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Flutter analysis of periodically supported curved panels

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Abstract

This paper presents the one-dimensional axial wave propagation in an infinitely long periodically supported cylindrically curved panel subjected to supersonic airflow. The aerodynamic forces are based on piston theory. For this study the structure is considered as an assemblage of a number of identical cylindrically curved panels each of which will be referred to as a periodic element. A high precision triangular finite element with certain wave boundary conditions (Floquet's principle) is introduced in flutter problems of the proposed structure for the first time. The airflow is assumed in the direction of the straight edges of the panel. It is assumed that the deflection function accounts for a phase lag term only and does not consider any attenuation terms. Aerodynamic damping has been neglected for brevity. For a given geometry a three-dimensional plot related to the phase constant, flutter frequency and pressure parameter has been obtained corresponding to the optimum periodic angle. The "flutter line"(line of instability) has been identified. The limiting values of flutter frequencies and pressure parameters of the "flutter line" are compared with the critical flutter condition of a single curved panel, using two methods—an exact approach and a finite element method. The critical flutter results for multi-supported (1-span, 2-span and 3-span) curved panels are obtained using the band discretization principle.

1. Introduction

The topic of panel flutter due to supersonic airflow has been a matter of interest due to its importance in supersonic aircraft and launch vehicle design. Liquid engines launch vehicles and aircraft fuselages may, as a first approximation be considered as cylindrical shells, curved panels stiffened by stringers and/or rings at regular intervals. These structures are strewn with periodic

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structures. By definition, a periodic structure is one that is made up of identical elements joined to its neighbours along their boundaries. In the present paper a method of analysis is proposed to predict the flutter characteristics of multi-span long curved panels supported on equally spaced transversely rigid supports for the first time using wave propagation method, combined with a FEM. The approach shows a considerable reduction in matrix size and consequent reduction in computer storage and/or time.

The problem of vibration and flutter analysis of flat panel [1], curved panel [2–7] and simply supported unstiffened and orthogonally stiffened shell [8,9] have been developed. The above reported works are related to flutter analysis of isolated curved panel. Extensive review of the aeroelasticity of plates and shells are reported in Refs. [10,11], but without periodic structure concept.

The aeroelastic stability of multi-bay periodically supported panels without curvature on transversely rigid supports, subjected to lengthwise supersonic fluid flow by the periodic structure theory (Floquet's principle) has been examined in Refs. [12,13] in conjunction with the piston theory aerodynamics of Ashley and Zartarian [14] and using relevant differential equation of motion of Dowell [15]. Wave dispersion relation has been reported in a finite periodic panel having arbitrary number of spans, subjected to supersonic airflow.

The high-precision triangular shell finite element of Cowper et al. [16] in conjunction with a wave approach of Mead [17] has been proposed for first time in this work to study the supersonic flutter analysis of infinitely long periodically supported curved panels (Fig. 1a) using a linearized piston theory. Aerodynamic damping has been neglected for brevity. The airflow is assumed in the direction of the axis. Here it is assumed that the propagation constant is purely imaginary [18–21], i.e., only propagating wave motions are studied. In a finite periodic panel the spatial attenuation from bay to bay is lowered if the number of panels are increased and vice versa [15]. In the limit therefore it may be concluded that in an infinitely long curved panel of equal spans the spatial attenuation is zero.

The optimum subtended angle corresponding to critical flutter velocity has been used in periodic flutter analysis. A "flutter line" similar to the case of a periodically supported flat panel [12,13] has been obtained in this paper. The relationship between the critical phase constant and critical pressure parameter for the periodic curved panel is presented. Finally, the critical flutter pressure parameter results for finite multi-supported curved panels have been obtained from "flutter line" using Sengupta's [22] discretization scheme.

2. Equation of motion

2.1. Flutter equation of a single curved panel

A thin isotropic cylindrically curved panel of length a, width b, thickness h and radius of curvature R subjected to a supersonic flow over its upper surface is considered (Fig. 1b).

