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Boundary feedback stabilization of the sine-Gordon equation without velocity feedback

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Abstract

This paper is concerned with the system governed by the sine-Gordon equation without damping. From a practical point of view, velocity may not be measured precisely. The global stabilization of the system governed by the sine-Gordon equation without damping is investigated in the case where any velocity feedback is not available. In such cases only position feedback cannot asymptotically stabilize the system. A parallel compensator is effective. The stabilizer is constructed by a proportional controller for the augmented system which consists of the controlled system and a parallel compensator. The asymptotic stability of the closed-loop system is proved.

1. Introduction

The sine-Gordon equation is used to model the dynamics of a Josephson junction driven by a current source [1,2]. For a single junction the governing equation is an ordinary differential equation similar to the pendulum equation. A coupled system of such equations appears when we consider a family of coupled junctions, and the continuous case is modelled by the sine-Gordon equation.

We consider the system governed by the sine-Gordon equation without damping on the domain $[0, 1]$. The linearized system has an infinite number of poles and zeros on the imaginary axis. From a practical point of view, velocity may not be measured precisely. In this paper we investigate the global stabilization of the system governed by the sine-Gordon equation without damping in the case where any velocity feedback is not available. In such cases only position feedback cannot

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asymptotically stabilize the system. A parallel compensator is effective. We construct the stabilizer by a proportional controller (P-controller) for the augmented system which consists of the controlled system and a parallel compensator. We show the asymptotic stability of the closed-loop system using LaSalle's invariance principle.

2. System description

The equation

$$z_{tt}(x,t) + \alpha z_t(x,t) - z_{xx}(x,t) + \beta \sin z(x,t) = 0, \quad x \in (0,1), \ t > 0, \tag{1}$$

is called the sine-Gordon equation. It is known that the system governed by Eq. (1) with initial and boundary conditions

$$z(x,0) = z^0(x), \qquad z_t(x,0) = z^1(x), \quad x \in (0,1), \tag{2}$$

$$z(0,t) = z_x(1,t) = 0, \quad t > 0, \tag{3}$$

is globally well posed and stable for $\alpha > 0$ in appropriate function spaces [2,3]. Moreover in the case where $|\beta| < \pi^2/4$, the system is globally asymptotically stable [3].

In this paper we consider stabilization of the system governed by the sine-Gordon equation without damping

$$z_{tt}(x,t) - z_{xx}(x,t) + \beta \sin z(x,t) = 0, \quad x \in (0,1), \ t > 0, \tag{4}$$

$$z(x,0) = z^0(x), \quad z_t(x,0) = z^1(x), \ x \in (0,1), \tag{5}$$

$$z(0,t) = 0, \quad z_x(1,t) = u(t), \ t > 0, \tag{6}$$

$$y(t) = z(1,t), \quad t > 0, \tag{7}$$

where β is a constant, $u(t)$ is a control input and $y(t)$ is the output.

The linear system without the term $\beta \sin z(x,t)$ has an infinite number of poles and zeros on the imaginary axis. The open-loop system given by Eqs. (4)–(6) is not asymptotically stable. In such cases only position feedback cannot asymptotically stabilize the system. In order to asymptotically stabilize the system given by Eqs. (5) and (6), velocity feedback or a parallel compensator such that

$$\frac{d\xi}{dt} = \alpha\xi(t) + bu(t), \quad \xi(0) = \xi_0, \tag{8}$$

is necessary.

In this paper we shall show that the stabilizer without velocity feedback can be constructed by a P-controller for the augmented system which consists of the controlled system and a parallel compensator.

3. Design of stabilizer

We first introduce a parallel compensator

$$\frac{\mathrm{d}\xi}{\mathrm{d}t} = -a\xi(t) + bu(t), \quad \xi(0) = 0, \tag{9}$$

where $a \geqslant 0$, $b > 0$. For the augmented system given by Eqs. (5), (6) and (9) we apply a controller

$$\begin{aligned} u(t) &= -k[y(t) + \xi(t)], \\ &= -ky_{\xi}(t), \quad k > 0. \end{aligned} \tag{10}$$

