, a linear model describing the global linear behaviour of the system, has been fit. Non-linearities are explicitly accounted for through an H^{∞} design criterion and an iterative procedure. By the latter, robustness specifications do not increase the controller dimensions. Validation of the active suspension shows good performance.

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