Available online at www.sciencedirect.com
SCIENCE DIRECT
Journal of Sound and Vibration 269 (2004) 439–454
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Letter to the Editor

Application of KS test in ball bearing fault diagnosis

Chinmaya Kar, A.R. Mohanty*

Mechanical Engineering Department, Indian Institute of Engineering Technology, Kharagpur, Kharagpur 721 302, India

Received 31 January 2003; accepted 28 March 2003

1. Introduction

Bearing fault diagnosis in machines has been a key concern for which many methods have been devised such as Fast Fourier Transform (FFT) band spectrum analysis, High Frequency Resonance Techniques (HFRT), Discrete Wavelet Transform (DWT). The first two methods mainly analyze the frequency domain of the signals. The FFT spectrum analysis [1] mainly suffers by the intervening extraneous noise that has to be averaged out. Application of HFRT [2] is limited by the use of vibration exciters and their controller. Hence, these methods are costly and time-consuming. The last and recently developed technique DWT [3] uses time-domain vibration signature and hence it is cost-effective and useful for transient and non-stationary signals. But here again large computing effort is required in decomposing the signal into very low resolution.

Hence, there is a great need of a simple but precise technique for quick analysis of time-domain signals. Kolmogorov & Smirnov (KS) test can be a useful tool in this direction. It has already been applied by Andrede et al. [4] in detecting the fatigue cracks in Gears. Prior to this, the technique had been used successfully in many areas [9,10].

This paper explores the possibility of applying KS test in diagnosing rolling element ball bearing faults. Vibration signatures of good and faulty bearings are taken and are compared statistically using KS test. The faulty bearings have scratch marks in inner race, outer race, balls and some combinations of them.

2. KS test

KS test is described in detail in Appendix A. In this application, the two sample KS test is used which tests the statistical similarity between the distribution of two samples. The statistic of such a test is denoted as *D-stat*. The *D-stat* will be determined from the following equation:

$$D\text{-}stat = \max|F_i(X_i) - R_j(X_i)|, \quad \forall i = 1,2,\ldots,N \text{ and } j = 1,2,\ldots,M. \tag{1}$$

*Corresponding author. Tel.: +91-3222-282944; fax: +91-3222-255303.
E-mail address: amohanty@mech.iitkgp.ernet.in (A.R. Mohanty).

doi:10.1016/S0022-460X(03)00380-8

Some papers [4,5,9] have introduced a probability density function (Q_{KS}) for *D-stat* so as to compute probability directly. The probability density function is as follows:

$$Q_{KS}(\lambda) = 2\sum_{i=1}^{\infty}(-1)^{i-1}\,\mathrm{e}^{-2i^2\lambda^2}, \tag{2}$$

where

$$\lambda = \left(\sqrt{N_e} + 0.12 + \frac{0.11}{\sqrt{N_e}}\right)D \quad \text{and} \quad N_e = \frac{MN}{M+N},$$

where D=*D-stat*, N_e is the effective number of data points and M and N are number of data points.

But the demerit of considering probability density function is that it will be difficult to differentiate between small changes. For example, in Ref. [4], there seems to be no difference between brand new gear, fatigue cracks F2 and F3 when compared with a gear under normal operating conditions (NO1). In Ref. [6], it has been recommended to convert the *D-stat* to chi-square test by applying following equation:

$$\chi^2 = 4D^2\frac{MN}{M+N}, \tag{3}$$

where χ^2 is the chi-square statistic.

Then all the procedure of chi-square test can be applied. Drew et al. [6], have described the expression for determining cumulative distribution function of *D-stat* by Birnbaums and then a short procedure for finding the values.

In this paper, the simple *D-stat* value and corresponding *p*-value (probability value) has been considered for decision-making. This will be more effective in order to detect very small defects. The *p*-value means probability value that denotes the significance level (and hence confidence level) for which the critical values of *D-stat* need not be calculated. Still for clarification purpose, the critical values are taken and compared. A significance level (α) of 5% is assumed here. The KS Test is also compared with a parametric test, i.e., Student's *t*-test in order to verify its efficiency. Chi-square test has not been considered as its efficiency decreases for testing continuous distribution. Also, the number samples are very large.

In Ref. [4], care has been taken so as to consider all data from the same starting point. In this study, the effect of time lagging on the result of KS Test has been discussed.

3. Experimental set-up and measurements

A ball bearing vibration test rig has been used to measure the vibration signals of different bearings. The ball bearing (6302 series) specifications are as follows. Ball diameter = 6.747 mm, pitch diameter = 28.7 mm, number of balls = 8 and the contact angle = 0°). Table 1 depicts the bearing numbers and the type of defects introduced artificially. Bearing-1 to -6 will be known as control group and bearing-7 to -13 will be known as treatment group. For the present study, the faults in the bearings were created artificially by an electrical etching process. The radial vibration signatures of the ball bearings were analyzed in an FFT analyzer.

Table 1
Bearing groups, numbers and corresponding defects

Group	Bearing number	Bearing characteristics	Bearing defects
Control	1	Good	No marks
	2	Inner race defect (on track)	1 mark
	3	Inner race defect (on track)	2 marks (180°apart)
	4	Outer race defect (on track)	1 mark
	5	Outer race defect (on track)	2 marks (180°apart)
	6	Ball defect (on surface)	2 marks (180°apart)
Treatment	7	Good	No marks
	8	Good	No marks
	9	Ball defect (on surface)	1 mark
	10	Inner race + ball defect	1 mark each
	11	Inner race + ball + outer race defect	1 mark each
	12	Inner race + outer race defect	1 mark each
	13	Outer race + ball defect	1 mark each

1024 samples with a time record of 0.2 s with a sampling frequency of 2.56 kHz are taken from each bearing. The speed of the motor driving the bearing spindle (and hence, the inner race) is 1800 r.p.m. (30 Hz). Thus for a record time of 0.2 s, the time history of six complete revolutions of the ball bearing is obtained, which is more than sufficient for the KS test to be at a higher confidence level. This will also facilitate in reducing the number of observations. In Fig. 1, the time-domain vibration signatures of the bearings are provided.

