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Fully coupled vibrations of actively controlled drillstrings

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Abstract

A fully coupled model for axial, lateral, and torsional vibrations of actively controlled drillstrings is presented. The proposed model includes the mutual dependence of these vibrations, which arises due to bit/formation and drillstring/borehole wall interactions as well as other geometric and dynamic non-linearities. The active control strategy is based on optimal state feedback control designed to control the drillstring rotational motion. It is demonstrated by simulation results that bit motion causes torsional vibrations, which in turn excite axial and lateral vibrations resulting in bit bounce and impacts with the borehole wall. It is also shown that the results are in close qualitative agreement with field observations regarding stick–slip and axial vibrations and that the proposed control is effective in suppressing them. However, care must be taken in selecting a set of operating parameters to avoid transient instabilities in the axial and lateral motions.

1. Introduction

During drilling operations for oil wells, severe vibrations occur that are detrimental to the service life of drillstrings and down-hole equipment. The causes of these vibrations include impact and friction at the borehole/drillstring and bit/formation interfaces, unbalance, eccentricity or initial curvature in the drill collar sections, various linear or non-linear resonances, and energy exchange between various modes of vibration (i.e., axial, lateral, and torsional). These vibrations are generally quite complex in nature. They are intimately coupled together both linearly and non-linearly, and occur simultaneously [1–4]. For example, the high bit speed level caused by stick–slip torsional motion can excite severe axial and lateral vibrations in the bottom hole assembly, which may cause bit bounce, excessive bit wear and reduction in the penetration rate [5]. Stick–slip vibrations are self-excited, and generally disappear as the rotary table speed is increased beyond a

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threshold value. However, increasing the rotary speed may cause lateral problems such as backward and forward whirling, impacts with the borehole wall and parametric instabilities [6]. Therefore, it is desirable to extend the range of safe drilling speeds. In order to achieve this, a proper understanding of the coupled dynamics of drillstrings is necessary. For this reason drillstring vibrations and ways to control them have received a lot of attention in recent years. Early works focused on individual excitation mechanisms [7–9]. Realizing the importance of coupling between different modes of vibrations some researchers developed models that account for coupling between two of these modes (see e.g., Refs. [1,3,10–12]). Recently, Tucker and Wang [13], presented an integrated continuum model which account for full coupling between axial, lateral, and torsional vibrations and energy exchange between them.

The control methods include operational guidelines to avoid, eliminate or reduce torsional vibrations as well as active control methods using feedback [4,5,14–18]. Although most proposed control methods have been shown to be successful in controlling torsional vibrations, the effects of this control on lateral and axial vibrations have not been studied extensively. In order to design and implement an effective control system, a coupled model is essential for identifying the critical speeds as well as predicting the behavior of the whole system [5,13]. Recently, the authors proposed an optimal control strategy, which is applied to a model that considers the coupling between torsional and lateral vibrations [19]. The model was demonstrated to be quite realistic with respect to stick–slip vibrations, which were effectively eliminated through the feedback control scheme. Furthermore, the importance of including the axial vibrations was demonstrated in Ref. [20]. Though the contact and cutting mechanics at the bit were not consistently accounted for, the study showed that the presence of axial vibrations significantly affects the response. In the current paper, this model is completed to include consistent contact and cutting dynamics at the bit. The current model is "complete" in the sense that it includes all relevant dynamics and their interactions with the environment. Fully coupled axial, lateral, and torsional motions of the drillstring are accounted for through a lumped parameter model. The dynamics of the drive system is included. Drillstring/borehole wall and bit/formation interactions are considered through realistic contact/impact models and the cutting action at the bit is included through empirical relationships. Simulation results confirm that all vibration phenomena are intimately coupled and affect each other. Also it is shown that an effective feedback controller can be designed to reduce the vibrations to an acceptable level and maintain the set of desired operating conditions (e.g., WOB, ROP, bit speed), which are determined based on the coupled dynamics.

2. Fully coupled dynamics

A rotary drilling system is used for drilling deep wells for oil or gas production. The equations of motion of such system are obtained by using a simplified lumped parameter model. The essential components of the system and the necessary geometry used for the model are shown in Fig. 1. The equations originally developed in reference [19] are extended to include axial motion. Furthermore, the weight-on-bit (WOB) results from the contact characteristics between the bit and the formation, which is more realistic than just using a prescribed function. Similarly, the torque-on-bit (TOB) results from the cutting conditions at the bit. The resulting coupled equations are given as

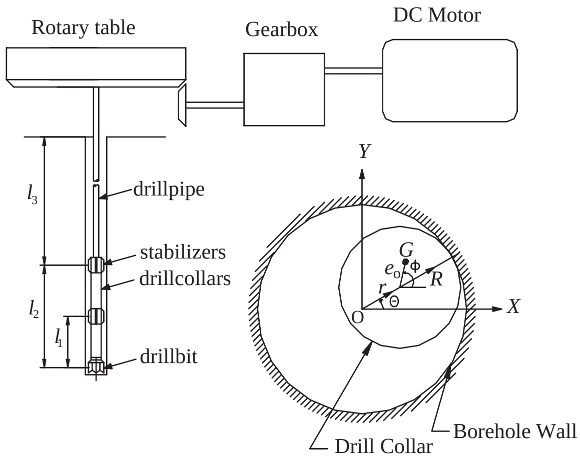

Fig. 1. Essential components of the system and the geometry used for modelling.

