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Journal of Sound and Vibration 267 (2003) 1184–1192

JOURNAL OF SOUND AND VIBRATION

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Letter to the Editor

Controlling chaotic instability of cutting process by high-frequency excitation: a numerical investigation

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Received 21 August 2002; accepted 29 January 2003

1. Introduction

Self-excited chatter is undesirable in machine tool system. Chatter is a kind of instability inherent to the interaction of complex cutting process and elastic machine tool structure. Research has established two major causes of chatter in dynamic machining system, namely regeneration effect and velocity dependence of cutting force. A linear analysis of chatter is sufficient to predict the onset of instability of cutting process. However, dynamics of fully developed chatter calls for a suitable non-linear model. Various non-linear models of cutting process are available in Refs. [1–3]. Recently, it has been understood that apparent random fluctuation of cutting force and other dynamic variables like displacement, acceleration etc. of machine tool structure can be explained in the light of chaos theory [1,4–6].

A vast body of literature exists on chatter control [7]. Various passive and active control techniques of controlling chatter have been proposed in literature. Passive techniques include the use of passive tuned–mass–damper, hydraulic damper, friction damper and impact damper etc. Use of negative feed-back control, based on the direct measurement of cutting force or relative displacement between tool and workpiece, in mitigating chatter have also been investigated. In the present article, a passive chatter control technique utilizing a very high-frequency force is investigated.

Results on the effect of high-frequency excitation on non-linear instability are available in literature. Theoretical and experimental research have been carried out to understand this stabilizing effect [8,9]. High-frequency excitation is termed as fast vibration in literature.

In what follows, a simplified non-linear model of chatter as described in Ref. [1] is considered to investigate the effect of high-frequency excitation on chatter dynamics. In this model, a negatively sloped non-linear cutting force–velocity relationship stands as a primary source of instability. A two-degree-of-freedom linear oscillator models elastic machine tool structure. As the resulting

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doi:10.1016/S0022-460X(03)00131-7

equations of motion are complex in nature, no analytical solution is attempted. Equation of motion is numerically simulated using Runge–Kutta–Merson algorithm with adaptive step size control. Effect of high-frequency excitation on the system dynamics is investigated in terms of vibration displacement, and the mechanism of the effect is explained theoretically using the notion of effective cutting force.

2. Theoretical model of cutting dynamics

2.1. Steady state model of cutting process

A phenomenological model of steady orthogonal cutting process is depicted in Fig. 1a. Cutting force F_c generated in the process is described by an average empirical relationship as described below. Two cutting force components F_x and F_y are related to each other by the following relationship:

$$F_y = KF_x, \tag{1}$$

where F_x and K are functions of cutting velocity v and uncut chip thickness h as given below:

$$F_x = F_{x0}\left\{\lambda_1\left(\frac{v}{v_0}-1\right)^2+1\right\}\frac{h}{h_0} \tag{2}$$

and

$$K = K_0\left\{\lambda_2\left(\frac{v}{v_0}-1\right)^2+1\right\}\left\{\lambda_3\left(\frac{h}{h_0}-1\right)^2+1\right\}, \tag{3}$$

where parameters

$$F_{x0}, K_0, h_0, v_0, \lambda_1, \lambda_2, \lambda_3$$

are related to cutting conditions. Chip flow velocity along chip tool interface is given by

