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Interaction due to mechanical sources in micropolar elastic medium with voids

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Abstract

The present paper is concerned with the dynamic problem of a homogeneous isotropic half-space with voids subjected to a set of normal point sources. The integral transforms have been inverted by using a numerical technique to obtain the normal force stress, normal displacement, tangential couple stress and volume fraction field in the physical domain for the two different sources. The expressions of these quantities have been given and illustrated graphically to depict the effect of micropolarity and voids.

1. Introduction

The linear theory of elastic materials with voids is one of the generalization of the classical theory of elasticity. This theory has practical utility to investigate various types of geological, biological and synthetic porous materials for which the elastic theory is inadequate. This theory is concerned with elastic materials consisting of a distribution of small pores (voids), in which the voids volume is included among the kinematics variables, and in the limiting case of volume tending to zero, the theory reduces to the classical theory of elasticity. The presence of voids is known to affect the estimation of the physical-mechanical properties of the composite and also weaken the bond as these pores get spread over a wide area. The intended applications are to the materials like rock, soil and to manufactured porous materials.

It is commonly accepted that the mechanical behavior of granular masses is strongly affected by their microstructure, namely the relative arrangement of voids and particles, i.e., the granular fabric. Therefore, parameters which characterized the granular are of paramount importance in a fundamental description of overall macroscopic stresses and deformation measures. The study of

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deformations in granular materials is important in many areas of sciences and technology, such as powder metallurgy and earthquake engineering. In recent years, dynamic compaction of powders has been used to manufacture advanced composites. A granular medium is composed of a large number of distinct particles as well as some heterogeneous inclusion and voids. The inclusions consists of materials with higher or lower acoustical impedance, and the voids may be filled with gas or liquid. At the boundaries of these discontinuities and mismatch and incident wave will produce both transmission and reflection waves of different modes. Wave propagation phenomenon in such a media not only depends on the microstructure, but on the existence of inclusion and voids.

Following some basic ideas of previous papers on granular materials by Goodman and Cowin [1,2], Nunziato and Cowin in Ref. [3] set-up a continuum theory in which porosity is modelled by assigning an additional degree of freedom to each particle of the material structure, namely, the fraction of volume which is possibly found void of matter. Lewis and Isaak [4] discussed the voids of minimum stress concentration. Later, Cowin and Nunziato [5] developed a theory of linear elastic materials with voids, for the mathematical study of the mechanical behavior of porous solids in which the skeletal or matrix material is elastic and the interstices are void of materials. They considered several applications of the linear theory by investigating the response of the materials to homogeneous deformations, pure bending of a beam and small amplitudes acoustic waves. Pouget and Maugin [6,7] established non-linear elastoacoutic equations for piezoelectric powders and discussed continuum approach to electroacoutic echoes in piezoelectric powders. Piezoelectric ceramics and composites have been extensively used in many engineering applications such as sensors, actuators, intelligent structures, etc. The problems of quasi-static plane strain and plane stress for a linear elastic material with voids was studied by Cowin [8]. Puri and Cowin [9] studied the behavior of plane harmonic waves in a linear elastic material with voids. Iesan [10] developed the basic theories of linear thermoelastic materials with voids. Chandersekharaiah [11] studied the plane wave in a rotating elastic solid with voids. Chandersekharaiah and Cowin [12], obtained the field equations governing two different continuum theories, namely the theory of thermoelasticity and Biot's theory of poroelasticity. The problem of complete solutions in the theory of isotropic elastic materials with voids was discussed by Chandrasekharaiah [13]. A domain of influence theorem in the linear theory of elastic materials with voids was discussed by Dhaliwal and Wang [14]. Scarpetta [15] proved some theorem of uniqueness for linear elastic materials with voids.

The particles of a classical elastic materials have transnational degree of freedom only, and transmission of the load across a differential element of the surface is described by a force vector only. The polycrystalline materials do not confirm this. These materials are fibrous and composite in nature and display size effects. These materials have additional micro-deformational degree of freedom, i.e., they possess a microstructured whose size cannot be neglected in comparison with length scales of interest. Various degrees of freedom of a microstructure were considered by different authors. Notable among them are Cosserat and Cosserat [16], Eringen and Suhubi [17] and Mindlin [18]. Each one has given an independent set of governing equations. The force at a point of a surface element of bodies of these materials is completely characterized by a stress vector and a couple stress vector at that point. In the classical theory of elasticity, the effect of couple stress is neglected. Eringen [19] has modified his earlier theory and renamed it as the "Linear Theory of Micropolar Elasticity".

Iesan [20] studied shock waves in micropolar elastic material with voids. Scarpetta [21] worked on the fundamental solutions in micropolar elasticity with voids. Marin [22] obtained the existence and uniqueness of solutions for boundary value problem in elasticity of micropolar materials with voids. Marin [23] discussed generalized solutions in elasticity of micropolar bodies with voids. Marin [24] derived a temporally evolutionary equation in micropolar elastic body with voids.

2. Formulation of the problem

We consider a homogeneous, isotropic, micropolar elastic half-space with voids. The cylindrical polar co-ordinate system (r, θ, z) having the z-axis pointing vertical into the medium is introduced. A normal Delta distribution or continuous point source is assumed to be acting at the origin of the cylindrical polar co-ordinates.

