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Non-parametric identification of non-linear oscillating systems

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Abstract

The problem of system identification from a time series of measurements is solved by using non-parametric additive models. Having only few structural information about the system, a non-parametric approach may be more appropriate than a parametric one for which detailed prior knowledge is needed. Based on non-parametric regression, the functions in the additive models are estimated by a penalized least-squares approach using backfitting. The optimal smoothing parameters are determined via generalized cross-validation, making this approach completely adaptive to the data. The procedure is applied to identify the non-linear restoring force of vibrationally excited helical wire rope isolators.

1. Introduction

A non-parametric method for non-linear dynamical system identification from time series of measurements is proposed for the case that only few structural information about the system is known. In contrast to the widespread approach of fitting parameters in given model equations to the data, e.g., NARMAX models [1,2], in this approach the functions involved in the equations are estimated themselves. The basic requirement is that the system can be described by an additive model. Additive models have been successfully applied to various data modelling problems and can be regarded as a generalization of linear regression to non-linear dependencies [3–5]. In this article these statistical concepts will be applied to the non-parametric identification of non-linear oscillating systems given by ordinary differential equations. It is shown that non-linearities involved in these models can be estimated from time series of observations without the requirement of explicitly providing their analytic form. As an example application, the modelling of a vibrationally excited helical wire rope isolator reveals that the change of the non-linear

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behavior of the restoring forces under changing excitation parameters can be modelled accurately within this concept.

Additive models are mapping p different input variables $x_1, \ldots, x_p$ additively to a single output variable y. In the presence of an observational error ε which is usually assumed to be normally distributed and independent of $x_1, \ldots, x_p$, the model equations are given by

$$y = f_1(x_1) + \cdots + f_p(x_p) + \varepsilon. \tag{1}$$

The unknown functions $f_1(x_1), \ldots, f_p(x_p)$ have to be estimated from observations of y given at some design points $(x_1, \ldots, x_p)$. In the following, a method is presented to estimate these functions efficiently and data-adaptively. The basic idea of this procedure is to utilize the additivity in order to built an iterative algorithm. This algorithm solves a one-dimensional problem in every iteration step by smoothing. The reduction to one variable during the iteration process therefore does not suffer from the so-called curse of dimensionality, which denotes the effect that high-dimensional spaces are filled sparsely [6].

The smoothing parameters of each function can be efficiently selected in additive models. This parameter indicates how rough the estimated function should remain. The optimal smoothing parameter is balancing the bias to the variance of the estimated curve, or more exactly, minimizing the mean-squared error. It can be estimated by generalized cross-validation [7]. Having once introduced additive models, it is quite intuitive to adjust the procedure to a specific problem. If, for example, some functions are already known and given by parametric models, a semi-parametric method can be set up simply by replacing the smoothers of these functions with their corresponding parameter estimation algorithms.

The remainder of the paper is structured as follows: In Section 2.1, smoothing of a single function is described putting most attention to smoothing splines. The problem of selecting the optimal smoothing parameter is discussed in Section 2.2. Generalizing these considerations to the multi-dimensional additive model, the method of backfitting is described in Section 2.3 and finally applied on a mechanical system to identify the restoring force of a shock absorber in Section 3.

2. Estimation of additive models

2.1. Linear smoothers and smoothing splines

Consider the simple one-dimensional model

$$y = f(x) + \varepsilon, \tag{2}$$

where ε is a normally distributed error which is independent of x, and the function $f(x)$ is assumed to be unknown. To estimate $f(x)$ from observed data, the data has to be smoothed to remove the wiggliness which is induced by the error. There are numerous methods of smoothing which are classified by linear and non-linear smoothers. The smoothers chosen here are always linear which is motivated by the fact that confidence bands can be calculated and that there are certain conditions in which backfitting converges, in contrast to non-linear smoothers, where there are no general convergence conditions. Linear smoothers are mapping the data points linearly to the corresponding estimated function values, or more formally: let $\mathbf{y} = (y_1, \ldots, y_N)^t$ be the data at the so-called design points $\mathbf{x} = (x_1, \dots, x_N)^t$ ($(\cdot)^t$ being the transposition), then a function estimate $\hat{\mathbf{f}} = (\hat{f}_1, \dots, \hat{f}_N)^t$ at $\mathbf{x}$ is

$$\hat{\mathbf{f}} = S\mathbf{y}, \tag{3}$$

where S is the $N \times N$ hat-matrix. A smoother is said to be non-linear if there does not exist a hat-matrix S such that Eq. (3) is satisfied.

