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System for vertical boom corrections on hilly fields

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Abstract

Classical vertical spray boom suspensions reduce the effect of roll on the spray distribution pattern by using gravity. On hilly fields, the boom is not positioned parallel to the ground, resulting in an uneven spray distribution pattern and causing harm to the boom when hitting the soil. This paper models mathematically systems that are able to follow the slope of the field. The structure of the models is studied, which turns out to be the same for the four investigated suspensions. The analytically determined models are validated. A controller is designed to create a slow active suspension, following the slope of the field and filtering soil undulations, causing unwanted boom roll. The controller is implemented on a laboratory set-up and on a real machine. It takes into account the dynamics of the suspension and adds damping. Despite the low power of the actuator, a good performance is achieved. Possible performance enhancement can be obtained by increasing the resonance frequency of the suspension and by making use of a more powerful actuator.

1. Introduction

To increase yield in agriculture, plants need to be protected against diseases and need to be provided with fertilizer. One of the most important methods to supply the plants with agro-chemicals is by using spray booms. Studies pointed out that the efficiency of the chemicals is highly correlated with the uniformity of the spray distribution pattern [1]. Spray boom motions, vertically mainly rolling and horizontally mainly yawing, an anti-phase movement of the boom tips, and jolting, and an in-phase movement of the boom tips, have a dramatic effect on the spray distribution pattern. This paper concentrates on boom roll, the effect of which is shown in Fig. 1.

On places where the boom goes up, the spray nozzles cover a larger area, resulting in a redistribution of spray liquid across the boom. Where the boom goes down, the area covered is

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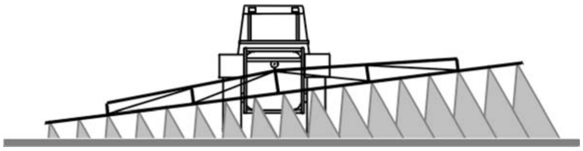

Fig. 1. Effect of boom roll on the spray distribution pattern.

smaller resulting in over application under the nozzle beneath or no application between two spray cones. To reduce the effect of boom roll, almost as many variants of passive suspensions exist as there are manufacturers. They all use gravity as a reference and move back the boom into its correct position. Commonly encountered suspensions are pendulum or trapezium based. As vertical suspensions attempt to position the boom perpendicular or at least at some angle with the direction of gravity, problems occur when driving over inclined fields. Manually operated and automatic systems [2–7] have been developed to place the boom parallel to the field. Concerning automatic systems, problems have been noted at high driving speeds. They seem to originate from bad tuning of the controller to the dynamics of the spray boom suspension combination and under powered actuators [8]. The objective of this paper is to show that systems that can follow the slope of the field are mathematically the same. The structure of the mathematical models is validated on a laboratory set-up. Based on a priori knowledge a model is identified and a controller is designed to make an automatic slope following system. The controller takes into account the dynamics of the suspension. As the automatic system attempts only to follow the slope of the field and adds some extra damping to the resonance frequency, it is called slow active. Vibrations, harmful to the spray distribution pattern, are filtered passively. Finally, a version of this type of controller is implemented on a real sprayer.

2. Mathematical modelling

For model-based linear controller design, linear models are required. Linear modelling of vertical suspension systems is justified as the angles of rotation are small i.e., smaller than 15°. An additional advantage of linear models is that they provide a clearer insight into the dynamic behaviour. Furthermore, many mathematical tools for linear systems are available to easily compare the filtering characteristics of the suspensions and simulation is less computer intensive.

2.1. Modelling procedure

Vertical boom suspensions are modelled by applying the Lagrange equation [9–11]:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j, \tag{1}$$

in which T is the kinetic energy, t the time, q_j a generalized co-ordinate and $\dot{q}_j$ a generalized velocity (derivative with respect to time of q_j). Operator d is used for full derivatives whereas operator ∂ is applied for partial derivatives. Q_j is the generalized force corresponding to q_j and is computed as the virtual work delivered by all external forces when only co-ordinate q_j is varied:

$$Q_j \delta q_j = \sum_{i=1}^{n} \mathbf{F}_i \frac{\partial \mathbf{r}_i}{\partial q_j} \delta q_j, \tag{2}$$

where $\mathbf{F}_i$ is an external force, $\mathbf{r}_i$ is the absolute position of the force $\mathbf{F}_i$ and n is the total number of external forces acting on the mechanism.

