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Letter to the Editor

Transverse vibrations of simply supported rectangular plates with two rectangular cutouts

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1. Introduction

This note presents a series of numerical experiments performed on vibrating simply supported rectangular plates with two rectangular holes with free edges; see Figs. 1–3. The geometric configurations under study constitute triply connected domains and, apparently no exact solutions appear to be possible like the case of Laplace's equation when dealing with steady state diffusion-type problems.

From the point of view of plates executing transverse vibrations the problem is of direct technological interest since holes are practiced in plates or slabs in order to allow for the passage of ducts, conduits, cables, etc. and the designer must (or should) know the effect of these perturbations upon the dynamic characteristics of the structural element.

The present study is an extension of previous studies [1–3] and shows that the algorithmic procedure previously developed is efficient and accurate in the case of triply connected configurations.

The methodology of solution is quite simple and straightforward: it constitutes in the deduction from the energy functional corresponding to the full plate the subsidiary energy functional corresponding to the two holes. The Rayleigh–Ritz method is then applied. The approach yields reasonable results as long as the holes are placed sufficiently apart from each other and their sizes are moderate when compared with the plate of smaller dimension (less than 20%). Possibly the approach may be also applicable if more than two holes are practiced but one should be extremely careful with the validity of the physic–mathematical model. This also applies if the holes degenerate into slits: the approach will not be valid in this case.

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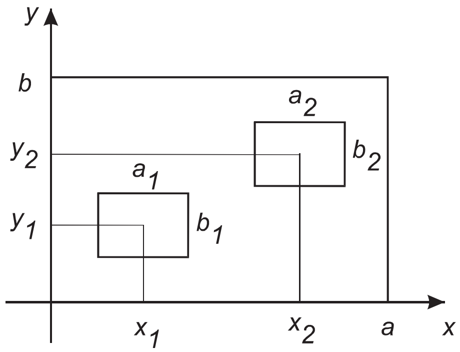

Fig. 1. Triply connected plate executing transverse vibrations.

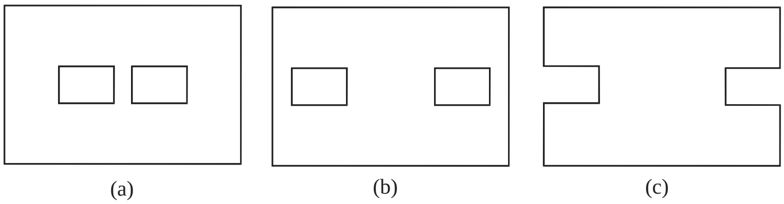

Fig. 2. Case of holes displacing along the middle horizontal axis of the plate.

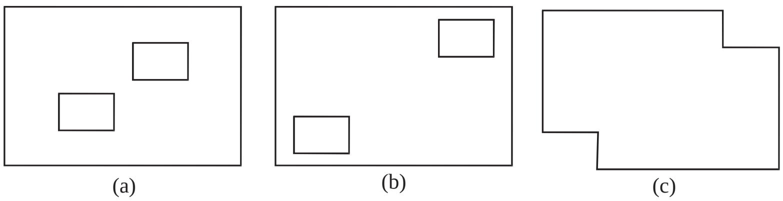

Fig. 3. Case of holes displacing along the plate diagonal.

Calculations are performed for isotropic, orthotropic and anisotropic plates. The lower four natural frequencies are determined. Excellent numerical stability is observed. As expected the frequencies are lower than those corresponding to a solid plate (no dynamic stiffening effect is observed for the situations under study).

2. Approximate analytical solution

For the rectangular plate under study, depicted in Fig. 1, the Rayleigh–Ritz variational approach requires minimization of the functional

$$J[W'] = U[W'] - T[W'], \tag{1}$$

Table 1
Values of the first four frequency coefficients in the case of an isotropic rectangular plate of aspect ratio 2/3 for two different sizes of the cutouts when they are displaced along the horizontal middle line (Fig. 2) and along the diagonal (Fig. 3)

