Available online at www.sciencedirect.com

SCIENCE DIRECT®

ACADEMIC PRESS

Journal of Sound and Vibration 268 (2003) 465–483

JOURNAL OF SOUND AND VIBRATION

www.elsevier.com/locate/jsvi

Dynamic analysis of solid propellant grains subjected to ignition pressurization loading

Shiang-Woei Chyuan*

Chung Shan Institute of Science and Technology, P.O.Box 90008-15-3 Lung-Tan, Tao-Yuan 325, Taiwan, ROC

Received 16 April 2002; accepted 20 November 2002

Abstract

Traditionally, the transient analysis of solid propellant grains subjected to ignition pressurization loading was not considered, and quasi-elastic–static analysis was widely adopted for structural integrity because the analytical task gets simplified. But it does not mean that the dynamic effect is not useful and could be neglected arbitrarily, and this effect usually plays a very important role for some critical design. In order to simulate the dynamic response for solid rocket motor, a transient finite element model, accompanied by concepts of time–temperature shift principle, reduced integration and thermorheologically simple material assumption, was used. For studying the dynamic response, diverse ignition pressurization loading cases were used and investigated in the present paper. Results show that the dynamic effect is important for structural integrity of solid propellant grains under ignition pressurization loading. Comparing the effective stress of transient analysis and of quasi-elastic–static analysis, one can see that there is an obvious difference between them because of the dynamic effect. From the work of quasi-elastic–static and transient analyses, the dynamic analysis highlighted several areas of interest and a more accurate and reasonable result could be obtained for the engineer.

---

1. Introduction

Viscoelasticity is concerned with materials, which exhibit strain rate effects in response to applied stresses. These effects are manifested by the phenomena of creep under constant stresses and stress relaxation under constant strain. This time-dependent behavior may have a significant effect on the stress distribution developed in a rocket motor, such as solid propellant grains made of viscoelastic materials subjected to prescribed loads. The stress or strain at a specific point in the

---

*Corresponding author. Fax: +886-347-13318.

E-mail address: yeaing@iris.seed.net.tw (S.-W. Chyuan).

0022-460X/03/$ - see front matter

doi:10.1016/S0022-460X(02)01554-7

material may vary significantly with time even though the applied forces are constant. In order to be able to predict the change in stress and strain distribution with time, a viscoelastic stress analysis method is needed and a powerful computational technique is indispensable. Recent developments in numerical techniques and computer simulation methods have resulted in a very prestigious progress in engineering analysis. With the increasing developments of digital computer power, finite element method [1,2] is considered to be one of the most powerful CAE design tools for engineers. Complex structural configurations can be modelled using finite elements and the response at any desired point of the structure can easily be determined. In the recent part, the finite element method has been evolving to be a widely accepted tool for the solution of pragmatic engineering problems.

Solid rocket motor structural design currently is based on the concept of a mechanically weak solid propellant grain cast into a stronger metallic or composite case. The outer case provides the essential structural resistance against service and operational loads, and the inner propellant grain's low strength is used for transmission of loads from the grain surface to the outer case. In general, solid rocket motors are subjected to diverse loading during shipment, storage and firing. It is well known that under these loading conditions, cracks can develop in solid propellants because of excessive loads. Therefore, in order to determine the integrity and the ultimate service life of solid rocket motors, studies should be conducted to evaluate the significance of the value and distribution of stress and strain. In a missile system, the structural configuration of solid rocket motor is one of the most complicated parts, and numerical techniques are necessary to simulate the physical behavior and to evaluate the structural integrity of the different designs, and, to minimize the cost of product development. During the last two decades, more and more attention has been paid to the design, manufacturing, and evaluation of solid propellant grains in order to meet service life and performance requirements [3]. Meanwhile, it is also recognized that aging studies (for example: mechanical aging [4,5], chemical aging [6,7], etc.) are extremely necessary to predict the service life of solid propellant grains and the thermal response under thermal shock loads [8,9] plays an important role. Therefore, the reduction of thermal response is one of the significant considerations in the primary design of solid propellant grains and some effective designs (for example: free flap design, P-groove design, stress reliever design, etc.) have been adopted [10]. For the time–temperature-dependent behavior of linear viscoelastic and incompressible polymer materials, concept of time–temperature shift principle, reduced integration and thermorheologically simple material (TSM) assumption were widely used in the linear visoelastic analysis [11–13]. In addition, the method of non-linear visocoelastic analysis of solid propellant grains was developed [14]. From the above statements concerning solid propellant grains, one can see that it is a very difficult and laborious task to predict the physical response during the design phase. Therefore, the use of computer simulation technique to analyze the structural behavior of solid rocket motor in the preliminary design stage is very important and necessary. Because the transient analysis of solid propellant grains subjected to ignition pressurization loading was not considered for simplifying the analytical task in the previous papers [3–14], the dynamic effect could not be simulated and modelled accurately. But it does not mean that the dynamic effect is not important and could be negligible arbitrarily, and this effect usually plays a very important role for some critical design. In order to study the dynamic response, diverse ignition pressurization loading cases were used and investigated in the present paper.

In engineering analysis, theoretical model was the first choice for researchers and scientists because of totally correct and unique solution. But in the pragmatic design problem, theoretical model was scarcely utilized to predict physical response because of the very complex geometrical design and loading transfer path. In addition, an analytical solution to a viscoelastic problem has only been possible for certain simple configurations and for material properties, which are represented by relatively simple viscoelastic models. For the more complex solid rocket motors, numerical or approximate methods have to be used. Therefore powerful numerical method was introduced to analyst to face the difficulty. Among different numerical approaches, finite element method (FEM) and boundary element method (BEM), and, the increasing developments of digital computer power have moved from being research tools for select groups to become powerful design tools for engineers. For boundary element method [15], the easy data preparation due to one dimension reduction makes it attractive for special practical use. For problems with singularities (for example: seepage flow problems, crack, etc.), it is well known that BEM accompanying dual integral formulation [16] became a very effective analytical model. However, the main drawback of BEM is that it is not easy to apply in the field of complicated design. In the structural analysis of complex geometrical solid propellant grains, BEM is difficult to apply. Therefore, FEM has become the most widely used numerical technique for analysis of missile system because the extremely complex structural configurations (e.g., cylinder head structure [17]) can be modelled using finite elements and the response at any desired point of the structure can easily be determined [18].

2. Finite element modelling

2.1. Fundamentals of finite elements and methods of eigenvalue extraction

Transient response analysis is the most general method for computing forced dynamic response. The purpose of a transient response analysis is to compute the behavior of a structure subjected to time-varying excitation. The transient excitation is explicitly defined in the time domain, and all the forces applied to the structure are known at each instant in time. The four basic components of a dynamic system are mass, energy dissipation (damper), resistance (spring), and applied load. As the structure moves in response to an applied load, forces that are a functions of both the applied load and the motion in the individual components are induced. The equilibrium equation representing the dynamic motion of the system is known as the equation of motion. This equation, which defines the equilibrium condition of the system at each point in time, is represented in matrix form [1,2]

$$[M]\{\partial^2 u(t)/\partial t^2\} + [C]\{\partial u(t)/\partial t\} + [K]\{u(t)\} = \{P(t)\}\ , \tag{1}$$

where $\{u(t)\}$ is displacement vector, $\{\partial u(t)/\partial t\}$ is the velocity vector, $\{\partial^2 u(t)/\partial t^2\}$ is the accelerator vector, $\{P(t)\}$ is the external load vector, $[M]$ is the mass matrix, $[K]$ is the stiffness matrix and $[C]$ is the damping matrix. Depending on the structure and the nature of the loading, two different numerical methods can be used for a transient response analysis: direct transient response analysis and modal transient response analysis. The direct method performs a numerical integration on the complete coupled equations of motion. The modal method utilizes the mode

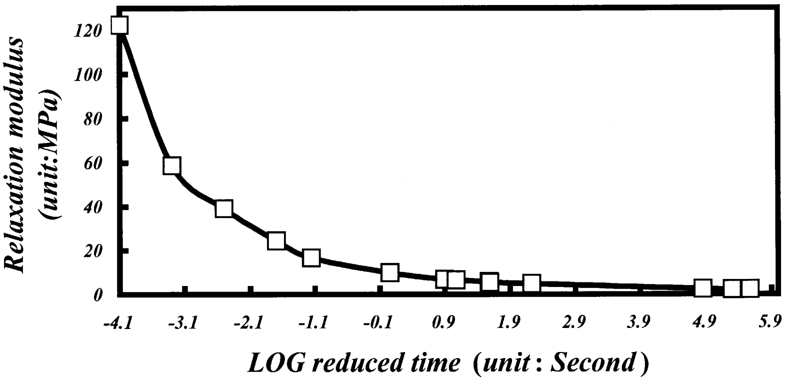


Fig. 1. The master curve of the relaxation modulus for the HTPB propellant (reference temperature: +25°C).