For each co-ordinate, Eq. (1) can be calculated. Generally, passive and especially systems enabling the slope of the field to be followed are closed kinematic chains, implying dependency between different co-ordinates. This dependency is derived by expressing the geometry of the system, resulting in as many algebraic constraint equations as there are redundant co-ordinates. The number of constraint equations is given by the difference between the total number of selected co-ordinates and the number of degrees of freedom of the mechanism. Generally, these algebraic equations need to be introduced into the differential equations, obtained from Eqs. (1) and (2) by Lagrange multipliers and lead to a set of constrained equations of motion [12]. As stated in the Introduction, rotation angles and displacements are small in vertical spray boom suspensions, which allows linearization of the algebraic and differential equations. Because of the linear equations, it is easy to express the dependency of one co-ordinate with respect to the other co-ordinates explicitly. Therefore, the algebraic equations can be directly substituted in the differential equations such that the use of Lagrange multipliers is avoided.

2.2. Modelling of a slow active pendulum suspension

Fig. 2 shows a sketch of a slow active pendulum suspension. When the spray boom is connected at its centre of gravity to the rod, linking the boom with the frame connected to the tractor, the rod always aligns with gravity, providing the passive suspension. In a hilly field, by changing the angle between the rod and the spray boom, again gravity can position the boom parallel to the field. In this way, the passive suspension is preserved.

Fig. 3 shows the vectors by which the closed kinematic chain can be described:

$$\mathbf{r}_1 + \mathbf{r}_2 - \mathbf{d} - \mathbf{r}_3 = 0, \tag{3}$$

in which the vectors $\mathbf{r}_i$ correspond with the links with fixed length and $\mathbf{d}$ corresponds with the length adjustable link incorporating the hydraulic cylinder.

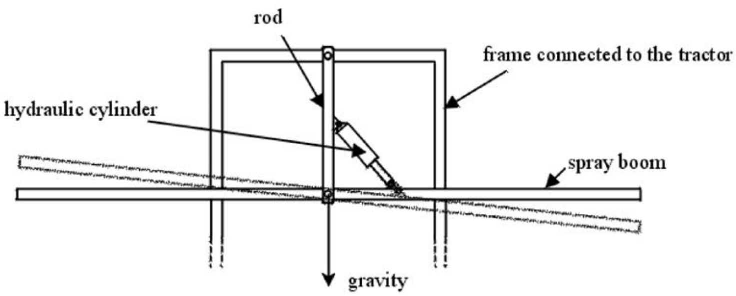

Fig. 2. Slow active pendulum suspension.

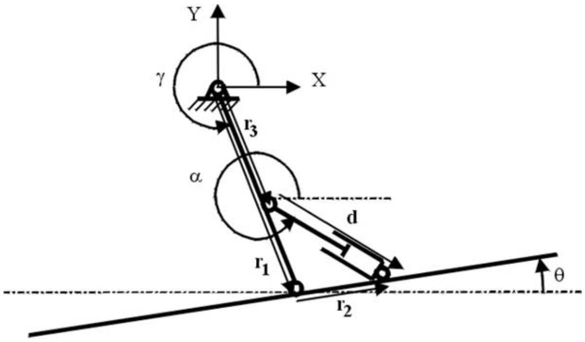

Fig. 3. Geometric description of the slow active pendulum suspension.

