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Modal parameter identification through Gabor expansion of response signals

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Abstract

A new method of identifying modal parameters by decomposing response signals with Gabor transform is presented in this paper to estimate natural frequencies, damping ratios and mode shapes of linear time invariant systems. According to Gabor expansion theory, responses of a multi-degree-of-freedom system can be decomposed into uncoupled signal components, each vibrating at a single natural frequency. From these uncoupled signals, modal parameters are subsequently extracted with common methods. The proposed method can process stationary and non-stationary responses and requires no input signal except for the response signals generated by unknown excitation acting on a system. In the sense of less restriction on the in–out signals, the approach based on time–frequency decomposition is very general. A simulation study on a simply supported beam under non-stationary excitation has demonstrated that the proposed method is effective in parameter estimation.

1. Introduction

Traditional methods of modal parameter identification have been well developed to fulfill general identification tests in laboratory and other fields, where excitation can be conveniently carried out on structures. With sampled input and output data, many procedures can be implemented in time or frequency domain, such as those methods extracting modal parameters on the basis of frequency response matrix, recursive model of time series or state space equations [1–5]. In laboratory and other almost ideal environment, these conventional methods are effective

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and irreplaceable. However, for large-scale structures or running machines, it is the general case that energy consumption in excitation is unusually high or no suitable position can be found to carry out the excitation, therefore implementation of tests becomes a big problem [6]. The recently proposed approaches of parameter identification under environmental excitation have bypassed some difficulties encountered in these circumstances, where environmental excitation is supposed to have sufficient energy and satisfy certain assumptions regarding signal stationarity, and modal parameters are extracted from response data only. When the environmental excitation is stationary white noise, the well-known NExT method can be employed to estimate modal parameters [7–9]. But in real world, being stationary is somewhat a mandatory requirement for environmental excitation. So, non-stationary process or structural responses to environmental excitation must be considered in modal parameter identification. Fortunately, the time–frequency analysis of signals, well developed in the last two decades, presents a new way to identify modal parameters from response data only. With time–frequency analysis both stationary and non-stationary signals can be processed and represented in time–frequency domain, which is essential to the parameter identification from non-stationary response data.

Signal processing in pure time or frequency domain is usually unable to separate characteristics mixed in a given signal, especially for those signals of variable frequencies [10–11]. However, the time–frequency analysis takes an arbitrary signal as the function of time and frequency and thus gives more accurate and sufficient information on characteristic evolution as well as energy distribution in the time–frequency plane. Moreover, time–frequency analysis methods have found many successful applications in industries [12]. In modal parameter identification, the bilinear time–frequency transform of signals has been employed to extract modal parameters [13]. Although the bilinear transform possesses many good properties, the cross term and negative energy distribution will decrease resolution and result in poor accuracy. In contrast to this kind of transform, the linear transforms, such as short-time Fourier transform, Gabor transform and wavelet transform, have no cross term and always give non-negative energy distribution. In wavelet analysis, adaptive windows are used in order to achieve the best time resolution [14]. As a result, frequency resolution is different between the lower and the higher frequency band. Compared with the wavelet transform, Gabor transform uses a fixed analysis window and consequently yields constant frequency resolution. Gabor transform can decompose an arbitrary signal into short-period waves that vibrate at a certain lattice in the time–frequency plane [15–16]. From these short-period waves, local information can be reconstructed to represent certain characteristics. In this paper, Gabor expansion is applied to decompose response signals and extract modal parameters.

In fact, the proposed approach based on Gabor expansion theory carries out estimation by decomposing every sampled response into uncoupled signal components and subsequently extracting modal parameters from these uncoupled components with common methods. This is distinct from the traditional approaches which extract parameters from input–output data. Moreover, this approach is suitable for stationary and non-stationary responses. We will discuss it in three sections. Section 2 gives an introduction of Gabor expansion theory, and in Section 3 the proposed method is fully explored. Section 4 gives a simulation study on the parameter estimation of a simply supported beam under non-stationary excitation.