The governing aeroelastic equation of motion for an elastic structural system undergoing small displacements neglecting aerodynamic damping can be expressed as

$$[M]\{\ddot{q}\} + [K]\{q\} = -\widehat{\lambda}[K_a]\{q\}, \tag{1}$$

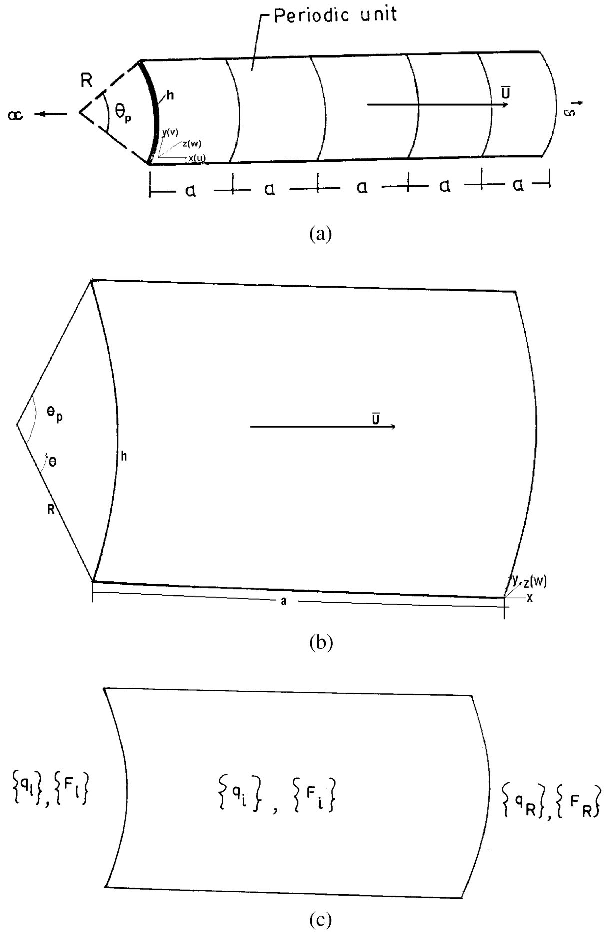

Fig. 1. (a) Infinite periodic curved panel subjected to supersonic flow. (b) Single curved panel (periodic unit) subjected to supersonic flow. (c) Generalized forces and displacements of a single periodic unit.

where $[M]$, $[K]$, and $[K_a]$ are the global mass, stiffness and aerodynamic stiffness matrices, respectively, of the total structural system. $\{\ddot{q}\}$ and $\{q\}$ are the global acceleration and displacement vectors. $\widehat{\lambda} = 2\widehat{q}/(\bar{M}_\infty^2 - 1)^{1/2}$ is the aerodynamic pressure parameter. $\widehat{q} = (1/2)\,\rho_a \bar{U}^2$ is the dynamic pressure of the free stream air, $\bar{U}$ is the free stream velocity of supersonic flow, $\bar{M}_\infty$ is the free stream Mach number and ρ_a is the free stream air density.

The basic shell element employed here is the conforming higher order arbitrary trianglar shaped shallow shell finite element of Cowper et al. [16]. The details of the element stiffness, consistent mass matrices and the asymmetric aerodynamic stiffness matrices are presented in Refs. [1,16,23]. Assuming the solution of the form

$$\{q\} = \{\bar{q}\}e^{i\omega t}, \tag{2}$$

Eq. (1) becomes

$$\left[\Omega^2[M] + ([K] + \Lambda[K_a])\right]\{\bar{q}\} = \{0\}, \tag{3}$$

where ω is the radian frequency; t is the time variable; q, $\bar{q}$ are the generalized global co-ordinates; $\Omega = \omega R\sqrt{\rho(1-\nu^2)/E}$ is the non-dimensional frequency; $\Lambda = \widehat{\lambda} a^3/D$; $D = Eh^3/12(1-\nu^2)$ is the bending stiffness; E is Young's modulus of elasticity and ν is the Poisson ratio.

