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Vibration analysis of a rotating system due to the effect of ball bearing waviness

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Abstract

This research presents an analytical model to investigate vibration due to ball bearing waviness in a rotating system supported by two or more ball bearings, taking account of the centrifugal force and gyroscopic moment of the ball. The waviness of rolling elements is modelled by the sinusoidal function, and it is incorporated into the position vectors of the race curvature center. The Hertzian contact theory is applied to calculate the elastic deflection and non-linear contact force, while the rotor has translational and angular motions. Both the centrifugal force and gyroscopic moment of the ball and the waviness of the rolling elements are included in the kinematic constraints and force equilibrium equations of a ball to derive the non-linear governing equations of the rotor, which are solved by using the Runge–Kutta–Fehlberg algorithm to determine the new position of the rotor. The proposed model is validated by the comparison of the results of the prior researchers. This research shows that the centrifugal force and gyroscopic moment of the ball plays the important role in determining the bearing frequencies, i.e., the principal frequencies, their harmonics and the sideband frequencies resulting from the waviness of the rolling elements of ball bearing. It also shows that the bearing vibration frequencies are generated by the waviness interaction not only between the rolling elements of one ball bearing, but also between those of two or more ball bearings constrained by the rotor.

1. Introduction

Waviness is defined as the geometric imperfection of a ball, inner or outer race in a ball bearing, and it is considered one of the important sources of machine vibration. It is always included in a ball bearing to varying degrees through the manufacturing process. Although the rolling elements

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are perfectly manufactured without waviness, it may be generated by load or operating conditions. Therefore, it is becoming important to investigate the dynamic characteristics of rotating systems due to the effect of ball bearing waviness in order to achieve high precision and sound operation of modern sophisticated rotating machines.

Wardle [1,2] calculated the relations between the amplitude of waviness and the excitation force of a ball bearing, and his results were validated through experiments. He also predicted the vibration frequencies due to the load–deflection non-linearity of the ball bearing. However, his model only included a ball bearing without considering a rotor, so that it could not predict the waviness interaction between two or more ball bearings to support the same rotor. Yhland [3] used a linear theory to calculate the stiffness matrix of the ball bearing with waviness, and he investigated the effect of waviness through his rotor dynamic model. Aktürk et al. [4,5] proposed a vibration model of a ball bearing with waviness considering three degrees of freedom. Recently, Jang and Jeong [6] proposed the excitation model of ball bearing waviness in a rigid rotor considering five degrees of freedom, and they investigated the vibration frequencies due to non-linear load–deflection characteristics. However, prior research did not consider the effect of the waviness considering the centrifugal force and gyroscopic moment of the ball. In addition to waviness, the centrifugal force and gyroscopic moment of the ball are important parameters to affect the characteristics of ball bearing vibration, so that, in some cases, the results of the prior research may be quite different from the results considering both the waviness of rolling elements and the centrifugal force and gyroscopic moment of the ball.

This research presents an analytical model to investigate vibration due to ball bearing waviness in a rotating system supported by two or more ball bearings, taking account of the centrifugal force and gyroscopic moment of the ball. The waviness of rolling elements is modelled by the sinusoidal function, and the centrifugal force and gyroscopic moment of the ball are included in the kinematic constraints and force equilibrium equations to produce the non-linear governing equations, which can be determined by using the Runge–Kutta–Fehlberg algorithm. The proposed model is validated by the comparison of the results of the prior researchers. This research characterizes the bearing frequencies, i.e., the principal frequencies, their harmonics and sideband frequencies, due to the waviness interaction between the rolling elements of two ball bearings constrained by the rotor as well as the effect of the centrifugal force and gyroscopic moment of the ball.