2. Gabor expansion

Suppose $f(t), g(t) \in L^2(R)$ and $\{g_{mn}(t) : g_{mn}(t) = g(t - ma)\exp(jnbt), a, b > 0, ab \leqslant 2\pi, m, n \in Z\}$ constitutes a frame, i.e., there exist constants A and B with $A \geqslant B > 0$ such that

$$B|\phi(t)|^2 \leqslant \sum_m \sum_n |\langle \phi(t), g_{mn}(t)\rangle|^2 \leqslant A|\phi(t)|^2 \quad \forall \phi(t) \in L^2(R), \tag{1}$$

where $\langle \phi(t), g_{mn}(t)\rangle = \int_R \phi(t)\tilde{g}_{mn}(t)\,\mathrm{d}t$, the inner product of $\phi(t)$ and $g_{mn}(t)$, $\tilde{g}_{mn}(t)$ is the complex conjugate of $g_{mn}(t)$, and $|\phi(t)| = \sqrt{\langle \phi(t), \phi(t)\rangle}$, the norm of $\phi(t)$ in $L^2(R)$. Gabor expansion of $f(t)$ under the frame $\{g_{mn}(t)\}$ can be expressed as

$$f(t) = \sum_m \sum_n C_{mn}\gamma_{mn}(t), \tag{2}$$

where $C_{mn} = \langle f(t), g_{mn}(t)\rangle$ are referred to as Gabor coefficients and $\{\gamma_{mn}(t)\}$ is the dual frame of $\{g_{mn}(t)\}$ with $\gamma_{mn}(t) = \gamma(t - ma)\exp(jnbt)$. Signal $g(t)$ is the analysis window function and $\gamma(t)$ the synthesis window function. Eq. (2) is the atomic decomposition of signal $f(t)$, which indicates that the superposition of locally vibrating waves at lattice (m, n) is equal to $f(t)$. This implies all information contained in signal $f(t)$ can be reconstructed from Gabor coefficients $\{C_{mn}\}$.

When Gabor coefficients corresponding to a certain characteristic are all distributed in a region Ω (in Fig. 1, the solid circle and rectangle represent different characteristic), one can fully recover the corresponding signal with Eq. (2). However, when Gabor coefficients are not fully covered by a given region (as shown in Fig. 1) or are contaminated to some extent by noise coefficients, the constructed signal from selected Gabor coefficients is different from the exact one. However, an optimal approximation can be obtained by performing an optimization procedure [16].

3. Parameter identification of LTI systems

Consider an LTI system of N degrees of freedom:

$$M\ddot{X} + C\dot{X} + KX = F(t). \tag{3}$$

Let $\{\phi_1, \phi_2, \ldots, \phi_N\}$ be the corresponding N vibration modes, $(0, 0, \ldots, 0)^T$ be the initial state of Eq. (3), and assume the excitation force $F(t) = (f_1(t), f_2(t), \ldots, f_N(t))^T$ has sufficient bandwidth. According to the vibration theory, the response $X(t) = (x_1(t), x_2(t), \ldots, x_N(t))^T$ of Eq. (3) can be

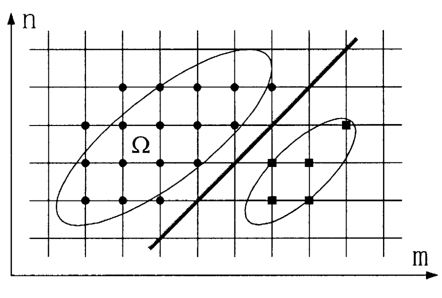

Fig. 1. Lattice and distributed coefficients.

written as

$$X(t) = \sum_r \frac{\phi_r}{\eta_r \omega_r m_r} \int_0^t \phi_r^{\mathrm{T}} F(\tau) \exp(-\xi_r \omega_r (t-\tau)) \sin(\eta_r \omega_r (t-\tau))\, \mathrm{d}\tau, \tag{4}$$

where $m_r = \phi_r^{\mathrm{T}} M \phi_r$, $\eta_r = \sqrt{1-\xi_r^2}$, ξ_r and ω_r are, respectively, the damping ratio and natural frequency of mode r. Eq. (4) implies that $X(t)$ is the superposition of vibration of N modes. Let $X_r(t)$ be the rth modal vibration, i.e.,

$$X_r(t) = \frac{\phi_r}{\eta_r \omega_r m_r} \int_0^t \phi_r^{\mathrm{T}} F(\tau) \exp(-\xi_r \omega_r (t-\tau)) \sin(\eta_r \omega_r (t-\tau))\, \mathrm{d}\tau, \quad r = 1, 2, 3, \ldots, N, \tag{5}$$

we have $X(t) = \sum_r X_r(t)$. Define Gabor operator $G : L^2(R) \to l^2(Z^2)$ as

