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Letter to the Editor

Free vibration of completely free coupled orthotropic rectangular plates

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1. Introduction

With the continued development of communication and other satellites, the problem of the vibration analysis of plates with completely free boundary conditions has become of increased importance. The traditional reference on plate vibration by Leissa [1] records some of the early work in this area. Recently, a number of solutions [2–6] have been presented for single completely free plates, using analytical and numerical methods, and considering isotropic and orthotropic materials. Further work [7–9] has been devoted to the study of coupled systems, involving typically combinations of basic components such as plates, beams, and shells. A framework has thus been established for the analysis of satellite problems, consisting of coupled structural systems having completely free support boundary conditions.

In this study the natural frequencies of vibration of completely free mechanically coupled rectangular plates are determined. The geometry consists of a pair of identical plates joined together by a pair of identical couplers. A finite element method (FEM) solution based on the Kirchoff classical thin shell theory is developed. Isoparametric rectangular elements are used, and an eigenvalue equation is set up. The versatile Jacobi iteration method is used to solve this equation. Results obtained from the approach are compared with previously published results for a single completely free orthotropic plate. Additional results are then presented for pairs of coupled orthotropic plates. The paper ends with an appropriate set of conclusions.

2. Finite element solution

The FEM [10] based on classical thin plate theory and isoparametric elements is used to set up the basic vibration equation

$$[K](x) = \lambda[M](x), \tag{1}$$

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where [K] and [M] respectively are the stiffness and mass matrices, while λ and (x) respectively are the eigenvalue and eigenvector. This equation represents a general eigenvalue problem in which λ gives the natural frequencies and (x) gives the mode shapes. For a completely free plate structure, lacking geometric constraints, the eigenvalue system obtained is of the semidefinite type.

Various methods are available to solve general eigenvalue problems, including those of Lanczos, Householder, and Jacobi [10]. As the cyclic Jacobi method [11] is particularly effective for semidefinite matrix systems it was adopted for the current study. In this method a sequence of transformations is performed on the system matrices until a diagonal form is obtained. The zero eigenvalues corresponding to the rigid body modes of the unsupported structural system are readily sorted out. A computer program labelled PLATEZ05 was developed to incorporate the current approach. Results from this program are presented in the following.

3. Validation problem

To validate the current approach a single completely free orthotropic plate is considered. The plate is square, 10 in to a side, with a thickness h of 1/16 in. The elastic modulus E_1 in the x (horizontal) direction of the plate is taken as 10×10^6 psi, while the modulus E_2 in the y direction is taken as various multiples of E_1. The mass density of the plate is $\rho = 0.2583 \times 10^{-3}$ lb s^2/in^4, while the effective Poisson ratio $\sqrt{v_1 v_2}$ is 0.25.

The first eight natural frequencies for this plate are given in Table 1 for three different values of the E_1/E_2 ratio. The table gives results from the PLATEZ05 program as well as results from an analytical solution given previously by Gorman [12]. The latter values include both symmetric and antisymmetric modes taken from Tables 1 and 2 of Ref. [12]. Eigenvalues corresponding to rigid body modes are not listed in these or subsequent tables of this paper. Close agreement of results from the two solutions is observed for all cases. It is noted that the PLATEZ05 solution predicts two modes of vibration that do not appear in the analytical solution.

Table 1
Natural frequencies for single free orthotropic plates (Hz)

E_1/E_2	4/3		3/2		3/1	
Mode	FEM	[12]	FEM	[12]	FEM	[12]
1	76.25	76.15	74.12	73.94	62.13	62.04
2	107.56	107.47	102.41	102.19	73.62	73.56
3	135.19	135.02	134.25	133.84	131.63	131.44
4	190.12	190.21	182.64	182.91	146.32	146.27
5	200.16	—	196.11	—	177.10	—
6	313.62	—	296.95	—	211.26	—
7	354.55	356.50	342.10	346.61	278.48	278.56
8	361.72	360.51	362.91	360.04	293.56	295.52

Table 2
Natural frequencies for free coupled orthotropic plates (Hz)

E_1/E_2	4/3	3/4	3/2	2/3	2/1	1/2	3/1	1/3
1	21.97	19.13	21.95	18.09	21.90	15.69	21.85	13.05
2	29.33	29.25	28.61	28.51	27.11	26.95	25.45	25.25
3	69.11	60.30	68.97	56.91	67.96	49.40	64.26	40.44
4	73.02	72.19	71.39	70.25	68.51	66.18	67.42	61.74
5	104.70	109.62	99.01	105.94	86.12	94.27	70.54	77.56
6	114.32	111.96	112.38	107.15	101.96	98.07	87.50	89.23
7	120.78	128.83	114.83	126.94	108.26	125.04	103.70	124.16
8	137.67	135.19	137.24	134.53	136.63	133.09	136.16	128.23

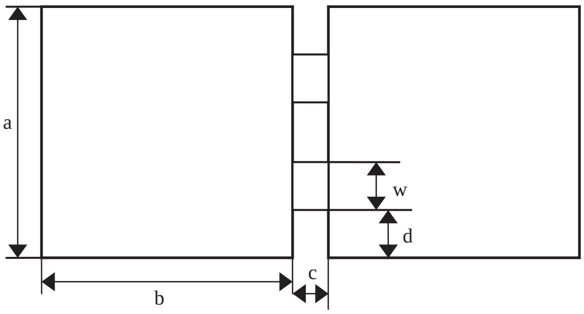

Fig. 1. Coupled rectangular plates.