$$(m+m_f)(\ddot{r}-r\dot{\theta}^2)+k(x,\phi,\dot{\phi})r+c_h|v|\dot{r}=(m+m_f)e_0[\dot{\phi}^2\cos(\phi-\theta)+\ddot{\phi}\sin(\phi-\theta)]-F_r, \quad (1)$$

$$(m+m_f)(r\ddot{\theta}+2\dot{r}\dot{\theta})+c_h|v|r\dot{\theta}=(m+m_f)e_0[\dot{\phi}^2\sin(\phi-\theta)-\ddot{\phi}\cos(\phi-\theta)]+F_\theta, \quad (2)$$

$$J\ddot{\phi}+k_T(\phi-\phi_{rt})+c_v\dot{\phi}+c_h|v|\dot{r}e_0\sin(\phi-\theta)-c_h|v|r\dot{\theta}e_0\cos(\phi-\theta)$$
$$=-T(x,\phi,\dot{\phi})+F_\theta[R-e_0\cos(\phi-\theta)]-F_re_0\sin(\phi-\theta), \quad (3)$$

$$(J_{rt}+n^2J_m)\ddot{\phi}_{rt}+k_T(\phi_{rt}-\phi)+c_{rt}\dot{\phi}_{rt}-nK_mI=0, \quad (4)$$

$$L\dot{I}+R_mI+K_mn\dot{\phi}_{rt}=V_c, \quad (5)$$

$$m_a\ddot{x}+c_a\dot{x}+k_ax=-F(x,\phi)+\bar{F}. \quad (6)$$

The equivalent system parameters for the lumped model are derived from the associated continuous model of the drillstring by using a Lagrangian approach, and are given in the Appendix A. In most real applications the upper part of the bottom hole assembly (BHA) is in permanent contact with the borehall wall. On the other hand, several stabilizers support the lower part. Therefore, the lateral vibrations in the upper segments of the drillstring are decoupled from the lower segments and mainly the transverse motion of the stabilized section contributes to the coupled dynamics at the bit. Thus, the stabilized section of the drill collars is modelled as a simply supported beam for transverse motion. The whole drillstring is assumed to be fixed at the top, and free at the bit for the axial and torsional motion. The drill collars are assumed to be rigid for torsional vibrations, in other words, the torsional deformations are assumed to take place only in the drill pipe. This can be justified since the drill collars are much stiffer than the drill pipe in torsion [16]. $F(x,\phi)$ is the WOB given as

$$F(x,\phi)=WOB, \quad (7)$$

and $\bar{F}$ is the amount of gravitational force released to produce the applied WOB (i.e., the desired value of WOB), and given as

$$\bar{F} = k_a x(0) + F_0. \tag{8}$$

It is assumed that the drillstring is lowered slowly until a desired WOB is achieved. Therefore, Eq. (8) simply states that initially a portion of the drillstring weight is supported by the initial drill pipe tension and by the initial contact reaction at the bit, which is the applied WOB, F_0. Impact between the bit and the formation in axial direction (i.e., bit bounce) is accounted for by using a quasi-static model (i.e., large mass impact) with a linear contact stiffness, which can be obtained from the elastic properties and the yield strength of the formation [21]. The instantaneous value of WOB is given from the contact condition at the bit as

$$WOB = \begin{cases} k_c(x - s) & \text{if } x \geqslant s, \\ 0 & \text{if } x < s, \end{cases} \tag{9}$$

where s is the formation surface elevation given as [3,11]

$$s = s_0 \sin(n_b \phi). \tag{10}$$

The TOB is related to the WOB and cutting conditions and given as [11]

$$T(x, \phi, \dot{\phi}) = F(x, \phi)[\mu f(\dot{\phi}) + \varsigma \sqrt{r_h \delta_c}], \tag{11}$$

where δ_c is the depth of cut per revolution given as

$$\delta_c = \frac{2\pi ROP}{\omega_d}. \tag{12}$$

The average rate of penetration, ROP, is a function of applied WOB F_0, average bit speed ω_d and rock/bit characteristics given as [11]

$$ROP = c_1 F_0 \sqrt{\omega_d} + c_2. \tag{13}$$

The continuous function of $f(\dot{\phi})$ is used to represent the effect of bit speed on TOB and given as

$$f(\dot{\phi}) = \tanh\dot{\phi} + \frac{\alpha_1 \dot{\phi}}{(1 + \alpha_2 \dot{\phi}^2)}. \tag{14}$$

The constants μ and ζ characterize the friction process and the cutting action at the bit, respectively. The dependence of the WOB and TOB on the bit angular displacement and velocity, was initially proposed by the authors [19] as an extension of harmonic functions of time used in earlier studies. Although this assumption has provided some insight towards understanding the nature of stick–slip vibrations, it was incomplete since the effects of rate of penetration and contact at the bit/formation interface were not included. These effects are supported by earlier experimental and analytical studies [3,4,11–13].

With this model WOB and TOB are not constants or prescribed harmonic functions of time. Instead they result from the bit motion and the bit/formation interactions. As a consequence, linear phenomena such as whirling, and simple and parametric resonance, which are based on the assumption of harmonic excitations, are no longer straightforward. As WOB is the result of contact between the bit and the formation, also other intermittent external excitations such as the radial and transverse contact forces are the results of impact of the drill collars with the borehole wall. When there is no contact the associated contact forces are zero. The impacts between the drill collars and the borehole wall are accounted for by using the momentum balance method. This impact model is chosen because it is capable of accounting for rolling with and without slip of the drill collars along the borehole wall. This model assumes instantaneous impact. The effects of impact are accounted for by calculating jump discontinuities in velocities from the associated impulse-momentum equations.