$$G(f) = \{C_{mn} : C_{mn} = \langle f(t), g_{mn}(t) \rangle, m, n \in Z\} \tag{6}$$

and substitute the response $x_s(t)$ of point s into Eq. (6) to obtain

$$G(x_s(t)) = G(\sum_r x_{sr}(t)) = \sum_r G(x_{sr}(t)) = \sum_r \{C_{mn}(x_{sr}(t))\}, \quad s = 1, 2, 3, \ldots, N, \tag{7}$$

where $\{C_{mn}(x_{sr}(t))\}$ are Gabor coefficients of the rth modal response $x_{sr}(t)$ of point s. Eq. (7) reveals that Gabor coefficients of response $x_s(t)$ are the summation of those of the N modes. This property makes Gabor transform superior to other high order transforms in response decomposition. Coefficients $\{C_{mn}(x_{sr}(t))\}$ are actually distributed over a certain region in the time–frequency plane, from which vibration signal of frequency ω_r can be reconstructed by selecting Gabor coefficients from $\{C_{mn}(x_{sr}(t))\}$. To guarantee the accuracy of reconstruction, energy of each vibration mode is supposed to be concentrated in the vicinity of frequency ω_r and modal frequencies are sufficiently spaced. Under this assumption, the reconstructed response corresponding to frequency ω_r will be lightly contaminated by other modes. Eq. (8) gives the formula of reconstructing $x_{sr}(t)$ from selected coefficients within the region $\Omega(\omega_r)$ in the time–frequency plane:

$$x_{sr}(t) = \sum_m \sum_n H_{mn}(\omega_r) C_{mn}(x_{sr}(t)) \gamma_{mn}(t), \quad r,\ s = 1, 2, 3, \ldots, N, \tag{8}$$

where $H_{mn}(\omega_r)$ is an index function defined by

$$H_{mn}(\omega_r) = \begin{cases} 1 & (m, n) \in \Omega(\omega_r), \\ 0 & \text{otherwise} \end{cases}$$

and $\Omega(\omega_r)$ is an area masking the selected Gabor coefficients.

Parameter estimation in the following sections is based on Eq. (8).

3.1. Frequency estimation

Natural frequencies can be estimated from Gabor coefficients of reconstructed signals. For the rth natural frequency ω_r, it can be given as

$$\omega_r = \frac{\sum_k \omega(k) H_{mk} C_{mk}(x_{sr}(t))}{\sum_k H_{mk} C_{mk}(x_{sr}(t))}, \quad r = 1, 2, 3, \ldots, N, \tag{9}$$

where $\omega(k)$ is the kth discrete frequency, s represents an arbitrary point and $t = ma$ is an arbitrary instant. For a stationary stochastic response, ω_r should be the expectation of all estimated samples, i.e.,

$$\omega_r = \frac{1}{M}\sum_{i=1}^{M}\omega_r(i), M \to \infty. \tag{10}$$

ω_r can also be estimated with traditional methods, but Eq. (9) is very general and suitable for frequency estimation of time-varying or non-linear systems.

3.2. Mode shape estimation (real mode)

With Eq. (5) the rth modal vibration of point s can be described as

$$x_{sr}(t) = \frac{\phi_{sr}}{\eta_r\omega_r m_r}\int_0^t \phi_r^{\mathrm{T}}F(\tau)\exp(-\xi_r\omega_r(t-\tau))\sin(\eta_r\omega_r(t-\tau))\,\mathrm{d}\tau, \quad r,s = 1,2,3,\ldots,N, \tag{11}$$

where ϕ_{sr} is the amplitude of mode r at point s. Suppose $x_{1r}(t_0)\neq 0$, then from Eq. (11) the rth normalized mode shape can be estimated as follows:

$$(1, \phi_{2r}/\phi_{1r}, \ldots, \phi_{Nr}/\phi_{1r})^{\mathrm{T}} = (1, x_{2r}(t_0)/x_{1r}(t_0), \ldots, x_{Nr}(t_0)/x_{1r}(t_0))^{\mathrm{T}}. \tag{12}$$