Following Eringen [25] and Iesan [20], the constitutive relations and the field equations in micropolar elastic solid with voids without body forces and body couples can be written as

$$t_{ij} = \lambda u_{k,k}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + K(u_{j,i} - \varepsilon_{ijk}\phi_k) + \delta_{ij}\beta^* q, \tag{1}$$

$$m_{ij} = \alpha\phi_{k,k}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i} \tag{2}$$

and

$$(\lambda + \mu)\nabla(\nabla.\mathbf{u}) + (\mu + K)\nabla^2\mathbf{u} + K\nabla \times \boldsymbol{\phi} + \beta^*\nabla q = \rho\frac{\partial^2\mathbf{u}}{\partial t^2}, \tag{3}$$

$$(\alpha + \beta + \gamma)\nabla(\nabla.\boldsymbol{\phi}) - \gamma\nabla \times (\nabla \times \boldsymbol{\phi}) + K\nabla \times \mathbf{u} - 2K\boldsymbol{\phi} = \rho j\frac{\partial^2\boldsymbol{\phi}}{\partial t^2}, \tag{4}$$

$$\alpha^*\nabla^2 q - \zeta^* q - \omega^*\frac{\partial q}{\partial t} - \beta^*\nabla.\mathbf{u} = \rho K^*\frac{\partial^2 q}{\partial t^2}, \tag{5}$$

where λ, μ, K, α, β, γ are material constants, ρ is the density, j is the micro inertia, $\mathbf{u}$ is the displacement vector, $\boldsymbol{\phi}$ is the micro rotation vector, t_{ij} is the component of force stress, m_{ij} is the component of the couple stress, q is the volume fraction field and α^*, β^*, ζ^*, ω^*, K^* are material constants due to the presence of voids.

For axial symmetric problem, we take

$$\mathbf{u} = (u_r, 0, u_z) \quad \text{and} \quad \boldsymbol{\phi} = (0, \phi_\theta, 0)$$

We define the non-dimensional quantities as

$$\begin{aligned} r' &= \frac{r}{h}, \quad z' = \frac{z}{h}z, \quad u_r' = \frac{u_r}{h}, \\ u_z' &= \frac{u_z}{h}, \quad t'^2 = \frac{\mu}{\rho h^2}t^2, \quad \phi_\theta' = \frac{\mu}{\rho j \bar{\omega}^2}\phi_\theta, \\ t_{ij}' &= \frac{t_{ij}}{\mu}, \quad m_{ij}' = \frac{\mu h}{\gamma K}m_{ij}, \quad q' = \frac{\beta^* K^*}{\alpha^*}q, \end{aligned} \tag{6}$$

where

$$\varpi^2 = K/\rho j, \quad c_1^2 = (\lambda + 2\mu + K)/\rho$$

and h is a parameter of dimension of length.

Due to axial symmetry about z-axis, the quantities are independent of θ. With these considerations and using dimensionless quantities given in expression (6), the system of equations (3)–(5) may be recast into the dimensionless form (after suppressing the primes) as

$$\frac{\partial e}{\partial r} + a_1\left(\nabla^2 - \frac{1}{r^2}\right)u_r - a_2\frac{\partial \phi_\theta}{\partial z} + a_3\frac{\partial q}{\partial r} = a_4\frac{\partial^2 u_r}{\partial t^2}, \tag{7}$$

$$\frac{\partial e}{\partial z} + a_1\nabla^2 u_z + a_2\frac{1}{r}\frac{\partial (r\phi_\theta)}{\partial r} + a_3\frac{\partial q}{\partial z} = a_4\frac{\partial^2 u_z}{\partial t^2}, \tag{8}$$

$$\left(\nabla^2 - \frac{1}{r^2} - b_1\right)\phi_\theta + b_2\left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right) = b_3\frac{\partial^2 \phi_\theta}{\partial t^2}, \tag{9}$$

$$\left(\nabla^2 - s_1 - s_2\frac{\partial}{\partial t}\right)q - s_4 e = s_3\frac{\partial^2 q}{\partial t^2}, \tag{10}$$

where

$$a_1 = \frac{(\mu + K)}{(\lambda + \mu)}, \quad a_2 = \frac{K^2}{(\lambda + \mu)\mu}, \quad a_3 = \frac{\alpha^*}{K^*(\lambda + \mu)}, \quad a_4 = \frac{\mu}{(\lambda + \mu)},$$

$$b_1 = \frac{2Kh^2}{\gamma}, \quad b_2 = \frac{h^2\mu}{\gamma}, \quad b_3 = \frac{j\mu}{\gamma},$$

$$s_1 = \frac{\zeta^* h^2}{\alpha^*}, \quad s_2 = \frac{\omega^* h}{\alpha^*}\sqrt{\frac{\mu}{\rho}}, \quad s_3 = \frac{K^*\mu}{\alpha^*}, \quad s_4 = \frac{\beta^{*2}K^*h^2}{\alpha^{*2}}$$

and

$$e = \frac{1}{r}\frac{\partial (r u_r)}{\partial r} + \frac{\partial u_z}{\partial z}, \quad \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}. \tag{11}$$