Projection according to the X- and Y-axis delivers, after linearization, the following algebraic equations:

$$r_{10}\gamma - \frac{\sqrt{2}}{2}d_0\alpha - \frac{\sqrt{2}}{2}d - r_{30}\gamma = 0 \quad (X\text{-axis}), \tag{4}$$

$$r_{20}\theta + \frac{\sqrt{2}}{2}d_0\alpha - \frac{\sqrt{2}}{2}d = 0 \quad (Y\text{-axis}), \tag{5}$$

in which r_{i0} is $|\mathbf{r}_i|$ and d_0 is $|\mathbf{d}|$. α, θ, γ and d signify small variations of the variables around a setpoint and for the setpoint value of α, $7\pi/4$ has been selected, for γ it is $3\pi/2$ and for θ it is 0. In order to calculate Eq. (1), the kinetic energy T and the generalized forces Q_j need to be determined. The kinetic energy is situated in the rotation of the boom and the motion of the centre of gravity of the boom:

$$T = \frac{I\dot{\theta}^2}{2} + \frac{mr_2^2\dot{\gamma}^2}{2}, \tag{6}$$

where I and m are, respectively, the moment of inertia and the mass of the boom. The generalized force Q_θ, associated with rotation θ, is obtained by fixing the displacement of the actuator d. By blocking the latter, only gravity is able to deliver work, resulting in

$$Q_\theta = -mgr_1(a_1 d + a_2\theta). \tag{7}$$

Parameters a_1 and a_2 are linearization constants for which an analytical expression can be found. In order not to overload the text, these expressions are omitted. Similarly, the generalized force Q_d, associated with the displacement d of the actuator, is computed by fixing the rotation angle θ. In this situation, gravity as well as the actuator deliver works, which can be linearized:

$$Q_d = F - mgr_1(a_3 d + a_4\theta), \tag{8}$$

where F is the force exerted by the actuator and a_3 and a_4 are linearization constants.

Application of Eq. (1) with Eqs. (6)–(8) results in the equations of motion:

$$(mr_1^2 a_1 a_2)\ddot{\theta} + (mr_1^2 a_1^2)\ddot{d} + (mgr_1 a_4)\theta + (mgr_1 a_3)d = F, \tag{9}$$

$$(I + mr_1^2 a_2^2)\ddot{\theta} + (mr_1^2 a_1 a_2)\ddot{d} + (mgr_1 a_2)\theta + (mgr_1 a_1)d = 0. \tag{10}$$

Generally, the piston of a hydraulic actuator is positioned by controlling the flow to the cylinder, in this way imposing a certain velocity or position on the cylinder. When an electrical actuator is used, manipulation of the voltage to the actuator, imposes again the speed or position of the moving member of the actuator. Therefore, for control purposes, the transfer function between actuator displacement d and rotation θ is required, which is obtained by performing the Laplace transform on Eq. (10) and rearrange the terms:

$$\frac{\theta}{d} = \frac{(mr_1^2 a_1 a_2)s^2 + (mgr_1 a_1)}{(I + mr_1^2 a_2^2)s^2 + (mgr_1 a_2)}, \tag{11}$$

in which s is the Laplace variable. For the considered configuration, i.e., $\alpha = 7\pi/4$ and the rod initially perpendicular to the boom, the linearization constant a_2 equals 1. Therefore, it can be concluded that by introduction of the actuator, of which the mass and moment of inertia are neglected, the natural frequency of the slow active pendulum suspension is the same as a regular pendulum suspension. Note that for Eq. (11), only Eq. (10) has been used. By Eq. (9) the required actuator force to position the boom parallel to the field can be calculated. However for control purposes this is not required.

A Bode plot of Eq. (11) is drawn in Fig. 4, calculated for a boom of 600 kg, moment of inertia of $33,000\ \mathrm{kg\ m^2}$ and a suspension with $r_1 = 0.7$ m, $r_2 = 0.5$ m, $r_3 = 0.2$ m and $\alpha = 7\pi/4$. It shows that the reaction speed of the actuator is limited to the natural frequency of the boom. Beyond the natural frequency, the action of the actuator is reduced by the inertia of the boom. At some frequency, i.e., the anti-resonance of the system, displacement of the actuator has almost no effect on the rotation angle θ.