4. Result and analysis

The KS test is carried out for control group bearings and the result of the *p*-values and *D-stat* values are given in Table 2.

4.1. Vibration signature analysis

The variation in the vibration signatures of control group bearings with defects is shown in Fig. 1. In a good bearing as in Fig. 1a, the amplitude level is very low, of the order of 0.005 m/s^2. Any defect in inner race or outer race gives rise to impulse and high amplitude levels. The amplitude of the impulse for two outer race defects is of the order of 0.4 m/s^2 (Fig. 1d) whereas the same for inner race defects is of the order of 0.15 m/s^2 (Fig. 1c). This proves the fact that any defect in the outer race will be more prominent. Bearing with two ball defects has an impulse with very less amplitude (Fig. 1f). Hence, the amplitude level has a specific distribution for any defect.

4.2. The *p*-value

The null hypothesis here is that both the bearings tested have the same population or they are statistically similar. The *p*-value means probability value, which can be defined as the likelihood of

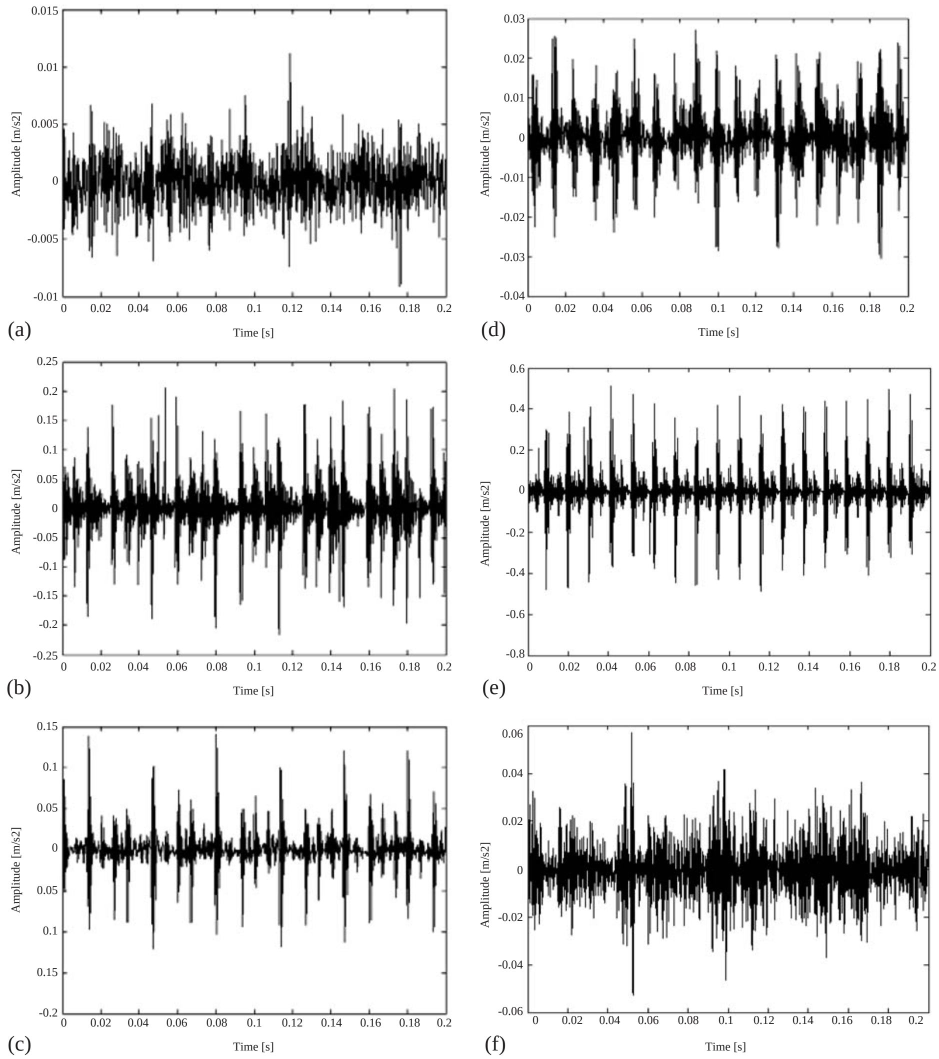

Fig. 1. The time-domain vibration signature of all control group bearings: (a) bearing-1, (b) bearing-2, (c) bearing-3, (d) bearing-4, (e) bearing-5 and (f) bearing-6.

an observed statistic occurring on the basis of the sampling distribution. The value indicates the statistical significance of the comparison and hence confidence level [5]. It has been observed that the *p*-value is one when a data is tested with itself. It signifies that the data sets belong to one family i.e., they both have the same distribution and hence, the *D-stat* value is zero. But when it is tested with others, its value is zero or close to zero. The low *p*-value rejects the null hypothesis that the data sets compared are statistically similar [7]. But the *D-stat* value signifies how much different are the data sets. High *D-stat* value indicates highly different data sets. When bearing-3 (with two defects in inner race) is compared with bearing-6 (with two numbers of ball defects), the resulted *p*-value is 0.0352 and *D-stat* is 0.0624. It implies that the significance level (α) of such a test is 3.52%. The convention of taking 'α' is 5% (confidence level of 95%) [8], and hence

Table 2
Result of KS Test amongst the control group bearings

	Bearing-1	Bearing-2	Bearing-3	Bearing-4	Bearing-5	Bearing-6
Bearing-1						
p-Value	1	0	0	0	0	0
D-stat	0	0.4205	0.3229	0.2156	0.4605	0.361
Bearing-2						
p-Value	0	1	0	0	0	0
D-stat	0.4205	0	0.2039	0.3005	0.1268	0.2273
Bearing-3						
p-Value	0	0	1	0	0	0.0352
D-stat	0.3229	0.2039	0	0.1444	0.2956	0.0624
Bearing-4						
p-Value	0	0	0	1	0	0
D-stat	0.2156	0.3005	0.1444	0	0.3746	0.1805
Bearing-5						
p-Value	0	0	0	0	1	0
D-stat	0.4605	0.1268	0.2956	0.3746	0	0.3259
Bearing-6						
p-Value	0	0	0.0352	0	0	1
D-stat	0.361	0.2273	0.0624	0.1805	0.3259	0

bearing-3 and bearing-6 are not statistically similar even if the *D-stat* is very low. The low *D-stat* may be due to the fact that accelerometer is placed near the outer race which will give a modulated signal for a defect in outer race where as signals due to the inner race and ball defects have larger transfer segments and hence very less distinct. But since this phenomenon holds good for all observations, it will not pose any serious difficulty.