With the aid of the expressions relating displacement components $u_r(r,z,t)$, $u_z(r,z,t)$ and rotational component $\phi_\theta(r,z,t)$ to the scalar potential function $\psi_1(r,z,t)$, $\psi_2(r,z,t)$ and $\tau(r,z,t)$ in dimensionless form as

$$u_r = \frac{\partial \psi_1}{\partial r} + \frac{\partial^2 \psi_2}{\partial r \partial z}, \quad u_z = \frac{\partial \psi_1}{\partial z} - \left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)\psi_2, \quad \phi_\theta = -\frac{\partial \tau}{\partial r}, \tag{12}$$

in Eqs. (7)–(10), we obtain

$$\left[(1 + a_1)\nabla^2 - a_4\frac{\partial^2}{\partial t^2}\right]\psi_1 + a_3 q = 0, \tag{13}$$

$$\left[a_1\nabla^2 - a_4\frac{\partial^2}{\partial t^2}\right]\psi_2 + a_2\tau = 0, \tag{14}$$

$$\left[\nabla^2 - b_1 - b_3\frac{\partial^2}{\partial t^2}\right]\tau - b_2\nabla^2\psi_2 = 0, \tag{15}$$

$$\left[\nabla^2 - s_1 - s_2\frac{\partial}{\partial t} - s_3\frac{\partial^2}{\partial t^2}\right]q - s_4\nabla^2\psi_1 = 0. \tag{16}$$

We define the Laplace and Hankel transforms as

$$\bar{f}(r,z,p) = \int_0^\infty f(r,z,t)\mathrm{e}^{-pt}\,\mathrm{d}t,$$

$$\hat{f}(\zeta,z,p) = \int_0^\infty \bar{f}(r,z,p)rJ_n(r\zeta)\,\mathrm{d}r. \tag{17}$$

Applying the Laplace and Hankel transforms defined by Eq. (17) on Eqs. (13)–(16), and then eliminating $\hat{\tau}$ and $\hat{q}$ from the resulting expressions, we obtain

$$\left[\frac{\mathrm{d}^4}{\mathrm{d}z^4} + A\frac{\mathrm{d}^2}{\mathrm{d}z^2} + B\right][\tilde{\psi}_1] = 0, \tag{18}$$

$$\left[\frac{\mathrm{d}^4}{\mathrm{d}z^4} + E\frac{\mathrm{d}^2}{\mathrm{d}z^2} + F\right][\tilde{\psi}_2] = 0, \tag{19}$$

where

$$A = -[(1+a_2)(2\zeta^2 + s_1 + s_2p + s_3p^2) + a_4p^2 - a_3s_4]/(1+a_1),$$

$$B = \zeta^4 + [((1+a_2)(s_1 + s_2p + s_3p^2) + a_4p^2 - a_3s_4)\zeta^2 + (s_1 + s_2p + s_3p^2)a_4p^2]/(1+a_1),$$

$$E = -[a_1(2\zeta^2 + b_1 + b_3p^2) + a_4p^2 - a_2b_2]/a_1,$$

$$F = [\zeta^4 + (a_1(b_1 + b_3p^2) + a_4p^2 - a_2b_2)\zeta^2 + (b_1 + b_3p^2)a_4p^2]/a_1. \tag{20}$$

The solution of Eqs. (18) and (19), satisfying the radiation conditions as z tends to infinity, are

$$\hat{\psi}_1 = A_1\mathrm{e}^{-\lambda_1 z} + A_2\mathrm{e}^{-\lambda_2 z}, \tag{21}$$

$$\hat{q} = R_1A_1\mathrm{e}^{-\lambda_1 z} + R_2A_2\mathrm{e}^{-\lambda_2 z}, \tag{22}$$

$$\hat{\psi}_2 = A_3\mathrm{e}^{-\lambda_3 z} + A_4\mathrm{e}^{-\lambda_4 z}, \tag{23}$$

$$\hat{\tau} = R_3A_3\mathrm{e}^{-\lambda_3 z} + R_4A_4\mathrm{e}^{-\lambda_4 z}, \tag{24}$$

where $\lambda_{1,2}^2$ and $\lambda_{3,4}^2$ are roots of Eqs. (18) and (19), respectively, given by

$$\lambda_i^2 = \left[\frac{-A + (-1)^{i+1}\sqrt{A^2 - 4B}}{2}\right], \quad i = 1, 2,$$
$$\lambda_i^2 = \left[\frac{-E + (-1)^{i+1}\sqrt{E^2 - 4F}}{2}\right], \quad i = 3, 4 \tag{25}$$

and

$$R_i = \left[\frac{(1 + a_1)(\zeta^2 - \lambda_i^2) + a_4 p^2}{a_3}\right], \quad i = 1, 2,$$
$$R_i = \left[\frac{a_1(\zeta^2 - \lambda_i^2) + a_4 p^2}{a_2}\right], \quad i = 3, 4. \tag{26}$$

3. Application

3.1. Instantaneous normal point source

Plane boundary is subjected to an instantaneous normal point force. Therefore, the boundary conditions in this case are

$$t_{zz} = -\frac{P_0\delta(r)\delta(t)}{2\pi r}, \quad t_{zr} = 0, \quad m_{z\theta} = 0, \quad \frac{\partial q}{\partial z} = 0 \quad \text{at } z = 0, \tag{27}$$

where P is the magnitude of force applied and $\delta()$ is Dirac's delta distribution.

