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Journal of Sound and Vibration 268 (2003) 897–916
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Generalized thermoelastic Lamb waves in a plate bordered with layers of inviscid liquid

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Received 16 April 2002; accepted 2 December 2002

Abstract

The propagation of thermoelastic waves in a homogeneous isotropic, thermally conducting plate bordered with layers of inviscid liquid or half-space of inviscid liquid on both sides is investigated in the context of generalized theories of thermoelasticity. Secular equations for the plate in closed form and isolated mathematical conditions for symmetric and antisymmetric wave modes in completely separate terms are derived. The results for coupled and uncoupled theories of thermoelasticity have been obtained as particular cases. The different regions of secular equations are obtained and special cases, such as Lame modes, thin plate waves and short wavelength waves of the secular equations are also discussed. The secular equations for thermoelastic leaky Lamb waves are also obtained and deduced. The amplitudes of displacement components and temperature change have also been computed and studied. Finally, the numerical solution is carried out for an aluminum-epoxy composite and aluminum materials plate bordered with water. The dispersion curves for symmetric and antisymmetric thermoelastic wave modes and amplitudes of displacement and temperature change in case of fundamental symmetric (S_0) and skew symmetric (A_0) modes are presented in order to illustrate and compare the theoretical results. The theory and numerical computations are found to be in close agreement. The results have been deduced and compared with the relevant publications available in the literature at the relevant stages of the work.

1. Introduction

Recently, resurgent interest in Lamb waves was partially initiated by its application of multisensors [1–3]. The density and viscosity sensing with Lamb waves is based on the principle

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0022-460X/03/$ - see front matter
doi:10.1016/S0022-460X(02)01639-5

that the presence of a liquid in contact with a solid plate changes the propagation velocity and the amplitude of the Lamb waves in the plate of free boundaries due, respectively, to the inertial and viscous effects of the liquid. Schoch [4] first investigated the effect of an inviscid liquid on the propagation of Lamb waves. When a plate of finite thickness is bordered with a half-space homogeneous liquid on both sides, part of Lamb wave energy in the plate is coupled into the liquid as radiation; most of the energy is still in the solid. This type of disturbance is called the leaky Lamb wave. Schoch derived the dispersion relation for leaky Lamb waves for an isotropic plate and an inviscid liquid. Incidentally, the dispersion equations also have an interface wave solution whose velocity is slightly less than the bulk sound velocity in the liquid and most of energy is in the liquid. It is often called the Scholte wave after Scholte [5]. Kurtze and Bolt [6] derived a dispersion equation for bending waves when a plate is in contact with an inviscid fluid based on the acoustic impedance concept. Watkins et al. [7] calculated the attenuation of Lamb waves in the presence of an inviscid liquid using an acoustic impedance method. Wu and Zhu [8] studied the propagation of Lamb waves in a plate bordered with inviscid liquid layers on both sides. The dispersion equations of this case were derived and solved numerically. It was also shown that the acoustic impedance approach is valid only when the plate thickness is much smaller than the wavelength of the transverse wave in the solid. Zhu and Wu [9] derived the dispersion equations of Lamb waves of a plate bordered with viscous liquid layer or half-space viscous liquid on both sides. Numerical solutions of the dispersion equations related to sensing applications are obtained.

The temperature of a deformable body can vary both with time and from point to point. This variation can be caused both by heat exchange with external medium and by the process of deformation itself during which a part of the mechanical energy is transformed into heat. The thermoelastic energy degradation is one of the causes of damping of elastic body vibrations. The classical theory of heat conduction predicts an infinite speed of heat transportation, which contradicts the physical facts. During the last three decades, non-classical theories have been developed to alleviate this paradox. Lord and Shulman [10] incorporated a flux-rate term in Fourier's law of heat conduction in order to formulate a generalized theory that admits finite speed for thermal signals. Green and Lindsay [11] included a temperature rate among the constitutive variables to develop a temperature rate dependent thermoelasticity that does not violate the classical Fourier's law of heat conduction when the body under consideration has a center of symmetry; this theory also predicts a finite speed of heat propagation. According to these theories, heat propagation should be viewed as a wave phenomenon rather than a diffusion phenomenon. A wave-like thermal disturbance is referred to as 'second sound' by Chandrasekharaiah [12]. These theories are also supported by experiments exhibited the actual occurrence of second sound at low temperatures and small intervals of time. Researchers such as [13–15] experimentally proved for solid helium that thermal waves (second sound) propagating with finite, though quite large, speed also exist, although for most of the solids, the corresponding frequency window namely, range of frequency of thermal excitations in which thermal waves can be detected is extremely limited. Sharma et al. [16], Sharma [17] and Sharma and Singh [18] studied the propagation of thermoelastic waves in homogeneous isotropic plates subjected to stress free insulated, stress free isothermal, rigidly fixed insulated and rigidly fixed isothermal boundary conditions in the context of Conventional-Coupled (CT), Lord–Schulman (LS), Green–Lindsay (GL) and Green–Nagdhi (GN) theories of thermoelasticity. The secular equations for the symmetric and antisymmetric wave modes in the plate have been derived in the compact form and solved numerically.

In the present paper, we present analysis of Lamb type wave propagation in a thermoelastic plate bordered with an inviscid liquid layer or half-space inviscid liquid on both sides. More general dispersion equations of thermoelastic Lamb type waves are derived and discussed in coupled and uncoupled theories of thermoelasticity. In uncoupled theory, i.e., in the absence of thermal variations, the analysis reduces to that of Wu and Zhu [8] in case of elastic plate and thermal part of the motion separates out from rest of the motion. The secular equations for different conditions of solution have been deduced from the present one. Numerical solution of the dispersion equations and displacement magnitudes for an aluminum-epoxy composite is also presented.

2. Formulation and solution of the problem

We consider an infinite homogeneous isotropic, thermally conducting elastic plate of thickness $2d$ initially at uniform temperature T_0. The plate is bordered both on the top and bottom with infinitely large homogeneous inviscid liquid layers of thickness h (If $h \to \infty$, it becomes the leaky Lamb wave type case). We take origin of the co-ordinate system (x, y, z) on the middle surface of the plate. The x–y plane is chosen to coincide with the middle surface and the z-axis normal to it along the thickness as illustrated in Fig. 1 below. We take x–z plane as the plane of incidence and we assume that the solutions are explicitly independent of y but implicit dependence is there so that transverse component v of displacement is non-vanishing.

In view of this the non-dimensional basic governing equations of motion and heat conduction, in the context of generalized thermoelasticity for the solid plate in the absence of heat sources and body forces are [16,17]

$$u_{,xx} + (1-\delta^2)w_{,xz} + \delta^2 u_{,zz} - (T + t_1\delta_{2k}\dot{T})_{,x} = \ddot{u}, \tag{1}$$

$$(1-\delta^2)u_{,xz} + \delta^2 w_{,xx} + w_{,zz} - (T + t_1\delta_{2k}\dot{T})_{,z} = \ddot{w}, \tag{2}$$

$$T_{,xx} + T_{,zz} - (\dot{T} + t_0\ddot{T}) = \in[\dot{u}_{,x} + \dot{w}_{,z} + t_0\delta_{1k}(\ddot{u}_{,x} + \ddot{w}_{,z})], \tag{3}$$

$$\delta^2(v_{,xx} + v_{,zz}) = \ddot{v}, \tag{4}$$

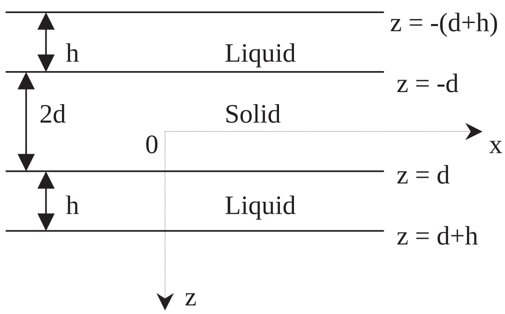


Fig. 1. Geometry of the problem.

where

$$x' = \omega^* x/c_1, \quad z' = \omega^* z/c_1, \quad t' = \omega^* t, \quad u' = \rho\omega^* c_1 u/\beta T_0, \quad T' = T/T_0, \quad t_1' = \omega^* t_1,$$
$$t_0' = \omega^* t_0, \quad w' = \rho\omega^* c_1 w/\beta T_0, \quad v' = \rho\omega^* c_1 v/\beta T_0, \quad \omega^* = C_e(\lambda + 2\mu)/K,$$
$$\in = \beta^2 T_0/\rho C_e(\lambda + 2\mu), \quad \sigma_{ij}' = \sigma_{ij}/\beta T_0, \quad \delta^2 = c_2^2/c_1^2, \quad c_1^2 = (\lambda + 2\mu)/\rho, \quad c_2^2 = \mu/\rho,$$
$$d = \omega^* d/c_1, \quad h' = h\omega^*/c_1, \quad \beta = (3\lambda + 2\mu)\alpha_t. \tag{5}$$