2. Method of analysis

2.1. Waviness model

Fig. 1 shows the rigid rotor supported by a pair of ball bearings. The following equations can represent the radial waviness of the inner and outer race, p_{ij} and p_{oj}, and the axial waviness of the inner and outer race, q_{ij} and q_{oj} [6,9].

$$p_{oj} = \sum_{l=1}^{O} A_{ol} \cos\left[-l(\omega_o - \omega_c)t + 2\pi l(j-1)/Z + \alpha_{ol}\right], \tag{1}$$

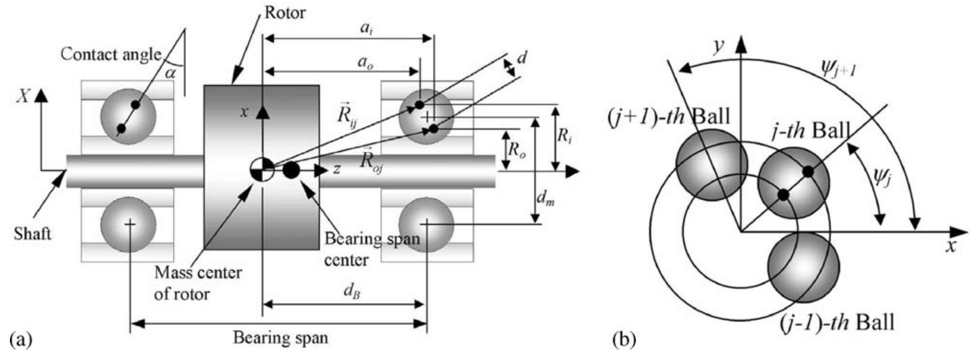

Fig. 1. (a) Rigid rotor supported by two ball bearings in $x-z$ plane, (b) ball bearing in $x-y$ plane.

$$p_{ij} = \sum_{l=1}^{O} A_{il}\cos\left[-l(\omega_i - \omega_c)t + 2\pi l(j-1)/Z + \alpha_{il}\right], \tag{2}$$

$$q_{oj} = \sum_{l=1}^{O} B_{ol}\cos\left[-l(\omega_o - \omega_c)t + 2\pi l(j-1)/Z + \beta_{ol}\right], \tag{3}$$

$$q_{ij} = \sum_{l=1}^{O} B_{il}\cos\left[-l(\omega_i - \omega_c)t + 2\pi l(j-1)/Z + \beta_{il}\right]. \tag{4}$$

In the above equations, l, Z, ω_c, ω_o, and ω_i are the waviness order, number of balls and the cage, outer race, and inner race rotating frequencies, respectively, and A_{ol}, A_{il}, B_{ol}, B_{il} and α_{ol}, α_{il}, β_{ol}, β_{il} are the amplitudes and initial phase angles of the inner and outer race in contact with the jth ball.

The phase angle of ball waviness in contact with the outer race is 180° ahead of the ball waviness in contact with the inner race, so that the ball waviness in contact with the inner and outer race, w_{ij} and w_{oj}, can be expressed as follows:

$$w_{ij} = \sum_{l=1}^{O} C_{jl}\left[\cos(l\omega_b t + \gamma_{jl})\right], \tag{5}$$

$$w_{oj} = \sum_{l=1}^{O} C_{jl}\left[\cos\left\{l\omega_b\left(t + \frac{\pi}{\omega_b}\right) + \gamma_{jl}\right\}\right], \tag{6}$$

where ω_b, C_{jl} and γ_{jl} are the ball spinning frequency, the amplitude and initial phase angle of the jth ball with waviness order l, respectively.

2.2. Kinematic constraint and force equilibrium equations of a ball considering waviness

The position vectors of the groove radius center of the inner and outer race in contact with the jth ball, $\vec{R}_{ij}$ and $\vec{R}_{oj}$, can be determined from the mass center of the rotor as shown in

Fig. 1 [6].

$$\vec{R}_{ij}(t) = R_i \cos\psi_j \vec{i} + R_i \sin\psi_j \vec{j} + a_i \vec{k}, \tag{7}$$

$$\vec{R}_{oj}(t) = R_o \cos\psi_j \vec{i} + R_o \sin\psi_j \vec{j} + a_o \vec{k}. \tag{8}$$

In the above equations, R_i, R_o, a_i and a_o are the radial and axial components of the position vectors of the groove radius center of the inner and outer race, and they can be expressed in terms of the pitch diameter d_m, the distance between the groove radius centers of inner and outer race d, and contact angle α. Also, ψ_j is the azimuth angle of the jth ball in the $x-y$ plane, and it can be expressed in terms of the cage rotating frequency [8,9].