Size of the cutouts	Cutouts position	Ω_1	Ω_2	Ω_3	Ω_4
	Fig. 2(a)–(c)				
$a_1/a = 0.1 = b_1/b$	$x_1/a = 0.40$; $y_1/b = 0.50$	31.097	61.167	98.082	110.33
$a_2/a = 0.1 = b_2/b$	$x_2/a = 0.60$; $y_2/b = 0.50$				
	$x_1/a = 0.20$; $y_1/b = 0.50$	31.691	60.597	98.097	109.62
	$x_2/a = 0.80$; $y_2/b = 0.50$				
	$x_1/a = 0.05$; $y_1/b = 0.50$	32.035	61.449	98.082	110.23
	$x_2/a = 0.95$; $y_2/b = 0.50$				
	Fig. 2(a)–(c)				
$a_1/a = 0.2 = b_1/b$	$x_1/a = 0.35$; $y_1/b = 0.50$	29.941	60.222	89.175	105.83
$a_2/a = 0.2 = b_2/b$	$x_2/a = 0.65$; $y_2/b = 0.50$				
	$x_1/a = 0.20$; $y_1/b = 0.50$	30.957	60.550	93.121	111.59
	$x_2/a = 0.80$; $y_2/b = 0.50$				
	$x_1/a = 0.10$; $y_1/b = 0.50$	31.558	60.003	94.800	109.47
	$x_2/a = 0.90$; $y_2/b = 0.50$				
	Fig. 3(a)–(c)				
$a_1/a = 0.1 = b_1/b$	$x_1/a = 0.40$; $y_1/b = 0.40$	31.207	61.152	97.199	110.24
$a_2/a = 0.1 = b_2/b$	$x_2/a = 0.60$; $y_2/b = 0.60$				
	$x_1/a = 0.20$; $y_1/b = 0.20$	31.730	61.167	97.503	110.31
	$x_2/a = 0.80$; $y_2/b = 0.80$				
	$x_1/a = 0.05$; $y_1/b = 0.05$	31.613	60.761	98.082	109.86
	$x_2/a = 0.95$; $y_2/b = 0.95$				
	Fig. 3(a)–(c)				
$a_1/a = 0.2 = b_1/b$	$x_1/a = 0.35$; $y_1/b = 0.35$	30.105	60.035	92.863	105.49
$a_2/a = 0.2 = b_2/b$	$x_2/a = 0.65$; $y_2/b = 0.65$				
	$x_1/a = 0.20$; $y_1/b = 0.20$	30.605	59.847	96.214	108.87
	$x_2/a = 0.80$; $y_2/b = 0.80$				
	$x_1/a = 0.10$; $y_1/b = 0.10$	30.285	58.355	96.003	106.55
	$x_2/a = 0.90$; $y_2/b = 0.90$				

where $U[W']$ is the maximum strain energy and $T[W']$ is the maximum kinetic energy for the (true) displacement amplitude W' of the plate.

As has been shown elsewhere, see for example Ref. [4], in the case of a plate of general anisotropy, that each term in Eq. (1) can be written

$$U[W'] = \frac{1}{2}\iint \left\{ D_{11}\left(\frac{\partial^2 W'}{\partial x'^2}\right)^2 + 2D_{12}\frac{\partial^2 W'}{\partial x'^2}\frac{\partial^2 W'}{\partial y'^2} + D_{22}\left(\frac{\partial^2 W'}{\partial y'^2}\right)^2 + 4D_{66}\left(\frac{\partial^2 W'}{\partial x'\partial y'}\right)^2 + 4\left[D_{16}\left(\frac{\partial^2 W'}{\partial x'^2}\right) + D_{26}\left(\frac{\partial^2 W'}{\partial y'^2}\right)\right]\left(\frac{\partial^2 W'}{\partial x'\partial y'}\right)\right\} \mathrm{d}x'\mathrm{d}y', \tag{2}$$

Table 2
Values of the first four frequency coefficients in the case of an isotropic square plate for two different sizes of the cutouts when they are displaced along the horizontal middle line (Fig. 2) and along the diagonal (Fig. 3)