If $\Lambda = 0$, the solution of Eq. (3) will yield the natural frequencies of vibration of the curved panel. For Λ greater than 0, the non-symmetric aerodynamic matrix comes into the picture and some of the eigenvalues become complex for a certain range of the dynamic pressure parameter Λ. The lowest value of Λ for which a pair of complex conjugate eigenvalues appear is identified as critical dynamic pressure Λ_{cr}.

2.2. Flutter analysis of periodic curved panel

Now a cylindrical panel of infinite length which is supported periodically along its length is considered (Fig. 1a). A periodic element is shown in Fig. 1b. Using the periodic FEM technique [20,21], the periodic element can be represented by a model with interior and boundary d.o.f. [25]. Each periodic element is connected to its neighbouring elements on its edges. Let $\{q_I\}$, $\{q_L\}$, $\{q_R\}$ be the d.o.f. and $\{F_I\}$, $\{F_L\}$, $\{F_R\}$ forces at the left, interior and right of the each periodic unit where subscripts I, L, R refer to the interior, left and right, respectively (Fig. 1c).

The linear undamped aeroelastic Eq. (3) of a single unit of the periodic element is given by

$$\left[\Omega^2[M] + ([K] + \Lambda[K_a])\right]\{\bar{q}\} = \{F\}, \tag{4}$$

where $\{\bar{q}\}$ and $\{F\}$ are the generalized displacements and forces, respectively, of the periodic element,

$$\{\bar{q}\} = \lfloor q_I \quad q_L \quad q_R \rfloor^{\mathrm{T}}, \tag{5}$$

$$\{F\} = \lfloor F_I \quad F_L \quad F_R \rfloor^{\mathrm{T}}. \tag{6}$$

The nodal forces $\{F\}$ are due to any external forces acting on the system and the forces of interaction between the periodic unit and its neighboring units. When an elastic body is placed in airflow, there is a possibility of instability due to self-excited oscillation termed as flutter. The oscillation at the instant of flutter is self-sustained; i.e., no external oscillation or forcing agency is required. For a free wave $\{F_I\} = \{0\}$. The force vector $\{F\}$ can be expressed in the form [17,20,21]

$$\{F\} = \begin{pmatrix} 0 \\ F_L \\ F_R \end{pmatrix}, \tag{7}$$

where the column on the right-hand side represents the forces on the element from the adjacent elements. The harmonic motion Eq. (4), using Eqs. (5) and (7) is given by

$$\left[([K] + \Lambda[K_a]) + \Omega^2[M]\right]\begin{Bmatrix} q_I \\ q_L \\ q_R \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_L \\ F_R \end{Bmatrix}. \tag{8}$$

Eq. (8) can be solved as a free vibration problem [20]. The matrix $[[K] + \Lambda[K_a]]$ is represented as $[KK]$. The $[KK]$ matrix is asymmetric which is partitioned into interior, left and right d.o.f. as

$$[KK] = \begin{bmatrix} KK_{I,I} & KK_{I,L} & KK_{I,R} \\ KK_{L,I} & KK_{L,L} & KK_{L,R} \\ KK_{R,I} & KK_{R,L} & KK_{R,R} \end{bmatrix}. \tag{9}$$

$[M]$ can be similarly derived as a symmetric matrix.