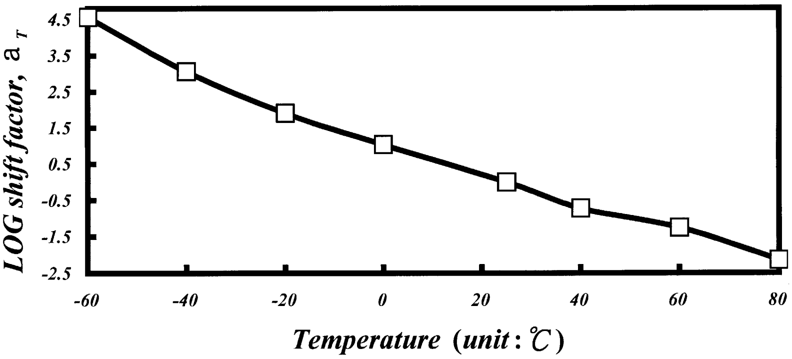


Fig. 2. The time-shift factor for the HTPB propellant (reference temperature: +25°C).

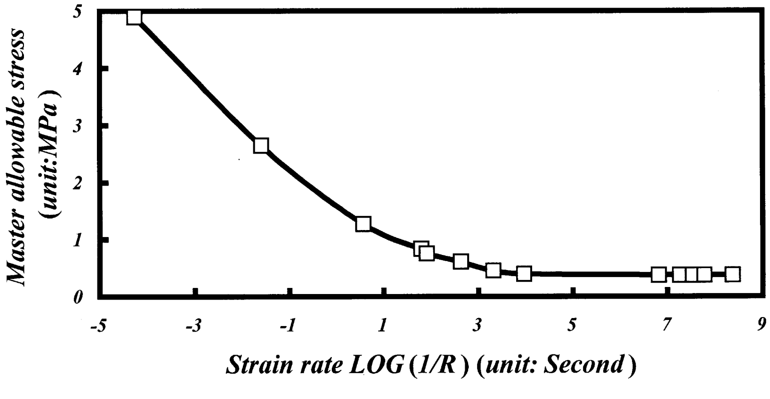


Fig. 3. The master curve of the allowable stress for the HTPB propellant (reference temperature: +25°C).

shapes of the structure to reduce and uncouple the equations of motion (when modal or no damping is used); the solution is then obtained through the summation of the individual modal responses. The choice of the approach is problem dependent. Although the direct method may be the most efficient because it solves the equations without first computing the modes for small models with a few time steps, larger models may be solved more efficiently in modal transient response because the numerical solution is a solution of a smaller system of uncoupled equations.

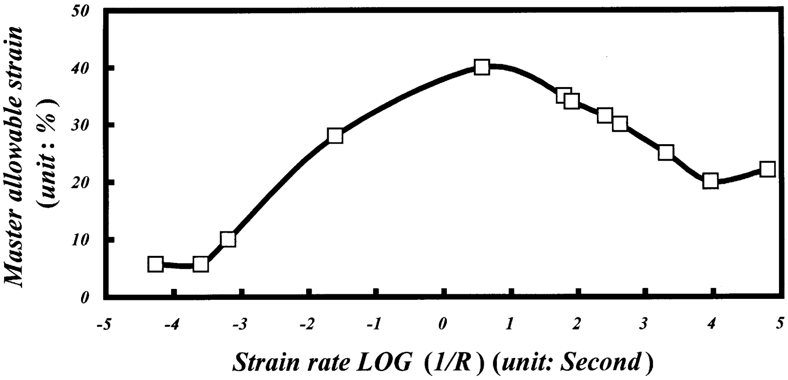


Fig. 4. The master curve of the allowable strain for the HTPB propellant (reference temperature: +25°C).

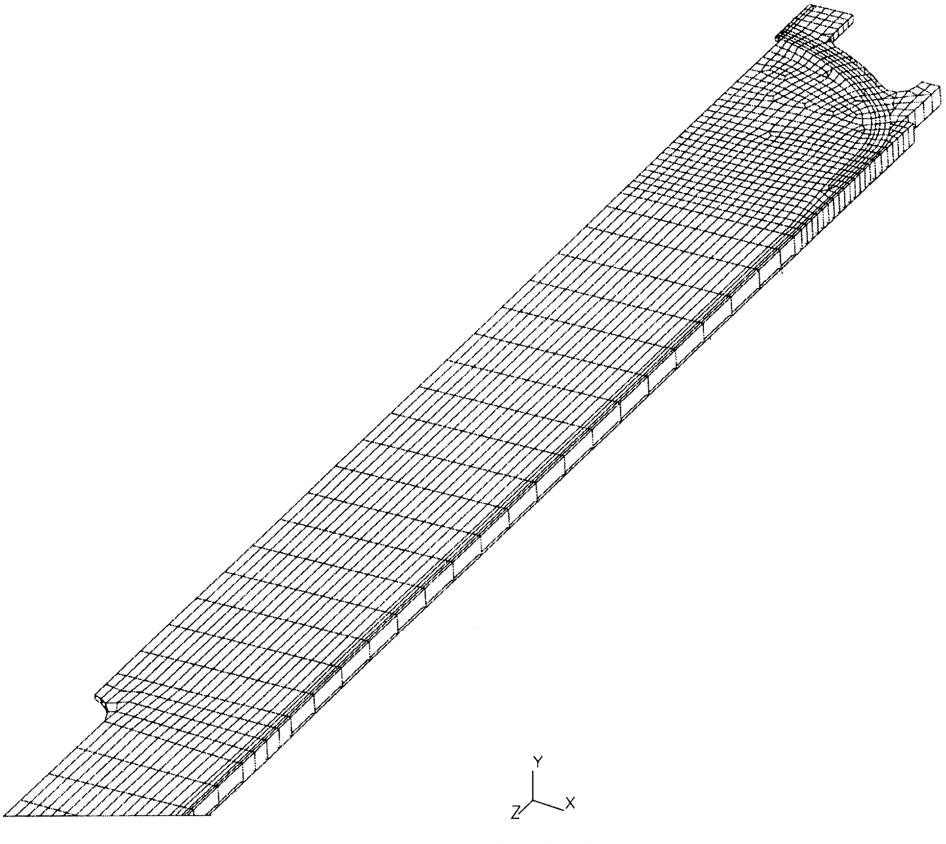


Fig. 5. The finite element model of solid rocket motor.

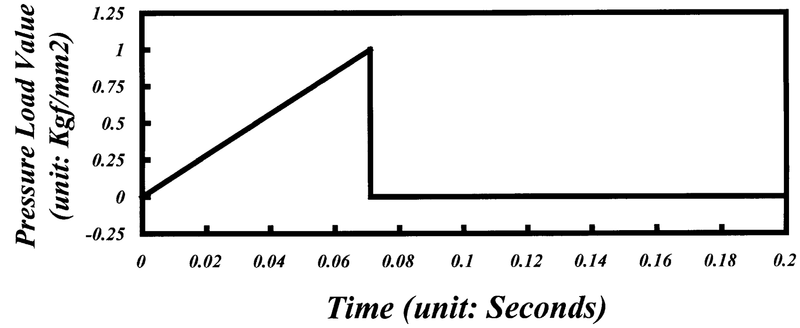


Fig. 6. Diagram of ignition pressurization loading case 1.