$$v_f = \frac{v}{R}, \tag{4}$$

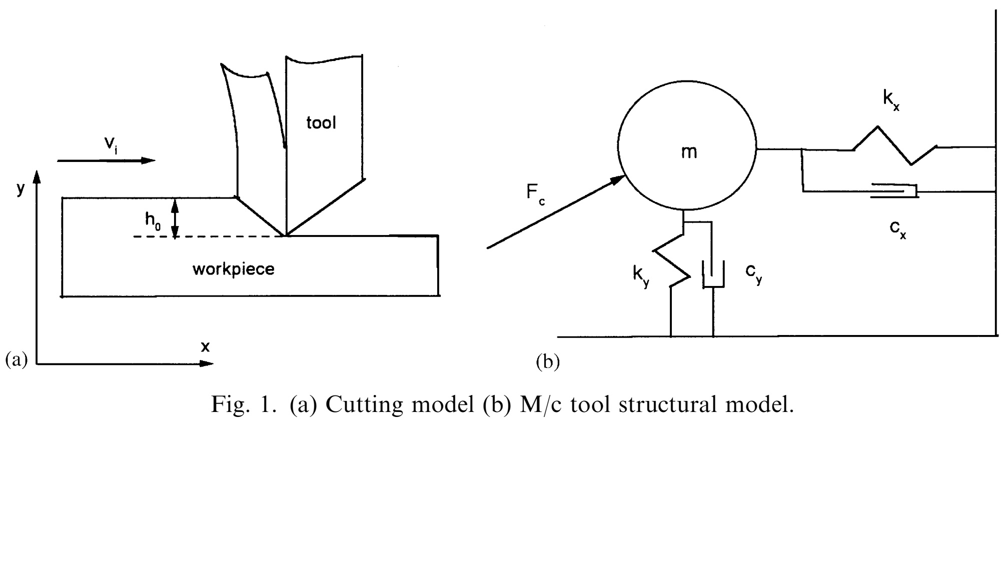

Fig. 1. (a) Cutting model (b) M/c tool structural model.

where

$$R = R_0\left\{\lambda_4\left(\frac{v}{v_0}-1\right)^2+1\right\}. \tag{5}$$

2.2. Dynamic model of cutting process

In Section 2.1, a steady state model of cutting process is discussed. For a large class of engineering materials, this model is valid only in some average sense under the assumption that there is no variation in cutting velocity and uncut chip thickness. However, in actual unsteady cutting process both cutting velocity v and h varies with time and is closely related to the dynamics of elastic machine tool structure. A simplified dynamic model of cutting involving the interaction of elastic machine tool structure and the dynamic cutting force is depicted in Fig. 1b. A two-degree-of-freedom oscillator describes dynamics of elastic machine tool structure, where dynamics in x and y directions are coupled through the cutting force. Dynamics of the system is described by the following differential equations:

$$mx'' + c_x x' + k_x x = F_x(t), \tag{6}$$

$$my'' + c_y y' + k_y y = KF_x(t), \tag{7}$$

where

$$F_x, F_y$$

are as described in Eqs. (1) and (2), except that in dynamic cutting model v, h, etc. are time-dependent quantities, and are given by

$$h(t) = h_i - y(t), \tag{8}$$

$$v(t) = v_i - x'(t), \tag{9}$$

$$v_f(t) = \frac{v(t)}{R(t)} - y'(t) \tag{10}$$

and

$$F_x = 0, \quad \forall\, h<0 \text{ or } v<0, \tag{11}$$

$$K(-v_f) = -K(v_f), \tag{12}$$

where h_i and v_i are nominal uncut chip thickness and cutting velocity as has been set by the operator. Eq. (11) is introduced to simulate the unusual cutting conditions in dynamic environment, i.e., when cutting tool comes out of the workpiece or resultant cutting velocity is reversed. Eq. (12) takes care of the fact that direction of friction force is reversed when resultant chip flow velocity is reversed.

Introducing the following non-dimensional parameters:

$$X = \frac{x}{h_0}, \quad Y = \frac{y}{h_0}, \quad \tau = w_0 t, \quad w_0 = \frac{v_0}{h_0},$$
$$\dot{X} = \frac{x'}{v_0}, \quad \dot{Y} = \frac{y'}{v_0}, \quad \ddot{X} = \frac{x''}{h_0 w_0^2}, \quad \ddot{Y} = \frac{y''}{h_0 w_0^2},$$
$$H_i = \frac{h_i}{h_0}, \quad H = H_i - Y, \quad V_i = \frac{v_i}{v_0}, \quad V = V_i - \dot{X},$$
$$K_{1,2} = \frac{k_{x,y}}{m w_0^2}, \quad C_{1,2} = \frac{c_{x,y}}{m w_0}, \quad F_0 = \frac{F_{x0}}{h_0 m w_0^2},$$

one can rewrite the equations of motion (6) and (7) as given below

$$\ddot{X} + C_1 \dot{X} + K_1 X = F, \tag{13}$$

$$\ddot{Y} + C_2 \dot{Y} + K_2 Y = KF, \tag{14}$$

where

$$F = F_0 H\{\lambda_1 (V-1)^2 + 1\} U(H) U(V),$$
$$K = K_0 \{\lambda_2 (V_f - 1)^2 + 1\}\{\lambda_3 (H-1)^2 + 1\} U(F) \mathrm{Sgn}(V_f),$$
$$V_f = V - R\dot{Y},$$

and

$$R = R_0 \{\lambda_4 (V-1)^2 + 1\},$$

with

$$U(x) = \begin{cases} 1 & \forall\, x > 0, \\ 0, & \text{otherwise} \end{cases}$$

and

$$\mathrm{Sgn}(x) = \begin{cases} 1 & \forall\, x > 0, \\ 0, & x = 0, \\ -1 & \forall\, x < 0. \end{cases}$$