The dot notation is used for time differentiation and comma denotes spatial derivatives. Here λ, μ are Lame's parameters, ρ is the density of the solid, ω^* is the characteristic frequency of the solid plate, $\in$ is the thermomechanical coupling constant, α_t, C_e are, respectively, the coefficient of linear thermal expansion and specific heat at constant strain of the solid plate, $T(x, z, t)$ is temperature change and c_1, c_2 are, respectively, the longitudinal and shear wave velocities in the solid plate and $\mathbf{u}(x, z, t) = (u, v, w)$ is the displacement vector.δ_{ik} is the Kronecker's delta with $k = 1$ for LS theory and $k = 2$ for GL theory. K is the thermal conductivity and t_0 and t_1 is thermal relaxation times. Eq. (4) corresponds to purely transverse (SH) wave, which uncoupled from the rest of the motion and does not depend on the thermal variations and thermal relaxation times. Hence, this equation will not be considered in the following analysis and the resulting problem will be a planar problem. In the solid, we take

$$u = \phi_{,x} + \psi_{,z}, w = \phi_{,z} - \psi_{,x}, \tag{6a}$$

where ϕ, ψ are the velocity potential functions of longitudinal and shear waves in the solid. In the liquid boundary layers, we have

$$u_1 = \phi_{1,x} + \psi_{1,z}, \quad w_1 = \phi_{1,z} - \psi_{1,x}, \tag{6b}$$

$$u_2 = \phi_{2,x} + \psi_{2,z}, \quad w_2 = \phi_{2,z} - \psi_{2,x}, \tag{6c}$$

where ϕ_j and ψ_j, $j = 1, 2$ are, respectively, the scalar velocity potential and vector velocity component along the y direction for the top liquid layer ($j = 1$) and for the bottom liquid layer ($j = 2$), u_j and w_j are, respectively, x and z components of the particle velocity in the layers of liquid. The potential functions ϕ, ψ, ϕ_j and temperature T all satisfy the non-dimensional basic governing equations

$$\nabla^2\psi - \frac{1}{\delta^2}\ddot{\psi} = 0, \tag{7}$$

$$\nabla^2\phi - \ddot{\phi} = T + t_1\delta_{2K}\dot{T}, \tag{8}$$

$$\nabla^2 T - (\dot{T} + t_0\ddot{T}) = \in\nabla^2(\dot{\phi} + t_0\delta_{1k}\ddot{\phi}), \tag{9}$$

$$\nabla^2\phi_j - \frac{1}{\delta_L^2}\ddot{\phi}_j = 0, \quad j = 1, 2, \tag{10}$$

where

$$\delta_L^2 = \frac{c_L^2}{c_1^2}, \quad c_L^2 = \frac{\lambda_L}{\rho_L}. \tag{11}$$

Here c_L is the velocity of sound in the liquid and λ_L is the bulk modulus.

We assume solutions of the form

$$\{\phi, \psi, T, \phi_1, \phi_2\} = \{\bar{\phi}(z), \bar{\psi}(z), \bar{T}(z), \bar{\phi}_1(z), \bar{\phi}_2(z)\}\mathrm{e}^{\mathrm{i}\xi(x-ct)}, \tag{12}$$

where $c = \omega/\xi$ is the non-dimensional phase velocity, ω is the frequency and ξ is the wave number. Upon using solutions (12) in Eqs. (7)–(10) and solving the resulting differential equations, the expressions for ϕ, ψ, T, ϕ_1 and ϕ_2 are obtained as

$$\left.\begin{aligned} \phi &= \textstyle\sum_{k=1}^{2}(A_k \sin m_k z + B_k \cos m_k z)\mathrm{e}^{\mathrm{i}\xi(x-ct)} \\ \psi &= (A_3 \sin \beta z + B_3 \cos \beta z)\mathrm{e}^{\mathrm{i}\xi(x-ct)} \\ T &= \mathrm{i}\omega^{-1}\tau_1^{-1} \textstyle\sum_{k=1}^{2}(\alpha^2 - m_k^2)(A_k\, sin\, m_k z + B_k \cos m_k z)\mathrm{e}^{\mathrm{i}\xi(x-ct)} \end{aligned}\right\} -d<z<d, \tag{13}$$

$$\phi_1 = A_4 \sin \gamma[z - (d+h)]\mathrm{e}^{\mathrm{i}\xi(x-ct)}, \quad d<z<d+h, \tag{14}$$

$$\phi_2 = A_5 \sin \gamma[z + d + h]\mathrm{e}^{\mathrm{i}\xi(x-ct)}, \quad -(d+h)<z<-d, \tag{15}$$

where

$$\alpha^2 = \xi^2(c^2 - 1), \quad \beta^2 = \xi^2\left(\frac{c^2}{\delta^2} - 1\right), \quad \gamma^2 = \xi^2\left(\frac{c^2}{\delta_L^2} - 1\right), \quad m_k^2 = \xi^2(a_k^2 c^2 - 1),$$

$$k = 1, 2, \quad \tau_0 = t_0 + \mathrm{i}\omega^{-1}, \quad \tau_0' = t_0\delta_{1k} + \mathrm{i}\omega^{-1}, \quad \tau_1 = t_1\delta_{2k} + \mathrm{i}\omega^{-1}, \tag{16}$$

$$a_1^2, a_2^2 = \{(1 + \tau_0 - \mathrm{i}\omega\epsilon\tau_0'\tau_1) \pm [(1 - \tau_0 - \mathrm{i}\omega\epsilon\tau_0'\tau_1)^2 - 4\mathrm{i}\omega\epsilon\tau_0\tau_0'\tau_1]^{1/2}\}/2.$$

The main difference between this case and the case of leaky Lamb waves is that the functions ϕ_1 and ϕ_2 here are chosen in such a way that the acoustical pressure is zero at $z = \pm(d+h)$, in other words ϕ_1 and ϕ_2 here are of standing wave solutions, for leaky Lamb waves they are of traveling waves. The boundary conditions at the solid–liquid interfaces $z = \pm d$ to be satisfied are:

(i) The magnitude of the normal component of the stress tensor of the plate should be equal to the pressure of the liquid. This implies that

$$\ddot{\phi} - 2\delta^2(\phi_{,xx} + \psi_{,xz}) = \frac{\omega^2\rho_L}{\rho}\phi_j, \quad j = 1, 2. \tag{17a}$$

(ii) The tangential component of the stress tensor should be zero, implying that

$$2\phi_{,xz} + \psi_{,zz} - \psi_{,xx} = 0. \tag{17b}$$

(iii) The normal component of the displacement of the solid should be equal to that of the liquid. This leads to

$$\phi_{,z} - \psi_{,x} = \phi_{j,z}, \quad j = 1, 2. \tag{17c}$$

(iv) The thermal boundary condition is given by

$$T_{,z} + HT = 0, \tag{17d}$$

where H is the heat transfer coefficient. Using Eqs. (13)–(15) in Eq. (17), the following eight equations in eight unknowns $A_1, A_2, A_3, A_4, A_5, B_1, B_2$ and B_3 can be readily obtained:

$$-\delta^2(\beta^2-\xi^2)[A_1s_1+B_1c_1+A_2s_2+B_2c_2]-2\mathrm{i}\xi\beta\delta^2[A_3c_3-B_3s_3]-\frac{\omega^2\rho_L}{\rho}s_4A_4=0,$$

$$-\delta^2(\beta^2-\xi^2)[-A_1s_1+B_1c_1-A_2s_2+B_2c_2]-2\mathrm{i}\xi\beta\delta^2[A_3c_3+B_3s_3]+\frac{\omega^2\rho_L}{\rho}s_4A_5=0,$$

$$2\mathrm{i}\xi[m_1c_1A_1-m_1s_1B_1+m_2c_2A_2-m_2s_2B_2]+(\xi^2-\beta^2)(A_3s_3+B_3c_3)=0, \tag{18}$$

$$2\mathrm{i}\xi[m_1c_1A_1+m_1s_1B_1+m_2c_2A_2+m_2s_2B_2]+(\xi^2-\beta^2)(-A_3s_3+B_3c_3)=0,$$

$$m_1c_1A_1-m_1s_1B_1+m_2c_2A_2-m_2s_2B_2-\mathrm{i}\xi(s_3A_3+c_3B_3)-\gamma c_4A_4=0,$$

$$m_1c_1A_1+m_1s_1B_1+m_2c_2A_2+m_2s_2B_2-\mathrm{i}\xi(-s_3A_3+c_3B_3)-\gamma c_4A_5=0,$$

$$(\alpha^2-m_1^2)[(m_1c_1+Hs_1)A_1+(-m_1s_1+Hc_1)B_1] \\ +(\alpha^2-m_2^2)[(m_2c_2+Hs_2)A_2+(-m_2s_2+Hc_2)B_2]=0,$$

$$(\alpha^2-m_1^2)[(m_1c_1-Hs_1)A_1+(m_1s_1+Hc_1)B_1]+(\alpha^2-m_2^2)[(m_2c_2-Hs_2)A_2+(m_2s_2+Hc_2)B_2]=0,$$

where $c_k=\cos m_kd$, $s_k=\sin m_kd$, $k=1,2$; $c_3=\cos\beta d$, $s_3=\sin\beta d$, $c_4=\cos\gamma h, s_4=\sin\gamma h$.