A prominent method for estimating functions is the kernel method. Here, a predefined kernel function $K_h(x)$ determines the weighting of adjacent points. The bandwidth h of the kernel function defines the amount of smoothing by widening or narrowing the shape of K_h. The so-called Nadaraya–Watson estimator [8,9]

$$\hat{f}(x) = \frac{\sum_{j=1}^{N} K_h(x - x_j) y_j}{\sum_{j=1}^{N} K_h(x - x_j)}, \tag{4}$$

is then an estimate for the unknown function $f(x)$. This smoother is linear because

$$S_{ij} = \frac{K_h(x_i - x_j)}{\sum_{k=1}^{N} K_h(x_i - x_k)}$$

is the corresponding hat-matrix. In order to obtain a consistent estimator, in which the bias and the variance asymptotically vanish, the choice of the kernel and the scaling behavior of h with respect to the amount of data N is restricted. These conditions and some mathematical details are discussed in Ref. [10]. If the kernel has no finite support, Eq. (4) is an $\mathcal{O}(N^2)$ computation time problem, which one usually wants to avoid. Therefore, kernel functions with finite support are commonly taken into account. Beside computation, the main disadvantage of kernel smoothers is the treatment of the boundary which is in the most cases drastically biased.

For these reasons, smoothing splines are used as an alternative. By assuming that the unknown function in Eq. (2) is at least twice continuously differentiable, an additional term is added to the usual least-squares functional which penalizes the roughness of the estimate. Hence, the complete functional

$$\mathcal{L}(g) = \underbrace{\sum_{i=1}^{N} \frac{(y_i - g(x_i))^2}{\sigma^2}}_{\textit{least-squares}} + \underbrace{\alpha \int \left[\frac{\mathrm{d}^2 g(x)}{\mathrm{d}x^2}\right]^2 \mathrm{d}x}_{\textit{penalty}}, \tag{5}$$

has to be minimized with respect to g in order to obtain an estimate of the function in Eq. (2). The smoothing parameter $\alpha \geqslant 0$ determines the amount of smoothing, ranging from zero (interpolation) to infinity (linear regression). The observational error σ is often unknown, but it can be absorbed in the smoothing parameter by using $\tilde{\alpha} = \alpha\sigma^2$. Therefore, $\sigma = 1$ can always be assumed without loss of generality. The minimization of Eq. (5) within the class of twice continuously differentiable functions and vanishing second derivative at the boundary leads to natural cubic smoothing splines. Reinsch showed that $\hat{f} = \arg\min_g \{\mathcal{L}(g)\}$ are piecewise cubic polynomials which are fitting together, such that the second derivative is continuous at the joints. An algorithm to calculate the parameters of these polynomials which takes only $\mathcal{O}(N)$ computation time is given in Refs. [5,11].

Like all linear smoothers, these splines can be regarded as kernel smoothers having a variable shape in some way. Silverman [12] gave an approximative kernel function for natural cubic splines and calculated the relation between the smoothing parameter α and the local bandwidth of the equivalent kernel. In comparing the kernel method with splines, the bias at the boundary is considerably reduced. Splines are very economic to store, since they are fully characterized by their value at the design points and the second derivative. Interpolation is possible by evaluating the spline function at the desired point. The disadvantages of smoothing splines are due to the assumptions that the function has to be twice continuously differentiable and the error model is Gaussian; spline smoothing will therefore fail if these conditions are violated. An overview of some other linear smoothers and their comparison are given in Refs. [3,4].