When a ball bearing operates at high speed, the centrifugal force and gyroscopic moment of the ball are not negligible, so that the contact angles of the inner and outer race are dissimilar and that the groove radius center of the inner race is not collinear with that of the outer race. Fig. 2 shows the position of the ball center and the race curvature center with and without including the centrifugal force and gyroscopic moment of the ball with respect to the same outer race curvature center. In Fig. 2, X_{zj}, X_{rj}, α_{ij}, α_{oj}, l_{ij} and l_{oj} are the axial and radial components of the position of the ball center, the contact angles of the inner and outer race, and the distances between the ball center and the groove radius centers of the inner and outer race, respectively. Considering the effect of ball waviness in contact with the inner and outer race, w_{ij} and w_{oj}, and ball oversize, h_j, the following equations determine l_{ij} and l_{oj}:

$$l_{ij} = r_i - (D + h_j)/2 - w_{ij},$$
$$l_{oj} = r_o - (D + h_j)/2 - w_{oj}, \tag{9}$$

where r_i and r_o are the groove radius of the inner and outer race.

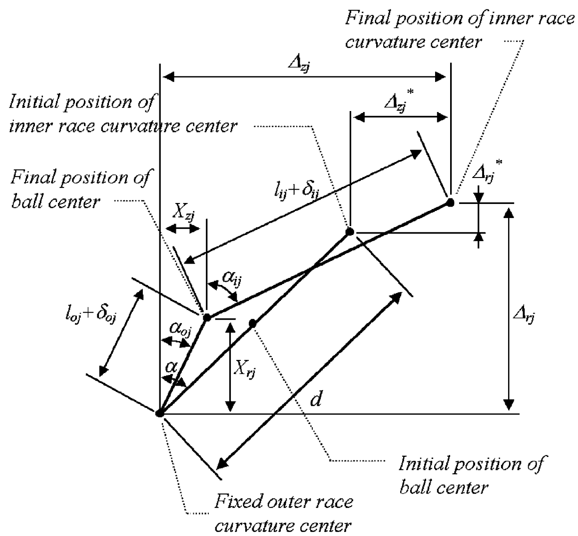

Fig. 2. Position of ball center and raceway curvature centers.

In the inner-race rotating type of ball bearing, the inner race has translational and angular motion, and the outer race is stationary during the rotation of the ball bearing. Under the angular motions of θ_x and θ_y and the translational motions of x, y and z, the position vector of the inner race groove radius center can be determined by transformation using the Euler angle. The difference between this vector and the position vector of the outer race groove radius center can be decomposed into the radial and axial directions. Introducing the waviness in Eqs. (1)–(4) to the position vectors of the groove radius centers of the inner and outer race, the distance between the two position vectors can be expressed as follows [6]:

$$\begin{aligned}\Delta_{rj} &= (R_i - R_o) + a_i(\theta_y \cos\psi_j - \theta_x \sin\psi_j) + x\cos\psi_j \\ &\quad + y\sin\psi_j + (p_{ij} - p_{oj}) + q_{ij}(\theta_y \cos\psi_j - \theta_x \sin\psi_j), \\ \Delta_{zj} &= (a_i - a_o) + R_i(\theta_x \sin\psi_j - \theta_y \cos\psi_j) + z + (q_{ij} - q_{oj}) \\ &\quad + p_{ij}(\theta_x \sin\psi_j - \theta_y \cos\psi_j),\end{aligned} \tag{10}$$