Size of the cutouts	Cutouts position	Ω_1	Ω_2	Ω_3	Ω_4
	Fig. 2(a)–(c)				
$a_1/a = 0.1 = b_1/b$	$x_1/a = 0.40$; $y_1/b = 0.50$	19.324	48.769	49.136	78.035
$a_2/a = 0.1 = b_2/b$	$x_2/a = 0.60$; $y_2/b = 0.50$				
	$x_1/a = 0.20$; $y_1/b = 0.50$	19.550	48.265	48.946	78.621
	$x_2/a = 0.80$; $y_2/b = 0.50$				
	$x_1/a = 0.05$; $y_1/b = 0.50$	19.707	48.816	49.035	77.722
	$x_2/a = 0.95$; $y_2/b = 0.50$				
	Fig. 2(a)–(c)				
$a_1/a = 0.2 = b_1/b$	$x_1/a = 0.35$; $y_1/b = 0.50$	19.066	46.324	47.300	74.957
$a_2/a = 0.2 = b_2/b$	$x_2/a = 0.65$; $y_2/b = 0.50$				
	$x_1/a = 0.20$; $y_1/b = 0.50$	19.121	47.009	47.778	75.402
	$x_2/a = 0.80$; $y_2/b = 0.50$				
	$x_1/a = 0.10$; $y_1/b = 0.50$	19.332	46.972	47.058	74.066
	$x_2/a = 0.90$; $y_2/b = 0.50$				
	Fig. 3(a)–(c)				
$a_1/a = 0.1 = b_1/b$	$x_1/a = 0.40$; $y_1/b = 0.40$	19.339	48.605	49.050	78.160
$a_2/a = 0.1 = b_2/b$	$x_2/a = 0.60$; $y_2/b = 0.60$				
	$x_1/a = 0.20$; $y_1/b = 0.20$	19.527	48.660	49.160	78.074
	$x_2/a = 0.80$; $y_2/b = 0.80$				
	$x_1/a = 0.05$; $y_1/b = 0.05$	19.402	48.316	49.347	77.644
	$x_2/a = 0.95$; $y_2/b = 0.95$				
	Fig. 3(a)–(c)				
$a_1/a = 0.2 = b_1/b$	$x_1/a = 0.35$; $y_1/b = 0.35$	18.902	46.988	48.308	77.417
$a_2/a = 0.2 = b_2/b$	$x_2/a = 0.65$; $y_2/b = 0.65$				
	$x_1/a = 0.20$; $y_1/b = 0.20$	18.863	47.980	48.511	79.667
	$x_2/a = 0.80$; $y_2/b = 0.80$				
	$x_1/a = 0.10$; $y_1/b = 0.10$	18.503	45.816	49.191	74.792
	$x_2/a = 0.90$; $y_2/b = 0.90$				

where the well-established Lekhnitskii's notation [4] for the flexural rigidities D_{ij} of the plate has been used, and

$$T[W'] = \frac{\rho\omega^2 h}{2} \iint W'^2 \, \mathrm{d}x' \, \mathrm{d}y'. \tag{3}$$

The integrals in expressions (2) and (3) extend over the actual area of the triply connected plate under study.

Taking the lengths of the sides of the rectangular plate to be a and b in the x and y directions respectively, and introducing the non-dimensional variables

$$W = W'/a; \quad x = x'/a; \quad y = y'/b \quad \text{and} \quad r = b/a, \tag{4}$$

Table 3
Values of the first four frequency coefficients in the case of an isotropic rectangular plate of aspect ratio 3/2 for two different sizes of the cutouts when they are displaced along the horizontal middle line (Fig. 2) and along the diagonal (Fig. 3)

Size of the cutouts	Cutouts position	Ω_1	Ω_2	Ω_3	Ω_4
	Fig. 2(a)–(c)				
$a_1/a = 0.1 = b_1/b$	$x_1/a = 0.40$; $y_1/b = 0.50$	13.910	27.324	43.050	48.855
$a_2/a = 0.1 = b_2/b$	$x_2/a = 0.60$; $y_2/b = 0.50$				
	$x_1/a = 0.20$; $y_1/b = 0.50$	14.074	27.128	42.417	49.058
	$x_2/a = 0.80$; $y_2/b = 0.50$				
	$x_1/a = 0.05$; $y_1/b = 0.50$	14.214	26.996	43.371	49.230
	$x_2/a = 0.95$; $y_2/b = 0.50$				
	Fig. 2(a)–(c)				
$a_1/a = 0.2 = b_1/b$	$x_1/a = 0.35$; $y_1/b = 0.50$	13.714	26.261	40.566	50.488
$a_2/a = 0.2 = b_2/b$	$x_2/a = 0.65$; $y_2/b = 0.50$				
	$x_1/a = 0.20$; $y_1/b = 0.50$	13.542	26.066	41.199	48.582
	$x_2/a = 0.80$; $y_2/b = 0.50$				
	$x_1/a = 0.10$; $y_1/b = 0.50$	13.722	25.800	40.136	48.363
	$x_2/a = 0.90$; $y_2/b = 0.50$				
	Fig. 3(a)–(c)				
$a_1/a = 0.1 = b_1/b$	$x_1/a = 0.40$; $y_1/b = 0.40$	13.871	27.175	43.199	48.996
$a_2/a = 0.1 = b_2/b$	$x_2/a = 0.60$; $y_2/b = 0.60$				
	$x_1/a = 0.20$; $y_1/b = 0.20$	14.097	27.183	43.332	49.027
	$x_2/a = 0.80$; $y_2/b = 0.80$				
	$x_1/a = 0.05$; $y_1/b = 0.05$	14.050	27.003	43.589	48.832
	$x_2/a = 0.95$; $y_2/b = 0.95$				
	Fig. 3(a)–(c)				
$a_1/a = 0.2 = b_1/b$	$x_1/a = 0.35$; $y_1/b = 0.35$	13.378	26.683	41.269	46.886
$a_2/a = 0.2 = b_2/b$	$x_2/a = 0.65$; $y_2/b = 0.65$				
	$x_1/a = 0.20$; $y_1/b = 0.20$	13.605	26.597	42.761	48.386
	$x_2/a = 0.80$; $y_2/b = 0.80$				
	$x_1/a = 0.10$; $y_1/b = 0.10$	13.457	25.933	42.667	47.355
	$x_2/a = 0.90$; $y_2/b = 0.90$				