2.2. Estimating the optimal smoothing parameter

Suppose the function in Eq. (2) is known and let $\hat{f}_\alpha$ be an estimate of f, depending on the smoothing parameter α. Then, the mean-squared error (MSE)

$$MSE(\alpha) = N^{-1} \sum_{i=1}^{N} (\hat{f}_\alpha(x_i) - f(x_i))^2, \tag{6}$$

is an appropriate measure to quantify the goodness of fit in dependence of the smoothing parameter. Thus, minimizing $MSE(\alpha)$ gives the optimal smoothing parameter, but the definition of the MSE still contains the unknown function f. The idea to save this concept is to estimate MSE first, and finally a minimization of the estimated MSE score with respect to α yields an estimate of the optimal smoothing parameter. This can be done by cross-validation: Let $\hat{f}_\alpha^{(-i)}$ be an estimated curve where the ith observation is left out in the smoothing procedure. An estimate of MSE is then given by the cross-validation score

$$CV(\alpha) = N^{-1} \sum_{i=1}^{N} (y_i - \hat{f}_\alpha^{(-i)}(x_i))^2.$$

As shown in Ref. [5], the cross-validation score is equivalent to

$$CV(\alpha) = N^{-1} \sum_{i=1}^{N} \left(\frac{y_i - \hat{f}_\alpha(x_i)}{1 - S_{ii}(\alpha)} \right)^2.$$

Hence $\hat{f}_\alpha^{(-i)}$ has not to be calculated explicitly, which reduces the computational costs drastically. But there is a weak point in this construction: consider a point y_i which influences $\hat{f}$ very strongly. Due to the calculation of CV, leaving out y_i will dominate the score. In order to correct this, these points should be weighted differently. An appropriate weighting leads to the generalized cross-validation

$$GCV(\alpha) = N^{-1} \sum_{i=1}^{N} \left(\frac{1 - S_{ii}(\alpha)}{1 - N^{-1}\,\mathrm{tr}\,S(\alpha)} \right)^2 (y_i - \hat{f}_\alpha^{(-i)}(x_i))^2,$$

or, again eliminating $\hat{f}_\alpha^{(-i)}$,

$$GCV(\alpha) = \frac{N^{-1}\sum_{i=1}^{N}(y_i - \hat{f}_\alpha(x_i))^2}{(1 - N^{-1}\,\mathrm{tr}\,S(\alpha))^2}. \tag{7}$$

In these expressions $S(\alpha)$ is the hat-matrix for smoothing splines and $\mathrm{tr}\,S(\alpha)$ its trace. Generalized cross-validation was introduced by Craven and Wahba [7]. Since the trace of a matrix is equivalent to the sum of all eigenvalues, GCV can be calculated very efficiently. If the design points are equally distributed on the interval $[a, b]$, $\mathrm{tr}\,S(\alpha)$ can be approximated by

$$\mathrm{tr}\,S(\alpha) \approx 2 + \sum_{v=3}^{N} [1 + C^{-4}\alpha(v - 1.5)^4]^{-1},$$

where $C = N^{1/4}\pi^{-1}(b-a)^{3/4}$. For a more detailed treatment of an approximative and an exact calculation of $\mathrm{tr}\,S$, see Refs. [5,13]. The minimum of the GCV-score can be found by the aid of standard non-linear minimization routines [14].

2.3. Additive models and the backfitting algorithm

An extension of model (2) to a p-dimensional model are additive models, which are of the form

$$y = f_1(x_1) + \cdots + f_p(x_p) + \varepsilon. \tag{8}$$

The functions $f_1, \ldots, f_p$ are unknown and the error ε is still assumed to be normally distributed and independent of $x_1, \ldots, x_p$. Let $\mathbf{y} = (y_1, \ldots, y_N)^t$ and $\mathbf{x_j} = (x_{j,1}, \ldots, x_{j,N})^t$ $(j = 1, \ldots, p)$ be the data of an additive model. By posing the twice differentiability of the unknown functions, the penalized least-squares functional

