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Letter to the Editor

Sliding control accounting for hardware limitation of mechanical actuators with deadzone

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1. Introduction

A deadzone as shown in Fig. 1(a) is one of the common non-smooth non-linear characteristics of physical components of control systems including hydraulic servo-valves and electric servomotors. Deadzones are usually poorly known and may vary with time. They often severely limit system performance, giving rise to undesirable inaccuracy or oscillations or even leading to instability.

Direct and indirect control methods have been used in continuous-time or discrete-time systems with a deadzone [1–3]. Adaptive non-linear control with a deadzone inverse as shown in Fig. 1(b) is a relatively recent method [4–6]. More recently, fuzzy logic control and neural networks are also introduced into the control design [7,8]. The deadzone inverse is attractive because it completely eliminates the non-linear effect of the deadzone. A disadvantage of this approach is the requirement of a jump in the control as indicated in Fig. 1(b). This requires an infinite rate of the control and is practically unrealizable when the control is mechanical such as a hydraulic actuator. In this paper, we present a sliding mode control explicitly accounting for the hardward constraint of mechanical actuators with a deadzone, resulting in a continuous control. It should be noted that there have been many studies of sliding controls. A good reference to start with is Ref. [9].

The paper is organized as follows. In Section 2, we present the control for a first order system with deadzone. In Section 3, we extend the study to higher order systems. Section 4 presents numerical simulations of the control and discusses the control performance as a function of system parameters.

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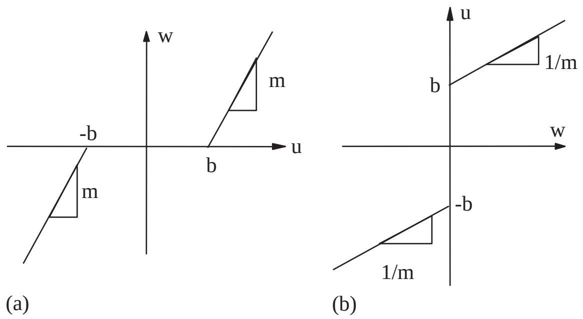

Fig. 1. (a) The symmetric deadzone function; (b) the inverse of the symmetric deadzone function.

2. A first order system example

The deadzone is a piecewise linear function given by

$$w(t) = \begin{cases} m[u(t) - b], & u \geqslant b, \\ 0, & -b < u < b, \\ m[u(t) + b], & -b \leqslant u, \end{cases} \tag{1}$$

where u is the input command, w is the actual output of the actuator, m is the slope and b is the width of the deadzone. We first consider a stable first order system. The equation of motion of the system is given by

$$\dot{x} = -ax + w, \tag{2}$$

where $a > 0$. Let x_d be a desired trajectory for x. The sliding function of the system is given by $s = x - x_d$. Following the standard steps of sliding mode control, and by considering the Lyapunov function $J = \frac{1}{2}s^2$, we obtain a nominal sliding mode control in terms of w when the deadzone is not in the loop:

$$w_n = ax + \dot{x}_d - \eta\, \mathrm{sgn}(s), \tag{3}$$

where $\eta > 0$. With a proper consideration of the boundary layer of the sliding surface, we can change the switching term sgn(s) to a saturation function [9]. For the sake of space, we shall only present the control in terms of the switching function sgn(s) in this paper.

We then put the deadzone in the loop, and derive the control in terms of u:

$$u(t) = \begin{cases} w_n(t)/m + b, & w_n(t) > 0, \\ [-b, b], & w_n(t) = 0, \\ w_n(t)/m - b, & w_n(t) < 0. \end{cases} \tag{4}$$

This control requires u to jump from $-b$ to b when w_n changes sign. When $-b < u < b$, the control output w is zero, and there is no influence of the control on the system dynamics. As a result, the tracking of the system response to x_d will be poor during this period of time. Nevertheless, one can easily show that the closed-loop system is stable as long as the open loop is stable.

Next, we modify the sliding mode control in such a way that $u(t)$ is a continuous function of time, and it will travel through the deadzone at its maximum speed $\dot{u}_m$ provided by the hardware.