Eqs. (18) will have a non-trivial solution iff the determinant of their coefficients is zero. After some algebraic manipulations of the determinant along with conditions $\gamma\neq 0$ and $\gamma h\neq(2n-1)\pi/2$, $n=1,2,3\ldots$ this leads to the following secular equations:

$$\left[\frac{T_1}{T_3}\right]^{\pm1}-\frac{m_1(\alpha^2-m_1^2)}{m_2(\alpha^2-m_2^2)}\left[\frac{T_2}{T_3}\right]^{\pm1}+\frac{\rho_Lc^2\xi^2(\beta^2+\xi^2)m_1(m_1^2-m_2^2)}{\gamma\rho\delta^2(\xi^2-\beta^2)^2(\alpha^2-m_2^2)}\frac{T_4}{[T_3]^{\pm1}}+\frac{4\xi^2\beta m_1H}{(\xi^2-\beta^2)^2m_2}\left[\frac{T_1T_2}{T_3}\right]^{\pm1} \\ \left\{\left[1+\frac{\rho_Lc^2(\beta^2+\xi^2)}{4\gamma\rho\delta^2\beta}\frac{T_4}{[T_3]^{\pm1}}\right]\left[\left[\frac{T_1}{T_3}\right]^{\mp1}-\frac{m_2(\alpha^2-m_1^2)}{m_1(\alpha^2-m_2^2)}\left[\frac{T_2}{T_3}\right]^{\mp1}\right]+\frac{(\xi^2-\beta^2)^2(m_1^2-m_2^2)}{4\xi^2\beta m_1(\alpha^2-m_2^2)}\right\} \\ =-\frac{4\xi^2\beta m_1(m_1^2-m_2^2)}{(\xi^2-\beta^2)^2(\alpha^2-m_2^2)}. \tag{19}$$

Here, the superscript + corresponds to skew symmetric and − refers to symmetric modes and $T_k=\tan m_kd$, $k=1,2$; $T_3=\tan\beta d$, $T_4=\tan\gamma h$.

If we let ρ_L approach to zero, Eqs. (19) reduce to the dispersion equation for Lamb type waves of free boundaries in a thermoelastic plate. Eq. (19) for $\rho_L\to 0$ reduces to

$$\left[\frac{T_1}{T_3}\right]^{\pm1}-\frac{m_1(\alpha^2-m_1^2)}{m_2(\alpha^2-m_2^2)}\left[\frac{T_2}{T_3}\right]^{\pm1}+\frac{4\xi^2\beta m_1H}{(\xi^2-\beta^2)^2m_2}\left[\frac{T_1T_2}{T_3}\right]^{\pm1}\left\{\left[\frac{T_1}{T_3}\right]^{\mp1}-\frac{m_2(\alpha^2-m_1^2)}{m_1(\alpha^2-m_2^2)}\left[\frac{T_2}{T_3}\right]^{\mp1}\right. \\ \left.+\frac{(\xi^2-\beta^2)^2(m_1^2-m_2^2)}{4\xi^2\beta m_1(\alpha^2-m_2^2)}\right\}=-\frac{4\xi^2\beta m_1(m_1^2-m_2^2)}{(\xi^2-\beta^2)^2(\alpha^2-m_2^2)}, \tag{20}$$

which is the secular equation for Lamb type plate waves in a stress free thermoelastic plate. For a stress free thermally insulated ($H \to 0$) thermoelastic plate the secular equation (20) becomes

$$\left[\frac{T_1}{T_3}\right]^{\pm 1} - \frac{m_1(\alpha^2 - m_1^2)}{m_2(\alpha^2 - m_2^2)}\left[\frac{T_2}{T_3}\right]^{\pm 1} = -\frac{4\xi^2\beta m_1(m_1^2 - m_2^2)}{(\xi^2 - \beta^2)^2(\alpha^2 - m_2^2)} \tag{21}$$

and for a stress free isothermal ($H \to \infty$) plate, we have

$$\left[\frac{T_1}{T_3}\right]^{\mp 1} - \frac{m_2(\alpha^2 - m_1^2)}{m_1(\alpha^2 - m_2^2)}\left[\frac{T_2}{T_3}\right]^{\mp 1} = -\frac{(\xi^2 - \beta^2)^2(m_1^2 - m_2^2)}{4\xi^2\beta m_1(\alpha^2 - m_2^2)}. \tag{22}$$

Eqs. (21) and (22) are the same as those obtained and discussed by Sharma et al. [16] and Sharma [17] in the case of stress free thermally insulated thermoelastic plate.

3. Discussion of the secular equation

3.1. Regions of the secular equation

From Eqs. (16), we have

$$\alpha^2 = \xi^2(c^2 - 1) = \omega^2 - \xi^2, \beta^2 = \xi^2\left(\frac{c^2}{\delta^2} - 1\right) = \frac{\omega^2}{\delta^2} - \xi^2, m_k^2 = \xi^2(a_k^2c^2 - 1) = a_k^2\omega^2 - \xi^2, k = 1, 2.$$

Here depending on whether $\xi^2 \geqslant \omega^2, \omega^2/\delta^2, a_1^2\omega^2, a_2^2\omega^2$ or $c^2 \leqslant 1, \delta^2, 1/a_1^2, 1/a_2^2$, we may have α, β, m_1, m_2 being real, zero or imaginary. Then the frequency equation (19) is correspondingly altered as follows.

3.1.1. Region I

For $\xi > \omega/\delta$ implying that $c < \delta, 1, 1/a_1, 1/a_2$ and consequently, we have $\alpha = i\alpha'$, $\beta = i\beta'$, $m_k = i\alpha_k$, $k = 1, 2$. In this case the secular equation is written from Eq. (19) by replacing circular tangent functions of α, β and m_k, $k = 1, 2$ with hyperbolic tangent functions of α', β' and α_k, $k = 1, 2$.

3.1.2. Region II

For $\omega/\delta > \xi > \omega$, it follows that $\delta < c < 1$, and the frequency equation in this case is obtained from Eq. (19) by replacing circular tangent functions of α and m_k, $k = 1, 2$ with hyperbolic tangent functions of α' andα_k, $k = 1, 2$.

3.1.3. Region III

For $\xi < \omega$, it follows that $c > 1$ and the frequency equation is given by Eq. (19).

3.2. Waves of short wavelength

Some information on the asymptotic behavior is obtainable by putting $\xi \to \infty$. If we take $\xi > \omega/\delta$, it follows that $\xi > \omega$ and that $c < \delta, 1$. In this case the roots of the secular equation lie in region I and then we replace α, β, m_1 and m_2 in the frequency equation (19) by $i\alpha', i\beta', i\alpha_1$ and $i\alpha_2$.

For $\xi \to \infty$, $\tanh \alpha_k d/\tanh \beta' d \to 1,\ k = 1,2,\ (\tanh \alpha_k d)(\tanh \beta' d) \to 1,\ k = 1,2,\ \tan \gamma h/\tanh \beta' d \to -\mathrm{i}$, so that the frequency equation reduces to

$$\left(2 - \frac{c^2}{\delta^2}\right)^2 [\alpha_1^2 + \alpha_1\alpha_2 + \alpha_2^2 - 1 + c^2] - 4\beta'\alpha_1\alpha_2(\alpha_1 + \alpha_2) = \pm \frac{\mathrm{i}c^4\rho_L\alpha_1\alpha_2(\alpha_1 + \alpha_2)}{\rho\delta^4(c^2/\delta_L^2 - 1)^{1/2}}, \tag{23}$$

which is the dispersion equation for thermoelastic Rayleigh waves of an infinite half-space solid bordered with an infinite half-space homogeneous liquid. The dispersion equation for thermoelastic Rayleigh waves of an infinite half-space solid bordered with a homogeneous liquid layer of thickness h is given as

$$\left(2 - \frac{c^2}{\delta^2}\right)^2 [\alpha_1^2 + \alpha_1\alpha_2 + \alpha_2^2 - 1 + c^2] - 4\beta'\alpha_1\alpha_2(\alpha_1 + \alpha_2) = \mp \frac{c^4\rho_L\alpha_1\alpha_2(\alpha_1 + \alpha_2)}{\rho\delta^4(c^2/\delta_L^2 - 1)^{1/2}} \tan \gamma h. \tag{24}$$

If ρ_L approaches to zero, Eqs. (23) and (24) reduce to

$$\left(2 - \frac{c^2}{\delta^2}\right)^2 [\alpha_1^2 + \alpha_1\alpha_2 + \alpha_2^2 - 1 + c^2] = 4\beta'\alpha_1\alpha_2(\alpha_1 + \alpha_2), \tag{25}$$

which is merely the Rayleigh surface wave equation [16,17] in a stress free thermally insulated thermoelastic half-space solid. In case of uncoupled theory of thermoelasticity ($\in \to 0$), we have $a_1^2 = 1,\ a_2^2 = \tau_0$ which leads to $\alpha_1^2 = \alpha^2 = c^2 - 1$ and $\alpha_2^2 = \tau_0 c^2 - 1$. Eqs. (23) and (24), respectively, reduce to

$$\left(2 - \frac{c^2}{\delta^2}\right)^2 - 4\beta'\alpha_1 = \pm \frac{\mathrm{i}c^4\rho_L\alpha_1}{\rho\delta^4}\left(\frac{c^2}{\delta_L^2} - 1\right)^{-1/2},$$

$$\left(2 - \frac{c^2}{\delta^2}\right)^2 - 4\beta'\alpha_1 = \mp \frac{c^4\rho_L\alpha_1}{\rho\delta^4}\left(\frac{c^2}{\delta_L^2} - 1\right)^{-1/2} \tan \gamma h,$$

which are equivalent to Eqs. (A.1) and (A.2) of Wu and Zhu [8] in non-dimensional form.