The resulting closed-loop system becomes

$$z_{tt}(x,t) - z_{xx}(x,t) + \beta \sin z(x,t) = 0, \quad x \in (0,1), \ t > 0, \tag{11}$$

$$z(x,0) = z^0(x), \quad z_t(x,0) = z^1(x), \ x \in (0,1), \tag{12}$$

$$z(0,t) = 0, \quad z_x(1,t) = -k[z(1,t) + \xi(t)], \ t > 0, \tag{13}$$

$$\frac{\mathrm{d}\xi}{\mathrm{d}t} = -[a + bk]\xi(t) - bkz(1,t), \quad \xi(0) = 0. \tag{14}$$

In this paper we use only standard notations, as in Ref. [4]. Sometimes, a function $z = z(x,t)$ will simply be denoted by $z(t)$, when the x-variable is not in consideration. Our analysis is based on two Hilbert spaces $L^2(0,1)$ and $H^1(0,1)$. Here $L^2(0,1)$ is the Lebesgue space of scalar-valued square-integrable functions $z(x)$ defined for $0 \leqslant x \leqslant 1$. The space $H^1(0,1)$ is the Sobolev space of scalar-valued square-integrable functions $z(x)$ defined for $0 \leqslant x \leqslant 1$ such that $z_x(x)$ is also square-integrable. The class of linear bounded operators from a space X into a space Y is denoted by $L(X, Y)$.

We introduce the real Hilbert state space $X = H_E^1(0,1) \times L^2(0,1) \times \mathbf{R}$ with the inner product

$$\begin{aligned} &\langle (z_1(x), v_1(x), \xi_1), (z_2(x), v_2(x), \xi_2) \rangle_X \\ &= \int_0^1 [z_{1x}(x) z_{2x}(x) + v_1(x) v_2(x)] \, \mathrm{d}x \\ &\quad + \frac{a}{b} \xi_1 \xi_2 + k[z_1(1) + \xi_1][z_2(1) + \xi_2], \end{aligned} \tag{15}$$

and the induced norm. Here $H_E^1(0,1) = \{z \in H^1(0,1), z(0) = 0\}$. Define a non-linear operator $\mathbf{A} : D(\mathbf{A}) \subset X \to X$ by

$$\mathbf{A}(z(x), v(x), \xi) = (v(x), z_{xx}(x) - \beta \sin z(x), -bkz(1) - (a + bk)\xi), \tag{16}$$

with

$$D(\mathbf{A}) = \{(z(x), v(x), \xi) \in H^1(0,1) \times H^1(0,1) \times \mathbf{R} | z(0) = 0, \ v(0) = 0, \ z_x(1) + kz(1) + k\xi = 0\}.$$

It is easily seen that $D(\mathbf{A})$ is dense in X.

Then the system given by Eqs. (11)–(14) can be written as a non-linear evolution equation on X:

$$\dot{Z}(t) = \mathbf{A}Z(t), \quad Z(0) = Z^0, \tag{17}$$

where $Z(t) = (z(x,t), \dot{z}(x,t), \xi(t))$.

We shall show that the operator $\mathbf{A}$ defined by Eq. (16) generates a non-linear strongly continuous semigroup on X.

The operator $\mathbf{A}$ is closed. For $Z_1, Z_2 \in D(\mathbf{A})$ by a simple calculation we have

$$\begin{aligned}
&\langle \mathbf{A}Z_1 - \mathbf{A}Z_2, Z_1 - Z_2 \rangle \\
&= -\beta \int_0^1 [\sin z_1(x) - \sin z_2(x)][\dot{z}_1(x) - \dot{z}_2(x)]\,\mathrm{d}x - \frac{1}{b}[\dot{\xi}_1 - \dot{\xi}_2]^2 \\
&\leqslant |\beta| \sqrt{\int_0^1 |\bar{z}(x)|^2\,\mathrm{d}x} \sqrt{\int_0^1 |\dot{\bar{z}}(x)|^2\,\mathrm{d}x} - \frac{1}{b}[(a+bk)\bar{\xi} + bk\bar{z}(1)]^2 \\
&\leqslant \frac{|\beta|}{2}\left[\int_0^1 |\bar{z}(x)|^2\,\mathrm{d}x + \int_0^1 |\dot{\bar{z}}(x)|^2\,\mathrm{d}x\right] - \frac{1}{b}[(a+bk)\bar{\xi} + bk\bar{z}(1)]^2 \\
&\leqslant \frac{|\beta|}{2}\left[\int_0^1 |\bar{z}_x(x)|^2\,\mathrm{d}x + \int_0^1 |\dot{\bar{z}}(x)|^2\,\mathrm{d}x\right] - \frac{1}{b}[(a+bk)\bar{\xi} + bk\bar{z}(1)]^2,
\end{aligned} \tag{18}$$

where $\bar{z} = z_1 - z_2$, $\bar{\xi} = \xi_1 - \xi_2$. Thus the operator $\mathbf{A} - (|\beta|/2)I$ is dissipative in X.