We propose the following control:

$$u(t) = \begin{cases} w_n(t)/m + b, & w_n(t) > \varepsilon \text{ or } 0 < w_n(t) < \varepsilon \text{ and } \dot{w}_n(t) > 0, \\ \mathrm{sgn}(\dot{w}_n(t))\dot{u}_m \cdot (t - t_0) + u(t_+), & 0 < w_n(t) \leqslant \varepsilon \text{ and } \dot{w}_n(t) < 0 \text{ and } t > t_+, \\ \mathrm{sgn}(\dot{w}_n(t))\dot{u}_m \cdot (t - t_0) + u(t_0), & w_n(t) = 0 \text{ and } t > t_0, \\ \mathrm{sgn}(\dot{w}_n(t))\dot{u}_m \cdot (t - t_0) + u(t_-), & -\varepsilon \leqslant w_n(t) < 0 \text{ and } \dot{w}_n(t) > 0 \text{ and } t > t_-, \\ w_n(t)/m - b, & w_n(t) < -\varepsilon \text{ or } -\varepsilon < w_n(t) < 0 \text{ and } \dot{w}_n(t) < 0. \end{cases} \tag{5}$$

Here, $\varepsilon > 0$ is a small number defining a boundary layer of the deadzone. t_+ is the time instant when $w_n(t_+) = \varepsilon$ entering the boundary layer from the positive side. t_- is the time instant when $w_n(t_-) = -\varepsilon$ entering the boundary layer from the negative side. t_0 is the time instant when $w_n(t_0) = 0$ entering the boundary layer from either side.

Note that in this control, the sign of the time derivative $\dot{w}_n(t)$ is needed. In real-time implementation, only the sign of the increment of $w_n(t)$ needs to be tracked. No differentiation is ever done.

2.1. Stability

We now consider the stability of the proposed control. In the top and bottom branches of the control in Eq. (5), the reaching condition $\dot{J} = s\dot{s} < 0$ is still satisfied and hence the system is stable. In the middle branch where $w_n(t) = 0$ and $-b \leqslant \mathrm{sgn}(\dot{w}_n(t))\dot{u}_m \cdot (t - t_0) + u(t_0) \leqslant b$, the system is stable by assumption. We only need to demonstrate the stability of the second and fourth branches. The system will actually stay in these two branches for a very short period of time.

Note that in these two branches, we have $0 < |w_n(t)| < \varepsilon$. From the equation of motion, we have

$$x\dot{x} \leqslant -ax^2 + \varepsilon|x|. \tag{6}$$

Consider an auxiliary system with $x\dot{x} = -ax^2 + \varepsilon|x|$ and a Lyapunov function for this system, $V(x) = \frac{1}{2}(|x| - d)^2$, where $d = \varepsilon/a$. One can readily show that $V(x) = 0$ at $|x| = d$ and $\dot{V}(x) \leqslant 0$ in the whole domain of x. The invariant set of the auxiliary system is $x = \pm d$. Hence, the auxiliary system is stable. Since the original system is bounded above by the auxiliary system, it is also bounded and stable in the Lyapunov sense.

2.2. Robust control

Assume that the deadzone parameters b and m are not known precisely, and that the range of these two parameters are known: $b \in [b_1, b_2]$, $m \in [m_1, m_2]$. Following Ref. [9], we propose a robust control as

$$u = \begin{cases} w_n(t)/\hat{m} + b_2, & w_n(t) > \varepsilon \text{ or } 0 < w_n(t) < \varepsilon \text{ and } \dot{w}_n(t) > 0, \\ \mathrm{sgn}(\dot{w}_n(t))\dot{u}_m \cdot (t - t_0) + u(t_+), & 0 < w_n(t) \leqslant \varepsilon \text{ and } \dot{w}_n(t) < 0 \text{ and } t > t_+, \\ \mathrm{sgn}(\dot{w}_n(t))\dot{u}_m \cdot (t - t_0) + u(t_0), & w_n(t) = 0 \text{ and } t > t_0, \\ \mathrm{sgn}(\dot{w}_n(t))\dot{u}_m \cdot (t - t_0) + u(t_-), & -\varepsilon \leqslant w_n(t) < 0 \text{ and } \dot{w}_n(t) > 0 \text{ and } t > t_-, \\ w_n(t)/\hat{m} - b_2, & w_n(t) < -\varepsilon \text{ or } -\varepsilon < w_n(t) < 0 \text{ and } \dot{w}_n(t) < 0, \end{cases} \tag{7}$$