Making use of Eqs. (1), (2), (6), (11) and (12) in the boundary conditions (27) and applying the transforms defined by Eq. (17) and substituting the valuesf $\hat{\psi}_1, \hat{\psi}_2, \hat{q}$ and $\hat{\tau}$ from Eqs. (21)–(24) in the resulting expressions, we obtain the expressions for the components of displacement, force stress, couple stress and volume fraction field as

$$\hat{u}_r(\zeta, z, p) = \zeta[-(\Delta_1 e^{-\lambda_1 z} + \Delta_2 e^{-\lambda_2 z}) + \lambda_3\Delta_3 e^{-\lambda_3 z} + \lambda_4\Delta_4 e^{-\lambda_4 z}]/\Delta, \tag{28}$$

$$\hat{u}_z(\zeta, z, p) = -[\lambda_1\Delta_1 e^{-\lambda_1 z} + \lambda_2\Delta_2 e^{-\lambda_2 z} - \zeta^2(\Delta_3 e^{-\lambda_3 z} + \Delta_4 e^{-\lambda_4 z})]/\Delta, \tag{29}$$

$$\hat{t}_{zz}(\zeta, z, p) = (H_1\Delta_1 e^{-\lambda_1 z} + H_2\Delta_2 e^{-\lambda_2 z} + H_3\Delta_3 e^{-\lambda_3 z} + H_4\Delta_4 e^{-\lambda_4 z})/\Delta, \tag{30}$$

$$\hat{t}_{zr}(\zeta, z, p) = (G_1\Delta_1 e^{-\lambda_1 z} + G_2\Delta_2 e^{-\lambda_2 z} + G_3\Delta_3 e^{-\lambda_3 z} + G_4\Delta_4 e^{-\lambda_4 z})/\Delta, \tag{31}$$

$$\hat{m}_{z\theta}(\zeta, z, p) = -\zeta(\lambda_3 R_3\Delta_3 e^{-\lambda_3 z} + \lambda_4 R_4\Delta_4 e^{-\lambda_4 z})/\Delta, \tag{32}$$

$$\hat{q}(\zeta, z, p) = (R_1\Delta_1 e^{-\lambda_1 z} + R_2\Delta_2 e^{-\lambda_2 z})/\Delta, \tag{33}$$

where

$$\Delta = -\lambda_3 R_3[G_4(H_1\lambda_2 R_2 - H_2\lambda_1 R_1) + H_4(G_2\lambda_1 R_1 - G_1\lambda_2 R_2)]$$
$$+ \lambda_4 R_4[G_3(H_1\lambda_2 R_2 - H_2\lambda_1 R_1) + H_3(G_2\lambda_1 R_1 - G_1\lambda_2 R_2)],$$

$$\Delta_i = (-1)^{i+1} P\lambda_j R_j[\lambda_3 R_3 G_4 - \lambda_4 R_4 G_3], \quad i = 1, j = 2; \quad i = 2, j = 1,$$

$$\Delta_i = (-1)^{i} P\lambda_j R_j[\lambda_1 R_1 G_2 - \lambda_2 R_2 G_1]; \quad i = 3, j = 4; \;\; i = 4, j = 3,$$

$$H_i = \frac{(\lambda + 2\mu + K)\lambda_i^2}{\mu} - \frac{\lambda\zeta^2}{\mu} + \frac{\alpha^* R_i}{K^*\mu}; \quad i = 1, 2,$$

$$H_i = \frac{-\zeta^2(2\mu + K)\lambda_i}{\mu}; \quad i = 3, 4, \;\; P = P_0/2\pi,$$

$$G_i = \frac{\zeta(2\mu + K)\lambda_i}{\mu}; \quad i = 1, 2,$$

$$G_i = -\frac{\zeta(\mu + K)\lambda_i^2}{\mu} - \zeta^3 - \frac{\zeta K^2 R_i}{\mu^2}; \;\; i = 3, 4. \tag{34}$$

Particular Case I: Neglecting the influence of the voids, i.e. ($\alpha^* = \beta^* = \zeta^* = \omega^* = K^* = 0$), the expressions for the displacement components, force stresses and couple stress are obtained in a micropolar elastic medium.

Particular Case II: If the effect of micropolarity is ignored, i.e. ($K = j = \alpha = \beta = \gamma = 0$), the expressions for the displacement components, force stresses and volume fraction field are obtained in a elastic medium with voids.

Particular Case III: If we let ($K = j = \alpha = \beta = \gamma = 0$) and ($\alpha^* = \beta^* = \zeta^* = \omega^* = K^* = 0$), then micropolarity and voids is not there and in this case the problem reduces to the problem of normal point load on a elastic half-space. The resulting expressions tally with those obtained by Achenbach [26] with the change of notations as same as used by the author, by putting $p = 1$ in this particular case.

3.2. Continuous normal point source

When the plane boundary is subjected to continuous normal point force, the boundary conditions are

$$t_{zz} = -\frac{P_0\delta(r)H(t)}{2\pi r}, \quad t_{zr} = 0, \quad m_{z\theta} = 0, \quad \frac{\partial q}{\partial z} = 0 \text{ at } z = 0, \tag{35}$$

where P is the magnitude of force applied and $H()$ is Heaviside distribution.

With the help of these boundary conditions (35), the expressions for the components of displacement, force stress, couple stress and volume fraction field are obtained by Eqs. (28)–(33) replacing Δ_i with $\Delta_i'(i = 1, \ldots, 4)$, where

$$\Delta_i' = \Delta_i/p. \tag{36}$$

Particular Case I: Neglecting the influence of the voids, i.e. ($\alpha^* = \beta^* = \zeta^* = \omega^* = K^* = 0$), the expressions for the displacement components, force stresses and couple stress are obtained in a micropolar elastic medium.

Particular Case II: If the effect of micropolarity is ignored, i.e. ($K = j = \alpha = \beta = \gamma = 0$), the expressions for the displacement components, force stresses and volume fraction field are obtained in a elastic medium with voids.