It seems that Eq. (24) can be obtained by multiplying the right side of Eq. (23) by a factor of i $\tan \gamma h$. This also seems to be true for the case of Lamb waves. The dispersion equations for leaky Lamb waves, i.e., Lamb waves in an isotropic plate bordered with an infinite half-space homogeneous liquid ($h \to \infty$) at both sides, are as follows:

$$\begin{aligned}&\left[\frac{T_1}{T_3}\right]^{\pm 1} - \frac{m_1(\alpha^2 - m_1^2)}{m_2(\alpha^2 - m_2^2)}\left[\frac{T_2}{T_3}\right]^{\pm 1} + \frac{\rho_L c^2\xi^2(\beta^2 + \xi^2)m_1(m_1^2 - m_2^2)}{i\gamma\delta^2\rho(\xi^2 - \beta^2)^2(\alpha^2 - m_2^2)}\left[\frac{1}{T_3}\right]^{\pm 1} + \frac{4\xi^2\beta m_1 H}{(\xi^2 - \beta^2)^2 m_2}\left[\frac{T_1T_2}{T_3}\right]^{\pm 1}\\ &\left\{\left[1 + \frac{\rho_L c^2(\beta^2 + \xi^2)}{4i\gamma\delta^2\beta\rho}\left[\frac{1}{T_3}\right]^{\pm 1}\right]\left[\left[\frac{T_1}{T_3}\right]^{\mp 1} - \frac{m_2(\alpha^2 - m_1^2)}{m_1(\alpha^2 - m_2^2)}\left[\frac{T_2}{T_3}\right]^{\mp 1}\right] + \frac{(\xi^2 - \beta^2)^2(m_1^2 - m_2^2)}{4\xi^2\beta m_1(\alpha^2 - m_2^2)}\right\}\\ &\quad = -\frac{4\xi^2\beta m_1(m_1^2 - m_2^2)}{(\xi^2 - \beta^2)^2(\alpha^2 - m_2^2)}.\end{aligned} \tag{26}$$

If we multiply a factor of $i \tan \gamma h$ to terms containing ρ_L in Eqs. (26), the dispersion equation (19) of Lamb waves in an isotropic thermoelastic plate bordered with a homogeneous layer of thickness h on both sides can be obtained. Since Lamb waves are special cases of Rayleigh waves

(d approaches infinity), this type of analogy seems reasonable and this result also agrees with Wu and Zhu [8] in case of elastokinetics in the absence of temperature change.

3.3. Lame modes

A special class of exact solutions called the Lame modes but evidently first identified by Lamb in 1917, can be obtained by considering the special case of $\xi = \beta$. The roots for this case are in region II and the frequency equations reduce to

Symmetric: $\tan \beta d \to \infty \Rightarrow \beta = n\pi/2d, (n = 1, 3, 5, \dots)$.

Antisymmetric: $\tan \beta d = 0 \Rightarrow \beta = n\pi/2d, (n = 0, 2, 4, \dots)$.

Here the frequency is given by

$$\omega = \sqrt{2}\beta\delta = n\pi\delta/\sqrt{2}d \quad (n = 0, 1, 2, 3 \dots).$$

Clearly, as expected, these modes do not depend upon the thermal effects and presence of the liquid.

3.4. Thin plate results

Let us consider the case when the transverse wavelength with respect to thickness of the plate is quite large, so that $2\pi/\beta, 2\pi/\alpha, 2\pi/m_1, 2\pi/m_2 \geqslant d$. Regions I and II yield the results of interest in this case. In region I the symmetric case has no root. For skew symmetric case on retaining the first two terms in the expansion of hyperbolic tangents and the frequency equation for $H \to 0$, reduces to

$$(\xi^2 - \beta'^2)^2 - \frac{4}{3}\xi^2\beta'^4 d^2 + \frac{\alpha'^2 d^2}{3}(\xi^2 + \beta'^2)^2 = \frac{\rho_L c^2 \xi^2(\xi^2 - \beta'^2)}{d\gamma\rho\delta^2}\tan \gamma h. \tag{27}$$

This on discarding the terms of higher order than $(c/\delta)^4$, leads to

$$c = 2\xi d\delta\sqrt{\frac{(1-\delta^2)}{3}}\left\{1 - \frac{\rho_L h}{\rho d}\right\}^{-1/2}, \tag{28}$$

which in the absence of liquid $\rho_L \to 0$ becomes

$$c = 2\xi d\delta\{(1-\delta^2)/3\}^{1/2}. \tag{29}$$

This result, with linear dependence of c on ξ agrees with that derived from classical plate theory in elastokinetics and of course pertains to the flexure vibration and represents only a single vibrational mode in limited frequency range in the over all frequency spectrum. No effect of thermomechanical coupling or thermal relaxation time has been observed on thin plates in this case. But the presence of liquid on both sides of the plate affects the phase velocity of flexural vibrational mode as a periodic function of liquid layers width. In region II the antisymmetric case has no roots and the secular equation in symmetric case becomes

$$\alpha'^2 - \alpha_1^2 - \alpha_2^2 + \frac{4\xi^2\alpha_1^2\alpha_2^2}{(\xi^2 - \beta^2)^2} = -\frac{\rho_L c^2 \xi^2(\xi^2 + \beta^2)\alpha_1^2\alpha_2^2 d \tan \gamma h}{\gamma\rho\delta^2(\xi^2 - \beta^2)^2}, \tag{30}$$

which for LS theory implies that

$$c = 2\delta\sqrt{1 - \frac{\delta^2}{1+\in}}\left[\frac{P}{2}\left(1 \pm \sqrt{1 - \frac{Q}{P^2}}\right)\right]^{1/2}, \tag{31a}$$

where

$$P = 1 + \frac{1}{4\delta^2\tau_0(1+\in-\delta^2)} \quad \text{and} \quad Q = \frac{1-\delta^2(1+\in)}{\delta^2\tau_0(1+\in-\delta^2)^2},$$

and for GL theory, we have

$$c = 2\delta\sqrt{1 - \frac{\delta^2}{1+(\tau_1/\tau_0)\in}}\left[\frac{R}{2}\left(1 \pm \sqrt{1 - \frac{S}{R^2}}\right)\right]^{1/2}, \tag{31b}$$

where

$$R = 1 + \frac{1}{4\delta^2\tau_0\left(1+(\tau_1/\tau_0)\in-\delta^2\right)} \quad \text{and} \quad S = \frac{1-\delta^2}{\delta^2\tau_0\left(1+(\tau_1/\tau_0)\in-\delta^2\right)^2}.$$

Thus, the phase velocity is approximately given by $c = 2\delta\sqrt{1-\delta^2/(1+\in)}$, which is the thin plate or plane stress analogue of the bar velocity of longitudinal rod theory in coupled thermoelasticity. In general, here the wave mode depends upon the thermoelastic coupling parameter and thermal relaxation times whose phase velocity is given by Eqs. (31) in case of generalized thermoelasticity. The phase velocity in this case also depends upon the thickness of the plate in addition to its periodic dependence on the thickness of the liquid layers. Thus in case of thin plates $\xi d \ll 1$, fundamental symmetric (S_0) mode becomes dispersionless, the phase velocity is equal to the group velocity and equal to $2\delta\sqrt{1-\delta^2/(1+\in)}$ approximately, thermal relaxation time being small. The fundamental skew symmetric (A_0) mode meanwhile, becomes the flexural or bending wave of the plate with its phase velocity approximately equal to $2\xi d\delta[(1-\delta^2)/3]^{1/2}$ and its group velocity equals to $4\xi d\delta[(1-\delta^2)/3]^{1/2}$. For a thin plate, A_0 mode is essentially a transverse mode, i.e., the z-component of the displacement dominates. On the contrary, for S_0 mode of a thin plate the x-component of the displacement dominates. For an ideal liquid (no viscosity) energy of Lamb waves can only be coupled into the liquid through the z-component of displacement at the plate surface. This explains why A_0 mode of a thin plate is the choice for biosensing applications. The above thin plate analysis reduces to that of Wu and Zhu [8] in elastokinetics, i.e., in case of uncoupled thermoelasticity when we set $\in = 0$, $\quad t_0 = 0 = t_1$ namely, in the absence of temperature change.