The impulse-momentum equations are obtained by integrating the equations of motion with respect to time during the contact duration, which is assumed to be very short. Noting that the system configuration is continuous, as the contact duration is assumed to approach zero, the following momentum balance equations are obtained

$$(m + m_f)\Delta\dot{r} = -P_r + (m + m_f)e_0\Delta\dot{\phi}\sin(\phi - \theta), \tag{15}$$

$$(m + m_f)r\Delta\dot{\theta} = P_\theta - (m + m_f)e_0\Delta\dot{\phi}\cos(\phi - \theta), \tag{16}$$

$$J\Delta\dot{\phi} = [R - e_0\cos(\phi - \theta)]P_\theta - e_0\sin(\phi - \theta)P_r, \tag{17}$$

where $\Delta\dot{r}$, $\Delta\dot{\theta}$, and $\Delta\dot{\phi}$ are the jump discontinuities in the velocities, and P_r and P_θ are the impulses of impact forces F_r and F_θ, respectively. The rest of the velocities remain unchanged. The response of the system before an impact is obtained by numerical integration of the equations of motion. At each integration time step, the contact conditions are checked using a contact algorithm. If an impact is detected to occur, the integration is stopped and the momentum balance equations are formed and solved for the jump discontinuities in velocities. Then, the integration is restarted again with the new initial conditions. The use of this method is explained in detail elsewhere [6].

3. Reduced order model for controller design

In control design a linear model is preferred. Such a model can be obtained by linearizing the governing Eqs. (1)–(6). In this case, the lateral and axial motions become uncontrollable with the drive motor, and need not be included in the model. Thus, the following set of equations represents the reduced order linear model, which can be used for control design:

$$J\ddot{\phi} + k_T(\phi - \phi_{rt}) + c_v\dot{\phi} = 0, \tag{18}$$

$$(J_{rt} + n^2J_m)\ddot{\phi}_{rt} + k_T(\phi_{rt} - \phi) + c_{rt}\dot{\phi}_{rt} - nK_mI = 0, \tag{19}$$

$$L\dot{I} + R_mI + K_mn\dot{\phi}_{rt} = V_c. \tag{20}$$

It should be noted that the reduced order model is only used to design a linear controller. The resulting control is then applied to the full model to evaluate the performance of the proposed controller. The problem can be considered as a set point control problem where the objective is to bring the table and bit speeds to their desired values within a prescribed period of time when the system is disturbed by an external influence (e.g., bit torque). A linear quadratic regulator (LQR) controller can then be designed as explained in Ref. [19].

The control voltage necessary to keep the torsional vibrations zero while maintaining a desired bit and rotary table speed is given by

$$V_c = K_m n\omega_d - K_1(\phi - \phi_{rt}) - K_2(\phi_{rt} - \omega_d t) - K_3(\dot{\phi}_{rt} - \omega_d) - K_4(\dot{\phi} - \omega_d) - K_5 I. \quad (21)$$

A closer look at the terms in the control equation reveals that it is indeed a generalized form of some traditional control strategies. The first term is the open loop voltage to be applied in case no feedback is used (open-loop control). The second term is essentially a torque feedback. The third and the fourth terms are classical integral and proportional feedback terms applied to the rotary speed (PI control), the fifth term is the bit speed feedback, and the last term represents the current feedback. Except for the bit speed, all other quantities required for calculating the control voltage can easily be measured or determined. The bit speed measurement requires downhole equipment and may be the most challenging task. It is becoming a common practice, however, to use an instrumented bit, which makes this measurement possible. Even in the absence of such measurements, a state estimator can be designed to estimate the bit speed from the other measurements since it is observable through the other states.

Table 1
Parameters used in the simulations

Drillstring	Drilling mud
$E = 210\,\text{GPa}$	$\rho_f = 1500\,\text{kg/m}^3$
$\rho = 7850\,\text{kg/m}^3$	$C_D = 1$
$d_o = 0.2286\,\text{m}$	$C_A = 1.7$
$d_i = 0.0762\,\text{m}$	$\mu_f = 0.2\,\text{N s/m}^2$
$e_0 = 0.0127\,\text{m}$	
$l_1 = 19.81\,\text{m}$	Borehole
$l_2 = 200\,\text{m}$	
$l_3 = 2000\,\text{m}$	$E = 210\,\text{GPa}$
$\bar{d}_o = 0.127\,\text{m}$	$\rho = 7850\,\text{kg/m}^3$
$\bar{d}_1 = 0.095\,\text{m}$	$d_h = 0.4286\,\text{m}$
$c_a = 4000\,\text{N s/m}$	
Rotary drive system	
$J_{rt} = 930\,\text{kg m}^2$	$R_m = 0.01\,\Omega$
$J_m = 23\,\text{kg m}^2$	$L = 0.005\,\text{H}$
$c_{rt} = 0$	$K_m = 6\,\text{V s}$
$n = 7.2$	
Controller gains	
$K_1 = -20,\ K_2 = 10,\ K_3 = 1.6,\ K_4 = 5.7,\ K_5 = 0.013$	
Weight and torque on bit	
$F_0 = 100\,\text{kN}$	$s_0 = 0.001\,\text{m}$
$k_c = 25000\,\text{kN/m}$	$\mu = 0.3,\ \zeta = 0.1$
$\alpha_1 = 2.0$	$\alpha_2 = 1.0$
$c_1 = 1.35\text{e}{-8}$	$c_2 = -1.9\,\text{e}{-4}$
$n_b = 1$	