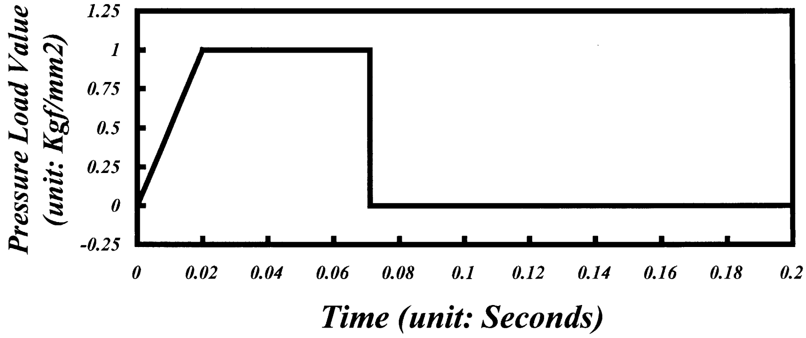


Fig. 7. Diagram of ignition pressurization loading case 2.

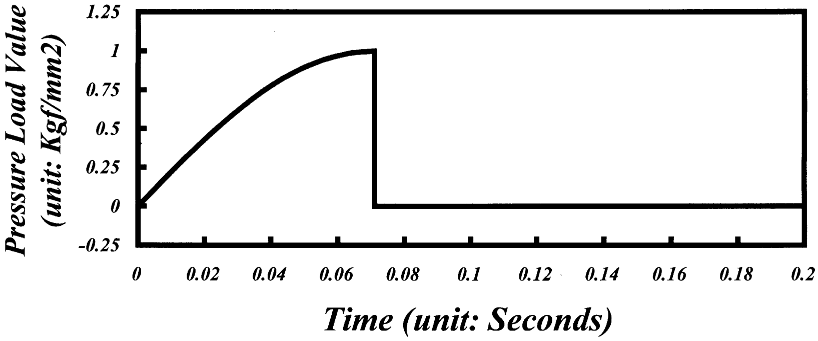


Fig. 8. Diagram of ignition pressurization loading case 3.

This result is certainly true if the natural frequencies and mode shape were computed during a previous stage of the analysis. In this paper, the modal method was chosen to implement the dynamic analysis work of solid propellant grains.

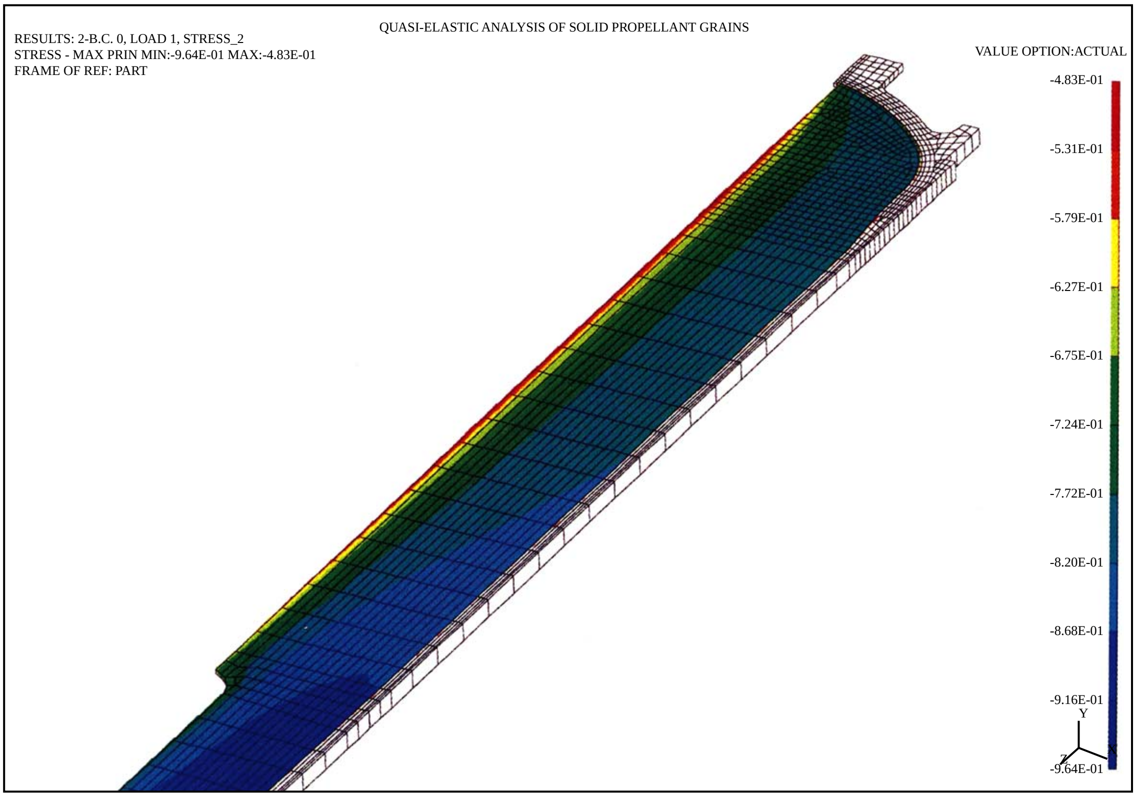


Fig. 9. The maximum principal stress distribution of HTPB propellant under ignition pressurization loading case—quasi-elastic–static analysis.

For modal transient response analysis, a good method for eigenvalue extraction is very important. Since there are a variety of real eigensolution methods, analyst must decide which is best for application. Because the Lanczos method [2] overcomes the limitations and combines the best features of the other approximate solution techniques (e.g., Inverse power method), and this method supports sparse matrix methods that substantially increase its speed and reduce disk requirements, the Lanczos method is the recommended choice for medium to large models like the finite element model of solid propellant grains used in this paper.

2.2. *Constitutive model and material properties of solid propellant grains*

All modern solid propellant grains utilize an elastomeric binder, which is filled with a quite high level of solid particles. The application of a load causes different mechanisms to take place in the binder, the filler or the interface between them such as the breakage of polymer chains, breakage and reformation of weak bonds, deformation and geometrical rearrangement of filler particles, interfacial debonding, also called dewetting, the formation of microvoids at or near the interface