In the above equations 'dot' denotes differentiation with respect to non-dimensional time τ. When a high-frequency excitation of amplitude f_e and frequency ω_f is imposed on the tool, Eqs. (13) and (14) are written as

$$\ddot{X} + C_1 \dot{X} + K_1 X = F + F_e \sin(\Omega_f . \tau), \tag{15}$$

$$\ddot{Y} + C_2 \dot{Y} + K_2 Y = KF, \tag{16}$$

where

$$F_e = \frac{f_e}{m h_0 w_0^2}$$

and

$$\Omega_f = \frac{\omega_f}{w_0}.$$

3. Numerical simulation

Eqs. (15) and (16) are non-linear equations, and are not amenable to closed form analytical solution. Therefore, Eqs. (15) and (16) are numerically integrated using the fourth order Runge–Kutta–Merson algorithm with adaptive step size control (a NAG library subroutine). The parameter values used in the article are listed below:

$$\lambda_1 = 0.3, \quad \lambda_2 = 0.7, \quad \lambda_3 = 1.5, \quad \lambda_4 = 2.2,$$
$$h_0 = 0.25\,\text{mm}, \quad v_0 = 6.6\,\text{m/s}, \quad K_0 = 0.36, \quad w_0 = 2.7 \times 10^4\,\text{s}^{-1}.$$

The above parameter values are approximately true for a wide range of low carbon steels. For further treatment, $V_i = H_i = 0.5$ are adopted to represent typical cutting condition when instability may arise. The resonant frequency of a clamped tool on lathe being in the range of 1–10 kHz, it is reasonable to choose $K_1 = 1$. Rigidity in y direction being less, a lower value of

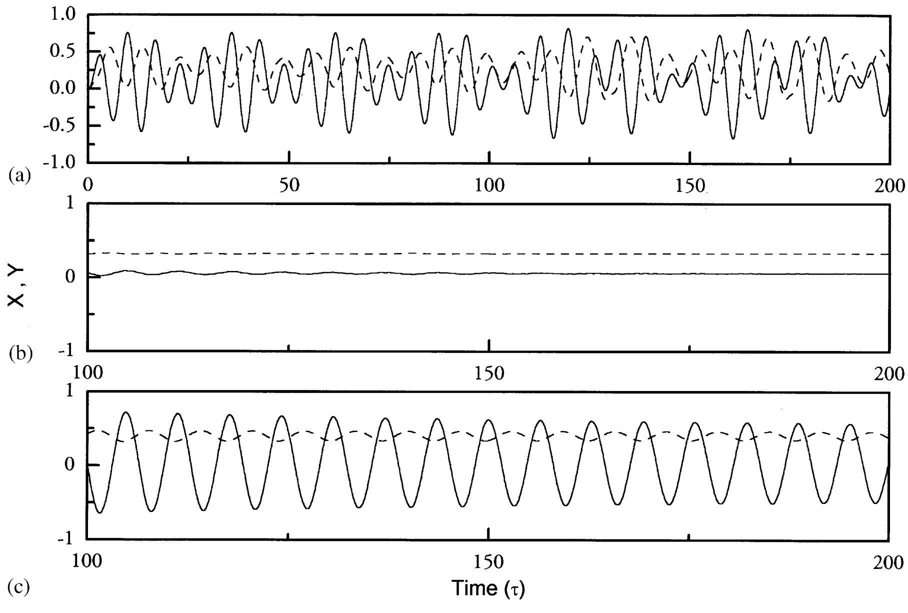

Fig. 2. X and Y vibration with and without fast excitation. ——, X vibration; – – –, Y vibration. $F_0 = 0.5$. (a) Without fast vibration; (b) with fast vibration, $F_e = 2000$, $\Omega_f = 1000$; (c) with fast vibration, $F_e = 3000$, $\Omega_f = 1000$.