Eqs. (2) and (3) above can be recast in a non-dimensional form. One gets for the functional for the whole system of Fig. 1,

$$\begin{aligned}
J_{nd} &= \frac{2J}{rD_{11}} \\
&= \iint \left\{ \left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \frac{2d_{12}}{r^2}\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} \right. \\
&\quad + \frac{d_{22}}{r^4}\left(\frac{\partial^2 W}{\partial y^2}\right)^2 + \frac{4d_{66}}{r^2}\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 \\
&\quad \left. + 4\left[\frac{d_{16}}{r}\left(\frac{\partial^2 W}{\partial x^2}\right) + \frac{d_{26}}{r^3}\left(\frac{\partial^2 W}{\partial y^2}\right)\right]\left(\frac{\partial^2 W}{\partial x \partial y}\right)\right\} \mathrm{d}x\,\mathrm{d}y \\
&\quad - \Omega^2 \iint W^2 \mathrm{d}x\mathrm{d}y, \qquad (5)
\end{aligned}$$

Table 4
Values of the first four frequency coefficients in the case of an orthotropic rectangular plate of aspect ratio 2/3 for two different sizes of the cutouts when they are displaced along the horizontal middle line (Fig. 2) and along the diagonal (Fig. 3)

Size of the cutouts	Cutouts position	Ω_1	Ω_2	Ω_3	Ω_4
	Fig. 2(a)–(c)				
$a_1/a = 0.1 = b_1/b$	$x_1/a = 0.40$; $y_1/b = 0.50$	30.019	63.746	79.128	114.80
$a_2/a = 0.1 = b_2/b$	$x_2/a = 0.60$; $y_2/b = 0.50$				
	$x_1/a = 0.20$; $y_1/b = 0.50$	30.144	63.691	78.652	114.60
	$x_2/a = 0.80$; $y_2/b = 0.50$				
	$x_1/a = 0.05$; $y_1/b = 0.50$	30.207	63.785	78.363	114.64
	$x_2/a = 0.95$; $y_2/b = 0.50$				
	Fig. 2(a)–(c)				
$a_1/a = 0.2 = b_1/b$	$x_1/a = 0.35$; $y_1/b = 0.50$	30.363	64.488	74.113	110.80
$a_2/a = 0.2 = b_2/b$	$x_2/a = 0.65$; $y_2/b = 0.50$				
	$x_1/a = 0.20$; $y_1/b = 0.50$	30.066	65.496	75.074	114.51
	$x_2/a = 0.80$; $y_2/b = 0.50$				
	$x_1/a = 0.10$; $y_1/b = 0.50$	29.988	63.496	74.847	111.08
	$x_2/a = 0.90$; $y_2/b = 0.50$				
	Fig. 3(a)–(c)				
$a_1/a = 0.1 = b_1/b$	$x_1/a = 0.40$; $y_1/b = 0.40$	30.074	63.660	78.855	114.69
$a_2/a = 0.1 = b_2/b$	$x_2/a = 0.60$; $y_2/b = 0.60$				
	$x_1/a = 0.20$; $y_1/b = 0.20$	29.863	63.683	79.011	114.72
	$x_2/a = 0.80$; $y_2/b = 0.80$				
	$x_1/a = 0.05$; $y_1/b = 0.05$	29.449	62.457	78.472	112.81
	$x_2/a = 0.95$; $y_2/b = 0.95$				
	Fig. 3(a)–(c)				
$a_1/a = 0.2 = b_1/b$	$x_1/a = 0.35$; $y_1/b = 0.35$	30.160	63.636	77.957	110.11
$a_2/a = 0.2 = b_2/b$	$x_2/a = 0.65$; $y_2/b = 0.65$				
	$x_1/a = 0.20$; $y_1/b = 0.20$	28.785	63.027	78.339	112.88
	$x_2/a = 0.80$; $y_2/b = 0.80$				
	$x_1/a = 0.10$; $y_1/b = 0.10$	27.589	59.417	76.769	108.53
	$x_2/a = 0.90$; $y_2/b = 0.90$				

where, as usual, $\Omega_i = \sqrt{\rho h/D_{11}}\omega_i a^2$ is the non-dimensional frequency coefficient and $d_{ij} = D_{ij}/D_{11}$ for $(i,j) = (1,2,6)$.