where

$$\eta \geqslant \beta[m_2(b_2 - b_1) + \max(\beta - 1, 1 - \beta^{-1})|w_n|], \tag{8}$$

$$\hat{m} = \sqrt{m_1 m_2}, \quad \beta^{-1} = \left(\frac{m_1}{m_2}\right)^{1/2} \leqslant \frac{m}{\hat{m}} \leqslant \left(\frac{m_2}{m_1}\right)^{1/2} = \beta. \tag{9}$$

The stability proof for the control far away from the deadzone follows the standard steps as outlined in Ref. [9]. For the branches where $|w_n(t)| < \varepsilon$, the proof steps are similar to the nominal case discussed above. We omit the detailed discussions of stability here for the sake of space.

3. Higher order systems

The control method presented above is also applicable to higher order systems. Here, we choose a stable second order system to demonstrate the application:

$$\ddot{x} + a_1\dot{x} + a_2 x = c_1\dot{w} + c_2 w. \tag{10}$$

The sliding surface is defined as

$$s = \left(\frac{\mathrm{d}}{\mathrm{d}t} + \lambda\right)\tilde{x}, \quad \tilde{x} = x - x_d, \quad \lambda > 0. \tag{11}$$

By considering the Lyapunov function $J = \frac{1}{2}s^2$, we can derive the nominal control:

$$w_n(t) = H(t) \otimes \{a_1\dot{x}(t) + a_2 x(t) + \ddot{x}_d(t) - \lambda[\dot{x}(t) - \dot{x}_d(t)] - \eta\,\mathrm{sgn}(s)\}, \tag{12}$$

where $H(t) = \mathscr{L}^{-1}(1/(c_1 p + c_2))$ is the inverse Laplace transformation of the transfer function, p is the Laplace transform variable, and $\otimes$ denotes the convolution integral in time domain. Note that this representation of $w_n(t)$ is convenient for proof of stability, not necessarily for implementation.

We then put the deadzone in the loop, and arrive at the control in terms of $u(t)$ having the same form as in Eq. (5). By assuming that the range of the deadzone parameters are known, and following the same steps as used before, we can derive a robust control having the same form as in Eq. (7).

The proof of stability of the controls for the second order system follows the same steps as for the first order system, and is omitted herein.

4. Numerical simulations

We have conducted extensive simulations to study the effectiveness of the controls. We present and discuss the numerical results in this section.

4.1. First order system

For the first order system, we have taken $a = 0.5$, $b = 1$, $m = 2$, $b_1 = 0.8$, $b_2 = 1.2$, $m_1 = 1.5$, $m_2 = 2.5$, $\hat{m} = 1.8$, and $\varepsilon = 0.5$. We have replaced the switching function $\mathrm{sgn}(s)$ to a saturation

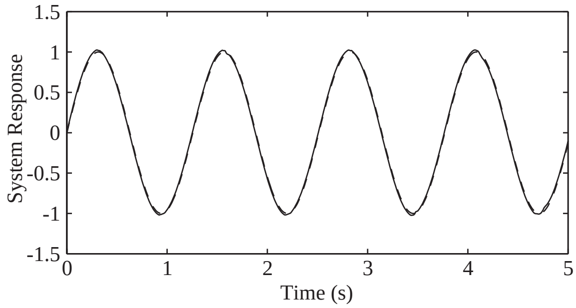

Fig. 2. Tracking performance of the first order system with a deadzone: $x_d = \sin 5t$, $\dot{u}_m = 100$. Solid line is the closed-loop system response, dashed line is the reference x_d.