Next, we show that $R(\lambda I - \mathbf{A}) = X$ for any $\lambda > \sqrt{|\beta|}$. It is sufficient to show that for any $\lambda > \gamma$ (γ is some positive constant) and any $(f, g, r) \in X$, there exists $(z, v, \xi) \in D(\mathbf{A})$ such that

$$(\lambda I - \mathbf{A})(z(x), v(x), \xi) = (f(x), g(x), r), \tag{19}$$

that is,

$$\lambda z(x) - v(x) = f(x), \quad kbz(1) + (\lambda + a + kb)\xi = r, \tag{20}$$

and z satisfies

$$\begin{aligned}
&-z_{xx}(x) + \beta \sin z(x) + \lambda v(x) = g(x), \\
&z(0) = 0, \quad z_x(1) = -kz(1) - k\xi.
\end{aligned} \tag{21}$$

If we define operators A and B by

$$\begin{aligned}
&Az = -z_{xx}(x), \\
&D(A) = \{z \in H^2(0,1) | z(0) = z_x(1) = 0\}, \\
&B^* z = z(1),
\end{aligned} \tag{22}$$

the operator A is unbounded and self-adjoint in $L^2(0,1)$. The operator B can be identified with $\delta(x-1)$ [4]. If we consider the space $V = H^1(0,1)$ and its dual space V'. Then it can be shown that $B \in L(\mathbf{R}, V')$, $A \in L(V, V')$ [5]. Since

$$z(x) = z(1) - \int_x^1 z_x(x)\,\mathrm{d}x,$$

$$z^2(x) \leqslant \left(1 + \frac{1}{\delta}\right) z^2(1) + (1+\delta)\int_0^1 z_x^2(x)\,\mathrm{d}x.$$

From this

$$\int_0^1 z^2(x)\,\mathrm{d}x \leqslant \left(1 + \frac{1}{\delta}\right) z^2(1) + (1+\delta)\int_0^1 z_x^2(x)\,\mathrm{d}x,$$

for any $\delta > 0$. We obtain

$$\langle (A + kBB^*)z, z\rangle = -\int_0^1 z_{xx} z \, dx + kz^2(1)$$
$$\geqslant (1 - k\delta)\int_0^1 z_x^2 \, dx + \frac{k\delta}{1+\delta}\int_0^1 z^2 \, dx.$$

If $0 < \delta < 1/k$, then it holds that for some $\alpha > 0$

$$\langle (A + kBB^*)z, z\rangle \geqslant \alpha \|z\|_V^2. \tag{23}$$

From Eqs. (20) and (21)

$$[\lambda^3 + (a+kb)\lambda^2 + \lambda(A + kBB^*) + (a+kb)A + akBB^*]z$$
$$+ (\lambda + a + kb)\beta \sin z = (\lambda + a + kb)(\lambda f + g) - kBr. \tag{24}$$

Define $\Gamma(\lambda)$ by

$$\Gamma(\lambda) = \lambda^3 + (a+kb)\lambda^2 + \lambda(A + kBB^*) + a(A + kBB^*) + kbA,$$

we have from Eq. (24)

$$\langle \Gamma(\lambda)z, z\rangle \geqslant [\lambda^3 + (a+kb)\lambda^2 + \alpha(\lambda + a)]\|z\|_V^2.$$

Thus $\Gamma(\lambda) \in L(V, V')$ is bounded invertible and self-adjoint in $L^2(0,1)$. Eq. (24) can be written as

$$z = -(\lambda + a + kb)\beta\Gamma^{-1}(\lambda)\sin z$$
$$+ (\lambda + a + kb)\Gamma^{-1}(\lambda)(\lambda f + g) - k\Gamma^{-1}(\lambda)Br = F(z). \tag{25}$$

For every z_1, z_2 in V and for any $\lambda > 0$, we have

$$\|F(z_1) - F(z_2)\| = (\lambda + a + kb)\beta\|\Gamma^{-1}(\lambda)(\sin z_1 - \sin z_2)\|$$
$$\leqslant \frac{(\lambda + a + kb)|\beta|}{\lambda^3 + (a+kb)\lambda^2}\|z_1 - z_2\|_H \leqslant \frac{|\beta|}{\lambda^2\sqrt{\lambda_1}}\|z_1 - z_2\|_V,$$

where λ_1 is the first eigenvalue of A [2].