Particular Case III: If the effect of micropolarity and voids is neglected, i.e. ($K = j = \alpha = \beta = \gamma = 0$) and ($\alpha^* = \beta^* = \zeta^* = \omega^* = K^* = 0$), the expressions for the displacement components and force stresses are obtained in a elastic medium. Again the resulting expressions telly with those obtained by Achenbach [26] with the change of notations as same as used by the author.

4. Inversion of transforms

We get expressions for displacement, microrotation and stress components solution in Eqs. (28)–(33). These expressions are functions of z, the parameters of the Laplace and Hankel transforms p and ζ, respectively, and hence of the form $\hat{f}(\zeta, z, p)$. To get the function $f(r, z, t)$ in the physical domain, first we invert the Hankel transform using

$$\bar{f}(r, z, p) = \int_0^\infty \zeta \hat{f}(\zeta, z, p) J_n(r\zeta)\, \mathrm{d}\zeta. \tag{37}$$

Now, for the fixed values of ζ, r and z, the $\bar{f}(r, z, p)$ in expression (37) can be considered as the Laplace transform $\bar{g}(p)$ of some function $g(t)$. Following Honig and Hirdes [27], the Laplace transformed function $\bar{g}(p)$ can be inverted as given below.

The function $g(t)$ can be obtained by using

$$g(t) = \frac{1}{2\pi\iota} \int_{C-\iota\infty}^{C+\iota\infty} \mathrm{e}^{pt} \bar{g}(p)\, \mathrm{d}p, \tag{38}$$

where C is an arbitrary real number greater than all the real parts of the singularities of $\bar{g}(p)$. Taking $p = C + \iota y$, we get

$$g(t) = \frac{\mathrm{e}^{Ct}}{2\pi} \int_{-\infty}^{\infty} \mathrm{e}^{\iota t y} \bar{g}(C + \iota y)\, \mathrm{d}y. \tag{39}$$

Now, taking $\mathrm{e}^{-Ct} g(t)$ as $h(t)$ and expanding it as Fourier series in $[0, 2L]$, we obtain approximately the formula

$$g(t) = g_\infty(t) + E_{D'} \tag{40}$$

where

$$g_\infty(t) = \frac{C_0}{2} + \sum_{k=1}^{\infty} C_k, \quad 0 \leqslant t \leqslant 2L \tag{41}$$

and

$$C_k = \frac{\mathrm{e}^{Ct}}{L} \Re\left[\mathrm{e}^{\frac{\iota k\pi t}{L}} \bar{g}\left(C + \frac{\iota k\pi}{L}\right)\right].$$

E_D is the discretization error and can be made arbitrarily small by choosing C large enough. The value of C and L are chosen according to the criteria out lined by Honig and Hirdes [27].

Since the infinite series in Eq. (41) can be summed up only to a finite number of N terms, so the approximate value of $g(t)$ becomes

$$g_N(t) = \frac{C_0}{2} + \sum_{k=1}^{N} C_k, \quad 0 \leqslant t \leqslant 2L. \tag{42}$$

Now, we introduce a truncation error E_T that must be added to the discretization error to produce the total approximation error in evaluating $g(t)$ using the above formula. Two methods are used to reduce the total error. The discretization error is reduced by using the 'Korrecktur'-method, Honig and Hirdes [27] and then 'ε-algorithm' is used to reduce the truncation error and hence to accelerate the convergence.

The 'Korrecktur'-method formula, to evaluate the function $g(t)$ is

$$g(t) = g_\infty(t) - \mathrm{e}^{-2CL} g_\infty(2L + t) + E_{D'}, \tag{43}$$

where

$$|E_{D'}| \ll |E_D|. \tag{44}$$

Thus, the approximate value of $g(t)$ becomes

$$g_{N_k}(t) = g_N(t) - \mathrm{e}^{-2CL} g_{N'}(2L + t), \tag{45}$$

where, N' is an integer such that $N' < N$.

We shall now describe the ε-algorithm which is used to accelerate the convergence of the series in Eq. (42). let N be a natural number an $S_m = \sum_{k=1}^{m} C_k$ be the sequence of partial sums of Eq. (42). We define the ε-sequence by

$$\begin{aligned} \varepsilon_{0,m} &= 0, \\ \varepsilon_{1,m} &= S_m, \\ \varepsilon_{n+1,m} &= \varepsilon_{n-1,m+1} + \frac{1}{\varepsilon_{n,m+1} - \varepsilon_{n,m}}, \quad n, m = 1, 2, 3, \ldots . \end{aligned}$$

It can be shown Honig and Hirdes [27] that the sequence $\varepsilon_{1,1}, \varepsilon_{3,1}, \ldots, \varepsilon_{N,1}$ converges to $g(t) + E_D - C_0/2$ faster than the sequence of partial $S_m, m = 1, 2, 3, \ldots$. The actual procedure to invert the Laplace transform consists of Eq. (45) together with the ε-algorithm.

