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Letter to the Editor

The relationship between the resonant and natural frequency for non-viscous systems

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1. Theory

The peak transmissibility of a real vibration system occurs at its resonant frequency f_r, and the natural frequency f_n would coincide with the resonant frequency if the damping were zero. For small values of damping, frequencies f_n and f_r are close but, from time to time, they become a matter of disputation among vibration engineers (what frequency is greater and how much). In books on vibration theory, the relationship between the two is analyzed only for viscous damping [1–4, etc.]. However, the loss factor may be governed by multiple energy dissipation mechanisms (in particular for rigid constructions, by hysteresis and structural dissipation) [5–15, etc.]. In this paper, a general close-form relationship between the resonant and natural frequencies is derived.

The resonant frequency is normally measured on sweep-sine shaker testing. Consider a single-degree-of-freedom vibratory model incorporating two rigid bodies connected with parallel spring and dashpot. One body simulates the shaker and moves harmonically with a displacement $Y_0 = y_0 \exp(\mathrm{i}\omega t)$, the differential equation for the displacement $Y = y \exp(\mathrm{i}\omega t)$ of the other body (with mass M) takes the form

$$M\ddot{Y} + K(Y - Y_0) = 0, \tag{1}$$

where $\omega = 2\pi f$ is the angular frequency and f is the frequency of vibration; $K = k(1 + \mathrm{i}\eta)$ is the complex spring constant combining the spring constant k and loss factor η. In particular for viscous damping, the loss factor equals $\eta = 2(c/c_c)(f/f_n)$ where the quantities c and $c_c = 2\sqrt{Mk}$ denote the coefficient of viscous damping and the critical damping [1]. For rigid structures, the total loss factor commonly includes two main components: the internal loss factor caused by hysteresis, and the structural loss factor resulting from the vibration energy absorption at junctures, edges, and adjacent structures. In particular for single walls, partitions, glazing, etc. the

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total loss factor is described [8,9] by equation

$$\eta = \frac{2\alpha}{\kappa_b L} + \eta_i. \tag{2}$$

Here, $\eta_i \approx$ const is the internal loss factor determined by hysteresis, $\kappa_b \propto \sqrt{f}$ is the wavenumber of bending waves, the length $L = \pi A/P$, where A and P are respectively the area and perimeter of the plate, and α is the average coefficient of vibration energy absorption at plate's edges; for building structures, the parameter α measures about 0.1–0.3. It follows from Eq. (2) and is experimentally confirmed [10] that, the structural loss determined by the first addend can be significant for smaller plates. At relatively low frequencies, the total loss factor $\eta \propto 1/\sqrt{f}$ and at high frequencies, $\eta \approx \eta_i \approx$ const. Using the partial solution $y = y_0(1 + i\eta)/[1 - (f/f_n)^2 + i\eta]$ of the linear differential equation (1), we calculate the transmissibility

$$T(f) = \left|\frac{y}{y_0}\right| = \sqrt{\frac{1+\eta^2}{[1-(f/f_n)^2]^2 + \eta^2}}. \tag{3}$$

At the frequencies close to the natural frequency and when $\eta \ll 1$, Eq. (3) can be reduced to a simpler form

$$T(f) \approx \sqrt{\frac{1}{4\varepsilon^2 + \eta^2}}, \tag{4}$$

with the variable $\varepsilon = f/f_n - 1$, provided that $|\varepsilon| \ll 1$. From Eq. (4), the peak transmissibility is attained at the minimum of the function $z(\varepsilon) = 4\varepsilon^2 + \eta^2$. The necessary condition is the linear algebraic equation

$$\frac{dz(\varepsilon)}{d\varepsilon} = 8\varepsilon + 2\eta\frac{d\eta}{d\varepsilon} = 0$$

with the only solution $\varepsilon_1 = -(\eta\, d\eta/d\varepsilon)/4$. Thus, the ratio of the resonant and natural frequencies is

$$\frac{f_r}{f_n} = 1 + \varepsilon_1 = 1 - \frac{f_n(\eta\, d\eta/df)}{4}, \tag{5}$$

where the derivative is calculated at frequency $f = f_n$. Let us now consider that the loss factor at frequencies close to the natural frequency is described by a simple power function

$$\eta = \beta f^p, \tag{6}$$

where $p = 1$ for the classical case of viscous friction, $p = -1/2$ for structural damping, and $p = 0$ for the hysteresis type of friction. Substituting formula (6) in Eq. (5) we obtain the clear practical result

$$\frac{f_r}{f_n} \approx 1 - \frac{p\eta^2}{4}. \tag{7}$$

From Eq. (7), the resonant frequency falls below the natural frequency in case of viscous friction and exceeds it if the structural loss prevails; under hysteresis, the resonant and natural frequencies coincide.

2. Conclusions

The general relationship between the resonant and natural frequencies was derived as a function of the loss factor η (that can be measured) in the practical case $\eta \ll 1$, and described by Eq. (5) or in the simplified case (6) by Eq. (7). The resonant frequency may exceed, equal, or fall below the natural frequency depending on the main type of friction in a single-degree-of-freedom vibratory system, however the difference between the two frequencies is minor.

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