2.3. Modelling of a slow active trapezium suspension

In a regular trapezium suspension, the boom rotates around a virtual rotation point, which is the point of intersection of the two elongated side arms. In the rest position the line through the centre of gravity of the boom and the virtual rotation point is in the line of action of the gravity force. By changing the

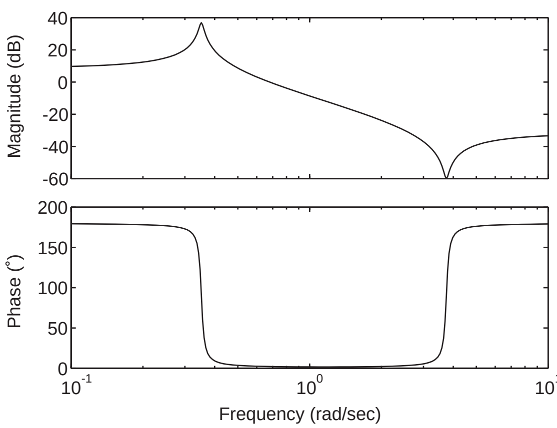

Fig. 4. Bode plot of a slow active pendulum suspension.

location of the virtual rotation point, the boom can be positioned parallel to the field. In Fig. 5, this is achieved by providing the trapezium with an extra bar and pivot point, controlled by an actuator.

The same procedure as for the pendulum suspension is used to model the slow active trapezium suspension. The transfer function between the actuator displacement and the boom rotation has the same shape as in Fig. 4, i.e., it is characterized by a second order system, having two complex conjugated zeros.

2.4. Modelling of the cable suspension

In the cable suspension, Fig. 6, the boom is attached to a cable, which is wrapped once over and fixed, by a clamp plate, on a pulley. If the electric motor is not activated, the pulley can be considered to be rigid. The cable is crossed once. Tilting the frame of the tractor for example to the right, results in a rotation of the pulley, causing the right end of the cable to roll up and the left end to unwind. This results in a left rotation of the boom, counteracting the rotational motion of the tractor. By activating the electric motor, again the cable will roll up at one end and unwind at the other end, putting the boom in a skewed position such that the slope of the field can be followed. This kind of suspension can be found on Inuma spraying machines.

To derive the equations of motions using Eq. (1), the kinetic energy T and the generalized forces Q_j are required. Neglecting the mass and moment of inertia of the pulley, and the mass of the cables, the kinetic energy is due to the rotation of the boom and the displacement of the centre of gravity of the boom, having co-ordinates x and y:

$$T = \frac{I\dot{\theta}^2}{2} + \frac{m(\dot{x}^2 + \dot{y}^2)}{2}. \tag{12}$$

However, the cable suspension has only two degrees of freedom and the objective is to find a relation between the rotation of the wheel γ and the absolute rotation of the boom θ. Therefore x and y need to be written as a function of γ and θ. To facilitate this, the direction cosines c_1 and c_2 of the two ends of the cable are introduced. A sketch of the cable suspension with all symbols is shown in Fig. 7. Analytical geometry helps to find the four dependencies between the six variables c_1, c_2, x, y, θ and γ. Two dependencies are found by expressing the radius of the pulley in two different ways:

$$R = \frac{|c_1(x + L\cos\theta) - y - L\sin\theta|}{\sqrt{1 + c_1^2}}, \tag{13}$$

$$R = \frac{|c_2(x - L\cos\theta) - y + L\sin\theta|}{\sqrt{1 + c_2^2}}. \tag{14}$$

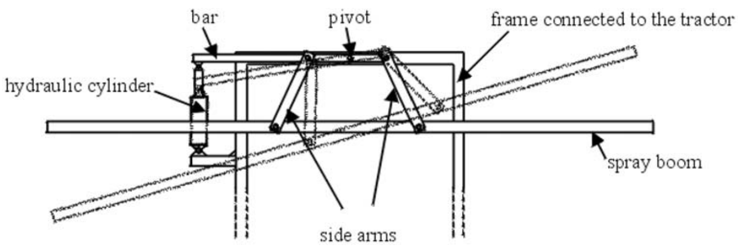

Fig. 5. Slow active trapezium suspension.