4.3. The D-stat value

In Fig. 2, the comparisons amongst the control group bearings are shown. Fig. 2a depicts the comparison between the good bearing (bearing-1) with all other control group bearings. All *D-stats* are greater than zero and distinct, thus conforming to the present paper's contention that all defects have different probability distribution and *D-stat* can be a good statistical parameter for ball bearing fault diagnosis. When bearing-3 and bearing-4 are compared separately with bearing-1, the *D-stats* have very small difference (i.e., 0.1073), but when they are compared with each other, the resulting *D-stat* is 0.1444 (Fig. 2d). A similar observation can be made when bearing-5 and bearing-6 (when compared separately with bearing-2, a *D-stat* difference of 0.1005 resulted while when compared with each other, the resulting *D-stat* was 0.3259). It can be inferred from these observations, all the bearings have to be tested with one another in order to reach any conclusion about any defect.

4.4. Other statistical parameters

Table 3 shows the comparison of different statistical parameters of control group bearings. Kurtosis can be a good parameter to measure defectiveness as there is a large variation for

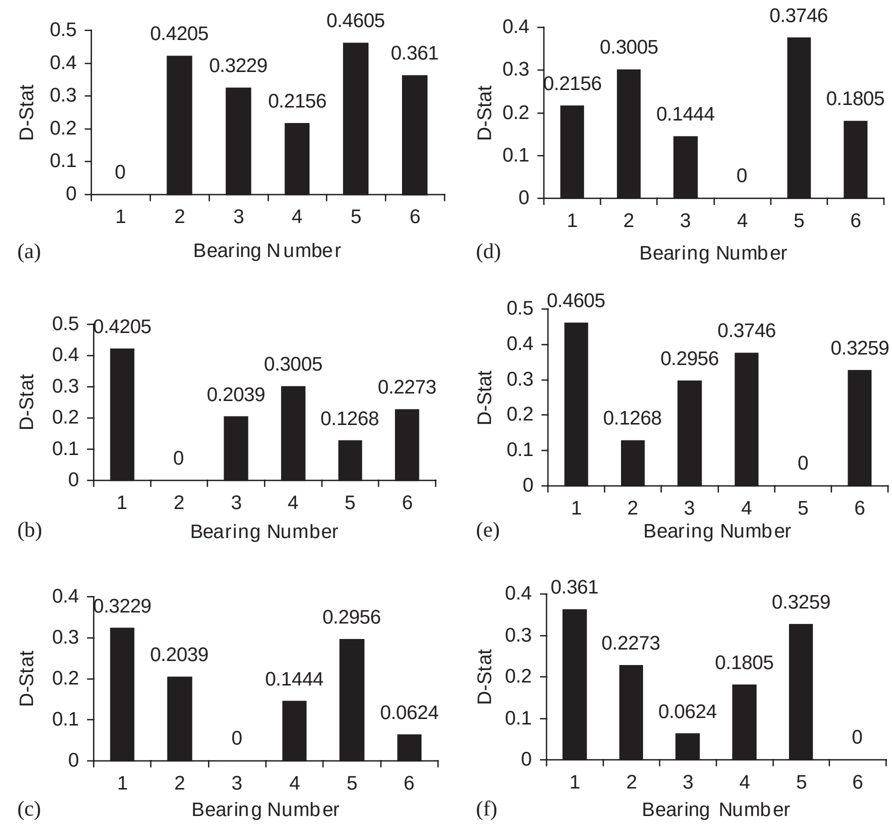

Fig. 2. Result of KS Test amongst the control group bearings: (a) bearing-1, (b) bearing-2, (c) bearing-3, (d) bearing-4, (e) bearing-5 and (f) bearing-6.

Table 3
Comparison of different statistical parameters

Bearing no.	Kurtosis	Skewness	Mean	Variance
1	3.7062	0.1049	0	0
2	5.5956	−0.0786	0.0003	0.0030
3	9.6151	0.0277	0	0
4	4.6037	−0.0942	−0.0003	0
5	6.8477	−0.0255	0	0.0168
6	3.5304	0.0766	0	0.0001

different bearings. But, bearing-6 (with two ball defects) has less kurtosis than bearing-1 (good). Similarly, bearing-3 (with two inner race defects) has kurtosis of 9.6151 while kurtosis for bearing-5 (with two outer race defects) is 6.8477. Hence, such irregularity in variation of kurtosis makes it impossible for judging defectiveness. All other parameters do not have any regular pattern for decision-making for defectiveness.

4.5. Critical values of D-stat

The critical values can be calculated from Ref. [8] by putting $N = 1024$ and $M = 1024$. The equation is

$$D\text{-}stat_{critical} = 1.36\sqrt{\frac{M+N}{MN}}, \quad (4)$$

for level of significance of 0.05.

Hence for all the data, $D\text{-}stat_{critical}$ is computed as 0.0601. If any two bearings have *D-stat* greater than this value, the null hypothesis will be rejected in favor of alternate hypothesis. Referring all the tables, it may be inferred that wherever the *D-stat* exceeds $D\text{-}stat_{critical}$, the significance level is below 0.05 and vice versa.