4. Uncoupled thermoelasticity

In case of uncoupled thermoelasticity, the thermomechanical coupling constant $\in = 0$, which leads to $a_1^2 = 1, a_2^2 = \tau_0$ so that $m_1^2 = \alpha^2$, $m_2^2 = \xi^2(\tau_0 c^2 - 1)$. Consequently, the secular equation

(19) reduce to

$$\left[\frac{T_1}{T_3}\right]^{\pm 1}+\frac{\alpha\rho_L c^2\xi^2(\beta^2+\xi^2)}{\gamma\rho\delta^2(\xi^2-\beta^2)^2}\frac{T_4}{[T_3]^{\pm 1}}+\frac{4\xi^2\beta\alpha H}{(\xi^2-\beta^2)^2 m_2}\left[\frac{T_1T_2}{T_3}\right]^{\pm 1}\left\{\left[\frac{T_1}{T_3}\right]^{\mp 1}\left[1+\frac{\rho_L c^2(\beta^2+\xi^2)}{4\gamma\delta^2\beta\rho}\frac{T_4}{[T_3]^{\pm 1}}\right]+\frac{(\xi^2-\beta^2)^2}{4\xi^2\beta\alpha}\right\}=-\frac{4\xi^2\beta\alpha}{(\xi^2-\beta^2)^2}. \tag{32}$$

If we let $H\to 0$, Eq. (32) becomes

$$\left[\frac{T_1}{T_3}\right]^{\pm 1}+\frac{\alpha\rho_L c^2\xi^2(\beta^2+\xi^2)}{\gamma\rho\delta^2(\xi^2-\beta^2)^2}\frac{T_4}{[T_3]^{\pm 1}}=-\frac{4\xi^2\beta\alpha}{(\xi^2-\beta^2)^2} \tag{33}$$

and for $H\to\infty$, Eq. (32) becomes

$$\left[\frac{T_1}{T_3}\right]^{\mp 1}+\frac{\rho_L c^2(\beta^2+\xi^2)}{4\gamma\delta^2\beta\rho}\frac{T_4}{[T_1]^{\pm 1}}=-\frac{(\xi^2-\beta^2)^2}{4\xi^2\beta\alpha}. \tag{34}$$

In the absence of liquid $\rho_L\to 0$ Eqs. (33) and (34), respectively, reduce to

$$\frac{T_1}{T_3}=\left[-\frac{4\xi^2\beta\alpha}{(\xi^2-\beta^2)^2}\right]^{\mp 1} \tag{35a}$$

and

$$\frac{T_1}{T_3}=\left[-\frac{(\xi^2-\beta^2)^2}{4\xi^2\beta\alpha}\right]^{\pm 1}. \tag{35b}$$

Eqs. (35) are the same as obtained by Sharma [17] and discussed in detail by Graff [19] in case of stress free boundary conditions in elastokinetics in the absence of temperature change. Eq. (33) is similar to the one obtained by Wu and Zhu [8] in the non-dimensional form (cf. Eqs. (5) and (6)) for Lamb waves in elastokinetics in such conditions. If we let ρ_L approach to zero, Eq. (33) recovers the dispersion equations for Lamb waves of free boundaries in an elastic plate. Also one observation of the dispersion equations (19), (32) and (33) is that, the thickness 'h' of the liquid layer is a parameter of the periodic tangent function. This reflects the periodic nature of the influence due to the presence of the liquid layers of varying thickness on both symmetric and skew symmetric modes of wave propagation in the plate, which has been also confirmed by numerical calculations discussed in the forthcoming section.

5. Displacement and temperature amplitudes

Using Eqs. (5) and (13), the amplitude w_{asy} of displacement z-component, the amplitude u_{sy} of displacement x-component and the amplitude T_{sy} of temperature may be calculated as

$$w_{asy}=\frac{\left[\begin{array}{c}(4\xi^2\beta s_3+T_4c_3)m_1m_2(c_1'c_2-c_1c_2')-(\xi^2-\beta^2)^2c_3(m_2c_2's_1-m_1c_1's_2)\\ -2\xi^2c_3'(\xi^2-\beta^2)(m_1c_1s_2-m_2c_2s_1)\end{array}\right]A_1e^{i\xi(x-ct)}}{4\xi^2\beta m_2c_2s_3+(\xi^2-\beta^2)^2c_3s_2+T_4m_2c_2c_3}, \tag{36}$$

$$u_{sy} = \frac{-\mathrm{i}\xi \begin{bmatrix} (4\xi^2\beta c_3 - T_4 s_3)(m_1 s_1 c_2' - m_2 s_2 c_1') + (\xi^2 - \beta^2)^2 s_3 (c_1 c_2' - c_1' c_2) \\ +2\beta c_3' (\xi^2 - \beta^2)(m_2 s_2 c_1 - m_1 s_1 c_2) \end{bmatrix} B_1 \mathrm{e}^{\mathrm{i}\xi(x-ct)}}{4\xi^2 \beta m_2 s_2 c_3 + (\xi^2 - \beta^2)^2 c_2 s_3 - T_4 m_2 s_2 s_3}, \tag{37}$$

$$T_{sy} = \frac{\mathrm{i}\omega^{-1}\tau_1^{-1} \begin{bmatrix} (4\xi^2\beta c_3 - T_4 s_3)((m_2 s_2 c_1' - m_1 s_1 c_2')\alpha^2 + m_1 m_2 (m_2 c_2' s_1 - m_1 c_1' s_2)) \\ -(\xi^2 - \beta^2)^2 s_3 ((c_1 c_2' - c_1' c_2)\alpha^2 + c_1' c_2 m_1^2 - c_1 c_2' m_2^2) \end{bmatrix} B_1 \mathrm{e}^{\mathrm{i}\xi(x-ct)}}{4\xi^2 \beta m_2 s_2 c_3 + (\xi^2 - \beta^2)^2 c_2 s_3 - T_4 m_2 s_2 s_3}, \tag{38}$$

where

$$T_4 = \frac{\rho_L}{\gamma\rho}\frac{\omega^2}{\delta^2}\frac{s_4}{c_4}(\xi^2 + \beta^2).$$

The expressions for $-w_{sy}, u_{asy}$ and T_{asy} can be obtained from Eqs. (36) to (38), respectively, by interchanging s_k and c_k, s_k' and c_k'; $k = 1, 2, 3$ and by changing T_4 by $-T_4$, A_1 with B_1 and vice versa.

If we ignore the thermal effect, the amplitudes w_{asy} of displacement z-component and the amplitude u_{sy} of displacement x-component in elastokinetics are given as

$$w_{asy} = \frac{[2\mathrm{i}\xi\beta m_1 c_1' s_3 + \mathrm{i}\xi(\xi^2 - \beta^2) c_3' s_1 - T_5 \xi m_1 (c_1' c_3 - c_3' c_1)] A_1 \mathrm{e}^{\mathrm{i}\xi(x-ct)}}{2\mathrm{i}\xi\beta s_3 - T_5 \xi c_3}, \tag{39}$$

$$u_{sy} = \frac{[(\xi^2 - \beta^2)\beta c_3' c_1 - 2\xi^2 \beta c_1' c_3 + \mathrm{i} T_5 (\xi^2 c_1' s_3 + m_1 s_1 c_3' \beta)] B_1 \mathrm{e}^{\mathrm{i}\xi(x-ct)}}{2\mathrm{i}\xi\beta c_3 + T_5 \xi s_3}, \tag{40}$$

where

$$T_5 = \frac{\rho_L}{\mathrm{i}\gamma\rho}\frac{\omega^2}{\delta^2}\frac{s_4}{c_4},$$

and the expressions for $-w_{sy}$, u_{asy} can be obtained from Eqs. (39) and (40), respectively, by interchanging s_k and c_k, s_k' and c_k'; $k = 1, 3$ and by changing T_5 with $-T_5$, A_1 with B_1 and vice versa.

6. Acoustic impedance approach

One observation is that w_{max} is more or less constant with respect to z for thin plates. Under such circumstances, the plate may be considered as a lump element instead of a distributed system. In other words the acoustic impedance concept could be used as a good approximation. For bending waves, the acoustic impedance of the plate Z_S is given by [6,20]

$$Z_S = \mathrm{i}\omega M - \mathrm{i}B\xi^4/\omega, \tag{41}$$

where M is the mass per unit area of the plate, which is equal to $\rho_S(2d)$, B is its bending stiffness, which is given by $[4\delta^2(1 - (\delta^2/1 + \in))](2/3d^3)$. The acoustic impedance of the water layer seen by the plate Z_L may be calculated from the acoustic pressure of the plate divided by the z-component of the particle velocity at the surface of the plate, which is $Z_L = \mathrm{i}\rho_L\omega \tan\gamma h/\gamma$. When the plate and the fluid layers are coupled together, the total acoustic impedance of the system Z is simply equal to the equivalent acoustic impedance of Z_S and two of Z_L in series, i.e., $Z_t = Z_S + 2Z_L$. Since the total impedance of the system Z_t is pure imaginary (any irreversible loss is neglected), the phase velocity of this case can be determined under the condition $Z_t = 0$ [6], which yields $M - B\xi^4/\omega^2 = -2\rho_L \tan \gamma h/\gamma$.

It is seen that the results derived by using dispersion relations and through acoustic impedance approach are found to be in close agreement. Therefore as pointed out by Wu and Zhu [8] in elastokinetics, it is not a good approximation to consider the thermoelastic plate as a lump element too. For applications in biosensing, it is useful to consider situation when a thin plate is bordered with liquid layer on one side and the other side is free. For thin plates, the acoustic impedance approach is valid one and the total impedance is sum of Z_S and Z_L. The phase velocity can be determined by letting $Z_S + Z_L = 0$.