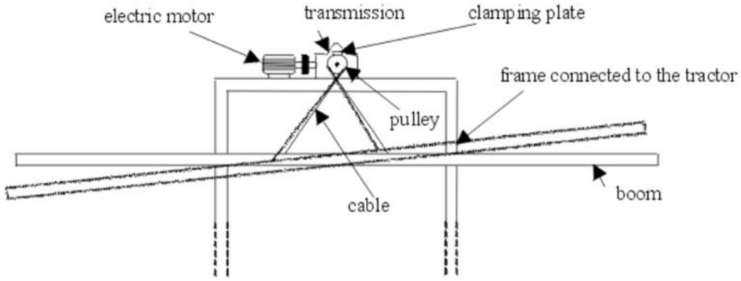

Fig. 6. Cable suspension.

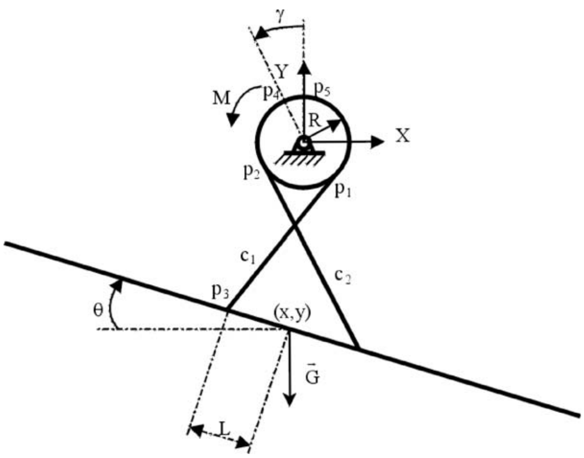

Fig. 7. Geometric description of the cable suspension.

Determination of the left and right cable halves as a function of c_1, c_2, x, y, θ and γ provides the required two additional algebraic equations:

$$\frac{l}{2} = \frac{|c_2(y - L\sin\theta) + x - L\cos\theta|}{\sqrt{1 + c_2^2}} + \left(\frac{\pi}{2} - \arctan\left(\frac{-1}{c_2}\right) + \gamma\right)R, \tag{15}$$

$$\frac{l}{2} = \frac{|c_1(y + L\sin\theta) + x + L\cos\theta|}{\sqrt{1 + c_1^2}} + \left(\frac{\pi}{2} + \arctan\left(\frac{-1}{c_1}\right) - \gamma\right)R. \tag{16}$$

To find the relation between γ and θ, only the generalized force Q_θ needs to be computed because Q_γ introduces the moment M required to turn the pulley which is, as F for the slow active pendulum suspension, not needed. Q_θ is determined by fixing the actuator, i.e., γ, and computing the work delivered:

$$Q_\theta = -mg(a_1\gamma + a_2\theta). \tag{17}$$

Elaboration of Eq. (1) and performing linearizations results again in a transfer function having a shape as in Fig. 4, i.e., a second order system with two complex conjugated zeroes.

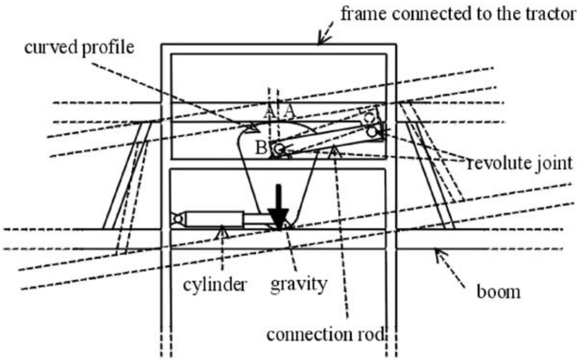

Fig. 8. Douven suspension.