$$\mathscr{L}_p(g_1, \ldots, g_p) = \sum_{i=1}^{N} \frac{(y_i - g_1(x_{1,i})\ldots - g_p(x_{p,i}))^2}{\sigma^2} + \sum_{j=1}^{p} \alpha_j \int \left[\frac{\mathrm{d}^2 g_j(x_j)}{\mathrm{d}x_j^2}\right]^2 \mathrm{d}x_j$$

can now easily be formulated. Here, σ is again the observation error and $\alpha_j \geqslant 0$ $(j = 1, \ldots, p)$ is the smoothing parameter for each function. An estimate for the unknown functions is thus the minimum of $\mathscr{L}_p$, but a direct minimization is quite clumsy and the optimal choice of the smoothing parameters is a difficult task. Fortunately, there is an iterative solution by the calculation of partial residuals for each function. The new function estimates are provided by stepwise smoothing of the partial residuals and the solution of $\min\{\mathscr{L}_p(g_1, \ldots, g_p)\}$ is obtained by cycling through the functions until the least-squares functional does not change. The algorithm of this procedure, called backfitting, is as follows:

1. Initialization of $\hat{f}_1, \ldots, \hat{f}_p$ (zero-function is appropriate).
2. Calculation of partial residuals: $\tilde{y}_{k,j} = y_j - \sum_{r \neq k} \hat{f}_r(x_{r,j})$ for all $j = 1, \ldots, N$, smoothing of $\tilde{\boldsymbol{y}}_k = (\tilde{y}_{k,1}, \ldots, \tilde{y}_{k,N})^t$ and replacing $\hat{f}_k$.
   Generalized cross-validation can be implemented at this point to find the optimal α_k.
   Cycle smoothing procedure until every $k = 1, \ldots, p$ has been smoothed.
3. Repeat (2) until the least-squares functional does not change.

The estimate of the function f_k is then $\hat{f}_k$. Convergence of this backfitting algorithm is assured if smoothing splines with fixed smoothing parameters are used in step (2) [3,4]. If generalized cross-validation is performed in every smoothing step, the convergence is not proven because the smoothing parameter depends non-linearly on the data which leads to non-linear smoothers. Nevertheless, using generalized cross-validation until the smoothing parameters and the least-squares functional do hardly change any more and switching to smoothing with fixed parameters can handle this problem.

Pointwise confidence bounds and effective degrees of freedom can be obtained using techniques similar to regression analysis [4].

3. Application

3.1. The data

The suggested method is applied on a mechanical system to identify the non-linear restoring force of helical wire rope isolators. The experiment was proposed by VTT Technical Research Centre of Finland within the framework of COST (European Cooperation in the Field of Scientific and Technical Research) [15]. It consists of a bottom plate, a top plate, and the wire rope isolators. These wire rope isolators are mounted between the top and the bottom plate. The bottom plate is excited by an electro-dynamic shaker and the top plate is the load for the isolators. A schematic configuration of this experiment is given in Fig. 1.

Measured are the acceleration $\ddot{x}_1$ of the bottom plate, the acceleration $\ddot{x}_2$ of the top plate, and the displacement $x = x_2 - x_1$ between the upper and the lower plate. The behavior of the isolators are recorded under several experimental conditions. The load mass m_2 is either 2.2 or 5.8 kg and there are two kinds of excitation: harmonic and white noise. The amplitude and frequency was varied for the harmonic excitation and different excitation levels were chosen for the random excitation.

Let $y = m_2\ddot{x}_2$ be the inertial force of the upper plate and $\dot{x}$ be the relative velocity between the two plates. Plotting y against x exhibits a hysteresis (Fig. 2, right column), which is smeared out in the case of white noise excitation (Fig. 2b). It is therefore likely to find a non-linear restoring force

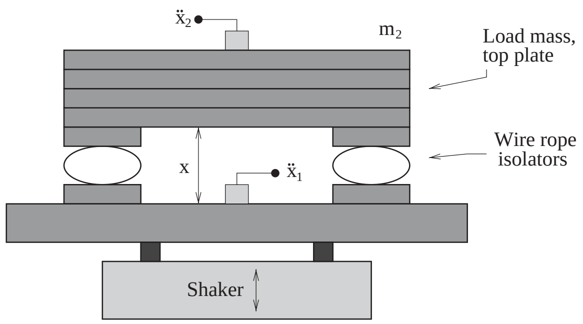

Fig. 1. Schematic configuration of the experiment.