$K_2 = 0.25$ is chosen. Damping values C_1 and C_2 are set to zero. Typical time history plots of tool vibrations in x and y directions are shown in Fig. 2a for $F_0 = 0.5$. From Fig. 2a, one observes chaotic oscillation, and detailed discussion on the nature of these oscillations is available in Ref. [1]. The effect of fast excitation on the vibration is plotted in Figs. 2b and c. From these figures, it is observed that for some parameter values of fast excitation, chaotic oscillation is completely suppressed. For stronger fast excitation, instability reappears again, but chaotic oscillation is replaced by regular periodic oscillation. However, it is to be understood that apart form the low-frequency oscillation, a high-frequency component is always present in the response, and this residual vibration can be made insignificantly small by choosing the frequency of excitation few orders higher than the natural frequency of the system.

So far as the proper selection of the amplitude and frequency of fast excitation is concerned it is worth noting that fast vibration is characterized primarily by its strength, quantified as $F_e\Omega_f^{-1}$. Numerically this signifies that an excitation with $F_e = 10$ and $\Omega_f = 10$ is as strong as that with $F_e = 100$ and $\Omega_f = 100$. To achieve significant effect of fast vibration on the class of systems under discussion, this strength factor must be greater than unity, while the frequency should be significantly higher than the natural frequency of vibration of the system. Due to fast vibration a high-frequency component is always present in the steady state response of the system. Selecting the frequency of excitation few orders higher than the natural frequency of the system can reduce this high-frequency component. However, this calls for very high amplitude of exciting force for maintaining the strength of the fast vibration at the desired level. Thus, a compromise between the high-frequency residual component and the frequency or force of fast vibration may sometimes be required in practice.

From Fig. 2b and c, one observes that due to fast excitation, the tool experiences substantial static-deflection in the y direction, and the static deflection increases with the strength of fast excitation. However, as shown in Fig. 3, applying an appropriate bias force F_{by} in y direction can reduce this static deflection.

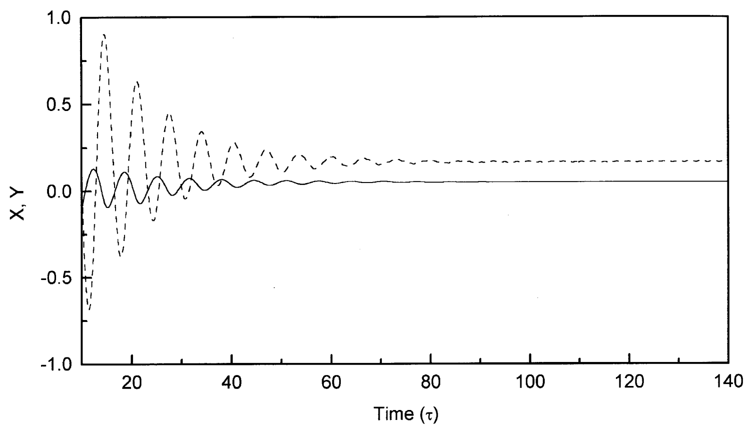

Fig. 3. Time history of X, and Y vibration with fast excitation and a bias force in y direction. $F_0 = 0.5$, $F_{by} = 0.14$, $F_e = 2000$, $\Omega_f = 1000$. ----, X vibration; ——, Y vibration.

From numerical simulation, it is understood that chaotic tool vibration may be suppressed by proper use of high-frequency excitation. In general with the increasing strength of fast vibration, tool vibration decreases initially; then it is completely suppressed for a range of parameter values, and vibration reappears again when fast vibration is very strong. In what follows, the mechanism of this effect is described in detail using the concept of effective cutting force.

4. Effective cutting force

The concept of effective cutting force stems from the theory of fast vibration [10]. According to this theory, any non-linear state dependent force undergoes non-trivial functional changes under the influence of fast vibration. This modification of force takes place due to the averaging effect (in a time scale comparable to the natural time scale of the system) of the fast sweeping of the states. The computational procedure of the effective cutting force is briefly discussed below.