4.6. KS Test compared to Student's t-test

To compare two sets of data, the most famous parametric statistical test is Student's *t*-test which judges the difference between the mean relatives to the spread or variability in order to verify the null hypothesis. Its value can be found mathematically as the ratio between the difference in the mean and the standard error:

$$t\text{-}stat = \frac{X_T - X_C}{\sqrt{\frac{\mathrm{Var}_T}{n_T} - \frac{\mathrm{Var}_C}{n_C}}}, \quad (5)$$

where subscript C refers to control group data (bearing-1) and T refers to treatment group data (bearing-2 to -6). For clarification of the difference in these tests, *t*-test is carried out only for bearing-1 with all other bearings. When bearing-1 is compared with bearing-2, the null hypothesis is not rejected as $H = 0$ with a *p*-value of 0.576 and *t-stat* value of 0.5593. Table 4 gives the result of the *t*-test. Even, test between bearing-1 & 3 results in not to reject the null hypothesis with a *p*-Value of 0.9422. The same is with all other bearings. Hence, the *t*-test fails to emphasize the difference between the signals. The negative values of *t-stat* are because of shifting of mean of bearing-4 and bearing-5 left to the mean of bearing-1.

4.7. Control group vs. treatment group

All the above sections dealt mainly with the importance of *D-stat* as a statistical parameter. But can these *D-stat* values be considered as standard for diagnosing bearing faults? To answer this

Table 4
Result of *t*-test of control group bearings with bearing-1 (good)

	Bearing-2	Bearing-3	Bearing-4	Bearing-5	Bearing-6
Bearing-1					
H	0	0	0	0	0
p-Value	0.576	0.9422	0.9824	0.3946	0.1458
t-stat	0.5593	0.0725	−0.022	−0.8515	1.4552

point, treatment group bearings (bearing-7 to -13) are compared with the previously taken control group bearings. Table 5 and Fig. 3 show the result of the KS Test. Any defective bearing can be detected when compared to bearing-1, i.e., a good bearing. Except bearing-7 and -8 (where *D-stat* values are less than $D\text{-}stat_{critical}$ or in other words, *p*-values greater than 0.05), all other bearings have very high values of *D-stat*. Bearing-9 to -13 can be conformed as defective bearing. And bearing-7 and -8 can be conformed as good bearings. This is an important observation as an expert system can very well be designed for bearing fault diagnosis. Fig. 4 shows the variation of *D-stat* of three good bearings, bearing-1, -7 and -8 when compared to all other control group bearings. It can be observed for a particular control group bearing, *D-stats* of all good bearings have nearly same values.

When bearing-4 (with one outer race defect) is compared with bearing-12 (with one defect each in inner race and outer race) and bearing-13 (with one defect each in outer race and ball), the *D-stat* value is very low with high *p*-values. It can be inferred that any defect in outer race dominates defects in inner race and ball as the outer race is very near to the accelerometer. Signal variation due to ball/inner race defect has too large transfer segment to be able to make a mark in the signature. Bearing-10 (inner race and ball defect) is compared with bearing-5 (two defects in outer race), the *D-stat* value is very less, but with a *p*-value less than 0.05. Hence these signatures have different distribution.

Fig. 5 shows the time-domain signatures of treatment group bearings. It can again be observed that good bearings (bearing-7 and -8) have amplitudes of the order of $0.005\,m/s^2$ where as for

Table 5
Result of KS Test of bearings from treatment group with bearing from control group

	Bearing-1	Bearing-2	Bearing-3	Bearing-4	Bearing-5	Bearing-6
Bearing-7						
p-Value	0.4091	0	0	0	0	0
D-stat	0.0361	0.4203	0.3268	0.2156	0.4624	0.3659
Bearing-8						
p-Value	0.4091	0	0	0	0	0
D-stat	0.039	0.414	0.3034	0.1922	0.4615	0.3415
Bearing-9						
p-Value	0	0	0	0	0	0
D-stat	0.1249	0.3824	0.2439	0.1268	0.4322	0.282
Bearing-10						
p-Value	0	0	0	0	0.0451	0
D-stat	0.4654	0.119	0.2868	0.3678	0.0605	0.319
Bearing-11						
p-Value	0	0	0	0	0	0
D-stat	0.359	0.1054	0.1766	0.2644	0.1385	0.2332
Bearing-12						
p-Value	0	0	0	0.2707	0	0
D-stat	0.2176	0.3307	0.1815	0.0439	0.3854	0.202
Bearing-13						
p-Value	0	0	0	0.5815	0	0
D-stat	0.2371	0.3259	0.1532	0.0341	0.3854	0.1805

