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Letter to the Editor

# On the number of modes required for statistical energy analysis-based calculations

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## 1. Introduction

Responses of multi-modal systems excited by high-frequency loads are usually estimated using statistical energy analysis (SEA) developed by Lyon [1] and others. In SEA, the average response behaviour is predicted based on the energy flow among the interconnected elements called subsystems. The average is taken over the frequency, the space as well as over an ensemble of systems.

It is sometimes wrongly stated that presence of large number of modes in the subsystems is necessary to apply SEA. But what is important is the presence of large number of modal pairs in the energy exchange between the subsystems and not the presence of large number of modes in each subsystem. Lyon has indicated this point [2]. Objective of this note is to explain this point and to compare the response of a panel, which has a few modes, estimated using SEA with the experimental results.

Let us look at the energy exchange between two subsystems having mode counts $N_1$ and $N_2$ and modal energies $E_{1m}$ and $E_{2m}$. The mean power flow denoted by $\pi_{12}$, is given by the relation

$$\pi_{12} = BN_1N_2\{E_{1m} - E_{2m}\}, \tag{1}$$

where $B$ is the power flow proportionality constant which is given by

$$B = (\pi/2)(1/\Delta\omega)\{\mu^2\omega^2 + (\alpha^2/\omega^2) + \gamma^2 + 2\mu\alpha\}. \tag{2}$$

In Eq. (2) $\mu$, $\alpha$ and $\gamma$ are the coupling parameters at frequency $\omega$ [1]. The proportionality constant $B$ is the expected value for the natural frequencies being any value in the frequency range $\Delta\omega$. Every mode in subsystem 1 will be exchanging energy with every mode of subsystem 2. Hence, there will be $N_1N_2$ pairs of modes involved in the energy exchange. The parameter $B$ is an average over different modal pairs. Therefore, the average value represents the behaviour if there are large number of modal pairs. This means that what is important is not the presence of large number of modes in each subsystem but large number of modal pairs in the connected subsystems. Hence,

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even if there is only one mode present in a particular subsystem, if the other subsystem has several modes in the frequency band, Eq. (1) can represent the mean power flow. It is important to note that the subsystem should have at least one mode in the frequency band, otherwise the estimated power flow will be zero. The presence of minimum of one mode in a subsystem is what is essential for the applicability of SEA, if the other subsystem has several modes.

This result is particularly important when response of a structure subjected to acoustic excitation has to be estimated. Normally the acoustic field will have large number of modes. Hence, the response of the structure can be estimated even if it has only a few modes.

If SEA can be applied even if the subsystem has only a few modes it is natural to think what difference it makes if there are many modes. The variance of the responses at different locations will be large if the subsystem has only a few modes. In other words the spatial average value of the response will not be a representative response parameter. Also, to determine the spatial average value of the response one has to consider large number of locations.

In summary, what is important for the applicability of SEA is the presence of large number of modal pairs in the subsystems. SEA can be applied to systems even if there is only one mode present in a particular subsystem, provided the interacting subsystem has several modes in the frequency band. Spatial variation of the responses will be lesser if many modes are present in the subsystem.

## 2. Experimental results

A plate having a few modes is subjected to reverberant acoustic field and the results are presented. The measured response shows very good agreement with the response estimated using SEA though only very few modes are present in the structure.

The structure considered is a honeycomb sandwich panel having dimensions $1.3 \times 1.1\,\mathrm{m}^2$. Face sheets are made of aluminium alloy having 0.19 mm thickness each. The thickness of the core is 25.4 mm. The measured mass of the panel is 4.3 kg and the mass of the panel without the concentrated masses is 2.75 kg. The modal density of the panel is estimated to be 0.014/Hz at 315 and 0.036/Hz at 4000 Hz [3]. The critical frequency of the panel is estimated to be 382 Hz when the speed of sound in air is 346 m/s [4].

The panel is hung in a reverberation chamber having dimensions of $10.33 \times 8.2 \times 13.0\,\mathrm{m}^3$ and subjected to diffuse acoustic field. The boundaries of the panel are free. The sound pressure level (SPL) is measured at three locations and the spatial average of the SPL is given in Table 1. The responses are measured at 11 randomly selected locations. The spatial average of the acceleration levels is given in Table 1 and Fig. 1. The results are given in terms of root mean square values in one-third octave bands from 315 to 2500 Hz.

Accelerometers having masses of 0.5 and 1.5 g are used. At 2500 Hz the average driving point impedance of the panel is estimated to be 464 N s/m, whereas the impedance due to the accelerometers having mass of 1.5 g is 23.6 N s/m. Hence, the mass loading of the accelerometers on the measured response is considered negligible. The useful frequency range of the accelerometers is 5–8000 Hz ($\pm 5\%$).