Using the Method of Direct Partition of Motion [10], one can show that the slow dynamics (involving a time scale comparable to the natural time scale of the system without fast excitation) of the system given by Eqs. (15) and (16) can be effectively described by the following autonomous equations:

$$\ddot{X}_s + C_1\dot{X}_s + K_1 X_s = \langle F \rangle, \tag{17}$$

$$\ddot{Y}_s + C_2\dot{Y}_s + K_2 Y_s = \langle KF \rangle. \tag{18}$$

In the above equations, X_s and Y_s represent slow components of X and Y motion, when the dynamics is viewed in a time scale comparable to the natural time period of vibration; and $\langle F \rangle$ and $\langle KF \rangle$ are the effective cutting forces non-trivially modified by the effect of fast excitation, and are computed after replacing

$$V = V_i - \dot{X}$$

in Eqs. (15) and (16) by

$$V = V_i - \dot{X}_s + \frac{F_e}{\Omega_f}\cos(\Omega_f \tau) \tag{19}$$

and computing the following average

$$\langle F, KF \rangle = \frac{1}{2\pi}\int_0^{2\pi} (F, KF)\, \mathrm{d}(\Omega_f \tau) \tag{20}$$

Eq. (20) is computed numerically [11] to obtain the effective cutting force. The effective cutting forces are plotted in Figs. 4 and 5. It is observed from Fig. 4 that the system is unstable because of the positive slope of cutting force, which introduces negative damping in the system. However, the slope of the effective cutting force $\langle F \rangle$ becomes negative due to the effect of fast vibration, and the system is stabilized in the x direction. It is also observed that the effective cutting force increases with the strength of fast excitation. From Fig. 5, similar observation is also made for cutting force in y direction, particularly in low x- and y-axis velocity region. However, in some high velocity regions, slope (with respect to y-axis velocity) of the effective cutting force $\langle KF \rangle$

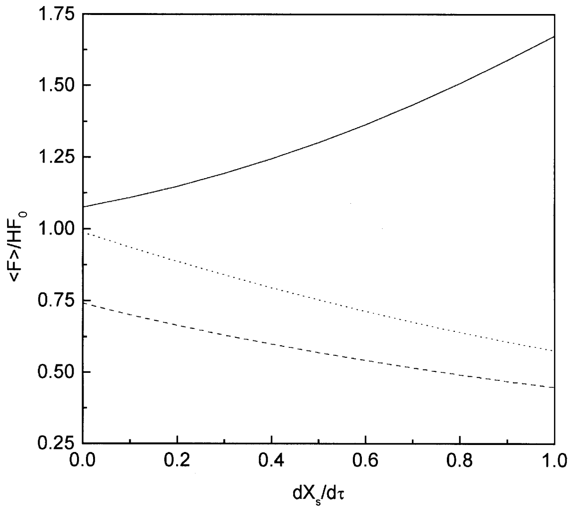

Fig. 4. Cutting force vs velocity without and with fast excitation. ——, Without fast excitation; - - - - - -, with fast excitation ($F_e = 2000$); · · · · · · · ·, with fast excitation ($F_e = 3000$). $\Omega_f = 1000$.