Expressing the displacement amplitude $W(x,y)$ in terms of a double Fourier series,

$$W(x,y) \cong W_a(x,y) = \sum_{n=1}^{N}\sum_{m=1}^{M} b_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{6}$$

and minimizing the governing functional with respect to the b_{mn}s, expression (5) yields an $(M \times N)$ homogeneous, linear system of equations in the b_{mn}s. A secular determinant in the natural frequency coefficients of the system results from the non-triviality condition. The present

Table 5
Values of the first four frequency coefficients in the case of an orthotropic square plate for two different sizes of the cutouts when they are displaced along the horizontal middle line (Fig. 2) and along the diagonal (Fig. 3)

Size of the cutouts	Cutouts position	Ω_1	Ω_2	Ω_3	Ω_4
	Fig. 2(a)–(c)				
$a_1/a = 0.1 = b_1/b$	$x_1/a = 0.40$; $y_1/b = 0.50$	19.894	43.292	50.800	78.605
$a_2/a = 0.1 = b_2/b$	$x_2/a = 0.60$; $y_2/b = 0.50$				
	$x_1/a = 0.20$; $y_1/b = 0.50$	19.925	42.824	50.542	99.816
	$x_2/a = 0.80$; $y_2/b = 0.50$				
	$x_1/a = 0.05$; $y_1/b = 0.50$	19.964	42.527	50.941	78.042
	$x_2/a = 0.95$; $y_2/b = 0.50$				
	Fig. 2(a)–(c)				
$a_1/a = 0.2 = b_1/b$	$x_1/a = 0.35$; $y_1/b = 0.50$	20.292	41.324	50.207	76.183
$a_2/a = 0.2 = b_2/b$	$x_2/a = 0.65$; $y_2/b = 0.50$				
	$x_1/a = 0.20$; $y_1/b = 0.50$	19.785	40.777	50.972	77.441
	$x_2/a = 0.80$; $y_2/b = 0.50$				
	$x_1/a = 0.10$; $y_1/b = 0.50$	19.699	40.105	49.527	74.214
	$x_2/a = 0.90$; $y_2/b = 0.50$				
	Fig. 3(a)–(c)				
$a_1/a = 0.1 = b_1/b$	$x_1/a = 0.40$; $y_1/b = 0.40$	19.894	43.285	50.808	78.910
$a_2/a = 0.1 = b_2/b$	$x_2/a = 0.60$; $y_2/b = 0.60$				
	$x_1/a = 0.20$; $y_1/b = 0.20$	19.738	43.308	50.894	79.175
	$x_2/a = 0.80$; $y_2/b = 0.80$				
	$x_1/a = 0.05$; $y_1/b = 0.05$	19.457	42.496	50.496	76.894
	$x_2/a = 0.95$; $y_2/b = 0.95$				
	Fig. 3(a)–(c)				
$a_1/a = 0.2 = b_1/b$	$x_1/a = 0.35$; $y_1/b = 0.35$	19.988	43.175	50.425	75.675
$a_2/a = 0.2 = b_2/b$	$x_2/a = 0.65$; $y_2/b = 0.65$				
	$x_1/a = 0.20$; $y_1/b = 0.20$	19.027	42.894	50.464	77.753
	$x_2/a = 0.80$; $y_2/b = 0.80$				
	$x_1/a = 0.10$; $y_1/b = 0.10$	18.214	40.378	49.480	73.691
	$x_2/a = 0.90$; $y_2/b = 0.90$				

study is concerned with the determination of the first four frequency coefficients, Ω_1–Ω_4, in the case of plates with two rectangular cutouts.