However, estimating mode shapes directly with Eq. (12) may result in large errors. To reduce estimation error, two finite sequences $\vec{x}_{sr}(L) = \{x_{sr}(t_0), x_{sr}(t_1), \ldots, x_{sr}(t_L)\}^{\mathrm{T}}$ and $\vec{x}_{1r}(L) = \{x_{1r}(t_0), x_{1r}(t_1), \ldots, x_{1r}(t_L)\}^{\mathrm{T}}$ can be constructed to give an estimation of ϕ_{sr}/ϕ_{1r}:

$$\phi_{sr}/\phi_{1r} = \vec{x}_{sr}(L)^{\mathrm{T}}\vec{x}_{1r}(L)/\sqrt{\vec{x}_{1r}(L)^{\mathrm{T}}\vec{x}_{1r}(L)} \tag{13}$$

Eq. (12) indicates the estimation of mode shapes is independent of exciting force. Therefore, one can obtain mode shapes as well by substituting the exciting force $F(\tau) = A(\tau)\sin(\eta_r\omega_r\tau + \varphi)$ into Eq. (11). Performing Fourier transform on both sides of Eq. (11) yields

$$x_{sr}(\omega) = \phi_{sr}h_r(\omega)\phi_r^{\mathrm{T}}F(\omega), \quad s = 1,2,3,\ldots,N, \tag{14}$$

where $x_{sr}(\omega) = \langle x_{sr}(t), \exp(\mathrm{j}\omega t)\rangle$, $F(\omega) = \langle F(t), \exp(\mathrm{j}\omega t)\rangle$, the Fourier transform of $x_{sr}(t)$ and $F(t)$, respectively, and $h_r(\omega) = \langle \exp(-\xi_r\omega_r t)\sin(\eta_r\omega_r t)/(\eta_r\omega_r m_r), \exp(\mathrm{j}\omega t)\rangle$. Eq. (14) implies that time–frequency filtering of force signal has the same effect as direct filtering of response data. Note that $x_{sr}(t)$ is the signal of frequency ω_r, it should be the same as that generated by $F(\tau) = A(\tau)\sin(\eta_r\omega_r\tau + \varphi)$. Therefore, normalized mode shapes can also be estimated from all the reconstructed $x_{sr}(t)$.

Generally, distribution of $x_{sr}(t)$ on the time–frequency plane is complicated due to the mixed environmental excitation, but mode shape estimation based on Eq. (8) is only affected by the validity of reconstructed signal. Hence, one can extract mode shapes with high accuracy as long as $x_{sr}(t)$ has sufficient signal-to-noise ratio (SNR) within the vicinity of ω_r. Sensor noise and disturbance from adjacent vibrating modes constitute the overall noise in $x_{sr}(t)$, which might cause large errors in estimated mode shapes when using Eq. (13). For stochastic responses, Gabor coefficients $\{C_{mn}(x_{sr}(t))\}$ of $x_{sr}(t)$ will distribute randomly over a certain area in the time–frequency plane. Note that $C_{mn}(x_{sr}(t)) = \langle x_{sr}(t), g_{mn}(t)\rangle$ represents a linear operator, $x_{sr}(t)$ should be reconstructed from those coefficients of high energy in the region $\Omega(\omega_r)$ so as to reduce disturbance from adjacent modes. However, according to the theory of stochastic vibration, correlation of responses contains magnitude and phase information of each mode when the environment excitation is stationary white noise, and in this case, each mode can be estimated indirectly by Gabor transform on given correlation data but with higher accuracy. As in the frequency estimation, mode shape estimation poses no restriction on signal stationarity except for the resolution of Gabor transform and SNR of sampled responses.

3.3. Damping estimation

Generally, damping ratio is more difficult to accurately estimate from response data than mode shape and frequency. In our approach, damping ratio is estimated on the assumption that there exists high-energy free vibration in responses or that free vibration can be constructed from response data. When excitation is stationary white noise, autocorrelation of response data has the form of free vibration. In this case, damping estimation is relatively simple. When response is non-stationary but has high-energy free vibration, one can reconstruct the free vibration according to Eq. (8).

Suppose there exists free vibration in $x_{sr}(t)$, it can be reconstructed and expressed as

$$x_{sr}(t) = (A_r \exp(-\xi_r \omega_r t) + n(t))\sin(\eta_r \omega_r t + \varphi_r), \tag{15}$$

where $n(t)$ denotes noise. Define a scalar function $\rho_r(t)$:

$$\rho_r(t) = \frac{1}{T}\int_t^{t+T} x_{sr}^2(\tau)\,d\tau, \tag{16}$$

where T is the length of integral interval. If $n(t) = 0$, $\rho_r(t)$ has the following form:

$$\rho_r(t) = (\alpha_r + \beta_r(T,t)\sin(2\eta_r\omega_r t + \theta_r))\exp(-2\xi_r\omega_r t), \tag{17}$$

where $\beta_r(T,t)\big|_{T\to\infty} \to 0$. Given a finite T, damping ratio ξ_r can be accurately estimated from Eq. (16) by curve fitting. This is a general method and it is still valid albeit $n(t) \neq 0$, but the accuracy is affected by $n(t)$.