where Δ_{rj} and Δ_{zj} are the radial and axial component of the distance between the groove radius centers of the inner and outer race. In the case of the outer-race rotating type of ball bearing, the outer race has translational and angular motion and the inner race is stationary during the rotation of the ball bearing. Similar expressions can be derived to determine the distance between the position vectors of the groove radius center of the inner and outer race. Applying the Pythagorean theorem to Fig. 2, the following equations can be obtained [7,9]:

$$\begin{aligned}&\left(\Delta_{zj} - X_{zj}\right)^2 + \left(\Delta_{rj} - X_{rj}\right)^2 - \left(l_{ij} + \delta_{ij}\right)^2 = 0, \\ &X_{zj}^2 + X_{rj}^2 - \left(l_{oj} + \delta_{oj}\right)^2 = 0,\end{aligned} \tag{11}$$

where δ_{ij} and δ_{oj} are the elastic deformation of the contact point between the ball and each race.

Fig. 3 shows the free-body diagram of the ball acted by the contact forces of the inner and outer race, f_{ij} and f_{oj} and the centrifugal force and gyroscopic moment of a ball, F_{cj} and M_{Gj}. In Fig. 3, D and ζ are the ball diameter and the angle between the spinning axis of the ball and the bearing centerline, and λ_{ij} and λ_{oj} are the constants determined by the race control theory [9]. Force equilibrium of a ball can result in the following equations [7,9]:

$$\begin{aligned}&f_{ij}\sin\alpha_{ij} - f_{oj}\sin\alpha_{oj} - \frac{\lambda_{ij}M_{Gj}}{D}\cos\alpha_{ij} + \frac{\lambda_{oj}M_{Gj}}{D}\cos\alpha_{oj} = 0, \\ &f_{ij}\cos\alpha_{ij} - f_{oj}\cos\alpha_{oj} + \frac{\lambda_{ij}M_{Gj}}{D}\sin\alpha_{ij} - \frac{\lambda_{oj}M_{Gj}}{D}\sin\alpha_{oj} + F_{cj} = 0.\end{aligned} \tag{12}$$

The contact force between the ball and race are expressed by using the Hertzian contact theory as follows:

$$f_{ij} = K_{ij}\delta_{ij}^{1.5}, \quad f_{oj} = K_{oj}\delta_{oj}^{1.5}, \tag{13}$$

where K_{ij}, K_{oj}, δ_{ij}, and δ_{oj} are the load–deflection constants and deflections of the contact point between the ball and each race.

The centrifugal force and gyroscopic moment of a ball can be expressed as follows [9]:

$$F_{cj} = 0.5 m_b d_m \omega_c^2, \tag{14}$$

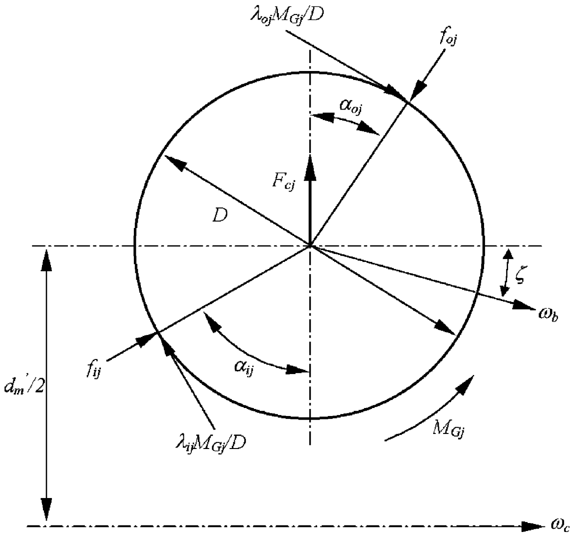

Fig. 3. Ball loading at arbitrary azimuth angle, ψ_j.