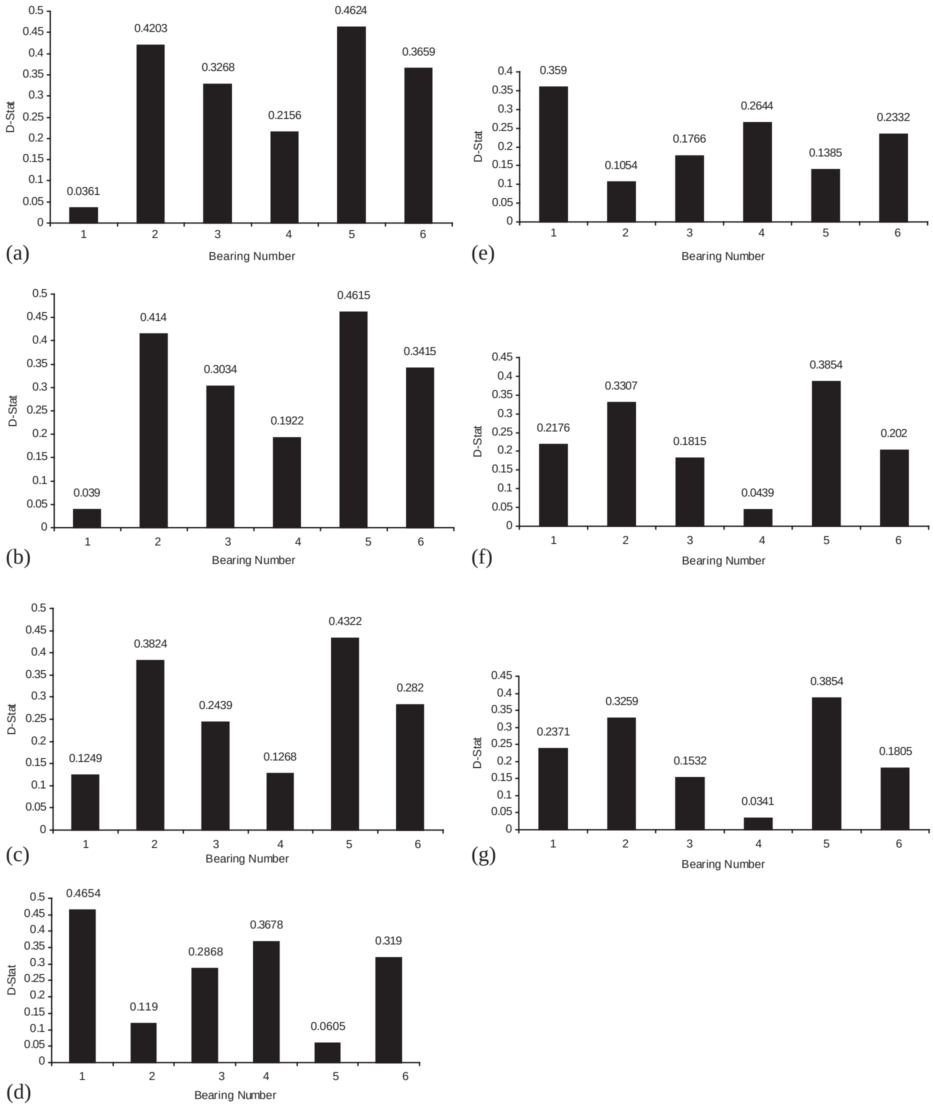

Fig. 3. The result of KS Test of each treatment group members with the control group members: (a) bearing-7, (b) bearing-8, (c) bearing-9, (d) bearing-10, (e) bearing-11, (f) bearing-12 and (g) bearing-13.

defective bearings, it is very high with presence of impulses. In Fig. 5e, with three simultaneous defects in ball, inner race and outer race (bearing-11), highest amplitude level of the order of $0.8\,\mathrm{m/s^2}$ is observed.

4.8. Effect of time lagging on D-stat

An interesting observation on time lagging effect on *D-stat* is shown in Table 6. Referring to Table 2, it is observed that the data when tested using KS Test with itself, the *D-stat* value is 0 with

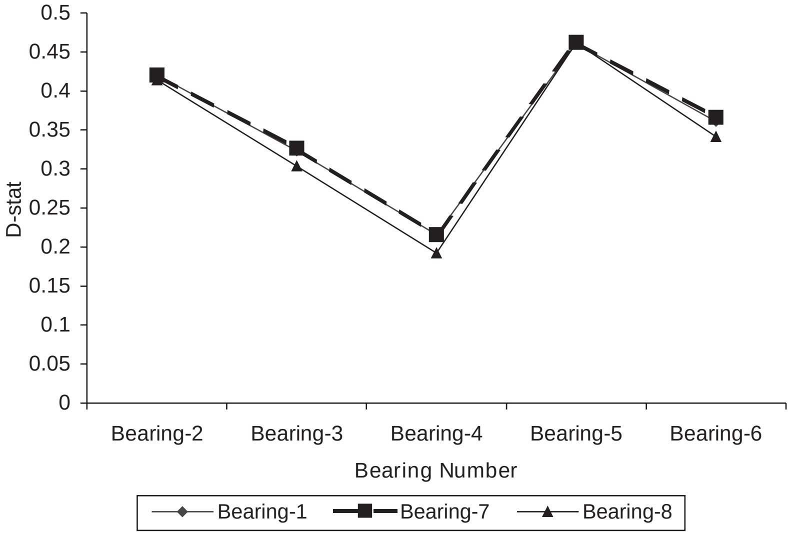

Fig. 4. Comparison of *D-stats* of three good bearings (bearing-1, bearing-7 and bearing-8) with other control group bearings.

a confidence interval of 100% (p-value being 1). In Table 6, three cases are taken. In case 1, time-domain signature of 1024 data points and 128 data points lagging (896 data points) are compared with $M = 1024$ and $N = 896$. In case 2, comparison is made with time signal of 1024 data points and the same time signal with 256 data points lagging (768 data points), where as case 3 compares 1024 data points with 512 data points lagging (512 data points). For each bearing, it has been found that the *D-stat* value is very small with very high confidence level. The deviation of p-value from 1.0 with high time lagging may be due to the fact that the data points are decreased (the value of N is gradually decreasing). But the *D-stat* and p-values can be approximated to the ideal values of 0% and 100%, respectively. These results may be due to the fact that KS Test is a non-parametric test in which the amplitudes are arranged in ascending order. It can be inferred that while considering six complete revolutions of ball bearings for a 0.2 s time record, data lagging will not play any significant role in the KS Test. This may be a great advantage of using KS Test for ball bearing fault detection.

5. Conclusions

The paper mainly discussed the applicability of Kolmogorov–Smirnov (KS) Test as a means of bearing fault diagnosis. The KS Test is carried out among the bearings to make a standard control group. *D-stat* is compared with all other conventional statistical parameters such as mean, kurtosis, skewness and variance. The paper also made an attempt to highlight the advantage of the KS Test when compared with t-test. At last, other bearings from treatment group are compared with the control group members so as to detect the defect, if any in the treatment group bearings.