As mentioned above, autocorrelation of response data has the form of free vibration when excitation is stationary white noise, however, performing correlation will weaken low-energy signals and result in free vibration only containing high-energy modal vibration. Therefore, if modal vibration energy is concentrated in the vicinity of modal frequencies, correlation can be done after the low-energy modal vibration is reconstructed, the validity of which is guaranteed by Eq. (14) which also implies that time–frequency filtering of response is equal to the weighting of frequency response function. Concentration of modal vibration energy is necessary otherwise correlation of reconstructed signal is meaningless.

In the next section, Eqs. (8), (9), (13) and (16) are used to estimate modal parameters of a simply supported beam under non-stationary excitation.

4. Simulation

Consider a simply supported beam of three degrees of freedom (Fig. 2). Natural frequencies and damping ratios of this model are, respectively, 28.33, 112.54, and 238.95 Hz and 0.0562,

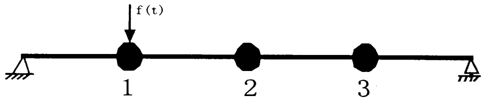

Fig. 2. A simply supported beam.

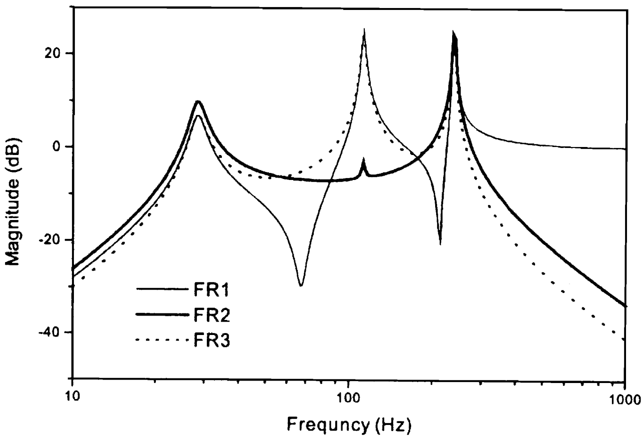

Fig. 3. Frequency response function.

0.0141, and 0.0067. Frequency response functions FR1–FR3 corresponding to each measurement point are given in Fig. 3.

Excite the beam at point 1 and measure acceleration responses at three points simultaneously with sampling rate 1000 Hz. Excitation is the fast sine sweep plus white noise and the duration is 1.28 s. Frequency band and amplitude of the fast sine sweep are 1–400 Hz and 1.0, respectively. The mean and variance of white noise are 0 and 1.0. Apparently, the acceleration response of each point is non-stationary. Fig. 4 gives the acceleration response of point 3, which contains overlapped free vibration of the three modes. Fig. 5 is the distribution of Gabor coefficients of the response of point 3, which clearly reflects the evolution of excitation and modal responses. Figs. 6 and 7 are, respectively, the distribution of Gabor coefficients of the responses of points 2 and 1. In Fig. 7, Gabor coefficients of the random response are not concentrated in the vicinity of the three modal frequencies due to the direct noise coupling in the acceleration response of point 1.

To estimate modal frequencies, Gabor coefficients corresponding to each modal frequency are collected from the distribution in Fig. 5 and the first order moment is computed according to Eq. (9). The estimated frequencies are listed in Table 1. In this example, as modal frequencies are constant, we can also reconstruct each modal vibration and then estimate all frequencies with spectrum analysis. Modal frequencies estimated from the distribution in Fig. 7 are also given in Table 1, which demonstrates frequency estimation is robust to noise contamination.