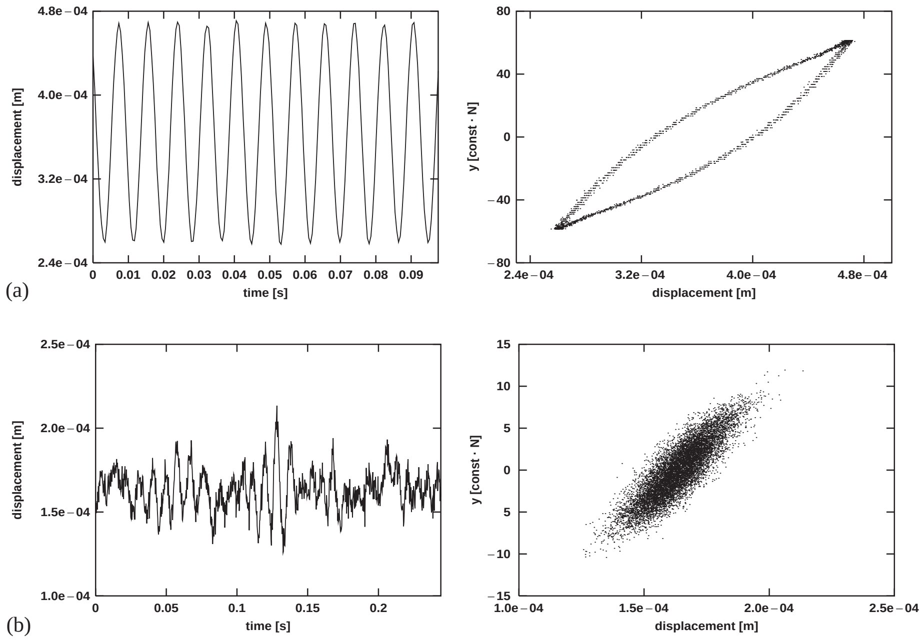

Fig. 2. Segment of the measured time series (left) and their hysteresis (right). (a) Sine excitation with load mass 2.2 kg, frequency 120 Hz, amplitude 3 V and (b) random excitation with load mass 2.2 kg, amplitude 4 V rms.

and a non-vanishing friction component. With these variables,

$$y + f(x, \dot{x}) = 0 \tag{9}$$

is the equation of motion of the upper plate. It is further assumed that the unknown function f is additive. Hence, $f(x, \dot{x}) = f_1(x) + f_2(\dot{x})$, where f_1 is the stiffness force and f_2 is a damping term. A possible coupling term $f_3(x\dot{x})$ will also be studied in the following.

In the case of white noise excitation, Kerschen et al. [16,17] proposed a model for the system. This model successfully explains the data. Therefore, only the harmonic excitation is considered in the following analysis.

3.2. Non-parametric analysis

Since the velocity $\dot{x}$ was not measured, it has to be estimated by numerical differentiation of the displacement x which is very sensitive to noise. For sufficiently large amplitudes of the excitation force, the observational noise is negligibly small, such that a high accuracy of $\dot{x}$ was achieved.

Fig. 3 shows the results of the analysis under different experimental conditions. In all cases, the restoring force and the damping are non-linear. The major difference is a change of the non-linearity with respect to symmetry. Whereas for the small load (Figs. 3(a) and (b)) both the restoring force and the friction component are almost point-symmetric, this property is lost for the larger load (Figs. 3(c) and (d)). This may be explained by a geometric distortion of the wire rope isolators towards a more elliptic shape for negative displacements (squeezing) and towards a more circular shape for positive displacements.