4. Results and discussion

The parameters used in the following simulations are shown in Table 1, and represent a typical case in oilwell drilling operations. The desired rotary table speed is chosen as 11.6 rad/s (110 rpm), which is within the common operating range for oilwell drilling. From a simple linear and uncoupled analysis for this setup, the critical frequencies for axial and torsional resonances are found to be 3.97 and 1.3 rad/s, respectively. The critical frequency for whirling resonance (due to bending) is found to be 6.65 rad/s. It is assumed that the rotary table and the bit are rotating at the desired speed when the bit is off bottom. When the bit starts to interact with the formation the system will inevitably be disturbed with the possibility of axial, torsional, and lateral vibrations, occurring simultaneously. Due to various linear and non-linear coupling terms, all critical frequencies will change. For instance, whirling frequency will be lower depending on the values of WOB and TOB. On the other hand, critical axial frequencies increase due to the contact stiffness. Therefore, determining a safe operating speed is not straightforward.

Figs. 2–6 show the response when there is no feedback control. Though the rotary table speed does not change appreciably, significant initial stick–slip behavior is seen at the bit as shown in Fig. 2. The qualitative agreement between this result and the field data observed in Ref. [14] is remarkable. The bit momentarily stops causing the TOB and the top torque to reach very high values (Fig. 4). When the bit starts slipping, the energy stored in the drillstring is released causing very large torsional and bending vibrations (Figs. 2 and 6). During this time the bit remains in contact with the formation as evidenced by non-zero WOB (see Fig. 3). It appears that some of the energy stored during stick phase is also transferred into axial vibrations, which in turn affects the magnitudes of WOB and TOB. Once the bit speed reaches a certain value (26.3 rad/s), very large axial vibrations are initiated as shown in Fig. 5. This is because of axial resonance since the critical axial frequency during contact is found to be 26.3 rad/s. As Fig. 3 clearly shows, this resonance causes the bit to lose contact with formation (bit bounce). Once the bit is off-bottom, the TOB and WOB are zero, and consequently, the critical frequencies for lateral and axial vibrations change. As can be seen from Fig. 6, the nature of the growth in the lateral vibrations, which led to several impacts between the collars and the borehole wall, suggest some form of transient instability, which may be caused by a momentary parametric resonance. Since there is no feedback, these large amplitude vibrations continue with continuous energy exchange between

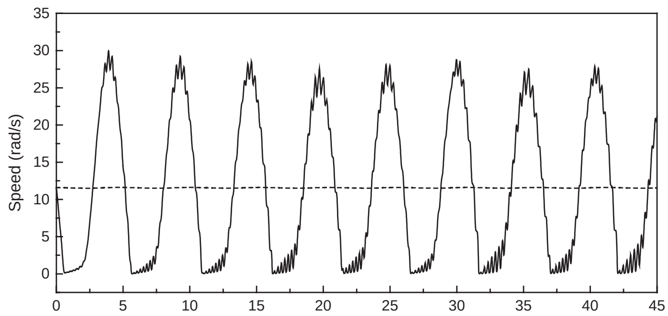

Fig. 2. Bit and rotary table speeds indicating stick–slip vibrations: —, bit speed; - - -, table speed.

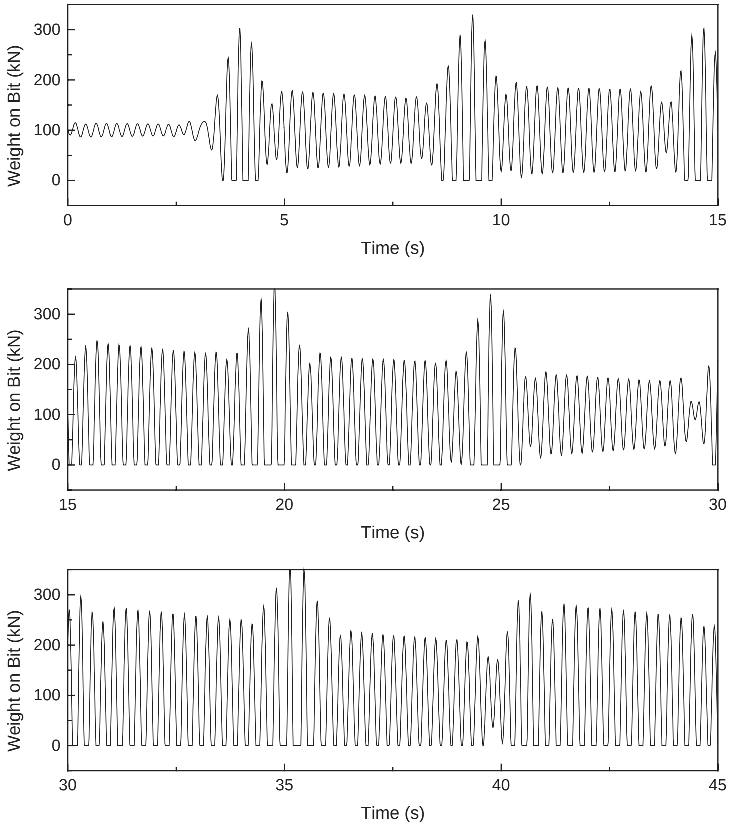

Fig. 3. WOB during indicating bit-bounce during stick–slip vibrations.

various modes of vibration. Clearly, this type of behavior will eventually lead to failure due to very large and often cyclic stresses present at different sections of the drillstring. These results show that controlling the rotary table speed alone will not help reducing the vibrations since in the above simulations this speed is almost constant. It appears that most of the downhole vibrations are driven by bit motion. Therefore, for an effective control of vibrations, the measurement and feedback of bit motion is required.