If $\lambda > \sqrt{|\beta|/\sqrt{\lambda_1}}$, then $F(z)$ is a strict contraction mapping in $V = H^2(0,1)$ and $F(z)$ has a unique fixed point z in V [6], which implies that Eq. (21) admits a solution. Thus $X \subset R(\lambda I - \mathbf{A})$ for any $\lambda > \sqrt{|\beta|/\sqrt{\lambda_1}}$.

The Crandall–Liggett theorem [7,8] gives the following existence and uniqueness result.

Theorem 1. *The operator* $\mathbf{A}$ *defined by Eq.* (16) *generates a unique nonlinear strongly continuous semigroup on* X. *Thus Eq.* (17) *admits a unique solution* $Z(t)$ *such that for each* $Z^0 \in D(\mathbf{A})$

$$Z \in C^1([0,T]; X) \cap C([0,T]; D(\mathbf{A})),$$

where $C^n([0,T]; X)$ *is the space of* n *times continuously differentiable functions from* $[0,T]$ *into* X.

Theorem 1 says that the closed-loop system given by Eqs. (11)–(14) is well posed. For asymptotic stability of the closed-loop system we can show the following theorem.

Theorem 2. *Suppose that* $|\beta| < \pi^2/4$. *Then the closed-loop system given by Eqs.* (11)–(14) *is asymptotically stable.*

Proof. Define energy-like (Lyapunov-like) functions for the system given by Eqs. (11)–(14)

$$E(t) = \frac{1}{2}\int_0^1 [z_t(x,t)^2 + z_x(x,t)^2]\,\mathrm{d}x + \beta\int_0^1 [1-\cos z(x,t)]\,\mathrm{d}x + \frac{a}{2b}\xi(t)^2 + \frac{k}{2}y_\xi(t)^2. \tag{26}$$

Here, since from the Poincaré inequality [2]

$$\int_0^1 (1-\cos z)\,\mathrm{d}x \leqslant \frac{1}{2}\int_0^1 z^2\,\mathrm{d}x \leqslant \frac{2}{\pi^2}\int_0^1 z_x^2\,\mathrm{d}x, \tag{27}$$

$E(t) \geqslant 0$ for $\beta \geqslant -\pi^2/4$.

Along the solution of the system given by Eqs. (11)–(14), we obtain

$$\begin{aligned}
\dot{E}(t) &= \int_0^1 z_t z_{tt}\,\mathrm{d}x + \int_0^1 z_x z_{xt}\,\mathrm{d}x \\
&\quad + \beta\int_0^1 z_t \sin z\,\mathrm{d}x + \frac{a}{b}\xi(t)\dot{\xi}(t) + k y_\xi(t)\dot{y}_\xi(t) \\
&= \int_0^1 z_t z_{xx}\,\mathrm{d}x - \beta\int_0^1 z_t\sin z\,\mathrm{d}x + \int_0^1 z_x z_{xt}\,\mathrm{d}x \\
&\quad + \beta\int_0^1 z_t \sin z\,\mathrm{d}x + \frac{a}{b}\xi(t)\dot{\xi}(t) + k y_\xi(t)\dot{y}_\xi(t) \\
&= z_t(1,t)z_x(1,t) - z_t(0,t)z_x(0,t) - \int_0^1 z_{xt}z_x\,\mathrm{d}x + \int_0^1 z_x z_{xt}\,\mathrm{d}x \\
&\quad + \frac{a}{b}\xi(t)\dot{\xi}(t) + k y_\xi(t)\dot{y}_\xi(t) \\
&= -k z_t(1,t)y_\xi(t) + \frac{a}{b}\xi(t)\dot{\xi}(t) + k y_\xi(t)\dot{y}(t) + k y_\xi(t)\dot{\xi}(t) \\
&= \frac{1}{b}\dot{\xi}(t)[a\xi(t) + bky_\xi(t)] \\
&= -\frac{1}{b}\dot{\xi}(t)^2.
\end{aligned} \tag{28}$$

We have a dynamical system on X with all orbits bounded. For the space $Y = D(\mathbf{A})$ with the graph norm, the operator $\mathbf{A}$ has a compact resolvent and the bounded set of Y is precompact in X [3]. So each orbit is precompact in X.