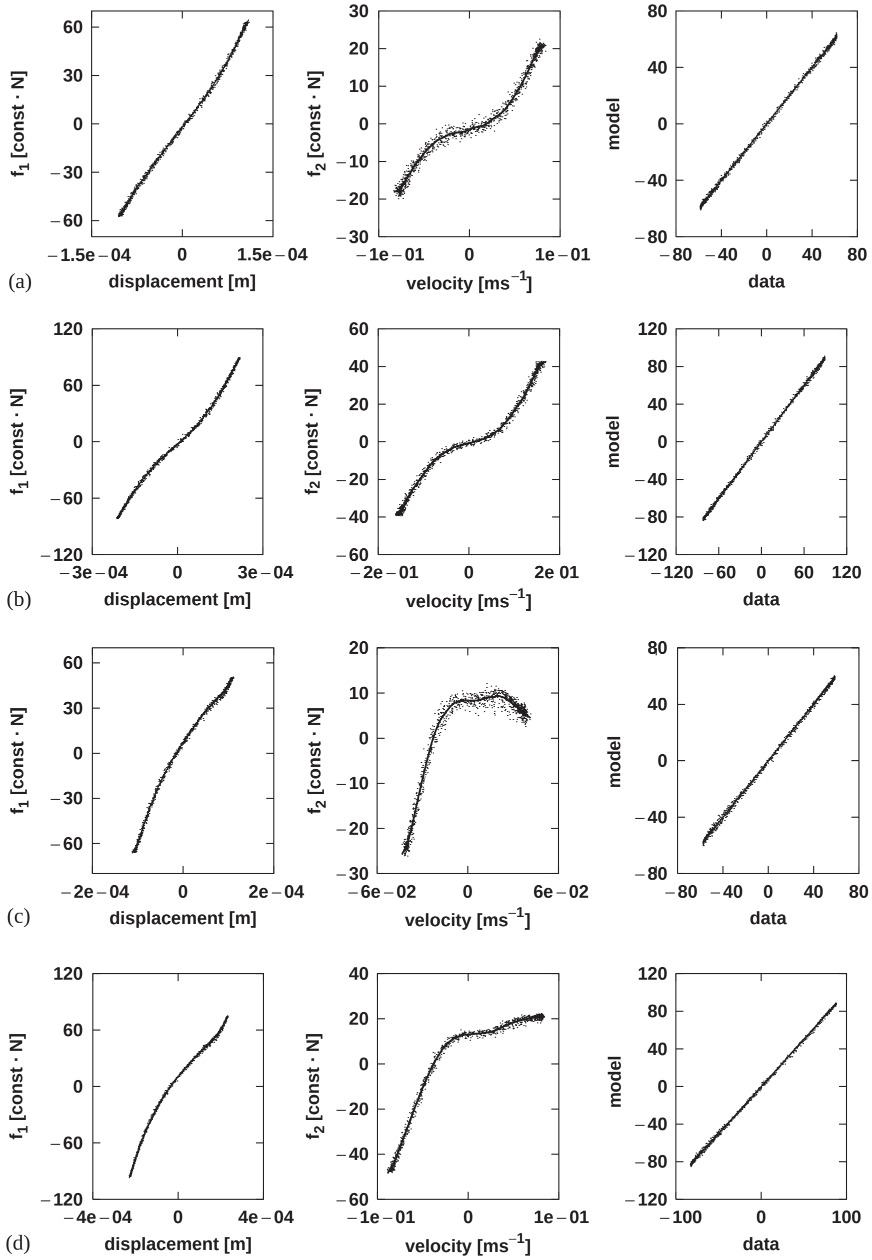

Fig. 3. Estimated restoring force f_1 and friction component f_2 at frequency ν, load mass m, and excitation amplitude U. (a) $\nu = 120$ Hz, $m = 2.2$ kg, $U = 3$ V; (b) $\nu = 120$ Hz, $m = 2.2$ kg, $U = 10$ V; (c) $\nu = 60$ Hz, $m = 5.8$ kg, $U = 1$ V; (d) $\nu = 60$ Hz, $m = 5.8$ kg, $U = 1.5$ V. Dots are indicating the partial residuals of the functions in the first two columns. A comparison between a model-based simulation and the data is shown in the third column.