3. Numerical results

All calculations were performed for simply supported rectangular plates of uniform thickness and for three different types of constitutive relations, i.e., isotropic, orthotropic and general anisotropic plates. For simplicity, in all cases the cutouts have been chosen to be of the same aspect ratio as the original whole plate. For each situation, three tables are presented, each with a

Table 6
Values of the first four frequency coefficients in the case of an orthotropic rectangular plate of aspect ratio 3/2 for two different sizes of the cutouts when they are displaced along the horizontal middle line (Fig. 2) and along the diagonal (Fig. 3)

Size of the cutouts	Cutouts position	Ω_1	Ω_2	Ω_3	Ω_4
	Fig. 2(a)–(c)				
$a_1/a = 0.1 = b_1/b$	$x_1/a = 0.40$; $y_1/b = 0.50$	14.628	26.394	43.527	44.246
$a_2/a = 0.1 = b_2/b$	$x_2/a = 0.60$; $y_2/b = 0.50$				
	$x_1/a = 0.20$; $y_1/b = 0.50$	14.707	26.035	59.089	65.316
	$x_2/a = 0.80$; $y_2/b = 0.50$				
	$x_1/a = 0.05$; $y_1/b = 0.50$	14.785	25.792	43.332	44.480
	$x_2/a = 0.95$; $y_2/b = 0.50$				
	Fig. 2(a)–(c)				
$a_1/a = 0.2 = b_1/b$	$x_1/a = 0.35$; $y_1/b = 0.50$	14.738	25.472	42.324	45.222
$a_2/a = 0.2 = b_2/b$	$x_2/a = 0.65$; $y_2/b = 0.50$				
	$x_1/a = 0.20$; $y_1/b = 0.50$	14.308	24.683	42.472	43.082
	$x_2/a = 0.80$; $y_2/b = 0.50$				
	$x_1/a = 0.10$; $y_1/b = 0.50$	14.347	24.042	55.808	62.964
	$x_2/a = 0.90$; $y_2/b = 0.50$				
	Fig. 3(a)–(c)				
$a_1/a = 0.1 = b_1/b$	$x_1/a = 0.40$; $y_1/b = 0.40$	14.605	26.402	43.378	44.347
$a_2/a = 0.1 = b_2/b$	$x_2/a = 0.60$; $y_2/b = 0.60$				
	$x_1/a = 0.20$; $y_1/b = 0.20$	14.644	26.386	43.316	44.488
	$x_2/a = 0.80$; $y_2/b = 0.80$				
	$x_1/a = 0.05$; $y_1/b = 0.05$	14.503	25.832	42.628	44.519
	$x_2/a = 0.95$; $y_2/b = 0.95$				
	Fig. 3(a)–(c)				
$a_1/a = 0.2 = b_1/b$	$x_1/a = 0.35$; $y_1/b = 0.35$	14.386	26.449	58.011	67.464
$a_2/a = 0.2 = b_2/b$	$x_2/a = 0.65$; $y_2/b = 0.65$				
	$x_1/a = 0.20$; $y_1/b = 0.20$	14.113	25.902	42.402	44.027
	$x_2/a = 0.80$; $y_2/b = 0.80$				
	$x_1/a = 0.10$; $y_1/b = 0.10$	13.707	24.488	41.027	43.503
	$x_2/a = 0.90$; $y_2/b = 0.90$				

different value of the aspect ratio b/a: 2/3, 1 (square plate) and 3/2 to make a total of nine tables. In each table, in turn, two different values for the sizes of the cutouts are taken as they are placed along the middle horizontal line of the plate and along its diagonal.

Tables 1–3 depict values for the first four frequency coefficients for an isotropic rectangular plate with its Poisson coefficient being $\mu = 0.3$. The same scheme is repeated in Tables 4–6 for an orthotropic rectangular plate, where $\mu_2 = 0.3$; $D_2/D_1 = 1/2$ and $D_k/D_1 = 1/2$. Finally in Tables 7–9, results for a rectangular plate of general anisotropy are depicted. In this case calculations were carried out taken $D_{12}/D_{11} = 0.3$; $D_{22}/D_{11} = D_{66}/D_{11} = 1/2$ and $D_{16}/D_{11} = D_{26}/D_{11} = 1/3$.

For the double Fourier series, Eq. (6), $N = M = 30$ has been used, that is to say a secular determinant of order 900 was generated for all situations. Although satisfactory convergence is

Table 7
Values of the first four frequency coefficients in the case of a rectangular plate of general anisotropy and aspect ratio 2/3 for two different sizes of the cutouts when they are displaced along the horizontal middle line (Fig. 2) and along the diagonal (Fig. 3)