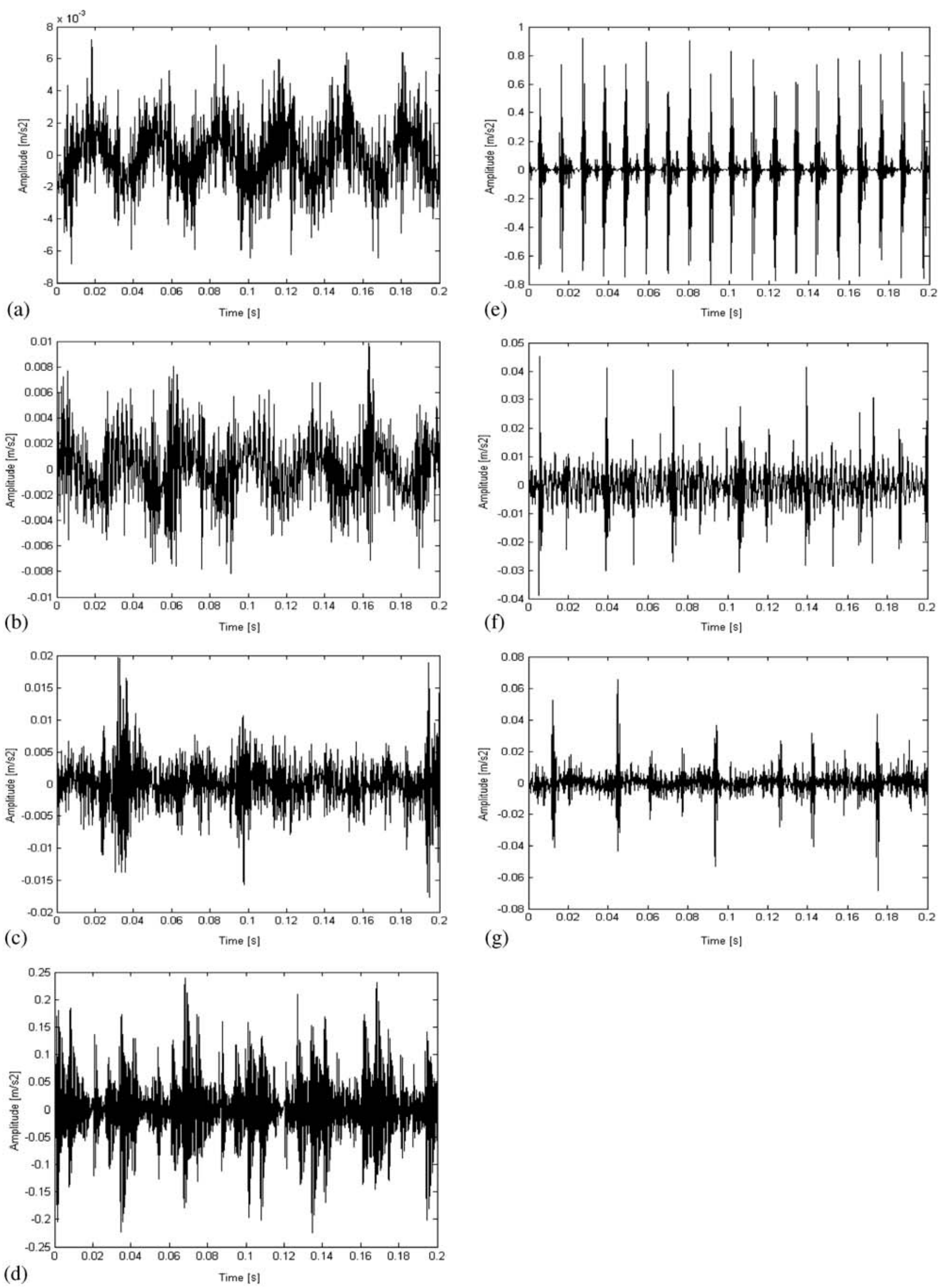

Fig. 5. The time-domain vibration signature of the treatment group bearings: (a) bearing-7, (b) bearing-8, (c) bearing-9, (d) bearing-10, (e) bearing-11, (f) bearing-12 and (g) bearing-13.

Table 6
Time lagging effect on *D-stat* of control group bearings for case 1: 128 data points lagging, case 2: 256 data lagging and case 3: 512 data lagging

	Bearing-1	Bearing-2	Bearing-3	Bearing-4	Bearing-5	Bearing-6
Case 1						
H	0	0	0	0	0	0
p-Value	1	1	1	1	1	1
D-stat	0.0087	0.0065	0.0123	0.0084	0.0118	0.0085
Case 2						
H	0	0	0	0	0	0
p-Value	1	1	0.9662	1	1	0.9987
D-stat	0.0147	0.0111	0.0235	0.0127	0.0143	0.018
Case 3						
H	0	0	0	0	0	0
p-Value	0.9534	0.9125	1	0.7934	0.9972	0.9999
D-stat	0.0277	0.0301	0.0163	0.0348	0.215	0.0181

It has been found that any defect in a ball bearing has a specific distribution, and hence can easily be distinguished from the good bearing with certainty. Also any time lagging in measurement affects *D-stat* marginally, thus proving *D-stat* to be an appropriate parameter for ball bearing diagnosis.

There may be some demerits of KS Test such as the values of data loses importance for it being non-parametric, it is more sensitive at the center than at the tails, etc. But it could be proved that *D-stat* can be a very good parameter to diagnose defects in bearings. The study can further be extended with inclusion of more number of cases. There is also a great scope of making an expert system for automatic decision-making process of bearing fault diagnosis.

Appendix A. KS test

There are many tests to signify the similarity between two distributions. These may be parametric tests (such as F-test, t-test, chi-square test, etc.) and non-parametric tests (such as sign test, rank-sum test, KS Test, etc.). In the parametric tests, the parameters such as mean and variance of the two distributions to be compared play important roles where as in the non-parametric tests, the empirical cumulative distributions functions (ECDFs) are compared which is calculated by arranging the data in ascending order. The KS Test is a non-parametric and distribution free goodness-fit-test. It is designed to test the null hypothesis in favor of the alternative hypothesis. It uses a statistic known as *D-stat* in order to test the hypothesis (assumptions).

Null hypothesis H_0 ($H = 0$): The two data sets are drawn from the same population or have same probability distribution.

Alternate hypothesis H_1 ($H = 1$): The two data sets are not drawn from the same population.