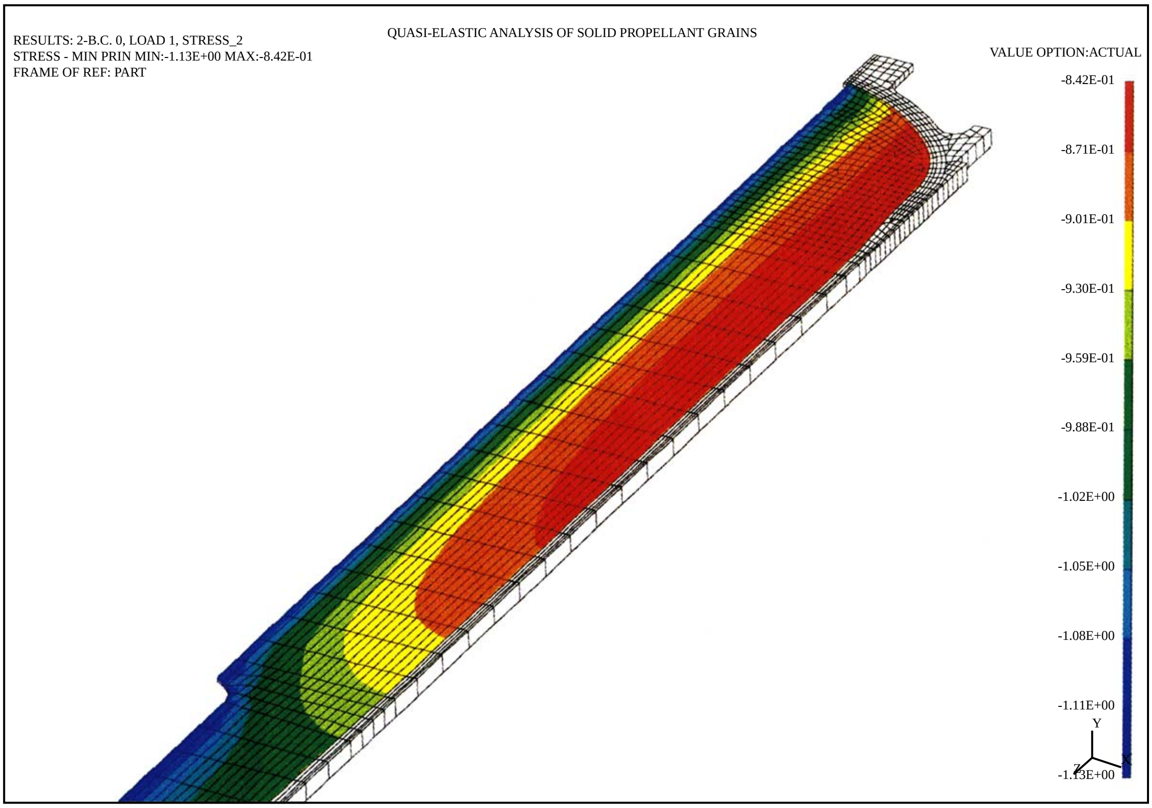


Fig. 10. The minimum principal stress distribution of HTPB propellant under ignition pressurization loading case—quasi-elastic–static analysis.

of the particles and surrounding matrix. Under these influences solid propellant grains exhibit very complex behavior including features associated with time and rate effects, temperature and superimposed pressure dependence, large deformations and large strains, stress softening during cyclic loading, and transition from incompressible to compressible behavior. Therefore, the attempt to represent all aspects of solid propellant grain behavior would result in a very complicated constitutive model and would require a wide range of tests to characterize the propellant grains. Thus, a number of previous investigations have been concerned with certain features only. In the present paper, a proper constitutive model [19,20] of solid propellant grains for engineering analysis is used.

$$\sigma(t) = \int_0^t E(t-\tau)\left[\partial\varepsilon(t)/\partial\tau\right]\,\mathrm{d}\tau, \tag{2}$$

where σ(t) is stress relaxation function, and E(t) is relaxation modulus. The dependence of the viscoelastic properities on the temperature is introduced in a form of an assumption that the solid propellant grain is a rheologically simple material. This assumption permits an introduction of the

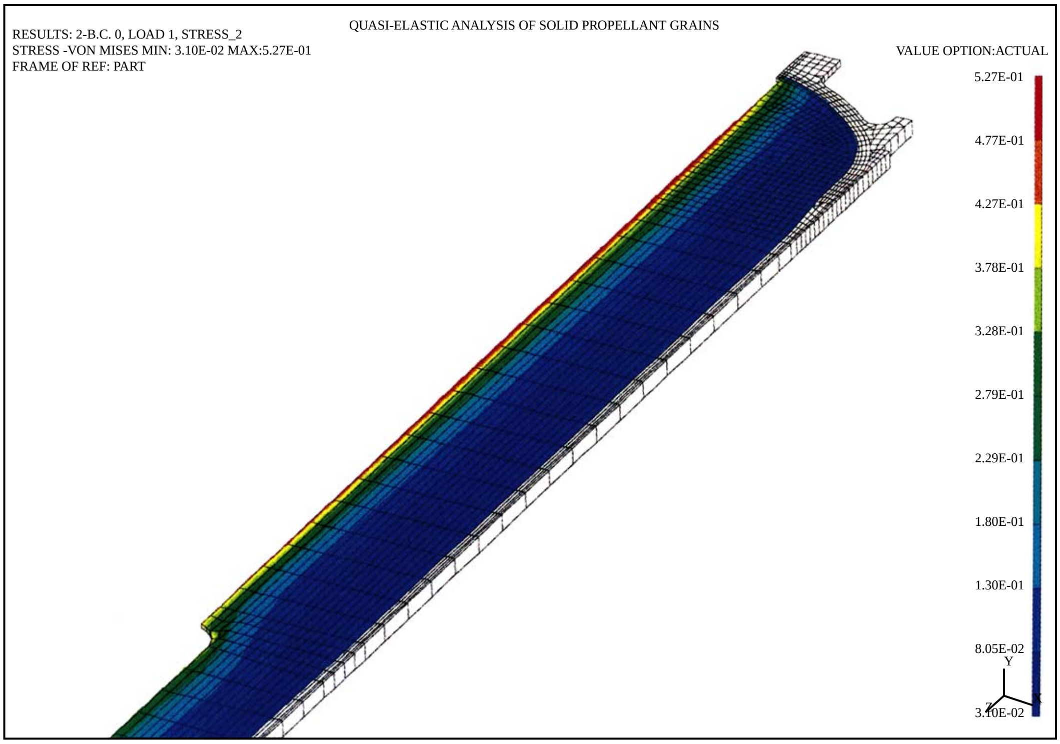


Fig. 11. The effective stress distribution of HTPB propellant under ignition pressurization loading case—quasi-elastic–static analysis.

reduced time ξ, which is defined as follows [21]:

$$\xi = \int_0^t \mathrm{d}\tau'/a_T, \tag{3}$$

where a_T is the time–temperature shift function. One of the most common functions relating the shift factor a_T and temperature has been proposed by Williams *et al.* [21] as follows:

$$\mathrm{Log}_{10}\, a_T = \mathrm{Log}_{10}(t/\xi) = [-k_1(T - T_0)]/[k_2 + (T - T_0)], \tag{4}$$

where k_1 and k_2 are material constants and T_0 is the reference temperature. This form is referred to as the WLF equation. This equation has been used to describe the temperature effect on the relaxation behavior of many polymers with fairly satisfactory results. The consequence of the existence of the reduced time is that a viscoelastic property at some arbitrary temperature T can now be related to the same function at a reference temperature T_0:

$$E(T, t) = E(T_0, \xi). \tag{5}$$

The effective propellant modulus, E_{eq}, to be used in the stress and strain analysis, shall be obtained from the master relaxation modulus curve at a temperature-reduced time corresponding to the time required to reach equilibrium, t^*, divided by the shift factor, a_{T}, for the surrounding

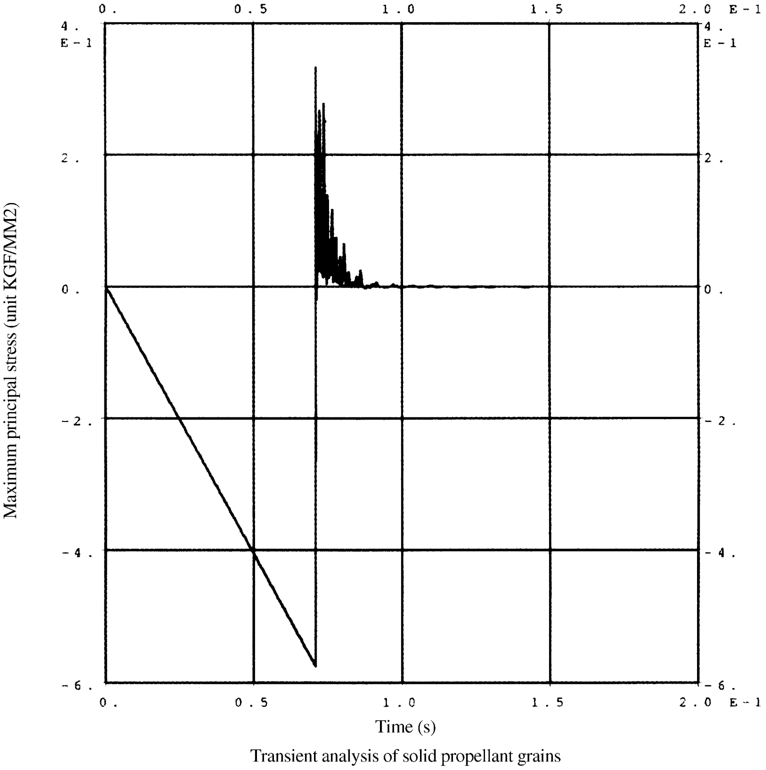


Fig. 12. Diagram of maximum principal stress for HTPB propellant under ignition pressurization loading case 1—transient analysis.