7. Numerical results and discussion

With the view of illustrating the theoretical result obtained in the preceding sections and comparing these in the various situations, we now present some numerical results. The material chosen for this purpose is aluminum-epoxy composite the physical data for which is given as [16]

$$\in = 0.073, \quad \lambda = 7.59 \times 10^{10}\,\mathrm{N\,m^{-2}}, \quad \mu = 1.89 \times 10^{10}\,\mathrm{N\,m^{-2}}, \quad T_0 = 23^\circ\mathrm{C},$$

$$\rho = 2.19 \times 10^3\,\mathrm{kg\,m^{-3}\,m}, \quad K = 2.508\,\mathrm{K/m\,s\,^\circ C}, \quad C_e = 961.4\,\mathrm{J\,kg^{-1}/^\circ C}, \quad \omega^* = 4.347 \times 10^{13}\,\mathrm{s^{-1}}.$$

The liquid taken for the purpose of numerical calculations is water, the velocity of sound in which is given by $c_L = 1.5 \times 10^3\,\mathrm{m/s}$. For such choice of engineering material the roots of the secular equations lie in regions I and II.

From Fig. 2 it is observed that the phase velocity of the lowest symmetric (i.e., fundamental mode) become dispersionless and gets significantly reduced and effected in the presence of liquid and remains closer to the velocity of thermoelastic Rayleigh waves in a solid half-space bordered with a liquid layers on both sides with increasing wave number. The energy transmission takes place mainly along the surface of the plate because the plate behaves as a semi-infinite medium in this situation. The lowest skew symmetric mode has zero velocity at vanishing wave number, which increases to become closer to the velocity of thermoelastic Rayleigh wave with increasing wave number but also gets reduced and affected due to the presence of liquid. The phase velocities of higher modes of propagation attain large values at vanishing wave number, which slash down to become steady and asymptotic to the reduced Rayleigh wave velocity with increasing wave number. The magnitude of velocity of higher modes is observed to develop at a rate, which is approximately n-time, the magnitude of the velocity of first mode ($n = 1$). In the absence of liquid ($\rho_L \to 0$), the dispersion curves for symmetric and skew symmetric modes of vibration in a stress free isothermal plate (Eq. (22)) is given in Fig. 4 for comparison purposes. From the comparison of the dispersion curves in Fig. 2 with those of Fig. 4, it is quite clear that due to the damping effect of the liquid on both sides of the plate, the phase velocity of fundamental mode decreases in case of symmetric S_0 modes from a value more than unity viz. 1.6741 and increases in case of skew symmetric A_0 modes from zero at vanishing wave number to attain the value 0.208183 which is

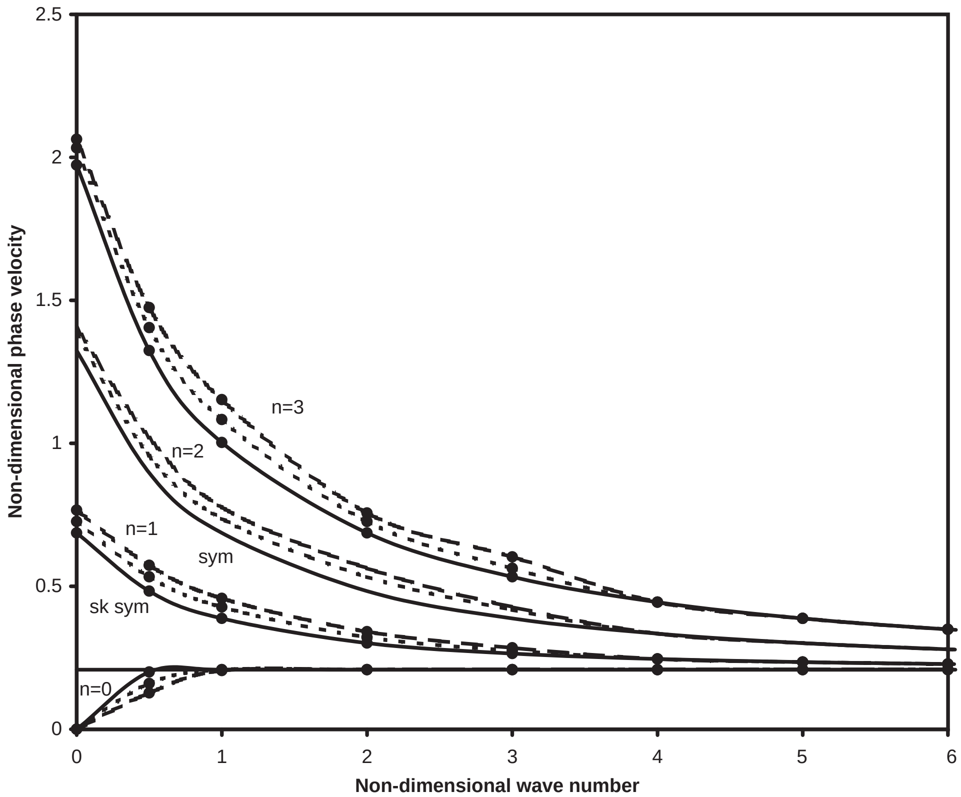


Fig. 2. Phase velocity profile of modified Lamb wave modes of vibrations of a plate bordered with liquid layers with wave number.

the reduced thermoelastic Rayleigh wave velocity. It is also observed that the phase velocity of the Lamb type plate waves falls significantly at vanishing wave number in the presence of the liquid layers as compare to that in the absence of liquid. The dispersion curves become more smoothen in this case than those in the absence of liquid because of the shock absorption nature of the liquid. The fundamental skew symmetric (A_0) mode is observed to be most effected and sensitive.

The phase velocity profile of symmetric and skew symmetric vibrations with respect to the thickness of liquid layers is given in Fig. 3 for a unit thickness plate. It is observed that values of phase velocity are quite high in case of thin plates (at vanishing thickness) except for the fundamental mode ($n = 0$), which slashes down significantly, but steadily to become asymptotically closer to thickness axis with increasing thickness of the liquid layers. Because some part of the energy carried by the waves will be leaked and coupled into the liquid when layers becomes half-space but major part still remains in the solid plate. The phase velocity of various modes of propagation becomes closer to the velocity of leaky Lamb waves with increasing thickness of the liquid. The behavior of S_0 and A_0 modes is again observed to be similar with varying thickness of the liquid as that of with wave number. The dispersion curves in Fig. 3 also confirm the periodic nature of the influence due to the presence of the liquid layers of varying thickness, an observation exhibited by dispersion equation (Fig. 4) in the absence of liquid. In case of thin plates $\xi d \ll 1$, S_0 mode becomes dispersionless; the phase velocity is equal to the group

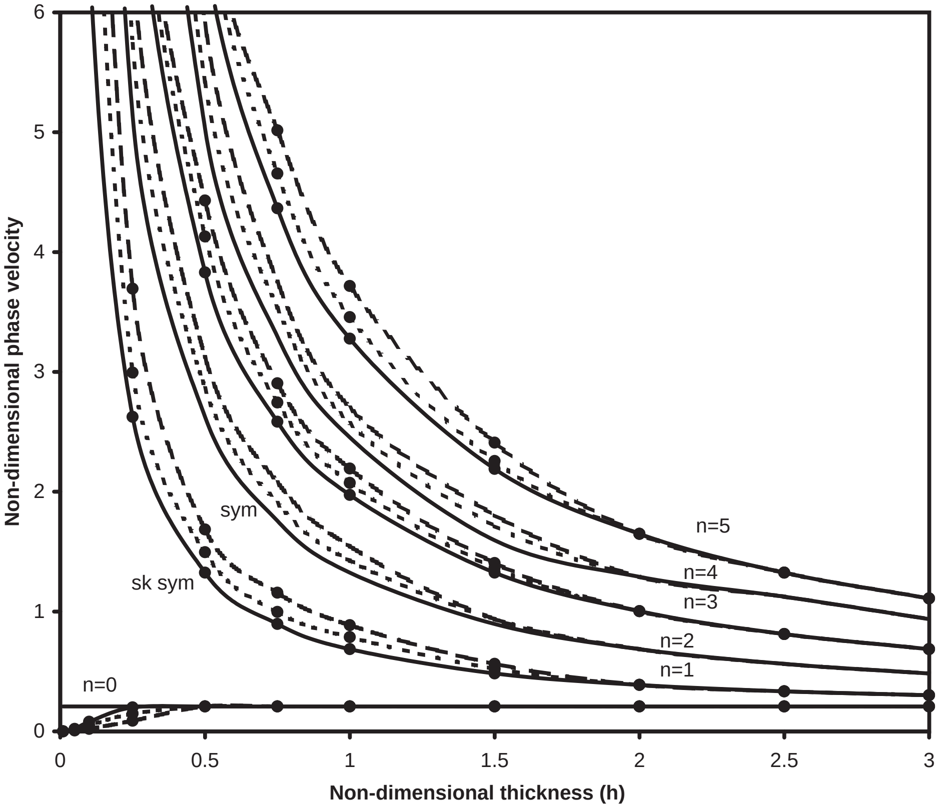


Fig. 3. Phase velocity profile of modified Lamb wave modes of vibrations of a plate with thickness of liquid layer.

velocity and equal to $2\delta\sqrt{1-\delta^2/(1+\in)}$ approximately, thermal relaxation time being small. A_0 mode meanwhile, becomes the flexural or bending wave of the plate, its phase velocity is approximately equal to $2\xi d\delta[(1-\delta^2)/3]^{1/2}$and its group velocity is equal to $4\xi d\delta[(1-\delta^2)/3]^{1/2}$. For a thin plate, A_0 mode is essentially a transverse mode, i.e., the z-component of the displacement dominates. On the contrary, for S_0 mode of a thin plate the x-component of the displacement dominates. For ideal liquid (no viscosity) energy of Lamb waves can only be coupled into the liquid through the z-component of displacement at the plate surface. It explains why A_0 mode of a thin plate is the choice for biosensing applications. For such applications, it is useful to consider situations when this plate is bordered with liquid layer on one side and other side is free. It is also noticed that in order to achieve the same phase velocity, it needs thicker water layer for thick plate than the thin plate. This is to say that for biosensing applications thinner plate has higher sensitivity.