It is called one-sample KS Test when a single data set (sample) is compared with any known continuous distribution such as normal distribution or exponential distribution. The *D-stat* for any value X_i can be defined as the absolute difference between the ECDFs of the data sets. ECDF can be defined as the proportions of number of data less than or equal to X_i. Mathematically, it can be expressed as

$$\begin{aligned}(ECDF_i) &= P(X_i \leqslant x) = i/N(\text{higher estimate probability})\\ &= (i-1)/N(\text{lower estimate probability}),\end{aligned}$$

where the data are arranged in ascending order and N the total number of data and X_i is the random variable at ith position. Hence from definition

$$D\text{-}stat = \max\{|F_0(X_i) - i/N|,\ |F_0(X_i) - (i-1)/N|\},$$

where $F_0(X_i)$ is the ECDF of a known distribution. Fig. 6 shows an example of one-sample KS Test. The random variable $X = [-5:2:5]$ and the null hypothesis assumes that the distribution is a normal distribution with 0 mean and 1 variance. The higher estimate of ECDF at $X_2 = 0.3334$ (2/6) and lower estimate of ECDF at $X_2 = 0.1667$ (1/6). *D-stat* is found to be 0.3413 with a p-value of 0.4033. It can be inferred that null hypothesis is rejected in favor of alternate hypothesis and chances of X having a normal distribution is less.

In two-sample KS Test, two samples are tested with a null hypothesis that the two samples are having same distribution. *D-stat* can be calculated as follows:

$$D\text{-}stat = \max|F_1(X_i) - F_2(X_i)| \quad \forall i = 1, 2, \ldots, N,$$

where $F_1(X_i)$ is the ECDF of first sample at X_i, and $F_2(X_i)$ the ECDF of second sample at X_i. Fig. 7 shows an example where $X = [-5:2:5]$ and $Y = [-6:3:9]$. The *D-stat* is found to be 0.3333 with a p-value 0.8614.

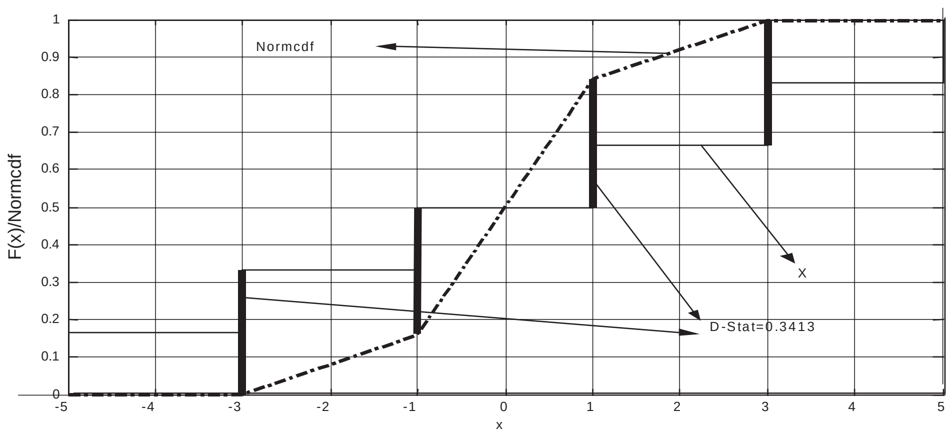

Fig. 6. Example of one-sample KS Test.

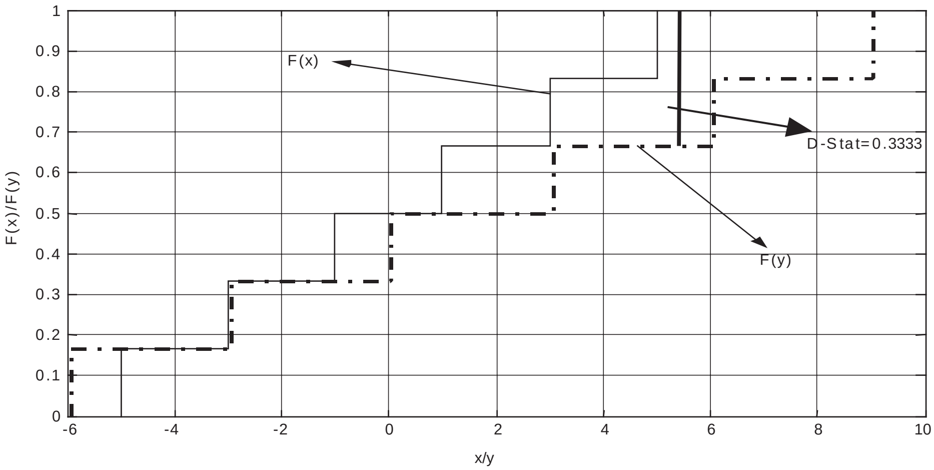

Fig. 7. Example of two-sample KS Test.

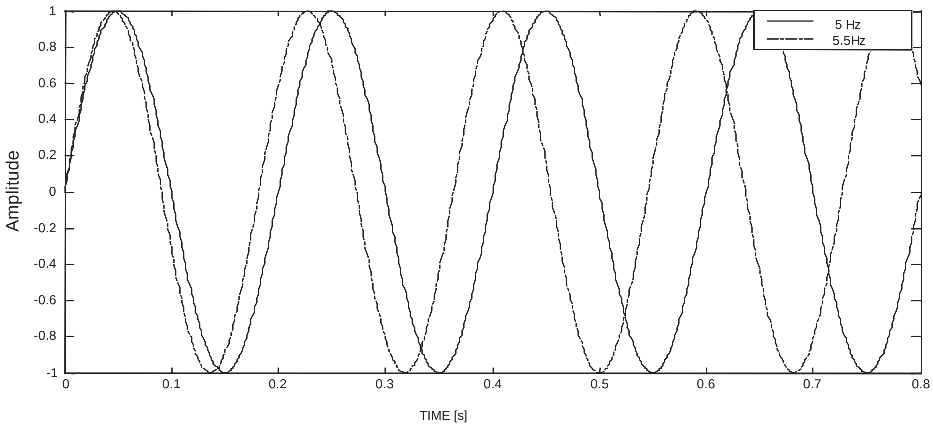

Fig. 8. The time-domain signature of two signals X and Y.

The KS Test can be a very effective tool in statistical signal processing, where two signals can be tested to verify that whether they have same type of distribution or not. Two signals $X = \sin(2\pi 5t)$ and $Y = \sin(2\pi 5.5t)$ are shown in the Fig. 8. Fig. 9 shows the ECDF of the sinusoidal function X (dash-dot line) and normal distribution function with 0 mean and 1 variance ($N \sim (0,1)$) (solid line). The resulting *D-stat* is shown in a thick solid line. Fig. 10 denotes the ECDF of the two signals and the resulting *D-stat* value. The solid line refers to the 5 Hz sinusoidal curve and the dash-dotted line refers to 5.5 Hz sinusoidal curve.