2.5. Suspension based on following a curved profile

Suspensions based on following a curved profile, can be found, for example, on Douven spray booms. Apart from some constructional details, Fig. 8 shows the functionality of this type of suspension. The boom is laid on a curved profile and is prevented from sliding away from this profile by a connection rod, linking the boom and the frame connected to the tractor via two revolute joints. The curved profile is also connected to the frame via a revolute joint. In order to avoid overloading the drawing, this joint coincides with one of the revolute joints (point B) of the connection rod. If for some reason, the boom moves into a slanted position, the rotation point A is not aligned with the gravity force, which creates a moment, retrieving the boom in its horizontal position.

Derivation of the equations of motion can be performed by applying Eq. (1). The kinetic energy is again due to the rotation of the boom and the motion of the centre of gravity as expressed by Eq. (12). The Douven suspension has also two degrees of freedom i.e., the absolute rotation of the boom θ and the displacement of the actuator d. The objective is to find a mathematical relation between these two. Therefore, x and y need to be expressed as a function of θ and d, which is performed by introducing extra co-ordinates and by using analytical geometry. It can be found that the transfer function between d and θ has again the same shape as in Fig. 4.

3. Validation and black box identification

To validate the structure of the model a small laboratory set-up has been constructed based on the cable suspension. The suspension itself is from a real spray boom put at our disposal by the compay INUMA. The actuator is a 12 V DC-motor with a power of approximately 70 W. The boom is replaced by a solid beam of 25 kg, having a length of 4 m and a moment of inertia of 33.3 kg m^2. In practice, spray booms may be considered as stiff structures in the vertical direction. Two ultrasonic sensors, mounted on the left and right side of the centre of gravity, measure the angle θ. Assessment of the model structure is performed by measurement of the frequency response function (FRF) between the rotation of the pulley γ, measured by a potentiometer and θ. The result is shown in Fig. 9.

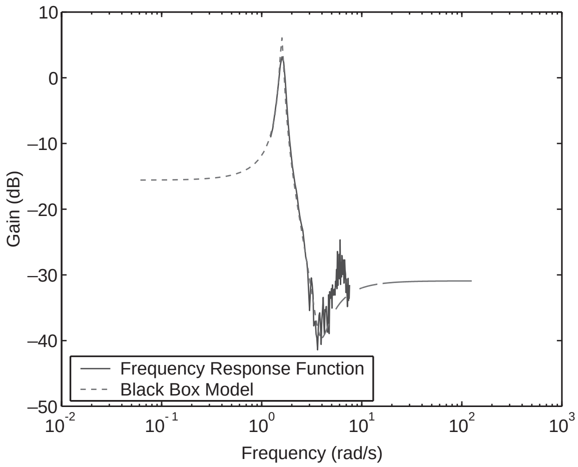

Fig. 9. Frequency Response Function and black box model of the set-up.

The set-up behaves clearly as a second order system, containing two complex conjugated zeroes. Filling in all the required parameters in the analytically derived model, can deliver a model for controller design. However, the analytically derived models do not contain damping which needs to be determined experimentally. Furthermore, the analytical model has an error in the natural frequency of approximately 15%. Therefore, the following model structure is put forward:

$$\frac{\theta(s)}{\gamma(s)} = \frac{(a_2 s^2 + a_1 s + a_0)}{(s^2 + b_1 s + b_0)}, \tag{18}$$

in which a_2, a_1, a_0, b_1 and b_0 are parameters to be determined. This is performed by the non-linear least-squares black box frequency domain identification method [13,14]. The result is also displayed in Fig. 9. A good correspondence between the black box model and the FRF is visible.