The last step is to evaluate the integral in Eq. (37). The method for evaluating this integral by Press et al. [28], which involves the use of Romberg's integration with adaptive step size. This, also uses the results from successive refinement of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

5. Numerical results and discussion

In this section, the numerical discussion for both the cases and all particular cases are reported. The analysis is conducted for a magnesium crystal like material. Following Eringen [29], the values of physical constants are

$$\lambda = 9.4 \times 10^{11}\ \text{dyn/cm}^2, \quad \mu = 4 \times 10^{11}\ \text{dyn/cm}^2,$$
$$K = 1 \times 10^{11}\ \text{dyn/cm}^2, \quad \rho = 1.74\ \text{g/cm}^3,$$
$$\gamma = 0.779 \times 10^{-4}\ \text{dyn}, \quad j = 0.2 \times 10^{-15}\ \text{cm}^2,$$

and the void parameters are

$$\alpha^* = 3.688 \times 10^{-4}\ \text{dyn}, \quad \beta^* = 1.13849 \times 10^{11}\ \text{dyn/cm}^2,$$
$$\zeta^* = 1.475 \times 10^{11}\ \text{dyn/cm}^2, \quad \omega^* = 0.0787\ \text{dyn/cm}^2,$$
$$K^* = 1.753 \times 10^{-15}\ \text{cm}^2, \quad h^2 = 1 \times 10^{-15}\ \text{cm}^2.$$

The computations were carried out for non-dimensional time $t = 0.1$ at $z = 1$ in the range $0 \leqslant r \leqslant 10$. The distribution of non-dimensional tangential couple stress $M_{z\theta}(= m_{z\theta}/P)$, non-dimensional normal displacement $U_z(= u_z/P)$, non-dimensional normal force stress $T_{zz}(= t_{zz}/P)$ and non-dimensional Volume fraction field $Q(= q/P)$ with non-dimensional distance $'r'$ have been shown in Figs. 1–8. The solid line predicts the variations of components for micropolar elastic solid with void (MESV) whereas the very small dashed line are for micropolar elastic solid (MES), small dashed line corresponds to the variations for elastic solid with voids (ESV) and large dashed line are for elastic solid (ES).

5.1. Instantaneous normal point source

The variations of normal displacement, volume fraction field, normal force stress and tangential couple stress with distance r for MESV, MES, EVS and ES when instantaneous normal point source is applied have been shown in Figs. 1, 2, 3, 4, respectively.

Fig. 1 depicts the variations of normal displacement U_z with r for all four theories (MESV, MES, ESV, ES) and it is observed that the behavior of U_z for MES is opposite to MESV, ESV and ES. The values of U_z decrease sharply as r lies between $0 \leqslant r \leqslant 3$ whereas for MES the values of U_z increase in the same range. Very near to the point of action of sources, the magnitude of values of U_z os larger for MESV and smallest for MES. The variations of volume fraction $Q(= q/P)$ with r for MESV and ESV have been shown in Fig. 2. The values of Q decrease sharply in the range $0 \leqslant r \leqslant 2.5$. The values for MESV is greater then that for ESV in the initial range $0 \leqslant r \leqslant 2.5$ and then in the range $7 \leqslant r \leqslant 10$. The variations of normal force stress T_{zz} with r have been shown in Fig. 3. The values of T_{zz} start with sharp decrease for the cases MESV, MES and ES whereas for the case of ESV it starts with small increase. The magnitude of value of T_{zz} is largest for ES and smallest for MES. Fig. 4. shows the variations of $M_{z\theta}$ with r. Starting with small decrease in the range $0 \leqslant r \leqslant 1.5$, the value of couple stress for MESV and MES begin to grownup with small variation. The behavior of variation of $M_{z\theta}$ for both the cases are same whereas their corresponding values are different.

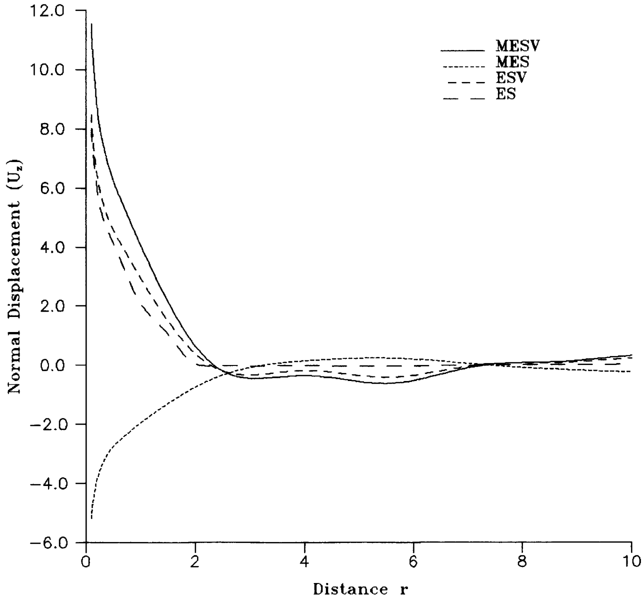

Fig. 1. Variations of normal displacement $U_x(= u_x/P)$ due to instantaneous source with distance r.

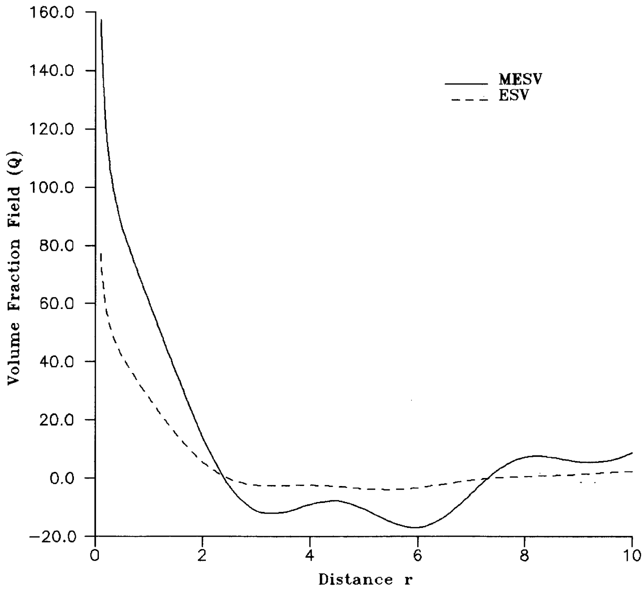

Fig. 2. Variations of volume fraction field $Q(= q/P)$ due to instantaneous source with distance r.