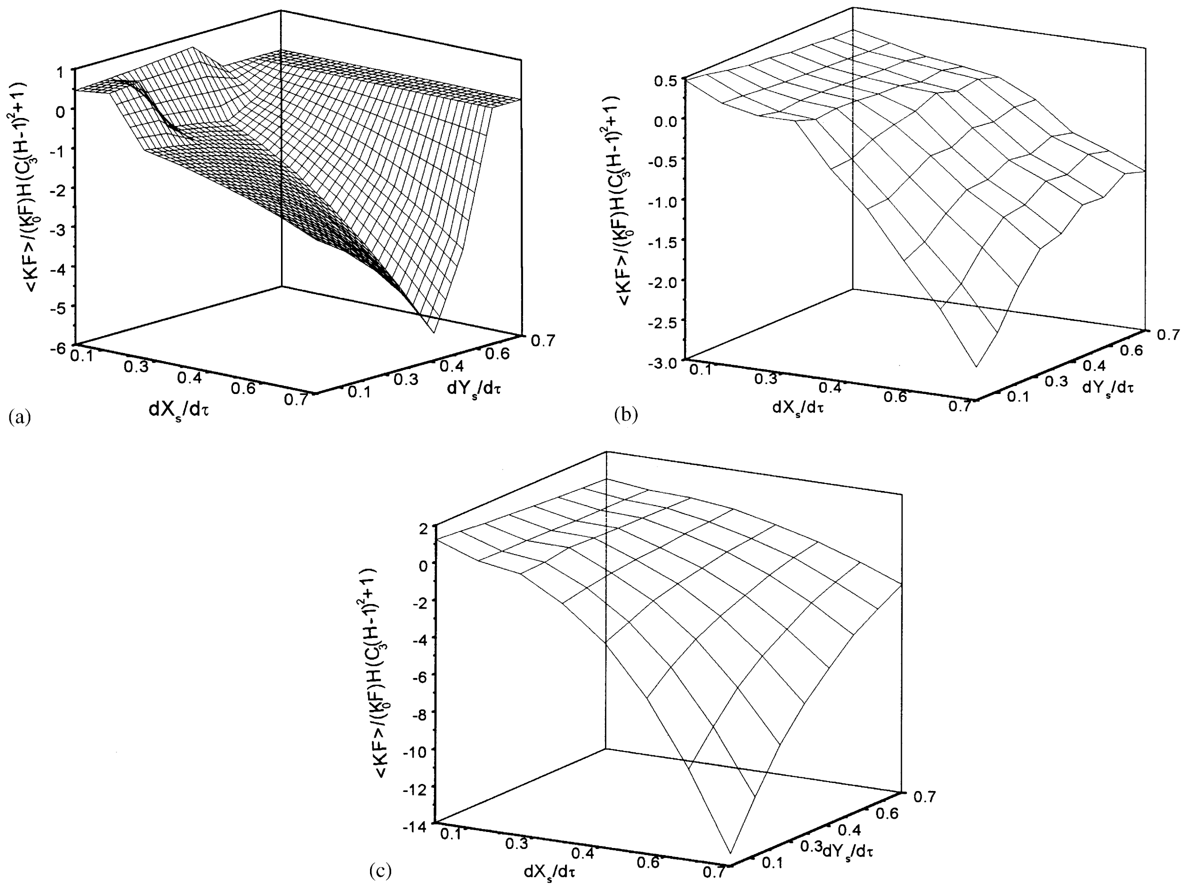

Fig. 5. Effective Y cutting force with and without fast excitation. $\Omega_f = 1000$. (a) $F_e = 0$, (b) $F_e = 2000$, (c) $F_e = 3000$.

may be positive, and instability is possible, if due to some reasons or other, velocity falls in that region. This accounts for the reappearance of instability for stronger fast excitation.

5. Conclusions

The effect of very high-frequency excitation on chaotic instability of machine tools during cutting is discussed. It is shown that machine tool chatter can be suppressed by using appropriate strong high-frequency excitation. For mathematically describing the chatter phenomenon, an average non-linear model of cutting process and a linear two-degree-of-freedom model of machine tool structure are considered. The effect of fast excitation on the stability of the system is described in light of the effective cutting force function.

References

[1] I. Grabec, Chaotic dynamics of the cutting process, International Journal of Machine Tools and Manufacture 28 (1) (1988) 19–32.
[2] J. Gradisek, E. Govekar, I. Grabec, A chaotic cutting process and determining optimal cutting parameter using neural networks, International Journal of Machine Tools and Manufacture 36 (1996) 1161–1172.
[3] J. Warminiski, G. Litak, J. Lipski, M. Wiercigroch, M.P. Cartmell, Vibrations in regenerative cutting process synthesis of nonlinear dynamical systems, Solid Mechanics and its Applications 73 (2000) 275–283.
[4] M. Wiercigroch, Chaotic vibrations of a simple model of the machine tool-cutting system, ASME Journal of Vibration and Acoustics 119 (1997) 468–475.
[5] M. Wiercigroch, A. Cheng, Chaotic and stochastic dynamics of orthogonal metal cutting, Chaos Solitons & Fractals 8 (1997) 715–726.
[6] I.N. Tansel, C. Erkal, T. Keramidas, The chaotic characteristics of three-dimensional cutting, International Journal of Machine Tools and Manufacture 32 (1992) 811–827.
[7] J.R. Pratt, A.H. Nayfeh, Design and modeling for chatter control, Nonlinear Dynamics 19 (1999) 49–69.
[8] J.J. Thomsen, Using fast vibrations to quench friction-induced oscillation, Journal of Sound and Vibration 228 (5) (1999) 1079–1102.
[9] B.F. Fenny, F.C. Moon, Quenching stick-slip chaos with dither, Journal of Sound and Vibration 237 (1) (2000) 173–180.
[10] I. Blekhman, Vibrational Mechanics: Nonlinear Dynamic Effects, General Approach, Applications, World Scientific Publishing company, Singapore, 2000.
[11] S. Chatterjee, T.K. Singha, S.K. Karmakar, Non-trivial effect of fast vibration on the dynamics of a class of non-linearly damped mechanical systems, Journal of Sound and Vibration 260 (4) (2003) 711–730.