Figs. 7–11 show the response when feedback control involving downhole measurements is activated. With some initial transient behavior, all vibration amplitudes are reduced to acceptable levels. This result is remarkable considering the fact that only torsional motion is being controlled directly. All the other modes of vibrations are reduced since their sources of excitation are eliminated or the magnitudes of excitation inputs are lowered. The performance of the controller may further be improved if the model parameters are tuned by utilizing an open loop or closed loop identification scheme [22].

Clearly, the choice of the desired rotary table speed influences the system response. Thus, by proper identification of a safe operating speed range, and by a proper selection of controller gains, a smooth drilling can be achieved. It should be noted however, that this is not a straightforward task since an accurate knowledge of downhole conditions is required. Extensive simulations can

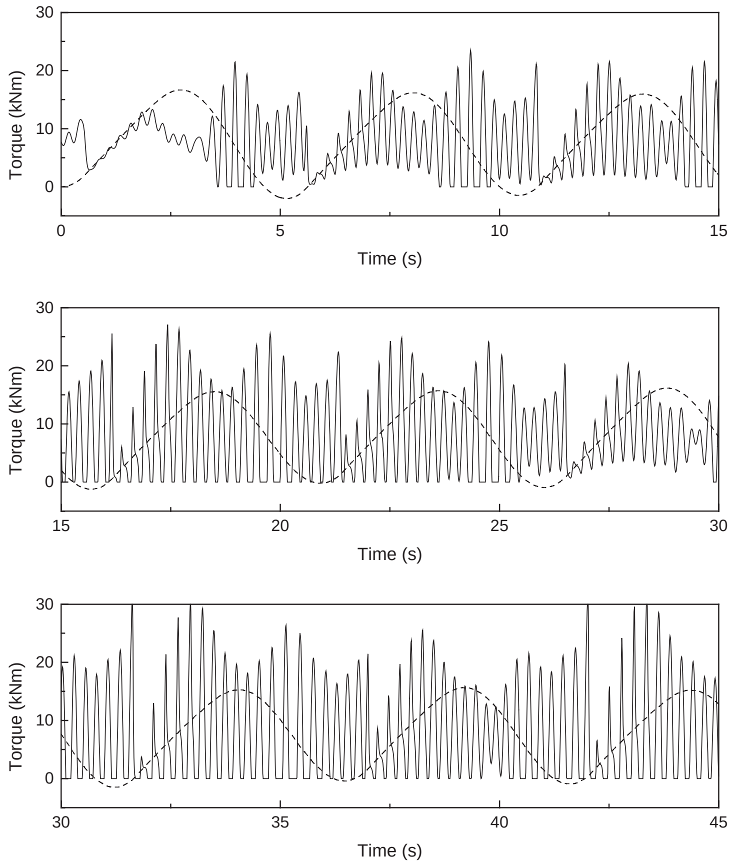

Fig. 4. TOB and top torque indicating stick–slip vibrations: —, TOB; - - -, top torque.

be carried out with fully coupled model investigating the dynamics for varying system parameters (e.g., varying formation stiffness and strength, cutting characteristics) to establish a safe operating range.

For the purpose of this paper one may examine the equations of motion and the contact conditions at the bit and come up with some helpful insight. It is obvious from the simulation results presented earlier that usually, at desired drilling conditions damaging coupled vibrations will occur and some sort of control is necessary to maintain a smooth drilling operation. It is possible, however, that the bit will encounter a different environment and otherwise smooth operation may cease to exist with the constant controller settings and operating conditions such as desired rotary table speed or applied WOB. For example, assume that the bit enters a softer formation. By keeping the applied WOB the same, it is clear that axial critical frequency will be lowered due to lower contact stiffness. If the bit speed reaches this new critical frequency, then, large amplitude axial vibrations will occur causing large fluctuations in the WOB and TOB, which in turn may cause bit bounce or stick–slip vibrations. Also, the fluctuations in the WOB may cause parametric instabilities in the lateral motion. Furthermore, variations in the cutting power of the bit by changing frictional and cutting parameters may cause variations in the TOB.