temperature; that is,

$$E_{eq} = E(t * / a_T). \quad (6)$$

For the hydroxy terminated polybutadiene binder (HTPB) propellant used in this analysis, the relaxation modulus versus the temperature-reduced time is shown by Fig. 1, and the time-shift factor for the HTPB propellant is shown by Fig. 2. The curves in Figs. 1 and 2 are plotted according to the experimental data in the environment under different temperatures. Propellant failure properties shall be used on master curves of maximum nominal stress, σ_{max}, and strain at maximum nominal stress, ε_{max}, versus temperature-reduced time, t/a_T. The tests used to generate these master curves shall be conducted in accordance with the JANNAF Tentative Standard Uniaxial Test Procedure and at the same temperatures for which relaxation modulus tests were conducted in Ref. [22]. The time–temperature shift factors, a_T, obtained from the relaxation modulus tests shall be utilized in constructing the master uniaxial faiure curves. The failure criterion for the solid propellant grains depends on its allowable stress and allowable strain, and the strength of the HTPB propellant used in the present paper is obtained from the master allowable stress and allowable strain curve in Figs. 3 and 4.

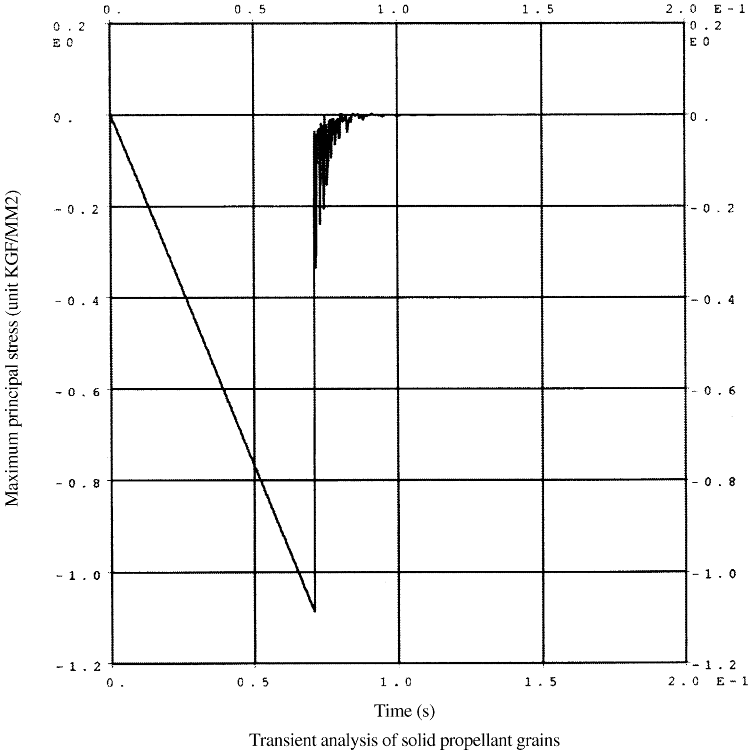
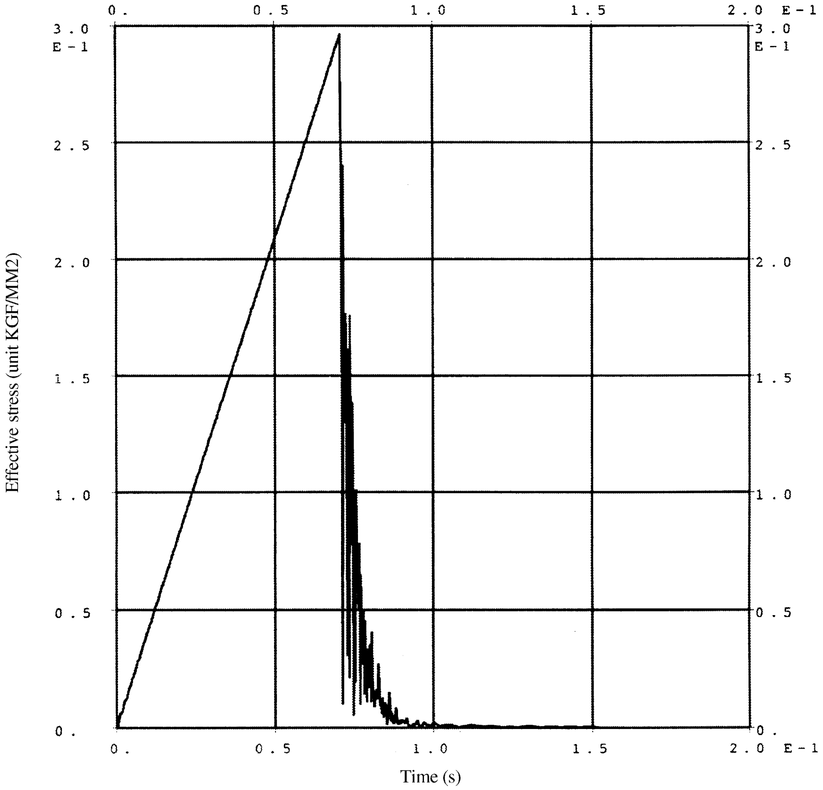


Fig. 13. Diagram of minimum principal stress for HTPB propellant under ignition pressurization loading case 1—transient analysis.

2.3. Finite element modelling of solid rocket motor and reduced integration

Because of the complexity of geometrical design and load path of solid rocket motor system, it is not easy to model the complicated stiffness distribution of this structure just using a simple analytical model. Therefore a 3-D solid model was chosen for this structure in order to predict the stress and strain response in detail. The mesh density on some critical areas is much higher than other sub-critical parts in order to reduce the number of degrees of freedom. Due to the symmetry of the geometry and loading, a model of a five-degree segment with axisymmetric boundary conditions on the cut faces was utilized for simplicity without loss of accuracy. Some appropriate finite element meshes (Fig. 5) with 1408 eight-node solid elements (CHEXA) and 2956 grid nodes were built for the stress and strain analysis, respectively, to acquire their corresponding convergent results. As the Poisson ratio v approaches to 0.5, a material becomes incompressible. For convenience, it is tempting to approximate incompressibility by using $v = 0.49$. But, near $v = 0.5$, stresses strongly dependent on v-stresses may double as v goes from 0.48 to 0.50. Also, structural equations become ill-conditioned as v approaches 0.5 and numerical trouble becomes more likely, the mesh "locks" finally (volumetric locking or shear locking). In the present paper, reduced integration [1] was adopted to simulate the nearly incompressible property of the solid propellant grains.



Transient analysis of solid propellant grains

Fig. 14. Diagram of effective stress for HTPB propellant under ignition pressurization loading case 1—transient analysis.

3. Ignition pressure loading and simulation cases of solid rocket motor

Generally, ignition pressurization in a solid rocket motor induces a compressive hydrostatic pressure through the grain, and there are two parts: abrupt fall and moderate decrease cases. The real pressurization profile is dependent on diverse design of solid rocket motor. In this paper, several simplified examples of ignition pressurization loading (see Figs. 6–8) were used for quasi-elastic–static and transient analysis.