Fig. 5 contains plots of the magnitude of non-dimensional amplitude of the z-component of the displacement (w) for three different values of liquid layer thickness in case of thermoelastic and elastic plates. Three curves namely, solid, broken line and dotted correspond respectively, to $h = 0.25$, 0.5 and 0.75 of the first branch (fundamental) skew symmetric (A_0) mode. Fig. 6 shows the amplitude of the non-dimensional x-component of the displacement (u) in the thermoelastic

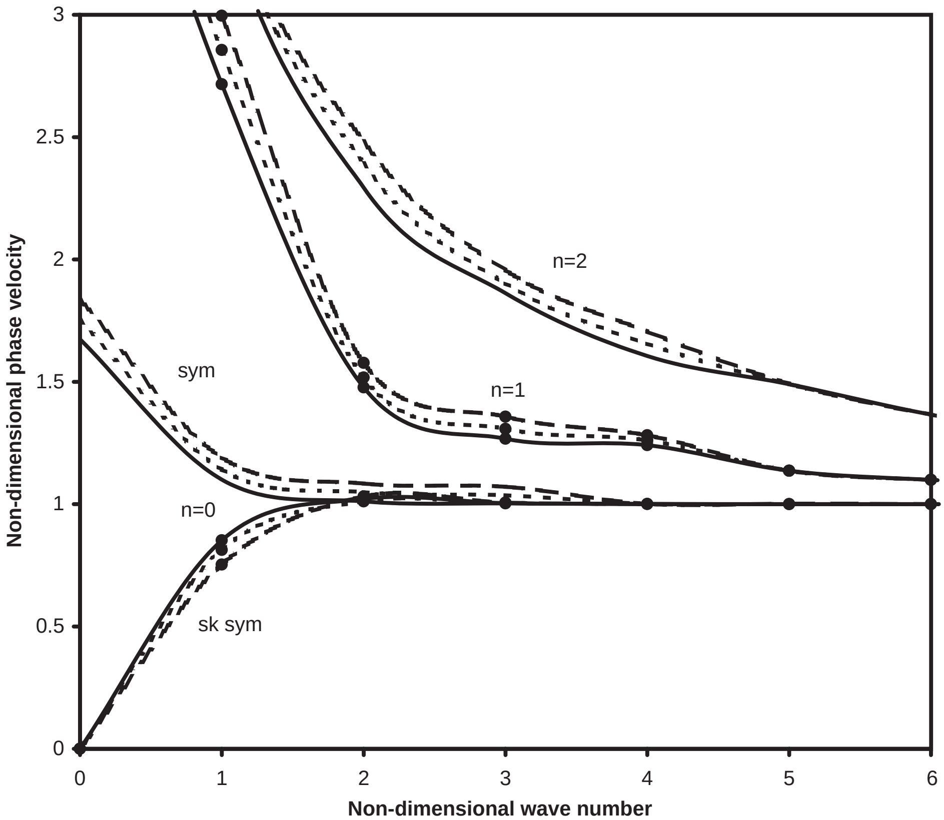


Fig. 4. Dispersion curves for symmetric and skew symmetric modes for stress free isothermal plate in the absence of liquid.

and elastic plates. The meaning of the curves is the same as in Fig. 5. As for as symmetric fundamental (S_0) mode is concerned, the amplitude of the x-component of displacement (u) looks like the amplitude of the z-component of displacement (w) for A_0 mode as shown by Fig. 5 for thermoelastic and elastic plates. The amplitude of z-component of displacement (w) looks like the x-component (u) of A_0 mode as shown in Fig. 6. The comparison of curves in Fig. 5 reveals that due to the thermal variations the amplitude of displacement waves increase significantly in thermoelastic plate as compared to that in elastic plate and hence these amplified signals are easy to detect in case of sensing applications. The damping effect becomes more and more prominent with the increase in liquid layer thickness. Similar observations have also been noticed in case of curves in Fig. 6.

Figs. 7a and b contains the plots of the magnitude of non-dimensional symmetric and skew symmetric temperature change (T) for three different values of the liquid layer thickness in case of a thermoelastic plate, respectively. Three curves (solid, broken line and dotted) again corresponds to $h = 0.25$, 0.5 and 0.75 of the first branches (fundamental) symmetric (S_0) and skew symmetric (A_0) modes. It is observed that the variation of the symmetric and skew symmetric temperature change (T), respectively, looks like the amplitudes of symmetric and skew symmetric x-component of strain.

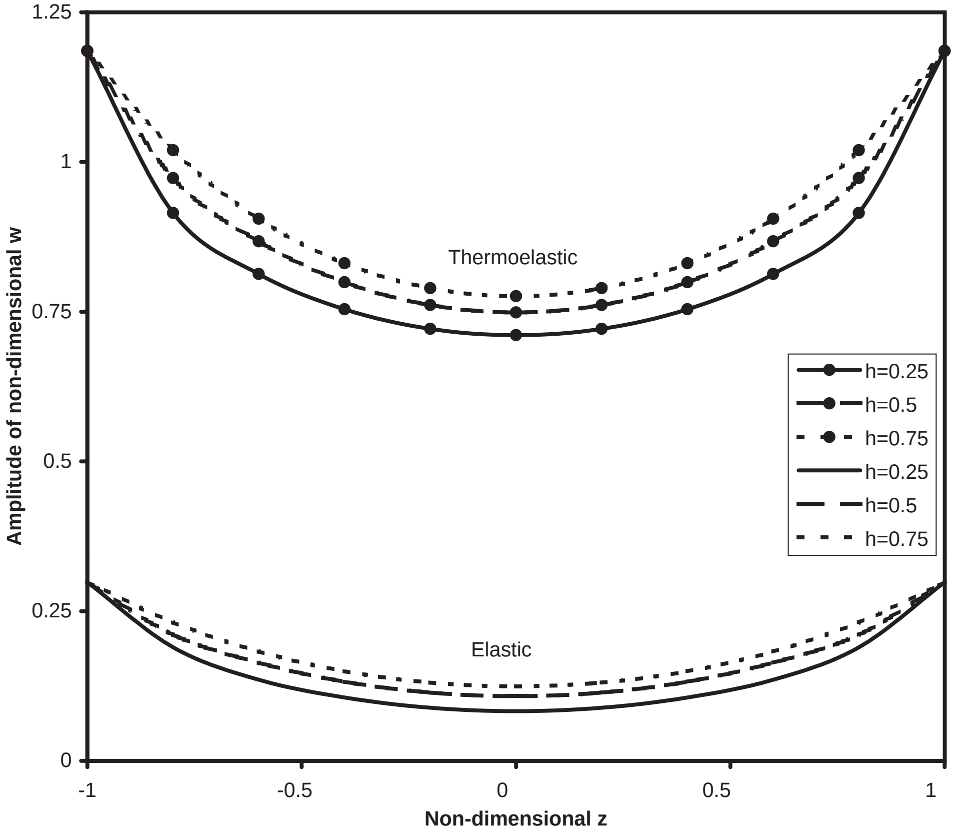


Fig. 5. Variation of amplitude of vertical displacement w w.r.t. z-co-ordinate and thickness (h) of liquid layer. The curves with ball represent thermoelastic case and those without ball are for elastic plate.