According to LaSalle's invariance principle [3,9], all solutions of Eqs. (11)–(14) asymptotically tend to the maximal invariant set of the following set:

$$S = \{(z, \dot{z}, \xi) | \dot{E} = 0\},$$

if the solution trajectories for $t \geqslant 0$ are precompact in X.

From $\dot{E} = 0$ it results in $\dot{\xi}(t) \equiv 0$. Since $\xi(0) = 0$, $\xi(t) \equiv 0$, $u(t) \equiv 0$. Thus system (11)–(14) reduces to

$$z_{tt}(x,t) - z_{xx}(x,t) + \beta \sin z(x,t) = 0,$$
$$z(0,t) = z(1,t) = z_x(1,t) = 0. \tag{29}$$

To prove that all solutions of the system given by Eqs. (11)–(14) asymptotically tends to zero, it is sufficient to show that system (29) has only the zero solution. This is the observability problem for the controlled system given by Eqs. (4)–(7).

Here introduce another energy-like functions such that

$$W(t) = \frac{1}{2}\int_0^1 [z_t(x,t)^2 + z_x(x,t)^2]\,\mathrm{d}x + \beta \int_0^1 [1 - \cos z(x,t)]\,\mathrm{d}x, \tag{30}$$

$$V(t) = W(t) + \varepsilon g(t), \quad \varepsilon > 0, \tag{31}$$

where $g(t)$ is a multiplier function given by

$$g(t) = \int_0^1 x z_t(x,t) z_x(x,t)\,\mathrm{d}x. \tag{32}$$

First from $\dot{W}(t) = 0$ it follows that $W(t) \equiv const$. Let us estimate $g(t)$. Since

$$\int_0^1 x z_t z_x\,\mathrm{d}x \leqslant \int_0^1 |z_t|\,|z_x|\,\mathrm{d}x \leqslant \frac{1}{2}\int_0^1 z_t^2\,\mathrm{d}x + \frac{1}{2}\int_0^1 z_x^2\,\mathrm{d}x,$$

it follows from Eq. (27) that

$$|g(t)| \leqslant g_\beta W(t) \quad \text{for any } t \geqslant 0, \tag{33}$$

where $g_\beta = 1$ for $\beta > 0$ and $g_\beta = (\pi^2 - 4|\beta|)/\pi^2$ for $0 > \beta > -\pi^2/4$.

Thus for $0 < \varepsilon < 1/g_\beta$ the function V satisfies

$$0 \leqslant (1 - \varepsilon g_\beta) W(t) \leqslant V(t) \leqslant (1 + \varepsilon g_\beta) W(t) \quad \text{for all } t \geqslant 0, \tag{34}$$

which implies that $V(0) \geqslant 0$ if $0 < \varepsilon < 1/g_\beta$.

On the other hand,

$$\dot{g}(t) = \int_0^1 x(z_{tt} z_x + z_t z_{xt})\,\mathrm{d}x = \int_0^1 x z_{xx} z_x\,\mathrm{d}x - \beta \int_0^1 x z_x \sin z\,\mathrm{d}x + \int_0^1 x z_t z_{xt}\,\mathrm{d}x.$$

For each term on the right side

$$\int_0^1 xz_{xx}z_x\,\mathrm{d}x = \frac{1}{2}\int_0^1 (xz_x^2)_x\,\mathrm{d}x - \frac{1}{2}\int_0^1 z_x^2\,\mathrm{d}x$$
$$= -\frac{1}{2}\int_0^1 z_x^2\,\mathrm{d}x,$$

$$\int_0^1 xz_t z_{xt}\,\mathrm{d}x = \frac{1}{2}\int_0^1 (xz_t^2)_x\,\mathrm{d}x - \frac{1}{2}\int_0^1 z_t^2\,\mathrm{d}x$$
$$= -\frac{1}{2}\int_0^1 z_t^2\,\mathrm{d}x,$$
$$-\int_0^1 xz_x \sin z\,\mathrm{d}x = \int_0^1 x(\cos z)_x\,\mathrm{d}x$$
$$= \cos z(1,t) - \int_0^1 \cos z\,\mathrm{d}x$$
$$= 1 - \int_0^1 \cos z\,\mathrm{d}x$$
$$= \int_0^1 (1-\cos z)\,\mathrm{d}x$$
$$\leqslant \frac{1}{2}\int_0^1 z^2\,\mathrm{d}x$$
$$\leqslant \frac{2}{\pi^2}\int_0^1 z_x^2\,\mathrm{d}x$$

(the last inequality is due to the Poincaré inequality [2]).