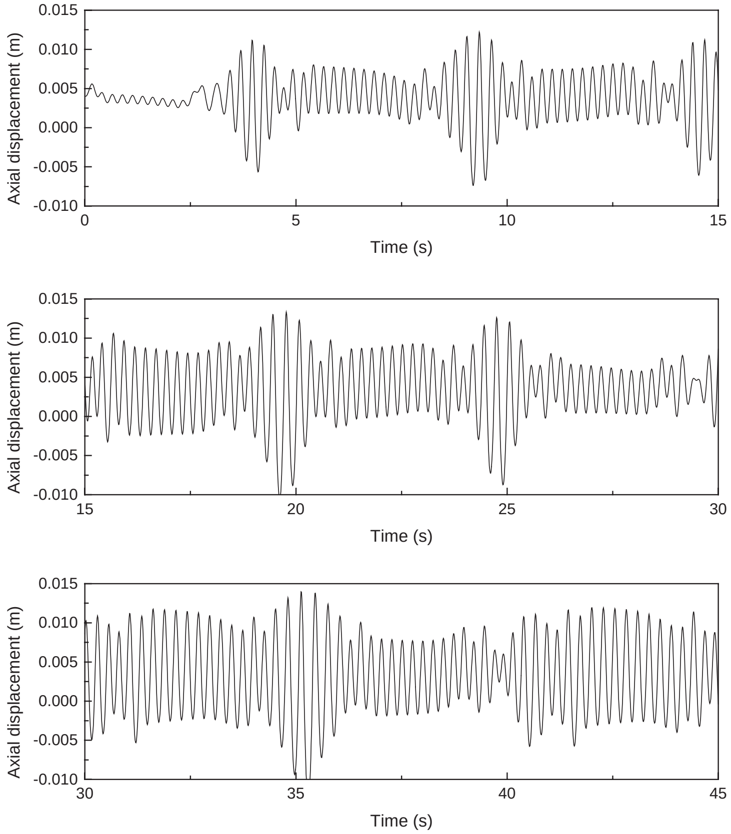

Fig. 5. Axial vibrations at the bit during stick–slip vibrations.

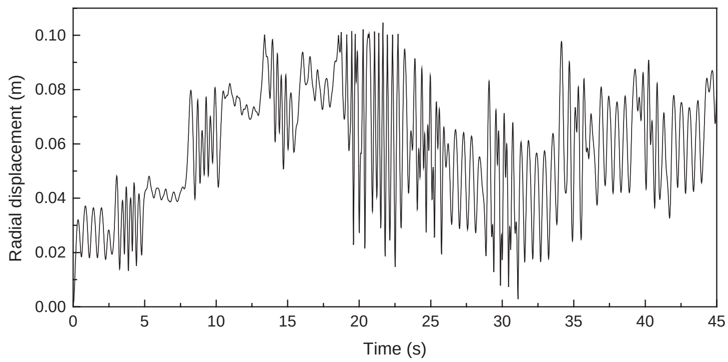

Fig. 6. Effect of axial and torsional vibrations on the lateral motion.

In closing, though the model presented here is relatively simple it seems to capture the basic characteristics of coupled drillstring dynamics. The utility of such analytical models lies in the explanation of various phenomena through simple observations and parametric studies. It is well

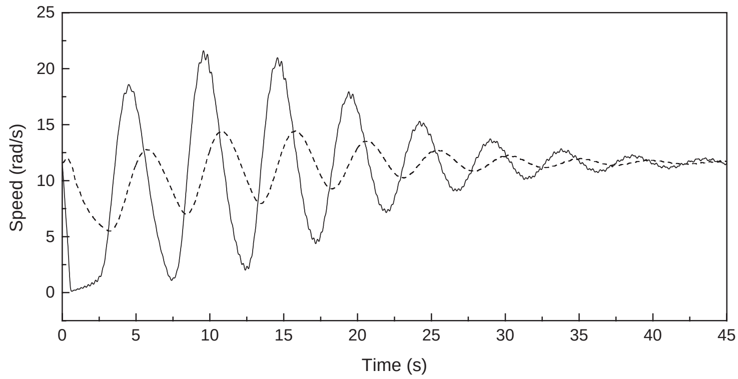

Fig. 7. Bit and rotary table speeds with active control; key as in Fig. 2.

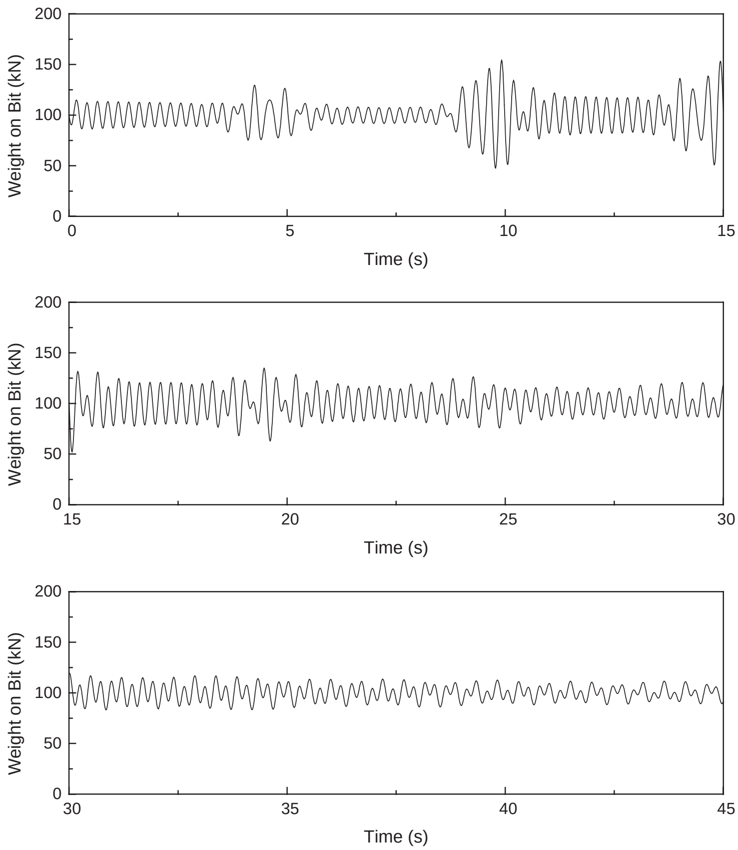

Fig. 8. WOB with active control showing a smooth drilling at the steady state.