This wave motion is characterized by relating the d.o.f. and equivalent nodal forces in one unit to the corresponding d.o.f. and forces in adjacent units. Using wave equation [17,20]:

$$\{q_R\} = e^{-i\mu_x}\{q_L\} \tag{10}$$

on the boundary of the cells, the quantity $\{q_R\}$ can be eliminated. Combining Eqs. (7), (8) and (10) results in the following:

$$\left(\left[\bar{K}(\mu_x)\right] - \Omega^2\left[\bar{M}(\mu_x)\right]\right)\begin{Bmatrix} q_I \\ q_L \end{Bmatrix} = \{0\}, \tag{11}$$

where

$$[\bar{K}(\mu_x)] = [W'][KK][W], \tag{12a}$$

$$[\bar{M}(\mu_x)] = [W'][M][W]. \tag{12b}$$

The matrix $[W]$ is explained in Ref. [20].

The boundary condition of the periodic element(single curved panel) is simply supported at all edges. Eq. (11) can be solved for different values of real phase constants μ_x and pressure parameter to find the corresponding eigenvalues (Ω, frequencies). μ_x is varied from 0 to π.

In aeroelastic analysis to model a single periodic curved panel, a 6×6 mesh of triangular elements has been chosen. The d.o.f. of a single curved panel (Eq. (3)) before applying wave boundary conditions (Eq. (10)) was 588. It is reduced to 504 d.o.f. (Eq. (11)) after applying wave boundary conditions (Eq. (10)).

3. Results and discussion

3.1. Finite element method (FEM) for isolated curved panels

A square cylindrical curved panel ($a/b = 1$) has been considered to compare the present triangular finite element results with Bismarck-Nasr [5]. The material properties are $E = 210 \times 10^9\ \mathrm{N/m^2}$, $\rho = 7800\ \mathrm{kg/m^3}$ and $\nu = 0.3$. The results are presented in terms of the non-dimensional dynamic pressure parameter Λ as a function of the maximum shell rise (H/h). The calulations were performed for a square ($a/b = 1$) curved panel with four edges simply supported and also with four edges clamped. The results are shown in Fig. 2. Bismarck-Nasr [5] and Dowell [3] results are obtained from Ref. [5, Fig. 2]. The present results show the same trend as that of Ref. [5]. The difference between the present results and those due to Bismarck-Nasr [5] is entirely the result of satisfying different in-plane boundary conditions.

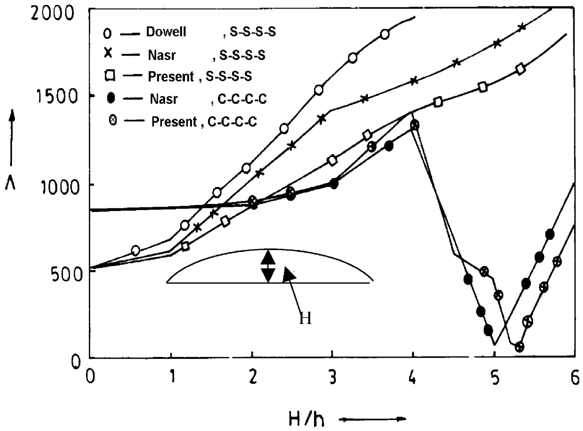

Fig. 2. Comparison of flutter pressure parameters of a single square ($a/b = 1$) with a curved panel with [3,5].

3.2. Optimum choice of a periodic element for flutter analysis

The numerical results are generated taking the dimension of a full circular cylindrical shell of axial length $a = 1.016\,\mathrm{m}$, radius $R = 0.508\,\mathrm{m}$ and thickness $h = 1.016\times10^{-3}\,\mathrm{m}$ [9]. The a/R and h/R ratios are 2 and 0.002, respectively. The binary flutter analysis of an unstiffened shell with simply supported ends is reported in Ref. [9]. The binary flutter pressure parameter is plotted for a range of the full circumferential mode number (n). It has been found from Ref. [9] that the circumferential full wave number (n) corresponding to the lowest binary flutter pressure parameter ($\Lambda = 21\,363$) is 18.