To estimate modal damping, free vibration is reconstructed from the distribution in Fig. 5. Figs. 8–10 are the reconstructed results which are lightly contaminated by noises. With the recovered free vibration, functions $\rho_1(t)$, $\rho_2(t)$, $\rho_3(t)$ are computed according to Eq. (16) and damping ratio of each mode is estimated by curve fitting. Table 1 gives the estimated results. Distribution in Fig. 7 is also used to give another estimation, but Table 1 shows that the estimated

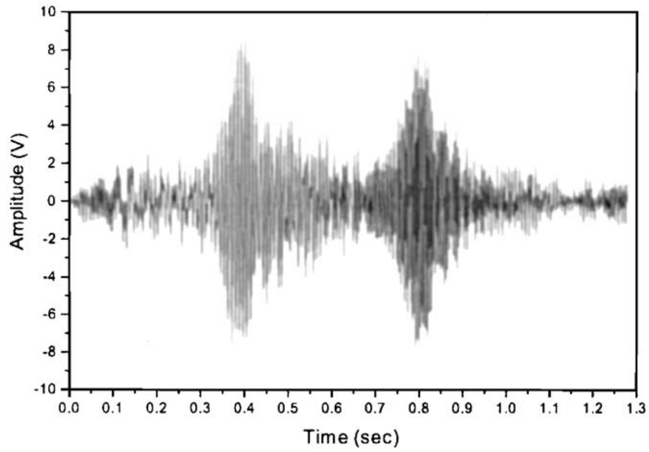

Fig. 4. Acceleration response of point 3.

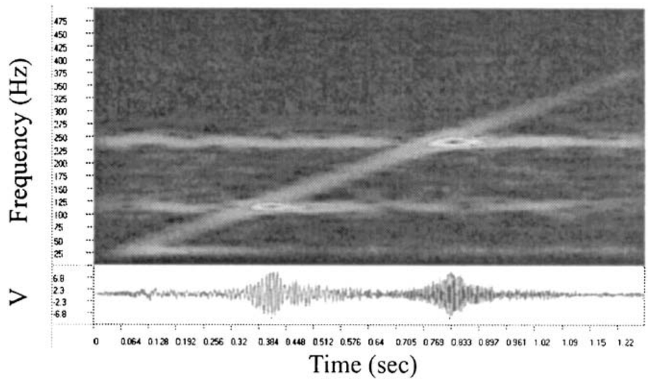

Fig. 5. Distribution of Gabor coefficients (point 3).

Table 1
Actual and estimated modal parameters

	Frequency (Hz)			Damping ratio		
	1	2	3	1	2	3
Actual	28.33	112.54	238.95	0.0562	0.0141	0.0067
Estimated (point 3)	28.5	112.8	239.5	0.0547	0.0144	0.0071
Estimated (point 1)	28.1	112.4	239.6	0.07	0.0147	0.0069

damping ratio of the first mode has large deviation whereas the others are relatively accurate. The reason is that the first modal vibration has low SNR and the others have high SNR. Here, SNR is defined as

$$\mathrm{SNR} = 10\log_{10}(E_s/E_n), \tag{18}$$

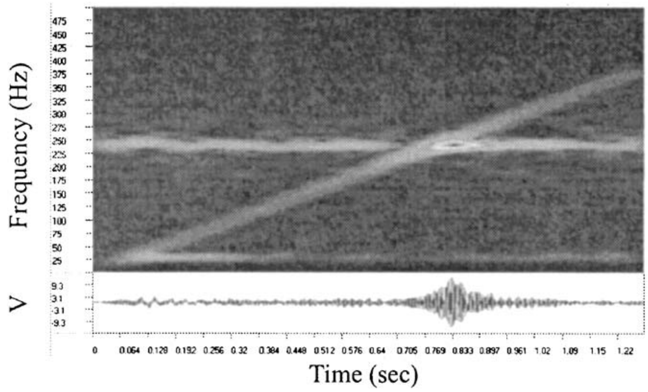

Fig. 6. Distribution of Gabor coefficients (point 2).

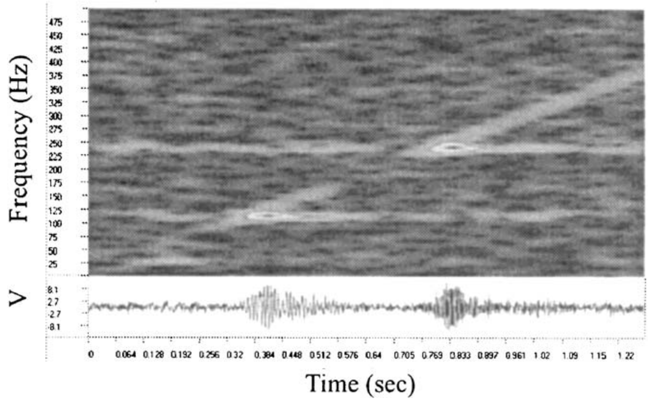

Fig. 7. Distribution of Gabor coefficients (point 1).