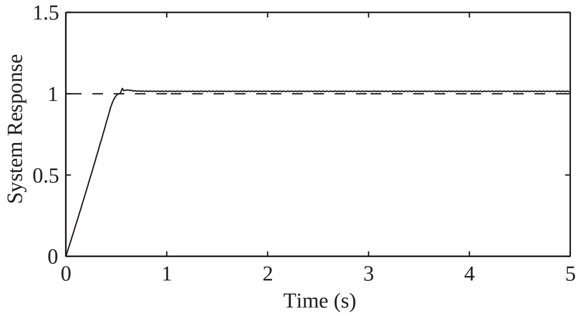

Fig. 3. Step response of the first order system with deadzone; $\dot{u}_m = 100$. Solid line is the closed-loop system response, dashed line is the step reference x_d.

function $sat(s/\phi)$ with $\phi = 0.1$ to avoid chattering in all the simulations. Fig. 2 shows the response of the system tracking a harmonic signal. Fig. 3 shows the step response of the system.

4.2. Second order system

For the second order system, we have taken $a_1 = 8$, $a_2 = 20$, $c_1 = 0.1$, $c_2 = 1$ and $\lambda = 5$ while all other parameters related to the deadzone and the hardware limit are the same as for the first order system. Fig. 4 shows the response of the system tracking a harmonic signal. Fig. 5 shows the step function.

4.3. Discussion

We have done more simulations to study the effect of various parameters. Here is a brief summary of this study. It should be noted that the present control is not intended to stabilize a unstable system because the control has no output in the deadzone. This is a topic for future research. In general, the system with deadzone cannot provide perfect tracking. This is a manifesto of the rate limitation of the hardware. The present study provides a way to evaluate the hardware limitation in terms of the transient and steady state tracking performance. We have also

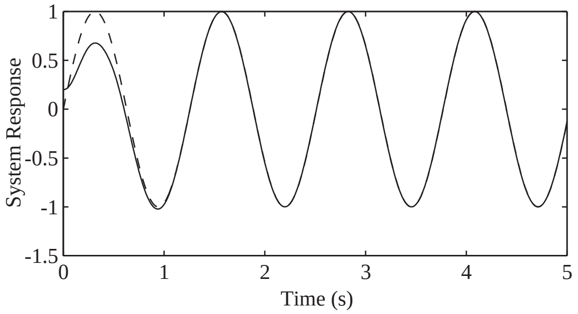

Fig. 4. Tracking performance of the second order system with deadzone: $x_d = \sin 5t$, $\dot{u}_m = 100$. Solid line is the closed-loop system response, dashed line is the reference x_d.

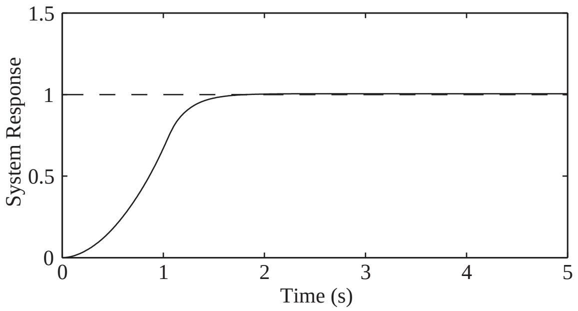

Fig. 5. Step response of the second order system with deadzone; $\dot{u}_m = 100$. Solid line is the closed-loop system response, dashed line is the step reference x_d.

imposed the same hardware rate limit away from the deadzone in the simulation. It has been found that when the rate saturation limit $\dot{u}_m$ is larger, the system tracks the reference much better.

We have also studied the effect of the boundary value ε. Generally, with a smaller ε, the system also tracks the reference better.

Another factor that limits the control performance is the knowledge of the unknown parameters. A narrower range of the uncertainty of these parameters will reduce the control magnitude for a given level of tracking performance. This is a well-known point in the robust sliding mode control.

5. Concluding remarks

A deadzone is one of non-smooth non-linear elements that exist in many real applications. In this paper, we have presented a continuous sliding mode control of such a non-linear dynamic system with a consideration of the hardware limit. A robust control has also been developed assuming that the bounds of the deadzone parameters are given. Stability of the control has also been proven. Simulations have been conducted to study the effectiveness of the control in tracking applications. A comprehensive study of the effects of various parameters on the system performance is not presented in this paper, and will be pursued in the future.

Acknowledgements

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