Size of the cutouts	Cutouts position	Ω_1	Ω_2	Ω_3	Ω_4
	Fig. 2(a)–(c)				
$a_1/a = 0.1 = b_1/b$	$x_1/a = 0.40$; $y_1/b = 0.50$	26.902	54.105	76.433	90.792
$a_2/a = 0.1 = b_2/b$	$x_2/a = 0.60$; $y_2/b = 0.50$				
	$x_1/a = 0.20$; $y_1/b = 0.50$	26.988	54.050	76.097	90.902
	$x_2/a = 0.80$; $y_2/b = 0.50$				
	$x_1/a = 0.05$; $y_1/b = 0.50$	26.980	54.066	76.121	90.363
	$x_2/a = 0.95$; $y_2/b = 0.50$				
	Fig. 2(a)–(c)				
$a_1/a = 0.2 = b_1/b$	$x_1/a = 0.35$; $y_1/b = 0.50$	26.949	54.589	70.550	88.894
$a_2/a = 0.2 = b_2/b$	$x_2/a = 0.65$; $y_2/b = 0.50$				
	$x_1/a = 0.20$; $y_1/b = 0.50$	26.574	54.714	73.269	89.621
	$x_2/a = 0.80$; $y_2/b = 0.50$				
	$x_1/a = 0.10$; $y_1/b = 0.50$	26.402	53.105	73.300	86.777
	$x_2/a = 0.90$; $y_2/b = 0.50$				
	Fig. 3(a)–(c)				
$a_1/a = 0.1 = b_1/b$	$x_1/a = 0.40$; $y_1/b = 0.40$	27.035	54.042	75.800	90.816
$a_2/a = 0.1 = b_2/b$	$x_2/a = 0.60$; $y_2/b = 0.60$				
	$x_1/a = 0.20$; $y_1/b = 0.20$	27.042	54.019	76.472	91.082
	$x_2/a = 0.80$; $y_2/b = 0.80$				
	$x_1/a = 0.05$; $y_1/b = 0.05$	25.667	53.449	74.097	91.089
	$x_2/a = 0.95$; $y_2/b = 0.95$				
	Fig. 3(a)–(c)				
$a_1/a = 0.2 = b_1/b$	$x_1/a = 0.35$; $y_1/b = 0.35$	27.386	53.222	74.566	90.722
$a_2/a = 0.2 = b_2/b$	$x_2/a = 0.65$; $y_2/b = 0.65$				
	$x_1/a = 0.20$; $y_1/b = 0.20$	26.480	53.175	76.519	90.199
	$x_2/a = 0.80$; $y_2/b = 0.80$				
	$x_1/a = 0.10$; $y_1/b = 0.10$	24.496	52.394	71.738	90.535
	$x_2/a = 0.90$; $y_2/b = 0.90$				

achieved for $N = M = 20$, such high values of M and N have been used taking advantage of the speed of modern desktop computers. As usual, special care has been taken to manipulate such large determinants and 80 bits floating point variables (IEEE-standard temporary reals) have been used to satisfy accuracy requirements.

It is worth noting that computations are very stable and all frequency coefficients uniformly converge as the number of terms in the Fourier series is increased. Typically, the values for the frequency coefficients differ by less than 0.5% when M and N are increased from 20 to 30.

As a general conclusion one may say that the mathematical model seems to be quite realistic and accurate, within the realm of the classical theory of vibrating plates. Even though from a mathematical viewpoint it may be possible, in principle, to obtain correct results, it may not be

Table 8
Values of the first four frequency coefficients in the case of a square plate of general anisotropy for two different sizes of the cutouts when they are displaced along the horizontal middle line (Fig. 2) and along the diagonal (Fig. 3)