Hence KS Test is very efficient deciding about the similarity in distributions.

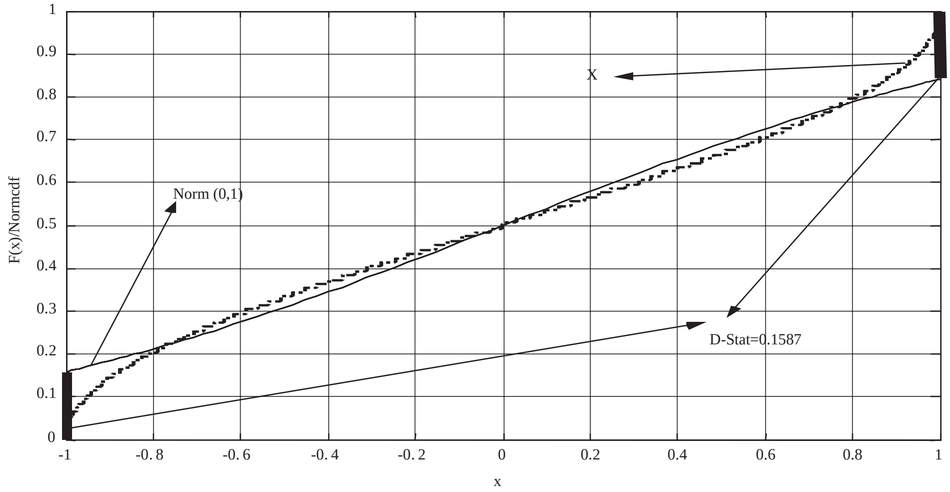

Fig. 9. ECDF of normal distribution and signal X.

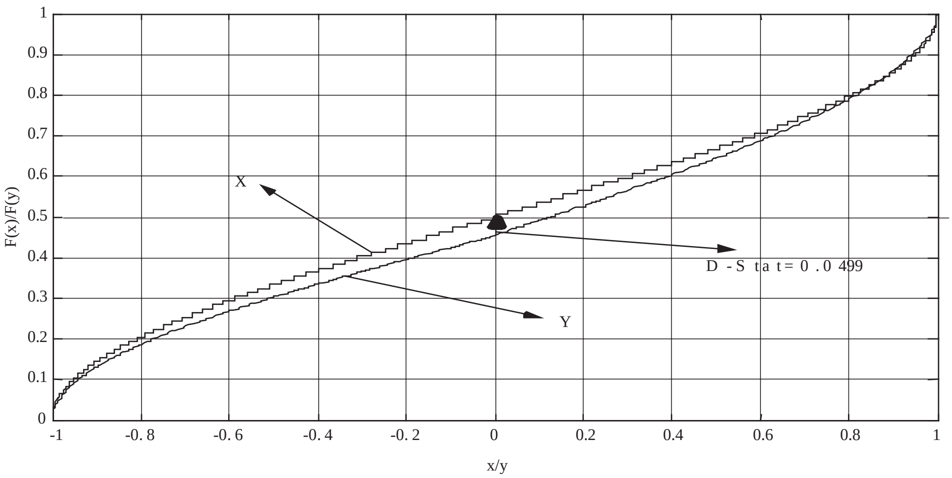

Fig. 10. ECDF of two signals X and Y.

Appendix B. Nomenclature

D-stat (D)	Kolmogorov statistic
H	Boolean decision on the hypothesis
H_0	null hypothesis ($H = 0$)
H_1	alternate hypothesis ($H = 1$)
X_i	random number at *i*th position
ECDF	empirical cumulative distributions function
$F_0(X_i)$	ECDF of a known distribution at X_i

$Fi(X_i)$	ECDF of sample 1 at X_i
$R_i(X_i)$	ECDF of sample 2 at X_i
N	number of data points in sample 1
M	number of data points in sample 2
T	treatment group data
C	data relating to control group
Var	variance
t	t-test statistic
N_e	effective number of data points
χ^2	chi-square test statistic
$Q_{KS}(\lambda)$	probability distribution of *D-stat* with parameter λ
α	significance level

References

[1] N. Tandon, B.C. Nakra, Vibration and acoustics monitoring techniques for the detection of defects in rolling element bearings—a review, The Shock and Vibration Digest 24 (1992) 3–11.
[2] H. Prasad, M. Ghosh, S. Biswas, Diagnostic monitoring of rolling element bearings by high frequency resonance technique, ASME Paper 81-DET-7, 1981, pp. 1–10.
[3] S. Prabhakar, A.R. Mohanty, A.S. Sekhar, Application of discrete wavelet transform for detection of ball bearing race faults, Tribology International 35 (2002) 793–800.
[4] F.A. Andrede, I.I. Esat, M.N.M. Badi, Gear condition monitoring by a new application of the Kolmogorov–Smirnov test, Proceeding of the Institute of Mechanical Engineers—Part C 215 (2001) 653–661.
[5] http://www.ipac.altech.edu
[6] J.H. Drew, A.G. Glen, L.M. Leemis, Computing the cumulative distribution function of a Kolmogorov–Smirnov statistic, Computational Statistics & Data Analysis 34 (1) (2000) 1–15.
[7] http://www.physics.csbsju.edu/stats/KS-test.html
[8] D. Zwillinger, S. Kossokko, CRC Standard Probability & Statistics Tables and Formulae, Chapman & Hall, CRC, London, Boca Raton, FL, 2000.
[9] W. Petrich, A. Staib, M. Otto, R.L. Somorjai, Correlation between the state of health of blood donors and the corresponding mid-infrared spectra of the serum, Vibrational Spectroscopy 28 (1) (2002) 117–129.
[10] N. Stollenwerk, F.R. Drepper, H. Siegel, Testing nonlinear stochastic models on phytoplankton biomass time series, Ecological Modeling 144 (2–3) (2001) 261–277.