$$M_{Gj} = I_p \omega_b \omega_c \sin\zeta, \tag{15}$$

where m_b and I_p are the mass and mass moment of inertia of a ball. In the case of the inner-race rotating type of ball bearing, the pitch diameter, cage rotating frequency and ball spinning frequency can be expressed as follows [9]:

$$\omega_c = \frac{\omega_i + \omega_o \left(\dfrac{d_m + D\cos\alpha_{oj}}{d_m - D\cos\alpha_{ij}}\right)\dfrac{\cos(\alpha_{ij}-\zeta)}{\cos(\alpha_{oj}-\zeta)}}{1 + \left(\dfrac{d_m + D\cos\alpha_{oj}}{d_m - D\cos\alpha_{ij}}\right)\dfrac{\cos(\alpha_{ij}-\zeta)}{\cos(\alpha_{oj}-\zeta)}}, \tag{16}$$

$$\omega_b = \frac{\omega_o - \omega_i}{D\left[\dfrac{\cos(\alpha_{ij}-\zeta)}{d_m - D\cos\alpha_{ij}} + \dfrac{\cos(\alpha_{oj}-\zeta)}{d_m + D\cos\alpha_{oj}}\right]}, \tag{17}$$

$$d_m = d'_m + 2X_{rj} - 2l_{oj}\cos\alpha, \tag{18}$$

where $d_m{}'$ is the free pitch diameter and ω_o and ω_i are the rotating speeds of the outer and inner race. Non-linear kinematic constraint equations in Eq. (11) and force equilibrium equations in Eq. (12) can be solved simultaneously by using the Newton–Raphson iteration method to determine X_{zj}, X_{rj}, δ_{ij}, and $\delta_{oj,}$. The sine and cosine of the contact angles are calculated as follows:

$$\cos\alpha_{oj} = \frac{X_{rj}}{l_{oj} + \delta_{oj}}, \quad \sin\alpha_{oj} = \frac{X_{zj}}{l_{oj} + \delta_{oj}},$$
$$\cos\alpha_{ij} = \frac{\Delta_{rj} - X_{rj}}{l_{ij} + \delta_{ij}}, \quad \sin\alpha_{ij} = \frac{\Delta_{zj} - X_{zj}}{l_{ij} + \delta_{ij}}. \tag{19}$$

According to the race control theory, ζ, λ_{ij}, and λ_{oj} of Eqs. (12) and (15) are expressed as follows:

$$\zeta = \tan^{-1}\left(\frac{d_m \sin \alpha_{oj}}{d_m \cos \alpha_{oj} + D}\right),$$
$$\lambda_{ij} = 0, \quad \lambda_{oj} = 2. \tag{20}$$

2.3. Equation of rotor motion with ball bearing excitation

In the case of the inner-race rotating type of ball bearing, the forces and moments acting on the inner race can be calculated by the following equations:

$$F_x = \sum_{k=1}^{N_B}\left[\sum_{j=1}^{Z}\left(f_{ij}\cos\alpha_{ij} + \frac{\lambda_{ij}M_{Gj}}{D}\sin\alpha_{ij}\right)\cos\psi_j\right],$$
$$F_y = \sum_{k=1}^{N_B}\left[\sum_{j=1}^{Z}\left(f_{ij}\cos\alpha_{ij} + \frac{\lambda_{ij}M_{Gj}}{D}\sin\alpha_{ij}\right)\sin\psi_j\right],$$
$$F_z = \sum_{k=1}^{N_B}\left[\sum_{j=1}^{Z}\left(f_{ij}\sin\alpha_{ij} - \frac{\lambda_{ij}M_{Gj}}{D}\cos\alpha_{ij}\right)\right],$$
$$M_x = \sum_{k=1}^{N_B}\left[\sum_{j=1}^{Z}\left(f_{ij}\tau_{ij} - \frac{\lambda_{ij}M_{Gj}}{D}e_{ij} + \frac{\lambda_{ij}M_{Gj}}{D}r_i\right)\sin\psi_j\right],$$
$$M_y = -\sum_{k=1}^{N_B}\left[\sum_{j=1}^{Z}\left(f_{ij}\tau_{ij} - \frac{\lambda_{ij}M_{Gj}}{D}e_{ij} + \frac{\lambda_{ij}M_{Gj}}{D}r_i\right)\cos\psi_j\right]. \tag{21}$$