The fitted model can be used for predictions, namely the measured displacement and the velocity are inserted into the corresponding function estimates in order to predict the observable y. These predictions are in good accordance with the data (Fig. 3, third column). The visual inspection of these plots are suggesting an almost perfect fit, but to give a more objective inspection of the model the residuals (the differences between the predictions and the measurements) were analyzed in more detail.

Estimating the power spectra of the data and the residuals (Fig. 4) reveals approximately flat spectra for the 120 Hz excitation (Fig. 4(a)). In case of the 60 Hz excitation (Fig. 4(b)) a small contribution of the first higher harmonics is present. A first explanation of the presence of the first higher harmonics might be the presence of a coupling between the variables. To study the effect of coupling, an additional function depending on $x\dot{x}$ was added to the model. The same analysis with the extended model shows no further improvement which is exemplarily displayed in Fig. 5. Therefore, coupling between the two variables does not play a significant role in this system. A

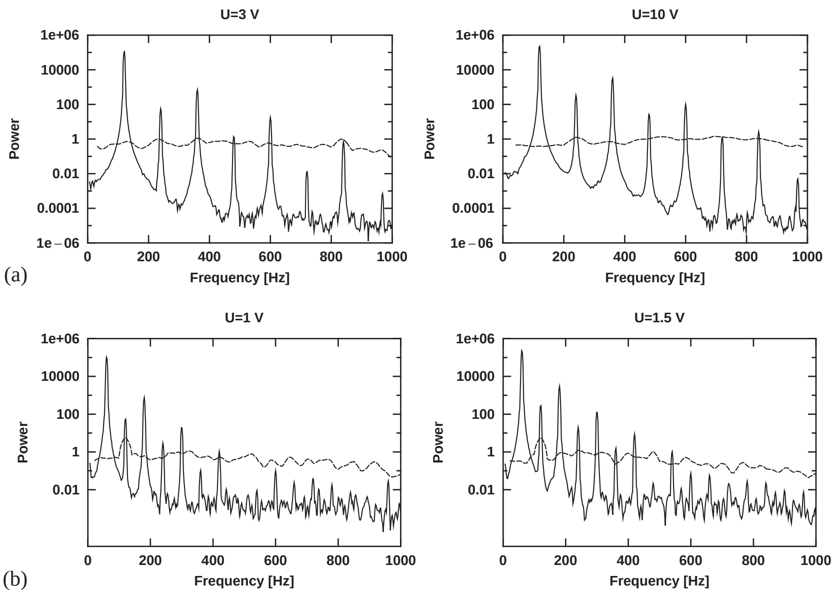

Fig. 4. Normalized spectra of the data (solid line) and the model residuals (dashed line) at (a) $\nu = 120$ Hz, $m = 2.2$ kg; (b) $\nu = 60$ Hz, $m = 5.8$ kg.

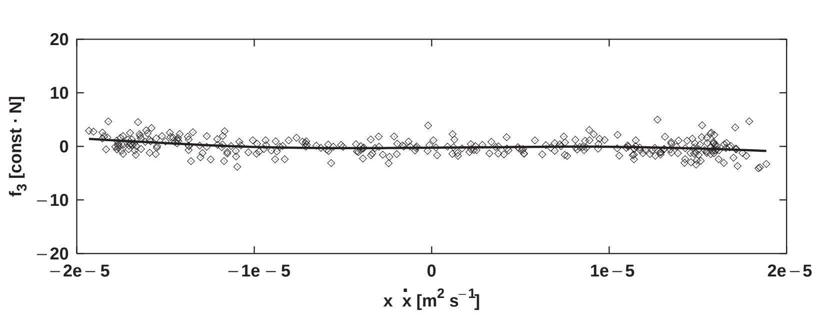

Fig. 5. Influence of coupling between x and $\dot{x}$, exemplarily shown for $\nu = 120$ Hz, $m = 2.2$ kg, $U = 10$ V. The estimate of function $f_3(x\dot{x})$ is fairly compatible with the zero function.