8. Conclusions

The propagation of thermoelastic waves in a homogeneous isotropic, thermally conducting plate bordered with layers of inviscid liquid or half-space of inviscid liquid on both sides, is investigated in the context of generalized theories of thermoelasticity. The results for uncoupled theory of thermoelasticity have been obtained as particular cases. The special cases such as Lame modes, thin plate waves and short wavelength waves of the secular equations are also discussed. For the waves of short wavelength it is observed that the dispersion equation of thermoelastic Lamb waves in an homogeneous isotropic plate bordered with a homogeneous layer of thickness h on both sides can be obtained by multiplying a factor of i tan γh to terms containing ρ_L in the dispersion equation for thermoelastic leaky Lamb waves. The periodic tangent function of liquid thickness reflects the periodic nature of the influence due to the presence of the liquid layers of varying thickness, which is also confirmed by our numerical calculations. This analysis includes thermal effects and hence is more general as compared to that of Wu and Zhu [8]. The phase velocity of the lowest symmetric (i.e., fundamental mode) become dispersionless and gets significantly reduced in the present situation and becomes closer to the velocity of thermoelastic

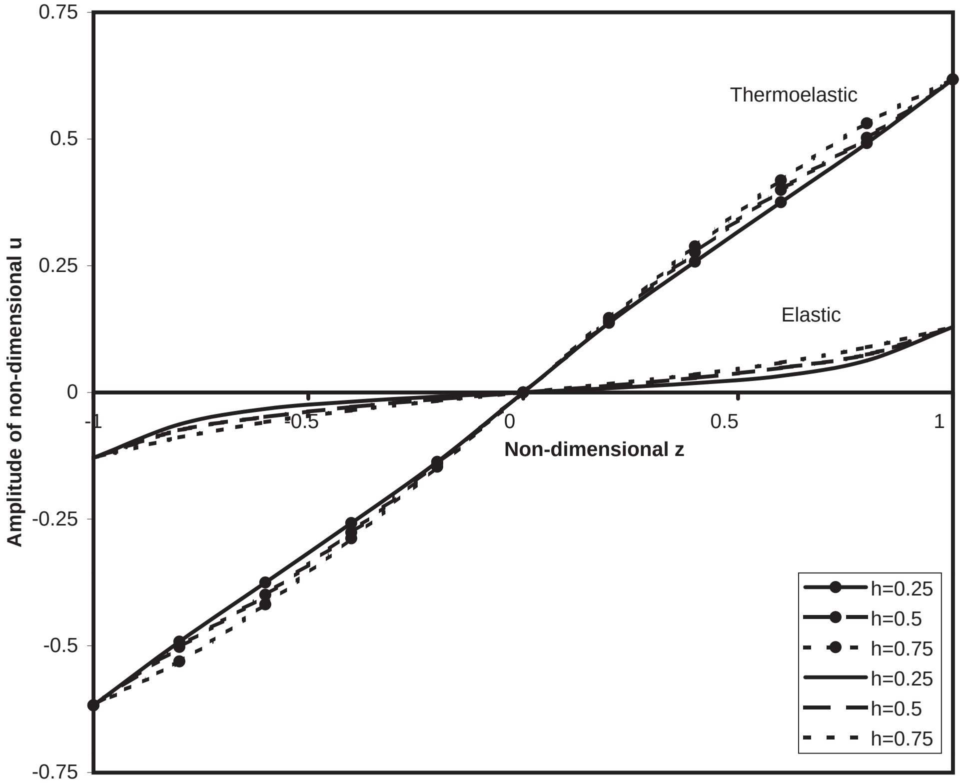
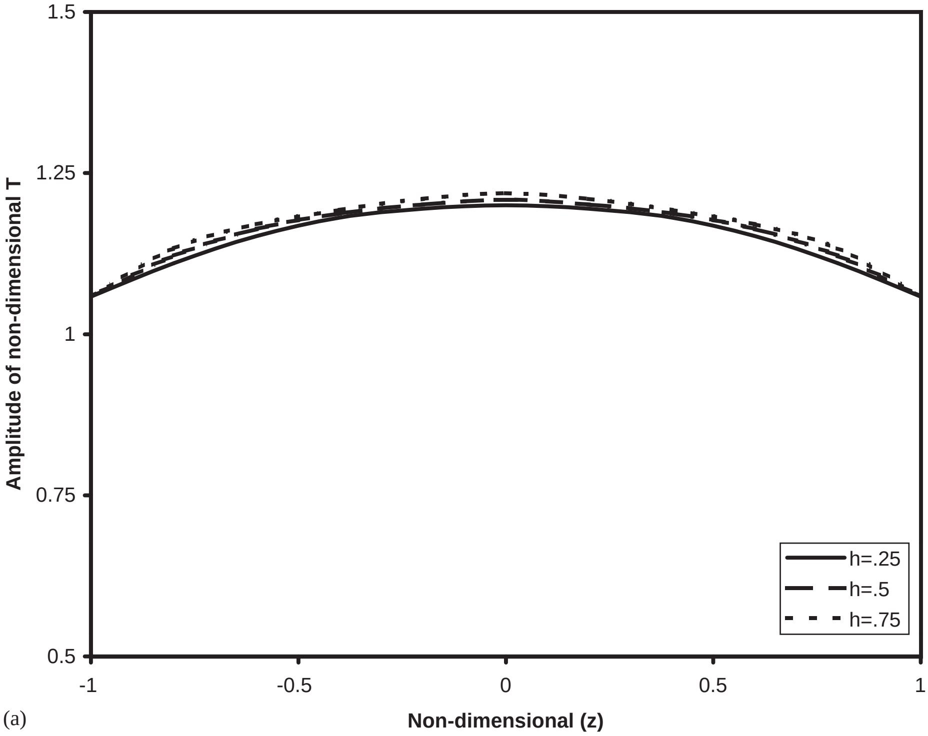


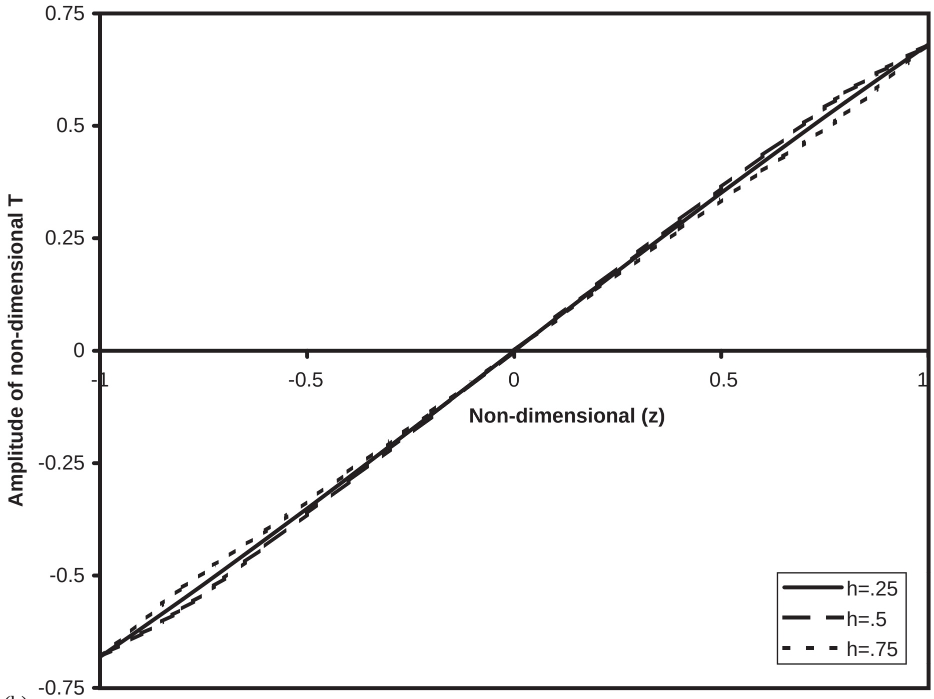
Fig. 6. Variation of amplitude of horizontal displacement u w.r.t. z-co-ordinate and thickness (h) of liquid layer. The curves with ball represent thermoelastic case and those without ball are for elastic plate.

leaky Lamb waves in a solid half-space bordered with liquid layers on both sides of the plate with increasing wave number. The asymptotic closeness of phase velocity to thermoelastic Rayleigh wave velocity at higher wave number is expected because in this case the energy transmission takes place mainly along the surface (interfaces) of the plate. The lowest skew symmetric mode has zero velocity at vanishing wave number, which increases to become closer to the velocity of thermoelastic leaky Lamb wave with increasing wave number and also gets affected due to the presence of liquid. The phase velocity of higher modes of propagation attains large values at vanishing wave number, which slash down to become steady and asymptotic to the reduced thermoelastic Rayleigh wave velocity with increasing wave number. Due to the thermal variations the amplitude of displacement waves increases significantly in a thermoelastic plate as compared to that in elastic plate. The damping effect becomes more and more prominent with the increase of liquid layer thickness. The behavior of temperature change amplitude is observed to resemble with the x-component of strain in the plate. The results for elastokinetics can be obtained s particular

Fig. 7. (a) Variation of amplitude of symmetric temperature change (T) w.r.t. z-co-ordinate and thickness (h) of liquid layer. (b) Variation of amplitude of skew symmetric temperature change (T) w.r.t. z-co-ordinate and thickness (h) of liquid layer.


1.5
1.25
1
0.75
0.5
Amplitude of non-dimensional T
h=.25
h=.5
h=.75
-1
-0.5
0
0.5
1
Non-dimensional (z)

(a)


0.75
0.5
0.25
0
-0.25
-0.5
-0.75
Amplitude of non-dimensional T
-1
-0.5
0
0.5
1
Non-dimensional (z)
h=.25
h=.5
h=.75

(b)

cases from the present analysis by setting the thermomechanical coupling and thermal relaxations parameters equal to zero.

Acknowledgements

The authors are thankful to the University Grants Commission, New Delhi for providing financial support via project grant no. F. 8-34/2001(SR-I) to complete this work. The suggestions of the referees for the improvement of this work are also thankfully acknowledged.

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