known in the drilling industry that initially what appears to be a smooth operation, more often than not, unexpectedly results in catastrophic failure. This is probably because of a lack of proper understanding of the coupled dynamics, or simply due to changing conditions in the drilling

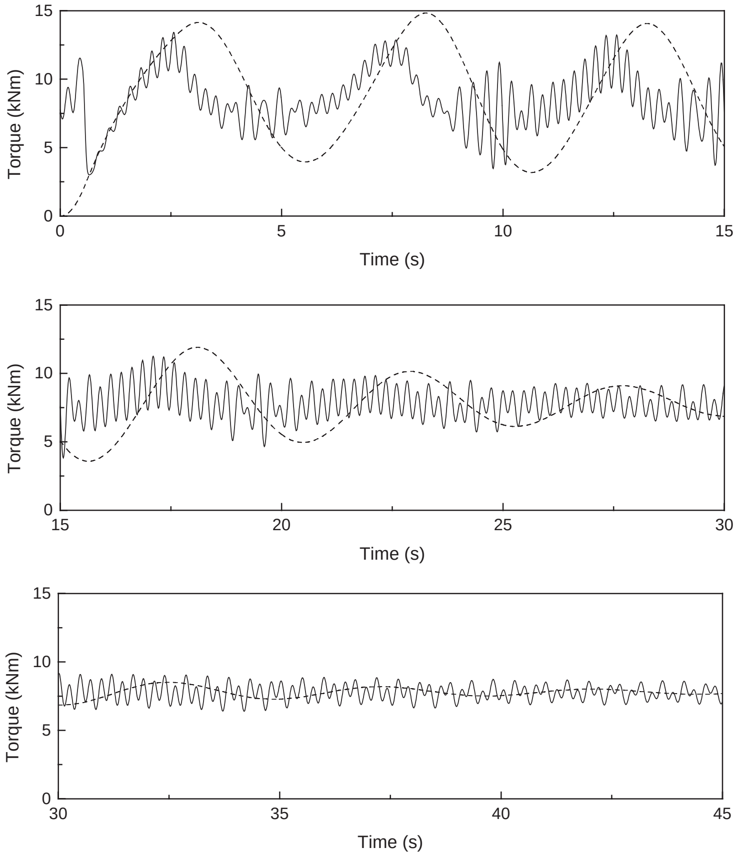

Fig. 9. TOB and top torque with active control showing a smooth drilling at the steady state; key as in Fig. 4.

environment. It is expected that the model presented here will be helpful in alleviating certain drilling problems by providing guidelines for design and operation through parametric studies. However, there is more work to be done in improving the cutting model, and considering the stochastic nature of the drilling environment.

5. Conclusions

A study of the coupled axial, torsional, and bending vibrations of an actively controlled drillstring has been presented. The proposed dynamic model includes the mutual dependence of axial, torsional, and lateral vibrations. Furthermore, the bit/formation and drillstring/borehole wall interactions are assumed to be related to drillstring axial, torsional, and lateral motions. Simulation results are in close qualitative agreement with field observations regarding stick–slip vibrations. These vibrations are self-excited, and they generally disappear as the rotary table speed is increased beyond a threshold value. However, since increasing the rotary speed may cause lateral problems such as backward and forward whirling, impacts with the borehole wall, and

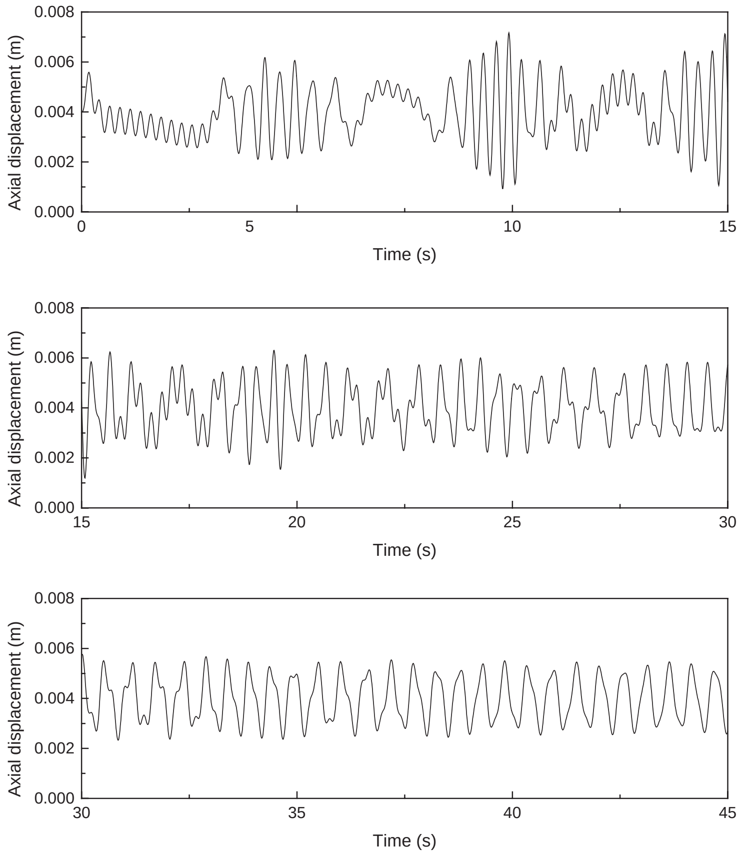

Fig. 10. Axial vibrations with active control.