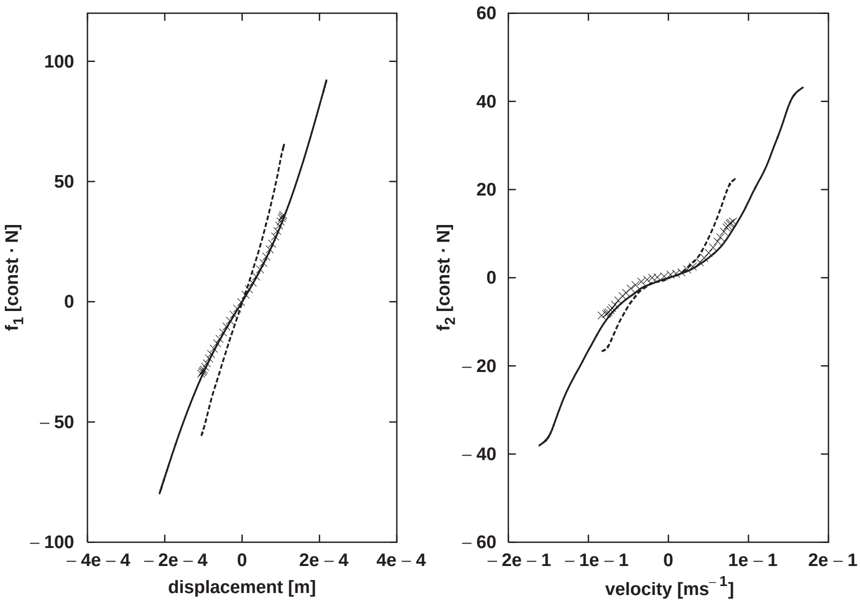

Fig. 6. Comparison of f_1 and f_2 at fixed mass $m = 2.2$ kg and frequency $\nu = 120$ Hz but for different amplitudes $U = 10$ V (solid line), $U = 3$ V (dashed line). The force of excitation amplitude $U = 3$ V (crosses) was rescaled.

second possible explanation is the presence of noisy design points which cannot be explained by the model. Nevertheless, most of the system features are captured by the identified models.

Finally, keeping the load mass fixed and varying the amplitude, one would expect that the functions do not differ in their overlapping part. A direct comparison of Figs. 3(a) and (b) shows different structural behavior (Fig. 6, full vs. dashed line). Rescaling the force of Fig. 3(a) by 0.55 and doing the same comparison yields a perfect match (Fig. 6, crosses). Therefore, within statistical variation of the function estimates, these functions do not differ if the force is rescaled. The necessity of rescaling is probably due to a change of amplification during the experiments in order to prevent amplifier saturation.

4. Discussion

A data adaptive method for the identification of non-linear additive models is proposed and applied to estimate functions in ordinary differential equations. If the model consists of only one unknown function, it can be estimated by smoothing the data. A criterion for selecting the optimal smoothing parameter can be formulated and efficiently computed if splines are used. More generally, having an additive model with more than one unknown component, the suggested function estimation procedure is the iterative backfitting algorithm.

The noise of the observable evaluated at the design points is always assumed to be Gaussian. But in contrast, the design points are considered as being free from any observational noise. Therefore, noisy design points can induce biased function estimates. For a too small signal-to-noise ratio this so-called errors-in-variables problem may become crucial. However, in such a case non-parametric methods still can be used as explorative tools to find initial guesses for parametric models which then can be refined in a subsequent parametric estimation step. For example, the parameters in ordinary differential equations can often be estimated taking the errors-in-variables problem into account. Such a two-step strategy has been successfully applied in Refs. [18,19].

If all conditions, i.e., additivity, normally distributed observations and high accuracy of the design points are fulfilled, the proposed method allows a data driven model identification which also is numerically efficient.

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