Now the above dimension ($a/R = 2$, $h/R = 0.002$) is considered for a curved panel with four edges simply supported. The same displacement functions which have been used for a full shell [24] have been chosen for the curved panel case. The axial (u), circumferential (v) and radial displacement (w) for curved panel are expressed as follows:

$$u = A_{mn}\sin(\lambda_c\theta)\cos\left(\frac{\lambda_m x}{R}\right)\mathrm{e}^{\mathrm{i}\omega t},$$
$$v = B_{mn}\cos(\lambda_c\theta)\cos\left(\frac{\lambda_m x}{R}\right)\mathrm{e}^{\mathrm{i}\omega t}$$
$$w = C_{mn}\sin(\lambda_c\theta)\cos\left(\frac{\lambda_m x}{R}\right)\mathrm{e}^{\mathrm{i}\omega t},$$
$$\theta = \frac{y}{R}, \quad \lambda_m = \frac{m\pi R}{a}, \quad \lambda_c = \frac{n_c\pi R}{b} = \frac{n_c\pi}{\theta_p}, \quad \theta_p = \frac{b}{R},$$
$$m = \text{number of axial half-waves.} \tag{13}$$

The circumferential half-wave ($= n_c$) is equal to 1. The binary (axial mode; $r = 1$, $s = 2$) flutter pressure parameter (Λ) is plotted for a range of periodic angle (θ_p). The results are shown in Fig. 3. The lowest pressure parameter($\Lambda = 21\,363$) corresponds to the periodic angle $\theta_p = 10^\circ$. These results are compared with an independent calculation using the present triangular FEM

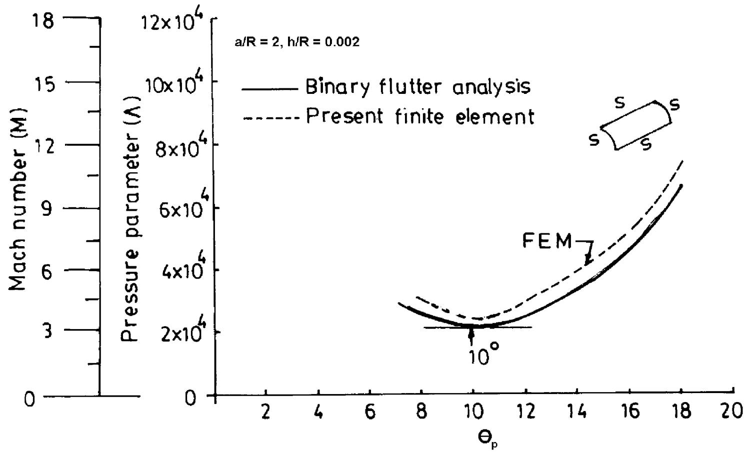

Fig. 3. Flutter dynamic pressure parameter versus periodic angle of a curved panel with all the edges simply supported.

(Fig. 3). In the FEM, for a single curved panel with all the edges simply supported, the lowest pressure parameter of value ($\Lambda = 23\,669$) is obtained. The difference between these two methods may be due to different inplane boundary conditions and neglecting aerodynamic damping. The lowest flutter pressure parameter (Λ) is observed at $\theta_p = \pi/18$ (10°) in finite element as well as binary flutter analysis. This periodic angle (corresponding to the lowest flutter pressure parameter) will now be considered for the supersonic flutter analysis of an infinite periodic curved panel coupled with Floquet's principle. The discussions are presented in the next section.

3.3. *Flutter analysis of an infinite periodic curved panel*

In this section, supersonic flutter analysis in an infinitely long periodically supported cylindrically curved panel is studied. The infinite curved panel is composed of curved panels (periodic elements) each of length a, thickness h and radius R and connected end to end with periodic simple non-deflecting supports (Fig. 1a). The dimension of each periodic unit (curved panel) is the same as that of Ref. [9], viz., $a/R = 2$, $h/R = 0.002$. The circumferential length of each panel is $R\theta_p$. θ_p is the angle subtended at the centre of circular cross-section of periodic curved panel. The periodic angle $\theta_p = 10^\circ$ considered in the computation corresponds to the lowest non-dimensional flutter dynamic pressure parameter (Λ) of a single curved panel simply supported at all edges.