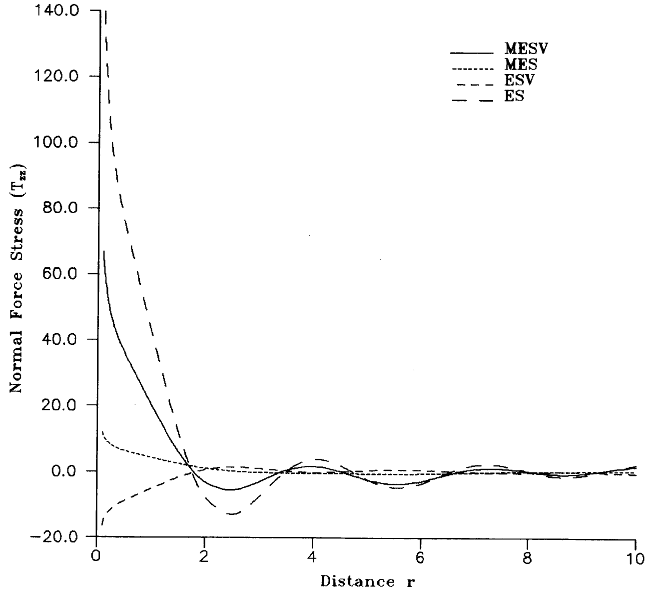

Fig. 3. Variations of normal force stress $T_{zz}(=t_{zz}/P)$ due to instantaneous source with distance r.

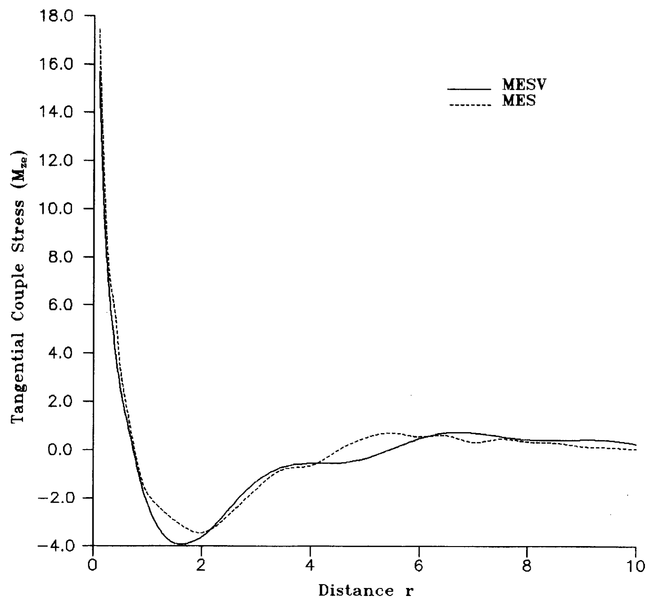

Fig. 4. Variations of tangential couple stress $M_{z\theta}(=m_{z\theta}/P)$ due to instantaneous source with distance r.

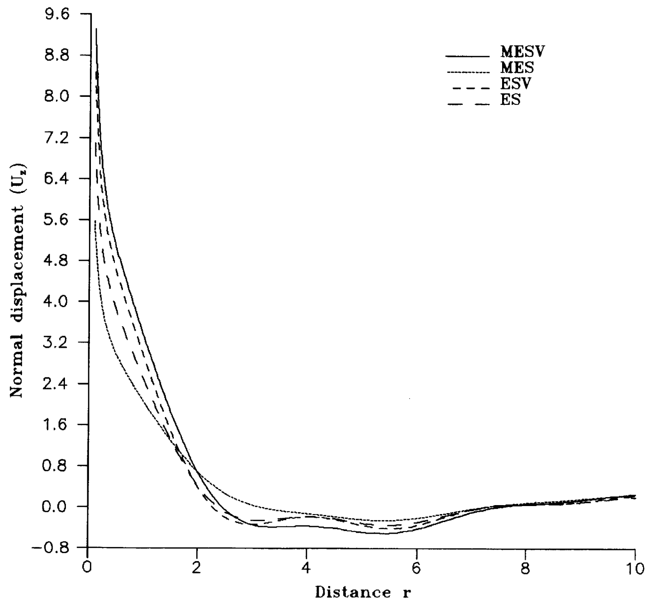

Fig. 5. Variations of normal displacement $U_z(= u_z/P)$ due to continuous source with distance r.