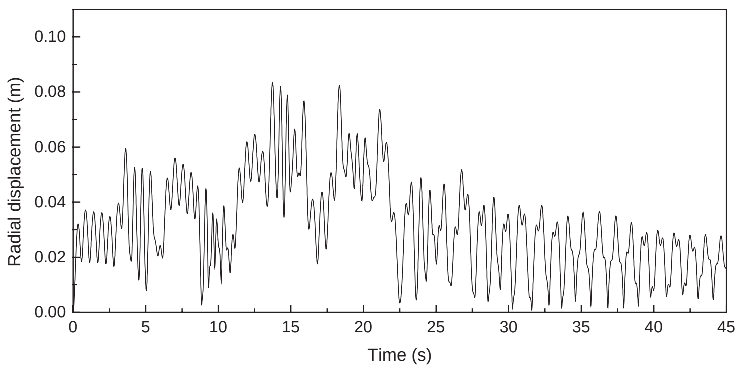

Fig. 11. Effect of controlled torsional motion on the lateral vibrations.

parametric instabilities it is desirable to extend the range of safe drilling speeds. An optimal state feedback control has been designed to control the drillstring rotational motion. It has been shown that the proposed control can be effective in suppressing stick–slip vibrations once they are initiated. Therefore, it is possible to drill at lower speeds, which are otherwise not possible without active control. Simulation results showed that controlling torsional motion helps in reducing all other vibration phenomena provided that a safe desired speed is selected based on a given applied WOB. Due to uncertainties and variations in environmental factors such as formation characteristics, a deterministic study may not be sufficient to establish with full confidence a safe range of operating conditions (e.g., applied WOB and desired bit speed). Therefore, future work should focus on the stochastic nature of the problem.

Appendix A. Lumped model parameters

Assuming a one-mode approximation for axial, lateral, and torsional vibrations, the equivalent lumped model parameters are obtained as

$$J = 2\rho I_A l_2 + (1/3)\rho I_p l_3, \tag{A.1}$$

$$m = \rho\pi(d_o^2 - d_i^2)l_1/8, \tag{A.2}$$

$$m_f = \pi\rho_f(d_i^2 + C_A d_o^2)l_1/8, \tag{A.3}$$

$$k(x, \phi, \dot{\phi}) = \frac{EI_A\pi^4}{2l_1^3} - \frac{T\pi^3}{2l_1^2} - \frac{F\pi^2}{2l_1}, \tag{A.4}$$

$$k_T = \frac{GI_p}{l_3}, \tag{A.5}$$

$$c_h = (2/3\pi)\rho_f C_D d_o l_1, \tag{A.6}$$

$$c_v = \frac{\pi\mu_f d_o^3 l_2}{2(d_h - d_o)}, \tag{A.7}$$

$$m_a = 2(m + m_f) + \rho\pi(\bar{d}_o^2 - \bar{d}_i^2)l_3/12, \tag{A.8}$$

$$k_a = E\pi(\bar{d}_o^2 - \bar{d}_i^2)/4l_3. \tag{A.9}$$

Appendix B. Nomenclature

C_A added mass coefficient
c_a effective damping for axial motion, N s/m
C_D drag coefficient
c_h hydrodynamic damping coefficient, N s^2/m^2
c_v viscous damping coefficient, N m s
c_{rt} equivalent viscous damping coefficient, N m s
c_1, c_2 penetration model empirical constants
d_h borehole diameter, m
d_i inside collar diameter, m

d_o outside collar diameter, m
$\bar{d}_i$ inside drill pipe diameter, m
$\bar{d}_o$ outside drill pipe diameter, m
e_0 eccentricity of the collars, m
E Young's modulus, Pa
F weight-on-bit (WOB), N
F_0 applied WOB, N
$\bar{F}$ gravitational force, N
F_h transverse contact force, N
F_r radial contact force, N
F_θ frictional contact force, N
G shear modulus, Pa
I current, A
I_A area moment of inertia, m^4
I_p polar area moment of inertia, m^4
J drillstring mass moment of inertia, $kg\,m^2$
J_m inertia of drive motor, $kg\,m^2$
J_{rt} inertia of the rotary table, $kg\,m^2$
k bending stiffness of collars, N/m
k_a effective axial stiffness, N/m
k_c contact stiffness, N/m
K_i controller gains
K_m motor constant, V s
k_T torsional stiffness, N m/rad
L motor inductance, H
m effective mass of collars, kg
m_a effective drillstring mass, kg
m_f added fluid mass, kg
n gear ratio
n_b bit factor
ROP rate of penetration, m/s
R_c collar radius, m
R_m armature resistance, Ω
T torque-on-bit (TOB), N m
s, s_0 formation surface profile, and its amplitude, m
v velocity of collar geometric center, m/s
v_c control voltage, V
α_1, α_2 coefficients for speed factor
δ depth of cut per revolution, m/rev
ϕ drill collar angular displacement, rad
ϕ_{rt} rotary table angular displacement, rad
μ, ζ cutting force factors
μ_f viscosity of drilling mud, $N\,s/m^2$
ρ drillstring material density, kg/m^3

P_f density of drilling mud, kg/m^3
ω_d desired rotary table speed, rad/s

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