In the present approach, we shall assume, for the undamped stable states only real values of the propagation constants. Solving Eq. (11) for each value of aerodynamic pressure parameter (Λ) we may determine the dispersion relationships existing between the phase μ_x (0 to π) and the real eigenvalues (ω^2) over the propagation band. μ_x is varied from 0 to π . It may be noted that within the interval of μ_x (between 0 to π), for all the values of μ_x other than 0 and π, the eigenvalue is complex and therefore the structure is unstable. The real eigenvalues has been converted into the present non-dimensional frequency (Ω).

3.3.1. Determination of "flutter line" and relationship among dynamic pressure parameter Λ, phase constant μ_x and the non-dimensional frequency Ω (real eigenvalues)

The three-dimensional graphical representation of the relationship between Λ, Ω and μ_x for undamped vibration is presented for the periodic angle $\theta_p = 10^\circ$ (Fig. 4). This periodic angle is the optimum angle corresponding to the lowest flutter dynamic pressure.

For zero dynamic pressure ($\Lambda = 0$), the natural frequencies of vibration of finite periodic structures are real (stable) and correspond to the free vibration natural frequencies of finite periodic structures. The first two propagation curves ($A-B$, $C-D$) corresponding to aerodynamic pressure $\Lambda = 0$ are shown in Fig. 4. Curve $A-B$ is the first propagation band. Curve $C-D$ is the second propagation band. As Λ increases, at pressure parameters $\Lambda = 23669.34$, the upper bounding frequency of the first band just joins up with the lower bounding frequency of the second propagation band at $\mu_x = 0$. The corresponding flutter frequency is $\Omega_f = 0.1923$. The point is represented by symbol 'X' in Fig. 4. This is the starting point (lower limiting point) of instability of the structure. This point X is the origin of the "flutter line". The pressure parameter $\Lambda = 23669.34$ is the lowest value for this particular panel dimension.

With further increase of Λ, this lower limiting point X moves towards $\mu_x = \pi$ and with higher frequency values. At pressure parameter $\Lambda = 60\,248$, the lower limiting point X is finally meeting at $\mu_x = \pi$. This point is represented by symbol 'Y' in Fig. 4. It may be noted the two different curves showing variation of real (stable) frequencies for $\mu_x = 0$ and π come closer to each other and finally coalesce into a single curve for infinite stiffness value for π. This is the upper limit of "flutter line" with flutter dynamic pressure $\Lambda = 60\,248$ at $\Omega = 0.203$ and $\mu_x = \pi$ (Fig. 4).

The flutter line denotes the critical conditions of onset of instability. It denotes the locus of the flutter points (corresponding to $\partial\mu_x/\partial\Omega = 0$) of the phase constant–frequency curves at different values of Λ [12,13].