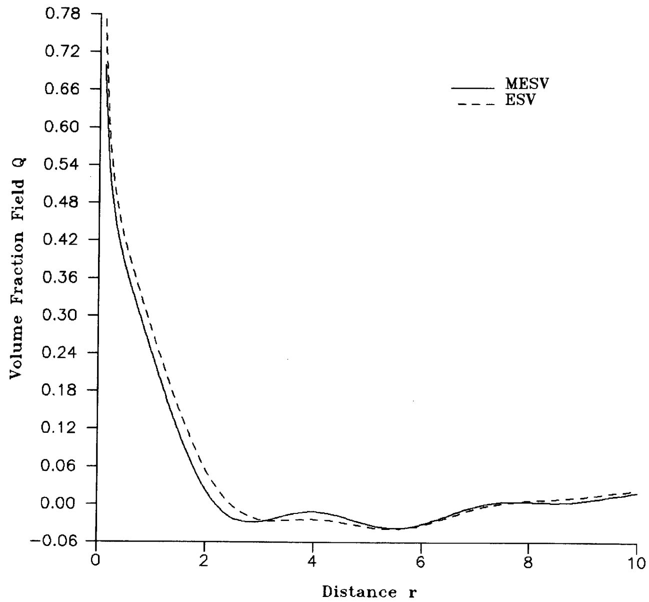

Fig. 6. Variations of volume fraction field $Q(= q/P)$ due to continuous source with distance r.

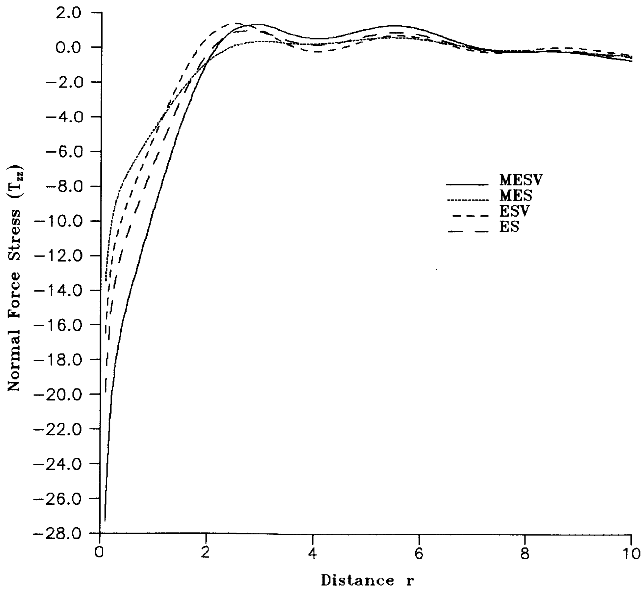

Fig. 7. Variations of normal force stress $T_{xx}(= t_{xx}/P)$ due to continuous source with distance r.

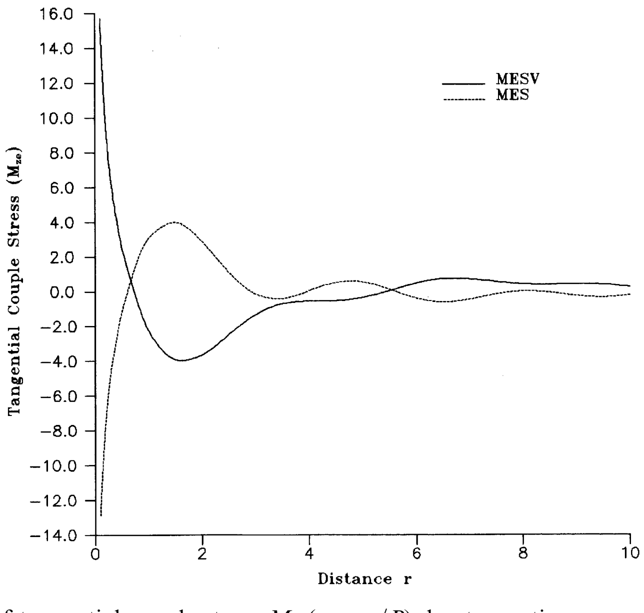

Fig. 8. Variations of tangential couple stress $M_{z\theta}(= m_{z\theta}/P)$ due to continuous source with distance r.

5.2. Continuous normal point source

The variations of normal displacement, volume fraction field, normal force stress and tangential couple stress with distance r for MESV, MES ESV and ES when continuous normal point source is applied have been shown in Figs. 5, 6, 7, 8, respectively.

Fig. 5 shows the variations of normal displacement U_z with r. The behavior of variation of U_z for all four case are same. The values of normal displacement start with sharp decrease and the approaches to zero as r increases. The values of U_z are greatest for the case of MESV and smallest for the case of MES in the range $0 \leqslant r \leqslant 3$. The variations of volume fraction $Q(= q/P)$ with r for MESV and ESV have been shown in Fig. 6. The values of Q decrease sharply in the range $0 \leqslant r \leqslant 3$. The values for ESV is greater then that for MESV in the range $0 \leqslant r \leqslant 3$. The variations of normal force stress T_{zz} with r have been shown in Fig. 7. Initially, with a sharp increase in values of force stress its value approaches to be vanished. In the range, $0 \leqslant r \leqslant 1$ the values are greater for MES and smallest for MESV. Fig. 8 depicts the variations of $M_{z\theta}$ with r. It is observed that the behavior of variation of values of couple stress is just opposite to each other in whole range. The values of $M_{z\theta}$ for the case of MESV start with sharp decrease whereas for the case of MES, start with sharp increase.

6. Conclusion

From the above numerical discussion a significant effect of void and micropolarity have been observed. The magnitude of variations of the normal displacement, normal force stress and tangential couple stress is observed for Delta distribution and continuous normal sources. The void effect is appreciable in both the cases. Very near to the point of application of source, it is observed that the components of displacement, force stress, couple stress and volume fraction filed have large values which become smaller and smaller with the increase in the value of distance 'r'.

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