The response of the panel is estimated using SEA. The average number of modes present in the structure is only one in 315 Hz one-third octave band. But there are many modes present in the

Table 1
Spatial average of the response of the plate

| $\frac{1}{3}$ octave band centre frequency (Hz) | SPL (dB) | Acceleration response (g) | |
|---|---|---|---|
| | | Theory | Experiment |
| 315 | 122.1 | 4.3 | 3.9 |
| 400 | 125.6 | 6.6 | 6.6 |
| 500 | 122.5 | 5.5 | 4.7 |
| 630 | 118.2 | 3.4 | 3.2 |
| 800 | 112.2 | 1.7 | 1.5 |
| 1000 | 112.5 | 1.6 | 1.6 |
| 1250 | 113.5 | 1.8 | 1.9 |
| 1600 | 112.8 | 1.9 | 1.8 |
| 2000 | 112.0 | 1.8 | 1.9 |
| 2500 | 110.7 | 1.5 | 1.9 |

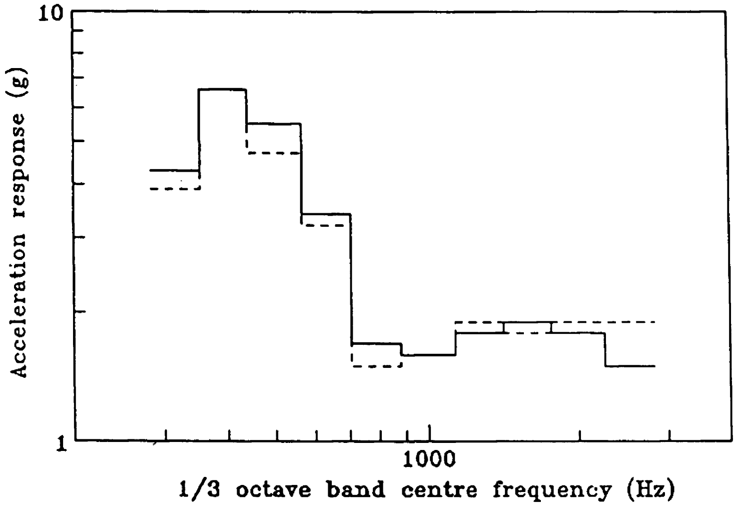


Fig. 1. Response of the panel to acoustic excitation: —, theory; - - -, experiment.

acoustic field. In this case there are 2400 modes in 315 Hz one-third octave band and the number of modes is still larger at higher frequencies. Though there are only very few modes in the structure there are many modes in the acoustic field. To estimate the response using SEA, radiation resistance and dissipation loss factor of the plate have to be determined. Any error in the above two parameters can lead to errors in the estimated responses. Hence, the measured values of radiation resistance and dissipation loss factor values are used for the calculations. Measured values of the radiation resistance of this plate are reported earlier [5]. Dissipation loss factor values are determined using the energy method [6]. For this, the plate is excited at a point by shaker and the corresponding acceleration responses are measured. From the measured input power and the spatial average of the acceleration responses the loss factor, denoted by $\eta_d$, at frequency $\omega$ is determined using the relation

$$\eta_d = \pi_{in}/\omega M < v^2 >_x, \tag{3}$$

where $\pi_{in}$ is the input power and $< v^2 >_x$ is the spatial average of the velocity response. Though there are only few modes in the lower frequency range, the loss factor determined using the energy method is a good estimate since the response is averaged over 11 locations and the experiment is done with three driving points. Since the experiment is conducted in air, the loss factor thus determined is not the dissipation loss factor. The loss factor thus obtained is a sum of a few loss factors as given by

$$\eta = \eta_1 + \eta_{12} - \eta_{21} E_2/E_1. \tag{4}$$

In Eq. (4) $\eta_1$ is the dissipation loss factor of the plate, $\eta_{12}$ is the radiation loss factor and $\eta_{21}$ is the reciprocal coupling loss factor which is from acoustic field to the structure. The parameter $E_1$ and $E_2$ are the energies of the plate and the acoustic field, respectively. The last term, that is $\eta_{21} E_2/E_1$, is found to be very much negligible compared to other terms. This is because the sound power radiated itself is very low and the coupling loss factor $\eta_{21}$ also is very small. The coupling loss factor $\eta_{21}$ can be determined from the reciprocal relation

$$\eta_{21} = \eta_{12} N_1/N_2, \tag{5}$$

where $N_1$ is the number of modes of the plate and $N_2$ is the number of modes of the acoustic field. As discussed earlier the number of modes of the acoustic field is 2400 in 31.5 Hz one-third octave band, and hence it can be seen from Eq. (5) that $\eta_{21}$ is very low. Therefore, the measured loss factor is practically the sum of the dissipation and the radiation loss factors called total loss factor [7]. Hence, the dissipation loss factor is determined by subtracting the radiation loss factor component [5] from the measured total loss factor. Measured radiation resistance values are used to determine the radiation loss factor. The dissipation loss factor at frequency $f$ is thus obtained as

$$\begin{aligned} &\text{for } f \leqslant 1250\,\text{Hz}, \quad \eta_d = 0.05, \\ &\text{for } f > 1250\,\text{Hz}, \quad \eta_d = 0.02. \end{aligned} \tag{6}$$

The energy average of the loss factor over the entire frequency band is about 0.033.

The spatial average response of the plate is now estimated using the above-measured parameters and are given in Table 1 and Fig. 1. It can be seen that there is a reasonably good match between the measured and the estimated responses of the plate, even at frequencies where the structure has only very few modes.

## 3. Conclusions

For the applicability of statistical energy analysis (SEA), what is important is the presence of large number of modal pairs in the interacting subsystems and not large number of modes in each subsystem. Even if there is only one mode present in a particular subsystem, if the interacting subsystem has several modes in the frequency band, SEA is applicable. The response measured on a plate subjected to acoustic excitation shows very good agreement with the responses estimated using SEA though there are only a few modes present in the plate.

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