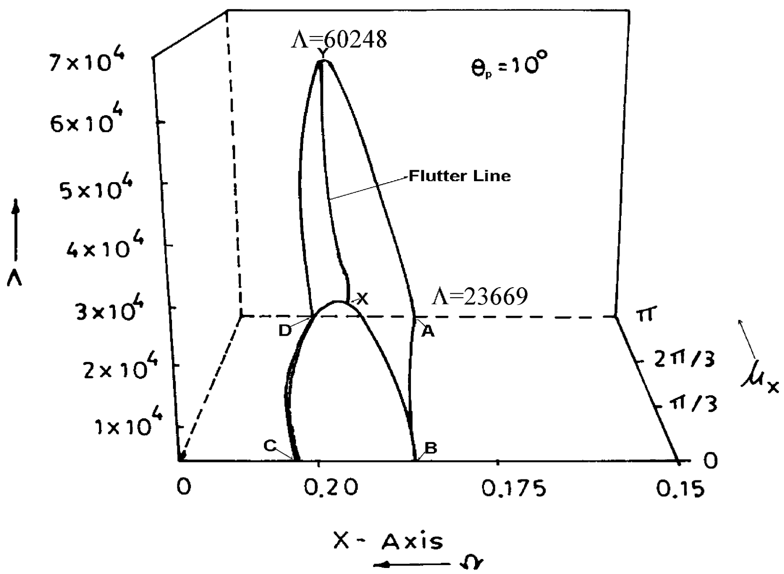

Fig. 4. The relationship among the non-dimensional dynamic pressure parameter Λ, the phase constant μ_x and the frequency Ω (real part of eigenvalues) of a periodically supported infinitely long curved panel.

3.3.2. Bounding mode of "flutter line

The variation of critical phase constant and pressure parameter is presented in Fig. 5 ($\theta_0 = 10°$). The flutter line (Fig. 5) originates at $\Lambda_{crit} = 23\,669$, $(\mu_x)_{crit} = 0$ and $\Omega_f = 0.1923$. The lower limiting point (X) of "flutter line" corresponds to the critical flutter conditions for a single curved panel with four edges simply supported. This is confirmed by the present finite element flutter and binary flutter analysis of the single isolated curved panel (Table 1). The bracketted values of Table 1 are the critical flutter condition ($\Lambda_{crit} = 21\,363$, $\Omega_f = 0.1917$) obtained from binary flutter analysis of a single curved panel ($a/R = 2$, $h/R = 0.002$) with four edges simply supported. The value $\Lambda_{crit} = 21\,363$ is also the same as the minimum critical pressure parameter of a full shell with simply supported ends [9] with circumferential full wave $n = 18$ and axial mode ($r = 1$, $s = 2$).

The upper limit point (Y) of "flutter line" (Fig. 5) represents $\Lambda_{crit} = 60248$ at $\Omega_f = 0.203$ and $(\mu_x)_{crit} = \pi$. This corresponds to the critical flutter condition of a single curved panel ($a/R = 2$, $h/R = 0.002$) with straight edges simply supported and curved edges clamped. This is verified by recomputing the flutter condition of a single curved panel (Table 2).

It has been observed that for zero dynamic pressure of the flow, the frequencies of vibration of finite periodic structures are real (stable) and correspond to the free vibration natural frequencies of finite periodic structures. In the infinite bay case, these free vibration frequencies are those that are the bounding frequencies of the propagation bands of the free waves in the panels [18,22,25]. It may be recalled that for the periodic beam vibration, the "propagation bands" are effectively the frequency ranges bounded by the frequencies that correspond to the simply supported single span (lower bounding mode) and the clamped–clamped single span (upper bounding mode).

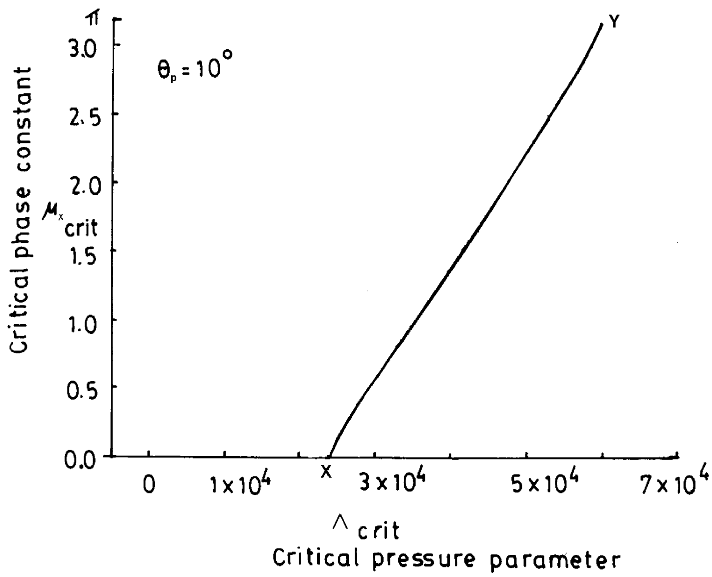

Fig. 5. The relationship between the critical phase constant and the critical non-dimensional dynamic pressure parameter (flutter curve).