4. Finite element analysis

4.1. Quasi-elastic-static analysis

Under diverse ignition pressurization loading cases (see Figs. 6–8), the effective propellant modulus ($E_{eq}=E(t^*/a_T)$) could be obtained from Fig. 1, and the shift factor ($\mathrm{Log}_{10}\, a_{\mathrm{T}}$) could be obtained from Fig. 2. Because the dynamic effect is not considered in quasi-elastic–static analysis,

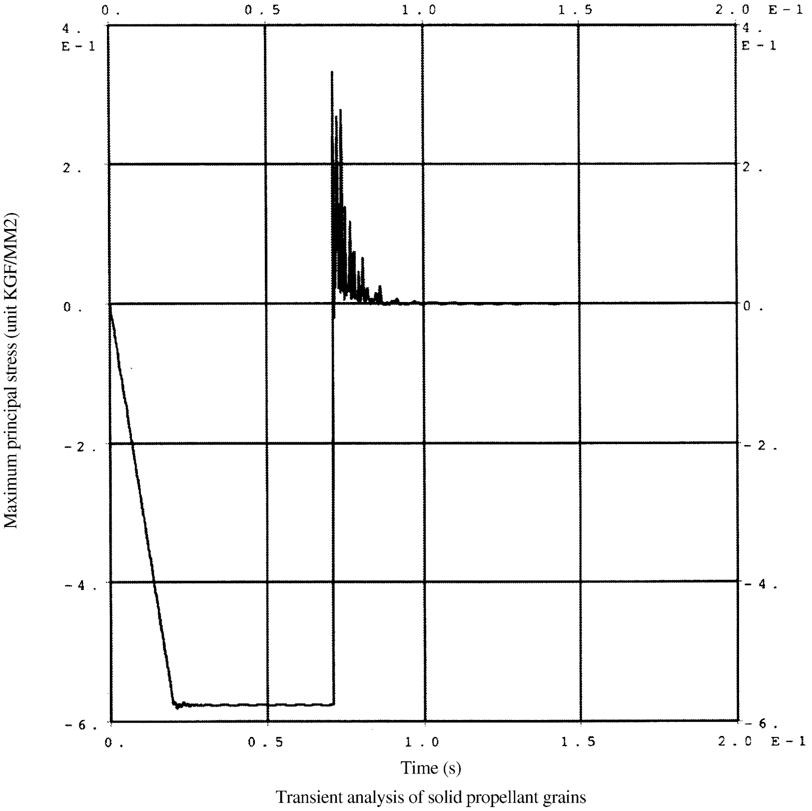


Fig. 15. Diagram of maximum principal stress for HTPB propellant under ignition pressurization loading case 2—transient analysis.

the stress and strain responses are all the same under the above diverse ignition pressurization loading cases shown by Figs. 6–8. From the quasi-elastic–static finite element simulation, the maximum principal stress (σ_{max}) of HTPB propellant is $-0.483\,\text{kgf/mm}^2$ (see Fig. 9), the minimum principal stress (σ_{min}) of HTPB propellant is $-1.13\,\text{kgf/mm}^2$ (see Fig. 10), the effective stress (σ_{eff}) of propellant is $0.527\,\text{kgf/mm}^2$ (see Fig. 11) and the critical area is located at the inner bore free surface of solid propellant grains.

4.2. *Transient analysis*

4.2.1. *Ignition pressurization loading case 1*

Under the ignition pressurization loading case 1 (see Fig. 6), the maximum principal stress of HTPB propellant is $0.3328739\,\text{kgf/mm}^2$ (see Fig. 12), the minimum principal stress of HTPB propellant is $-1.089191\,\text{kgf/mm}^2$ (see Fig. 13), the effective stress of propellant is $0.2966045\,\text{kgf/mm}^2$ (see Fig. 14), and the critical area is also located at the inner bore free surface of solid propellant grains. Comparing the effective stress of transient analysis (see Fig. 14) and quasi-elastic–static analysis (see Fig. 11), one can see that there is an obvious difference between them

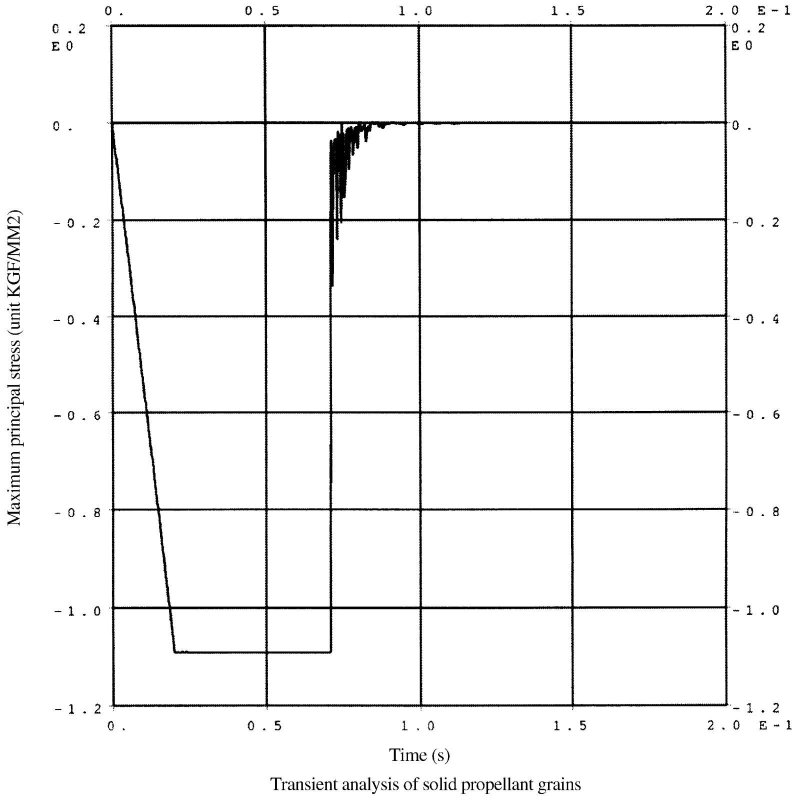


Fig. 16. Diagram of minimum principal stress for HTPB propellant under ignition pressurization loading case 2—transient analysis.

because of the dynamic effect. In addition, there is some oscillation when the ignition pressurization loading fell abruptly and the effective stress responses obtained from the quasi-elastic–static analysis are higher than those from transient analysis simulation.

4.2.2. Ignition pressurization loading case 2

Under the ignition pressurization loading case 2 (see Fig. 7), the maximum principal stress of HTPB propellant is 0.3329476 kgf/mm^2 (see Fig. 15), the minimum principal stress of HTPB propellant is -1.092028 kgf/mm^2 (see Fig. 16), the effective stress of propellant is 0.2996516 kgf/mm^2 (see Fig. 17), and the critical area is also located at the inner bore free surface of solid propellant grains. Comparing the effective stress of transient analysis (see Fig. 17) and quasi-elastic–static analysis (see Fig. 11), one can see that there is also a conspicuous disparity between them because of dynamic effect.