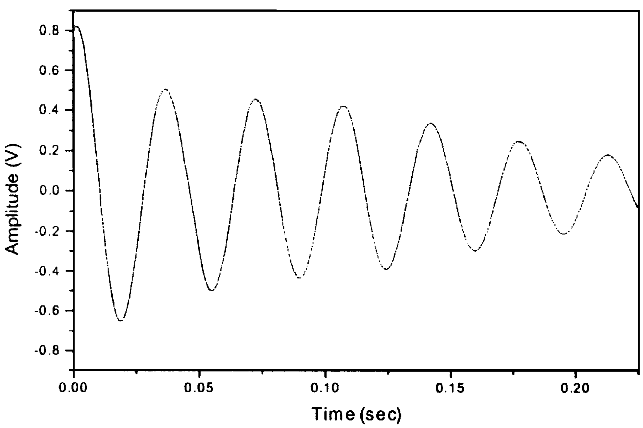

Fig. 8. Free vibration of the first mode.

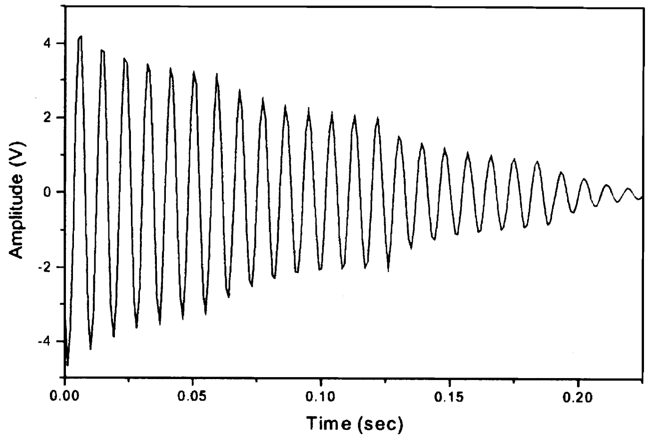

Fig. 9. Free vibration of the second mode.

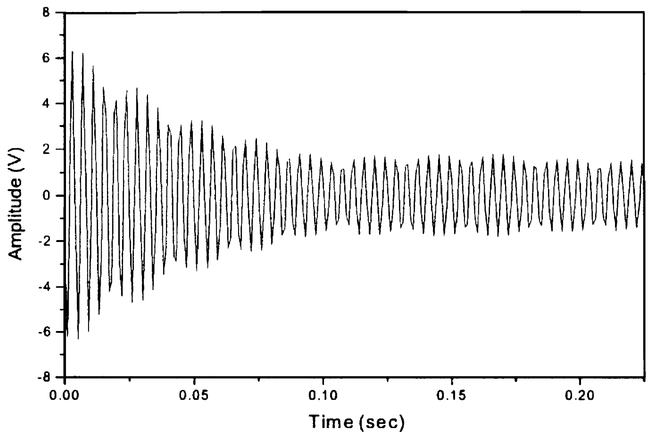

Fig. 10. Free vibration of the third mode

where E_s and E_n represent, respectively, the energy of the exact free vibration and noise in the recovered free vibration. At the measurement point 3, SNR corresponding to the first mode is 15.8 dB and the third mode 17.3 dB. However, at the measurement point 1, SNR is 0.6 dB for the first mode and 10.1 dB for the third mode.

Estimation of mode shapes needs response data of the three points. To estimate the first mode shape, one needs to reconstruct vibration signals corresponding to the first modal frequency from the distributions in Figs. 5–7, respectively. Because the response of point 1 has low SNR, signal reconstruction should be based on high-energy coefficients distributed in Fig. 5. Here, point 3 is chosen as the reference point, and the first normalized mode shape is computed according to Eq. (9). Fig. 11 gives the actual and estimated first mode shape, which illustrates the effect of heavy noise on mode shapes. Modes 2 and 3 can be estimated in the same way, and they are given, respectively, in Figs. 12 and 13. It is worth mentioning that mode 2 is

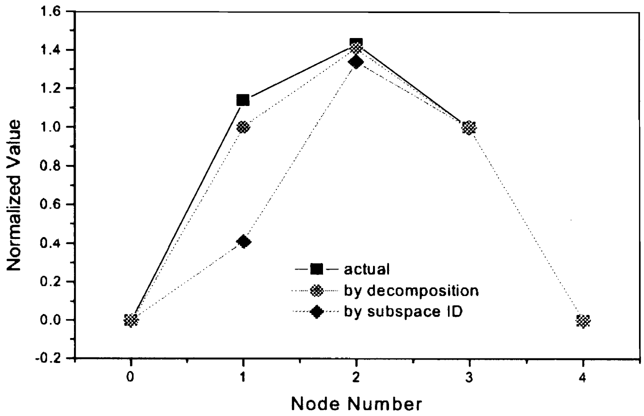

Fig. 11. The first mode shape.