4. Completely free coupled plates

The title problem of completely free coupled orthotropic rectangular plates (Fig. 1) is now considered. The geometry represents a mechanical model for pairs of satellite panels that are connected by two integral coupling plates. In the analysis the connectors were modelled by the same plate element used for the panels. Results are determined for systems with the following values of the geometric and material parameters (Fig. 1); $a = b = 18$ in, $c = d = w = 3$ in, $h = 0.1875$ in, $E_1 = 10.03 \times 10^6$ psi, $\sqrt{v_1 v_2} = 0.25$, $\rho = 0.2583 \times 10^{-3}$ lb s^2/in^4. Results are given for a number of different values of the E_1/E_2 ratio.

The first eight natural frequencies for this plate structure are given in Table 2 for eight different values of the E_1/E_2 ratio. The '1' direction here is parallel to the direction of the couplers. The values of the table are the ones obtained from the PLATEZ05 program. It is seen that the fundamental frequency corresponding to a specified aspect ratio $E_1/E_2 > 1$ is larger than the fundamental frequency corresponding to the inverse E_1/E_2 ratio. Representative mode shapes are given in Fig. 2.

A set of coupled isotropic plates with different coupler stiffnesses is considered next. Results are determined for systems with the following values of the geometric and material parameters: $a = b = 18$ in, $c = d = w = 3$ in, $h = 0.1875$ in, $E_p = 10.03 \times 10^6$ psi, $v = 0.334$, $\rho = 0.2583 \times 10^{-3}$ lb s^2/in^4. Results are given for a number of different values of the E_c/E_p ratio, where E_c represents the coupler elastic modulus, and E_p the plate modulus.

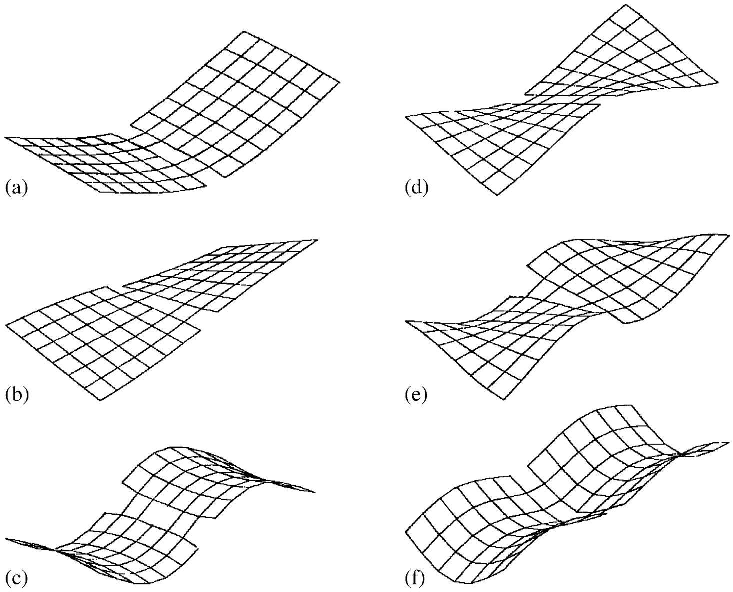

Fig. 2. Sample mode shapes for coupled plates: (a) mode 1, (b) mode 2, (c) mode 3, (d) mode 4, (e) mode 5, (f) mode 6.

Table 3
Effect of modulus of connectors on natural frequencies (Hz)

E_c/E_p	1/20	1/15	1/12	1/10	1/8	1/5	1/2	1/1
1	7.92	9.02	9.95	10.76	11.81	14.17	18.93	22.07
2	24.60	25.47	26.07	26.50	26.99	27.87	29.34	30.44
3	61.16	62.17	64.12	64.92	65.76	67.10	68.70	69.45
4	72.79	72.91	73.04	73.15	73.31	73.73	74.86	75.86
5	99.30	102.15	104.18	105.71	107.43	110.51	115.03	117.45
6	106.62	107.16	107.67	108.16	108.85	110.64	115.19	118.59

The first six natural frequencies for this plate are given in Table 3 for six different values of the E_c/E_p ratio. The values of the table are the results from the PLATEZ05 program. It is seen that the fundamental frequency increases steadily as the aspect ratio E_c/E_p is increased.

5. Conclusions

A solution for the natural frequencies of completely free mechanically coupled rectangular plates has been presented using the finite element method. A validation study indicates that the current method gives results for single plates that agree closely with previously published results. For coupled systems consisting of orthotropic plates larger fundamental frequencies are obtained when the larger stiffness is in the direction of the connectors. Increase of the stiffness of the connectors in coupled plates leads to an increase in the natural frequency. Overall, the results of the study provide confirmation of previous work on single completely free orthotropic plates, and give useful data for the specific geometry of coupled satellite panels.

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