4. Controller design

The controller design is based on the root-locus. It tries to minimize the difference in distance measured by the two ultrasonic sensors. For more details refer to Ref. [8]. The order and the structure of the controller has been selected similar to Eq. (18). By placing the poles and the zeroes such that the root locus is attracted in the left half-plane, damping is added to the suspension. It turns out that a good selection of the poles and zeroes of the controller results in a lead compensator. As the latter is sensible to high frequency noise, an additional first order filter is added. The performance of the controller is validated on a six degrees of freedom shaker [15]. The latter simulates the rolling motions of the tractor and the variations in the slope of the field. Underneath each ultrasonic sensor, a plate is connected. In this way, the boom tries, by the action of the controller, to follow the slow motions of the shaker, representing variations in slope of the field. A swept sine is applied to the shaker because it shows clearly the performance of the slow active suspension. The results are displayed in Fig. 10. Δ is the difference in distance measured by

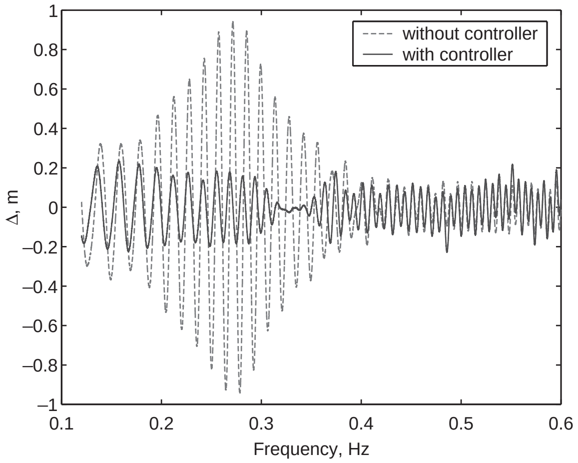

Fig. 10. Validation results.

the ultrasonic sensors. Up to the resonance frequency, the boom attempts to follow the shaker. Higher frequencies are considered as field undulations. They are not followed but filtered by the passive suspension.

5. Practical implementation

An automatic system to follow the slope of the field is also implemented on a real Inuma spray boom. The suspension system is exactly the same as the one used in the laboratory. However, the controller is adjusted to the dynamics of the spray boom. It is impossible to put the sprayer on the shaker. To validate the controller, the sprayer is put in a slanted position and the motions of the boom recorded until equilibrium is reached. These experiments are performed with controller, without controller and without controller but with a pair of dampers installed on the machine (Fig. 11). It is clear that only the system with controller is able to put the boom parallel to the field. The system with controller adds extra damping but not as much as if a pair of dampers is installed. This has basically to do with the low power of the actuator (70 W). The low power of the actuator also caused problems while turning over a slanted field. In this situation the boom has to turn from one side to the other. Due to the low power of the actuator and the low natural frequency of the spray boom, this operation takes time, such that the driver has to slow down dramatically. To overcome this problem, a more powerful actuator should be installed but the natural frequency of the boom should also be increased.

6. Conclusions

Modifying existing pendulum and trapezium suspensions to systems able to follow the slope of the field, does not change the natural frequency. All investigated slow active systems have mathematically the same structure. A controller is designed taking into account the dynamics of

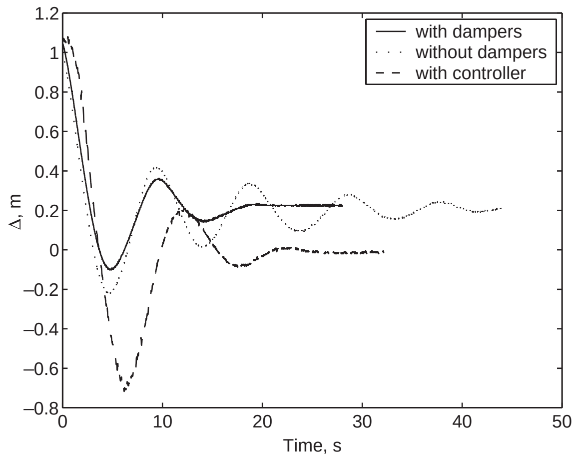

Fig. 11. Validation of the controller on a real spray boom.

the spray boom. It is implemented on a laboratory set-up and on a real sprayer. The controller adds damping and even with a 70 W actuator a reasonable performance is achieved. To improve the performance, the natural frequency of the suspension system must be increased and a more powerful actuator should be installed.

Acknowledgements

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