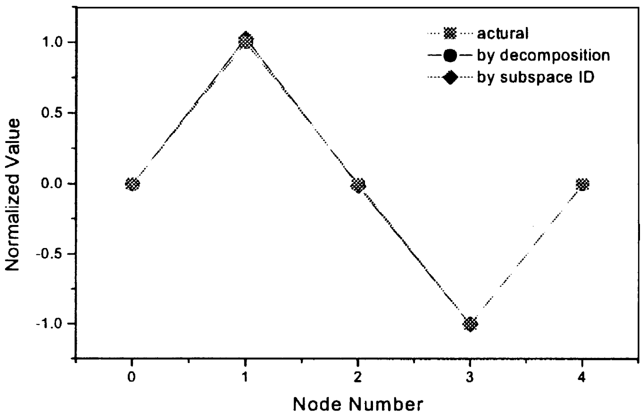

Fig. 12. The second mode shape.

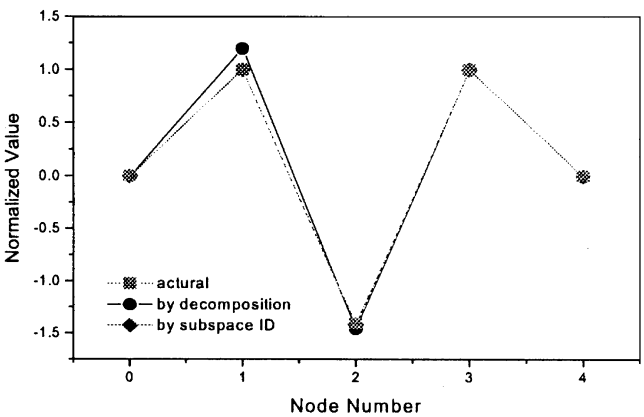

Fig. 13. The third mode shape.

Table 2
Estimated parameters by a subspace identification procedure

Order	1	2	3
Frequency (Hz)	28.32	112.74	238.9
Damping ratio	0.098	0.0145	0.0076

extracted from free vibration while mode 3 from random responses. As shown in Fig. 12, the second mode is almost accurately estimated, which is attributed to the high SNR of responses of points 1 and 3.

In the above estimation, modal parameters are extracted from non-stationary response signals, and especially modal damping and mode shapes are estimated in time domain from reconstructed free vibration or random responses after response signals are decomposed in time–frequency domain.

For the purpose of comparison, a subspace identification procedure is also employed to give another estimation. Estimated parameters are given in Table 2 and mode shapes are shown in Figs. 11–13, respectively.

As shown in the table and figures, the subspace identification procedure also gives a good estimation except for the large errors in damping ratio and mode shape of the first order.

5. Conclusion

With Gabor transform, the response of an N-degree-of-freedom system can be decomposed into atomic vibration in time–frequency domain, from which modal vibration corresponding to each modal frequency can be reconstructed. This makes modal parameter estimation of a multi-degree-of-freedom system almost like that carried out on a single-degree-of-freedom system. Moreover, as Gabor transform poses no restriction on signal stationarity, estimation based on this transform is suitable for general responses, which implies that the proposed method is effective in extracting modal parameters from response data only. For time-varying or non-linear systems, time-varying frequencies can also be estimated with time–frequency decomposition due to the general estimation formulae. In the simulation study, frequency estimation has proved robust to noise contamination while damping and mode estimation is liable to noise contamination. To guarantee the validity and accuracy of parameter estimation, modal frequencies are supposed to be sufficiently spaced and response energy is assumed to be concentrated in the vicinity of modal frequencies. This will limit the application of the proposed approach to some extent in general cases. Therefore, further study should solve the problem of identifying modal parameters of systems that are heavy damped or have dense modal frequencies, and one way out might be incorporating model based parametric estimation in the time–frequency analysis.

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