4.2.3. Ignition pressurization loading case 3

Under the ignition pressurization loading case 3 (see Fig. 8), the maximum principal stress of HTPB propellant is 0.3329776 kgf/mm^2 (see Fig. 18), the minimum principal stress of HTPB

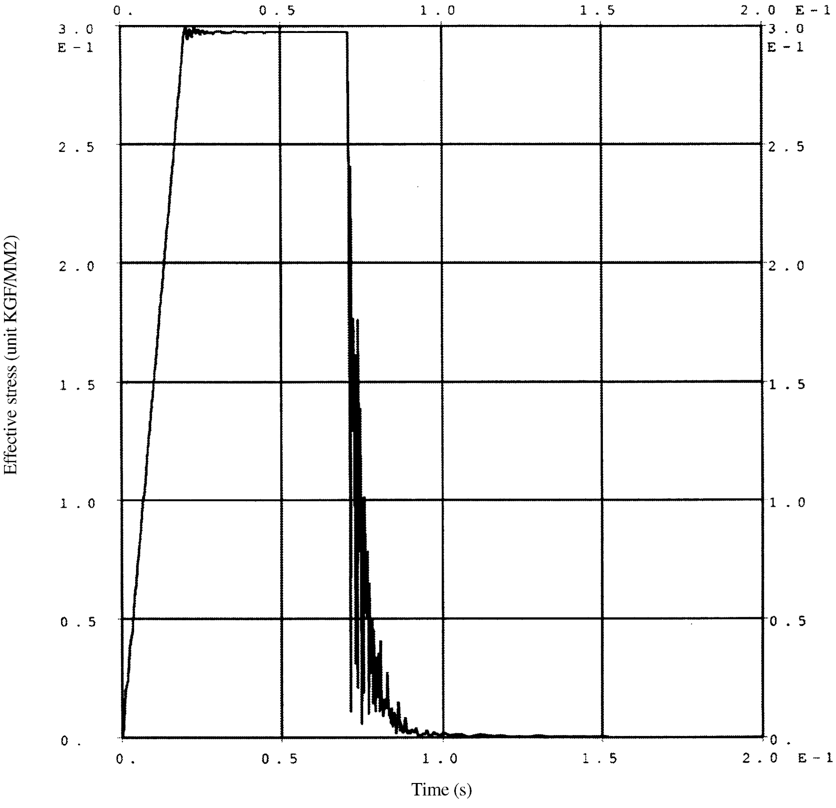


Fig. 17. Diagram of effective stress for HTPB propellant under ignition pressurization loading case 2—transient analysis.

propellant is $-1.091144\,\text{kgf/mm}^2$ (see Fig. 19), the effective stress of propellant is $0.2971872\,\text{kgf/mm}^2$ (see Fig. 20), and the critical area is also located at the inner bore free surface of solid propellant grains. Comparing the effective stress of transient analysis (see Fig. 20) and quasi-elastic–static analysis (see Fig. 11), one can see that there is also an apparent discrepancy between them because of the dynamic effect.

5. Results and Discussions

1. Results show that the dynamic effect is important for structural integrity of solid propellant grains under ignition pressurization loading. Comparing the effective stress from transient analysis and quasi-elastic–static analysis, one can see that there is an obvious difference between them because of the dynamic effect. From the work of quasi-elastic–static and transient analyses, the dynamic analysis highlighted several areas of interest and a more accurate and reasonable result could be obtained for the engineer.
2. Numerical results presented in this article show that there is some oscillation when the ignition pressurization loading fell abruptly. In addition, the effective stress responses obtained from

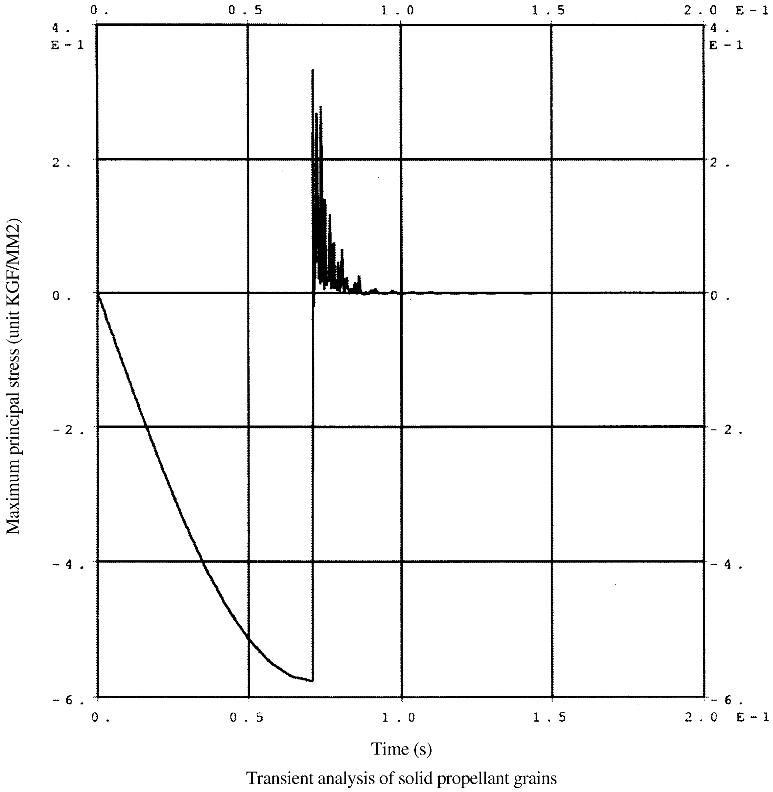


Fig. 18. Diagram of maximum principal stress for HTPB propellant under ignition pressurization loading case 3—transient analysis.

the quasi-elastic–static analysis are higher than those from transient analysis simulation. Because tensile stress might produce surface cracks, production of grain defects could be predicted by our dynamic analysis.

3. For solid propellant grains system, a more drastic assumption, which sometimes may be convenient and sufficiently accurate, is to treat the analysis as though the material was incompressible. Since materials clearly are never completely incompressible, this incompressible material assumption is artificial and less accurate than linear compressibility. The behavior of a material may be taken as that of an incompressible material if the effect of compressibility is unimportant in the application considered. Whether or not the effect of compressibility is negligible may be difficult to establish, however. It should not be expected that an incompressible model will describe real material behavior as accurately as a compressible model. In addition, Poisson's ratio value plays a very import role in structural integrity of solid rocket motor. Therefore, how to get an exact ν value becomes a crucial task. Although material test is the best method to get this, a good method [23] for calculating the viscoelastic Poisson's ratio is also useful for engineering analysis.
4. Because the configuration of solid rocket motor is too complex to predict the critical area and failure mode in the design phase using a simple analytical model, the finite element simulation becomes the best method to obtain the stress distribution under diverse loading cases. Without

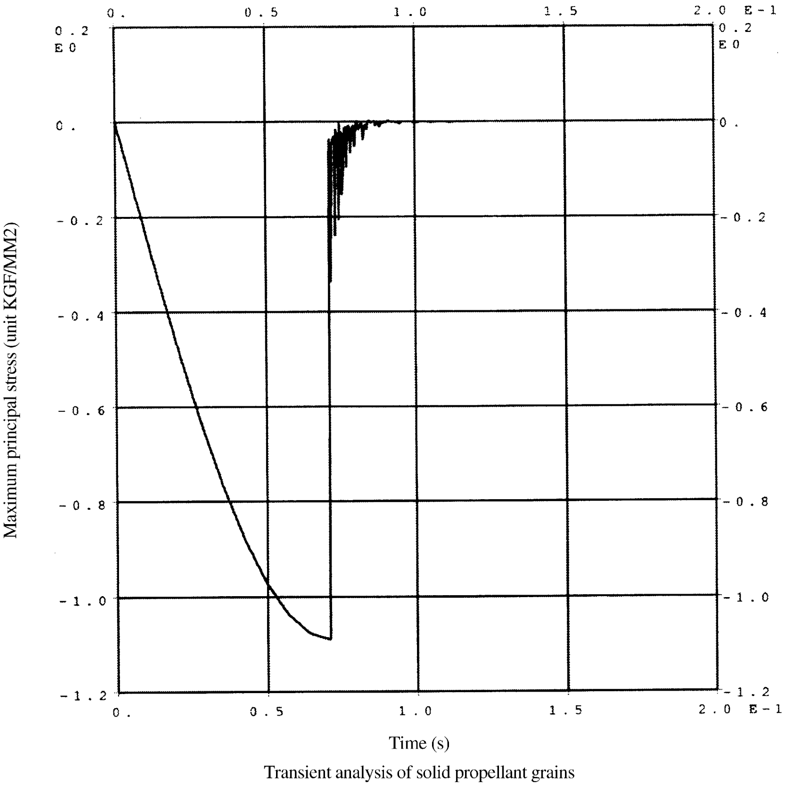


Fig. 19. Diagram of minimum principal stress for HTPB propellant under ignition pressurization loading case 3—transient analysis.

the numerical analysis, designers need much more time to get a feasible design type accompanied by many experimental data. Therefore, the finite element analysis can reduce project span time and save the total project cost.