Table 1
Comparison of lowest bounding flutter frequency and flutter dynamic pressure parameter of "flutter line" at phase constant '0' with single curved panel results

Periodic angle (θ_p)	Finite element method with periodic structure theory	
	Flutter frequency (Ω_f)	Dynamic pressure (Λ)
π/18 (10°)	0.1923 (0.1917)	23669.34 (21363.21)

Values between brackets obtained from binary flutter analysis of full shell [9] binary flutter analysis of single curved panel with four edges simply supported.

Table 2
Comparison of upper bounding flutter frequency and flutter dynamic pressure parameter of "flutter line" at phase constant π with single curved panel results

Periodic angle (θ_p)	Finite element method with periodic structure theory	
	Flutter frequency (Ω_f)	Dynamic pressure (Λ)
π/18 (10°)	0.2030 (0.2023)	60248.26 (60244.53)

Values between brackets are results of a single curved panel with straight edges simply supported and curved edges clamped using present finite element method.

Table 3
Critical flutter parameters for one-dimensional multi-bay panels: for simply supported edges, ignore rows marked by +; for clamped edges, ignore rows marked by * ($\theta_o = 10°$, optimum periodic angle corresponding to lowest flutter dynamic pressure)

Number of spans		Critical pressure parameter (Λ_{crit})	Critical phase constant $(\mu_x)_{crit}$
1	*	23669	0
	+	60248	π
2	*	23669	0
		43461	π/2
	+	60248	π
3	*	23669	0
		36923	π/3
		50050	2π/3
	+	60248	π

3.3.3. *Critical flutter parameter results of finite multi-span curved panels*

The finite 1-span, 2-span and 3-span critical non-dimensional pressure parameters are obtained from "flutter line" (Fig. 5) by using Sengupta's [22] discretization principle. For the finite structure of N_x bays the critical dynamic pressure parameters can easily be obtained [12,13] by finding the phase lags μ_x equal to $j\pi/N_x$ ($j = 0$ to $N_x - 1$) for clamped ends or $j = 1$ to N_x for simply supported ends, for the first propagation band. The results are shown in Table 3.

4. Conclusion

In this work it is shown that the Floquet's principle which had been so extensively used for free vibration analysis of periodic structures [18,25] and flutter of one-dimensional multi-bay periodic flat panels under supersonic flow [12,13], is also applicable to the analysis of wave propagation and flutter of multi-bay periodic curved panels under supersonic flow. The triangular FEM [16] with Floquet's principle for non-attenuating waves in periodic structure has been proposed for the supersonic flutter analysis of infinite periodic curved panels. The 3-D plot related to $\Omega - \mu_x - \Lambda$ is found corresponding to the optimum subtended angle where pressure parameter is lowest. Finally, the "flutter line" (the point where $\partial\mu_x/\partial\Omega = 0$) is observed in the $\Omega - \mu_x - \Lambda$ plot. The lower limiting points of "flutter line" have been identified to the critical flutter condition of single curved panel of finite length with four edges simply supported. The upper limit corresponds to the critical flutter condition of single curved panel with straight edges simply supported and curved edges clamped. This is verified by computing the critical flutter condition of single curved panel with these end boundary conditions in a finite element. These bounding flutter results are similar to the critical flutter condition of the flat panel case [12,13]. Finally, the critical flutter parameters of multi-supported curved panels (1-span, 2-span and 3-span) of finite length are presented. These results can be read off from the "flutter line" (Fig. 5). This is perhaps the first application of wave propagation method to flutter analysis of periodically supported curved panels.

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