Size of the cutouts	Cutouts position	Ω_1	Ω_2	Ω_3	Ω_4
	Fig. 2(a)–(c)				
$a_1/a = 0.1 = b_1/b$	$x_1/a = 0.40$; $y_1/b = 0.50$	18.082	36.660	49.972	60.941
$a_2/a = 0.1 = b_2/b$	$x_2/a = 0.60$; $y_2/b = 0.50$				
	$x_1/a = 0.20$; $y_1/b = 0.50$	18.082	36.425	49.566	60.480
	$x_2/a = 0.80$; $y_2/b = 0.50$				
	$x_1/a = 0.05$; $y_1/b = 0.50$	18.089	36.402	49.902	59.449
	$x_2/a = 0.95$; $y_2/b = 0.50$				
	Fig. 2(a)–(c)				
$a_1/a = 0.2 = b_1/b$	$x_1/a = 0.35$; $y_1/b = 0.50$	18.308	35.636	47.621	60.949
$a_2/a = 0.2 = b_2/b$	$x_2/a = 0.65$; $y_2/b = 0.50$				
	$x_1/a = 0.20$; $y_1/b = 0.50$	17.746	35.191	48.910	58.691
	$x_2/a = 0.80$; $y_2/b = 0.50$				
	$x_1/a = 0.10$; $y_1/b = 0.50$	17.613	34.660	47.832	56.347
	$x_2/a = 0.90$; $y_2/b = 0.50$				
	Fig. 3(a)–(c)				
$a_1/a = 0.1 = b_1/b$	$x_1/a = 0.40$; $y_1/b = 0.40$	18.128	36.644	49.816	60.863
$a_2/a = 0.1 = b_2/b$	$x_2/a = 0.60$; $y_2/b = 0.60$				
	$x_1/a = 0.20$; $y_1/b = 0.20$	18.097	36.652	50.230	60.808
	$x_2/a = 0.80$; $y_2/b = 0.80$				
	$x_1/a = 0.05$; $y_1/b = 0.05$	17.191	36.277	48.496	60.910
	$x_2/a = 0.95$; $y_2/b = 0.95$				
	Fig. 3(a)–(c)				
$a_1/a = 0.2 = b_1/b$	$x_1/a = 0.35$; $y_1/b = 0.35$	18.402	35.949	49.503	60.847
$a_2/a = 0.2 = b_2/b$	$x_2/a = 0.65$; $y_2/b = 0.65$				
	$x_1/a = 0.20$; $y_1/b = 0.20$	17.683	35.980	50.347	60.035
	$x_2/a = 0.80$; $y_2/b = 0.80$				
	$x_1/a = 0.10$; $y_1/b = 0.10$	16.371	35.589	46.910	60.308
	$x_2/a = 0.90$; $y_2/b = 0.90$				

meaningful, from a structural mechanics viewpoint, to extend the procedure to a larger number of holes.

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Table 9
Values of the first four frequency coefficients in the case of a rectangular plate of general anisotropy and aspect ratio 3/2 for two different sizes of the cutouts when they are displaced along the horizontal middle line (Fig. 2) and along the diagonal (Fig. 3)

Size of the cutouts	Cutouts position	Ω_1	Ω_2	Ω_3	Ω_4
	Fig. 2(a)–(c)				
$a_1/a = 0.1 = b_1/b$	$x_1/a = 0.40$; $y_1/b = 0.50$	13.621	23.300	37.097	42.980
$a_2/a = 0.1 = b_2/b$	$x_2/a = 0.60$; $y_2/b = 0.50$				
	$x_1/a = 0.20$; $y_1/b = 0.50$	13.699	23.042	36.660	42.464
	$x_2/a = 0.80$; $y_2/b = 0.50$				
	$x_1/a = 0.05$; $y_1/b = 0.50$	13.769	22.910	36.175	43.214
	$x_2/a = 0.95$; $y_2/b = 0.50$				
	Fig. 2(a)–(c)				
$a_1/a = 0.2 = b_1/b$	$x_1/a = 0.35$; $y_1/b = 0.50$	13.613	22.613	38.292	39.832
$a_2/a = 0.2 = b_2/b$	$x_2/a = 0.65$; $y_2/b = 0.50$				
	$x_1/a = 0.20$; $y_1/b = 0.50$	13.175	21.933	35.621	41.238
	$x_2/a = 0.80$; $y_2/b = 0.50$				
	$x_1/a = 0.10$; $y_1/b = 0.50$	13.199	21.441	34.605	40.113
	$x_2/a = 0.90$; $y_2/b = 0.50$				
	Fig. 3(a)–(c)				
$a_1/a = 0.1 = b_1/b$	$x_1/a = 0.40$; $y_1/b = 0.40$	13.613	23.292	36.941	43.019
$a_2/a = 0.1 = b_2/b$	$x_2/a = 0.60$; $y_2/b = 0.60$				
	$x_1/a = 0.20$; $y_1/b = 0.20$	13.738	23.371	36.589	43.136
	$x_2/a = 0.80$; $y_2/b = 0.80$				
	$x_1/a = 0.05$; $y_1/b = 0.05$	13.292	22.558	36.417	43.058
	$x_2/a = 0.95$; $y_2/b = 0.95$				
	Fig. 3(a)–(c)				
$a_1/a = 0.2 = b_1/b$	$x_1/a = 0.35$; $y_1/b = 0.35$	13.433	23.105	36.308	41.074
$a_2/a = 0.2 = b_2/b$	$x_2/a = 0.65$; $y_2/b = 0.65$				
	$x_1/a = 0.20$; $y_1/b = 0.20$	13.316	22.894	35.574	42.636
	$x_2/a = 0.80$; $y_2/b = 0.80$				
	$x_1/a = 0.10$; $y_1/b = 0.10$	12.714	21.816	35.246	41.722
	$x_2/a = 0.90$; $y_2/b = 0.90$				

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