6. Conclusions

An elaborate and extensive structural analysis of solid propellant grains considering the dynamic effect under ignition pressure loading case was carried out using a commercial analysis FEA software package [24,25] with a CAE pre-post processor [26]. A 3-D solid model was adopted to obtain detailed analysis results. In order to simulate the dynamic response, a transient finite element model accompanied by concepts of time–temperature shift principle, reduced integration and thermorheologically simple material assumption was used. Results show that the dynamic effect is important for structural integrity of solid propellant grains under ignition pressurization loading. From the work of quasi-elastic–static and transient analyses, the dynamic analysis highlighted several areas of interest and a more accurate and reasonable result could be obtained for the engineer. Recommendation resulting from this work has been forwarded to the

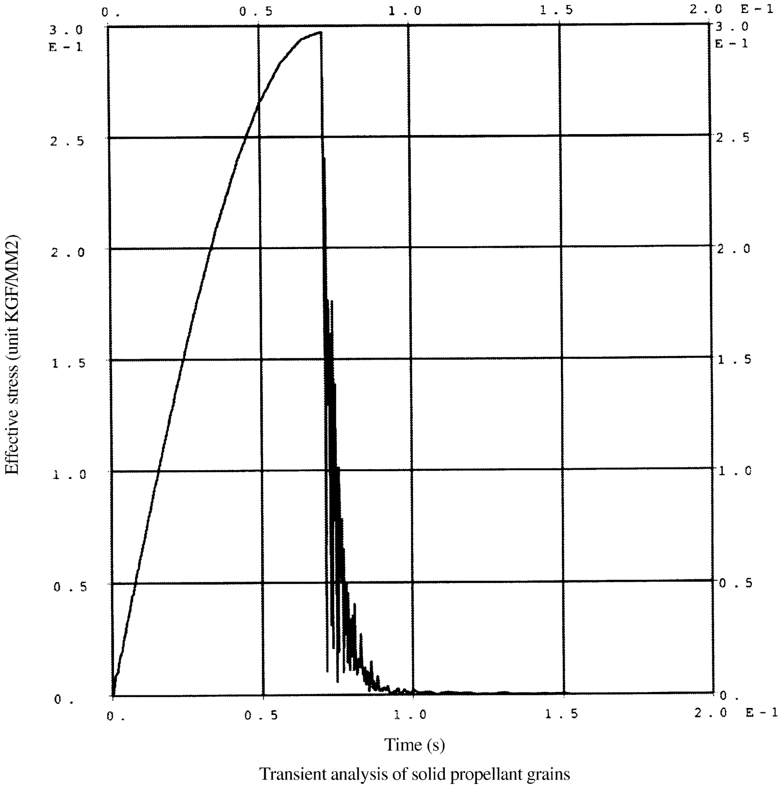


Fig. 20. Diagram of effective stress for HTPB propellant under ignition pressurization loading case 3—transient analysis.

designer successfully, for incorporation of modifications, and to other proper areas for design evaluation.

References

[1] O.C. Zienkiewicz, K. Morgan, Finite Elements and Approximation, Wiley, New York, 1983.

[2] K.J. Bathe, Finite Element Procedures in Engineering Analysis, Prentice-Hall, Englewood Cliffs, NJ, 1982.

[3] G.J. Svob, K.W. Bills Jr., Predictive surveillance technique for air-launched rocket motors, Journal of Spacecraft. 21 (2) (1984) 162–167.

[4] K. W. Bills Jr., Structural Design Monograph for Thermal Cycling of Tactical Rocket Propellants, NWC Tech. Memo 3365, December 1977.

[5] S.W. Chyuan, Life assessment of solid-propellant structure under thermal cycling (structure over test) using structural design nomograph, Proceedings of The Fifth ROC Symposium on Fracture Science, 1998, pp. 31-38. (in Chinese).

[6] A.G. Christiansen, L.H. Layton, R.L. Carpenter, HTPB propellant aging, Journal of Spacecraft. 18 (3) (1981) 211–215.

[7] W.M. Chang, S.W. Chyuan, N.C. Shieh, Aging life prediction of polymer material structure using MSC/NASTRAN and Layton Equation, MSC Taiwan Users' Conference Proceedings, 1994. (in Chinese).

[8] S.W. Chyuan, Finite element simulation on solid-propellant structure under thermal shock loads, Proceedings of The 14th National Conference on Mechanical Engineering, The Chinese Society of Mechanical Engineers Chung-Li, Taoyuan, Taiwan, ROC, December 1997, pp. 100-107. (in Chinese).
[9] S.W. Chyuan, Modeling of Transient Thermal Mechanical Response using Finite Element Method (MSC/NASTRAN), MSC Taiwan Users' Conference Proceedings, 1993. (in Chinese).
[10] J.T. Chen, S.-Y. Leu, Finite element analysis design and experiment on solid propellant motors with a stress reliever, Finite Elements in Analysis and Design. 29 (1998) 75–86.
[11] Solid propellant grain structural integrity analysis, NASA SP 8073, 1973.
[12] S.L. Lin, I.C. Tsai, N.C. Shieh, MSC/NASTRAN Stress Analysis Application in Linear Viscoelastic Material Problem, MSC Taiwan Users' Conference Proceedings, 1989. (in Chinese).
[13] S.W. Chyuan, A study of loading history effect for thermoviscoelastic solid propellant grains, Computers and Structures. 77 (2000) 735–745.
[14] S.W. Chyuan, Nonlinear thermoviscoelastic analysis of solid propellant grains subjected to temperature loading, Finite Elements in Analysis and Design. 38 (2002) 613–630.
[15] C.A. Brebbia, J.C.F. Telles, L.C. Wrobel, Boundary Element Techniques, Springer, Berlin, 1984.
[16] J.T. Chen, H.-K. Hong, S.W. Chyuan, Boundary element analysis and design in seepage problems using dual integral formulation, Finite Elements in Analysis and Design. 17 (1994) 1–20.
[17] S.W. Chyuan, Finite element simulation of a twin-cam 16-valve cylinder structure, Finite Elements in Analysis and Design. 35 (2000) 199–212.
[18] S.W. Chyuan, C.Y. Chiou,. Application and modeling of finite element method on structural mechanics and thermal analysis, Abstracts of Invited Lectures and Short Communications Delivered at the Sixth International Colloquium on Numerical Analysis and Computer Science with Applications, Plovdiv, Bulgaria, August 13-17 1997, pp. 31-33.
[19] M.L. William, Structural analysis of viscoelastic materials, Journal of the American Institute of Aeronautics and Astronauts. 5 (1964) 785–808.
[20] W.N. Findley, J.S. Lai, K. Onoran, Creep and Relaxation of Nonlinear Viscoelastic Materials, North-Holland, Amsterdam, 1976.
[21] L.W. Morland, E.H. Lee, Stress analysis of linear viscoelastic materials with temperature variation, Transactions of the Society of Rheology 4 (1960) 233–263.
[22] Minimum Standard Structural Analysis Procedures for Solid Rocket Grains under Thermal and Pressurization Loading, JANNAF Solid Propellant Structural Integrity Handbook, CPIA 230, 1987.
[23] M. Gao, A new method for calculating the viscoelastic poisson ratio for solid propellant, Journal of Propulsion Technology. 2 (1993) 69–73.
[24] G. Sitton, MSC/NASTRAN Basic Dynamic Analysis User's Guide (Version 69), The MacNeal-Schwendler Corporation, Los Angeles, CA, 1997.
[25] J.T. Chen, S.L. Lin, C.Y. Chiou, S.W. Chyuan, J.Y. Hwang, W.R. Harn, W.T. Chin, Finite Element Analysis and Engineering Applications Using MSC/NASTRAN, Northern Gate Publication, Taipei, Taiwan, 1996.
[26] I-DEAS User's Guide, Finite Element Modeling, SDRC, Milford, OH, 1990.