Using these relations we obtain

$$\dot{g}(t) = -\frac{1}{2}\int_0^1 z_x^2\,\mathrm{d}x - \frac{1}{2}\int_0^1 z_t^2\,\mathrm{d}x + \beta\int_0^1 (1-\cos z)\,\mathrm{d}x. \tag{35}$$

First let us consider the case where $\beta > 0$. For $\delta > 0$ it holds that

$$\dot{g}(t) = -\frac{1}{2}\int_0^1 z_x^2\,\mathrm{d}x - \frac{1}{2}\int_0^1 z_t^2\,\mathrm{d}x$$
$$- \delta\beta\int_0^1 (1-\cos z)\,\mathrm{d}x + (1+\delta)\beta\int_0^1 (1-\cos z)\,\mathrm{d}x$$
$$\leqslant -\left[\frac{1}{2} - \frac{2(1+\delta)\beta}{\pi^2}\right]\int_0^1 z_x^2\,\mathrm{d}x - \frac{1}{2}\int_0^1 z_t^2\,\mathrm{d}x$$
$$- \delta\beta\int_0^1 (1-\cos z)\,\mathrm{d}x.$$

If $0<\beta<\pi^2/4$, there exists $\delta>0$ such that

$$C_\beta = \min\left\{1 - \frac{4(1+\delta)\beta}{\pi^2},\ \delta\right\} > 0, \tag{36}$$

and

$$\dot{g}(t) \leqslant -C_\beta W(t).$$

Next consider the case where $\beta<0$. For $\delta>0$ it holds that

$$\dot{g}(t) \leqslant -\frac{1}{2}\int_0^1 z_x^2\,\mathrm{d}x - \frac{1}{2}\int_0^1 z_t^2\,\mathrm{d}x - \delta\beta\int_0^1 (1-\cos z)\,\mathrm{d}x.$$

If $\beta<0$, there exists $\delta>0$ such that

$$C_\beta = \min\{1,\delta\} > 0, \tag{37}$$

and

$$\dot{g}(t) \leqslant -C_\beta W(t).$$

Thus if $\beta<\pi^2/4$, there exists a positive constant C_β such that

$$\dot{g}(t) \leqslant -C_\beta W(t) \quad \text{for all } t\geqslant 0. \tag{38}$$

Lastly, since from Eqs. (34) and (38)

$$\begin{aligned} \dot{V}(t) &= \dot{W}(t) + \varepsilon\dot{g}(t) \\ &\leqslant -\varepsilon C_\beta W(t) \\ &\leqslant -\frac{\varepsilon C_\beta}{1+\varepsilon g_\beta} V(t) = -K_\varepsilon V(t), \end{aligned} \tag{39}$$

we have

$$0 \leqslant V(t) \leqslant V(0)\mathrm{e}^{-K_\varepsilon t} \quad \text{for all } t>0.$$

Again from Eq. (34) we obtain

$$0 \leqslant W(t) \leqslant \frac{1}{1-\varepsilon g_\beta} V(0)\mathrm{e}^{-K_\varepsilon t} \quad \text{for all } t>0. \tag{40}$$

This implies that $W(t) \equiv const. = 0$, from which it follows that $z_t(x,t)\equiv 0$, $z_x(x,t)\equiv 0$. Moreover, since $z(0,t)=0$ for all $t\geqslant 0$ and

$$|z(x,t)| = \left|\int_0^x z_x\,\mathrm{d}x\right| \leqslant \int_0^1 |z_x|\,\mathrm{d}x \leqslant \sqrt{\int_0^1 z_x^2\,\mathrm{d}x},$$

we conclude that $z(x,t)\equiv 0$. We have proved the theorem.

4. Conclusion

In this paper we have considered global asymptotic stabilization of the system governed by the sine-Gordon equation without damping on the domain [0, 1], in the case where any velocity feedback is not available. The linearized system has an infinite number of poles and zeros on the imaginary axis. In the case where any velocity feedback is not available, a parallel compensator plays an important role. The stabilizer has been constructed by a P-controller for the augmented system which consists of the controlled system and a parallel compensator. The asymptotic stability of the closed-loop system has been proved using LaSalle's invariance principle.

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