In the above equations, N_B is the number of ball bearings, and e_{ij} and τ_{ij} can be expressed as follows:

$$e_{ij} = R_i\cos\alpha_{ij} + a_i\sin\alpha_{ij}, \quad \tau_{ij} = R_i\sin\alpha_{ij} - a_i\cos\alpha_{ij}. \tag{22}$$

Applying these forces and moments to the force and moment equilibrium conditions, the equations of motion can be derived as follows:

$$m\ddot{x} + F_x = 0, \qquad m\ddot{y} + F_y = 0, \qquad m\ddot{z} + F_z = 0,$$
$$I_z\ddot{\theta}_x + I_r\Omega\dot{\theta}_x + M_x = 0, \qquad I_z\ddot{\theta}_y - I_r\Omega\dot{\theta}_x + M_y = 0, \tag{23}$$

where m, I_r, I_z, and Ω is the mass of the rotor, the radial mass moment of inertia, the polar mass moment of inertia and the rotating speed of the rotor, respectively.

3. Results and discussion

3.1. Analysis model and numerical procedure

This research investigates vibration resulting from ball bearing waviness in a rigid rotor supported by a pair of ball bearings, as shown in Fig. 1. The analysis model has a pair of inner-

Table 1
Specification of spindle system

Parameter	Value
Radial mass moment of inertia, I_r	3.985×10^{-3} (kg m^2)
Polar mass moment of inertia, I_z	7.534×10^{-3} (kg m^2)
Mass, m	8.6×10^{-1} (kg)
Bearing span	8.0×10^{-2} (m)
Number of bearing	2

Table 2
Specification of ball bearing

Parameter	Value
Number of ball, Z	16
Ball diameter, D	22.23×10^{-3} (m)
Free pitch diameter, d_{mo}	125.26×10^{-3} (m)
Axial preload, P_z	10 (kN)
Groove radius of inner race, r_i	11.63×10^{-3} (m)
Groove radius of outer race, r_o	11.63×10^{-3} (m)
Diametral clearance, P_d	0.43×10^{-3} (m)
Waviness amplitude, A	1×10^{-6} (m)

race rotating type ball bearings. Tables 1 and 2 show the specification of the spindle system and the ball bearing. It is assumed that the radial and axial waviness at the left and right ball bearings have 0° and 180° phase differences, respectively, and that the mass center of the rotor coincides with the span center of the ball bearing. Therefore, when two ball bearings have the same waviness amplitude, the axial force in the pair of ball bearings cancel each other and the axial vibration does not exist.

Fig. 4 shows the numerical procedure to calculate the ball bearing vibration due to the effect of waviness. Once the initial contact angles and elastic deformations of each ball are calculated under the application of preload, the waviness of each race is introduced to the position vectors of the groove radius center with respect to the mass center of the rotor, and the ball waviness is introduced by considering the kinematic constraints between the ball and each race. The simultaneous algebraic equations of the kinematic constraints in Eq. (11) and the force equilibrium equations in Eq. (12) are solved to calculate the elastic deformation and the position of each ball by using the Newton–Raphson iteration method. The permissible error of the Newton–Raphson iteration method is 10^{-15}% and the time step is 10^{-5} s. Then, bearing forces and moments acting on the rotor are calculated by using Eq. (21), and the equations of motion in Eq. (23) are solved by using the Runge–Kutta–Fehlberg method. Initial time step is 10^{-12} s, and the permissible integration error is 10^{-5}%. Also small numerical damping, which is 10^{-5} of the stiffness, is introduced to the equations of translational motion only in order to prevent the divergence of the solution [10]. The data with constant time step are obtained through linear interpolation, and Fourier transformation is performed to investigate the characteristics of bearing forces and displacements.

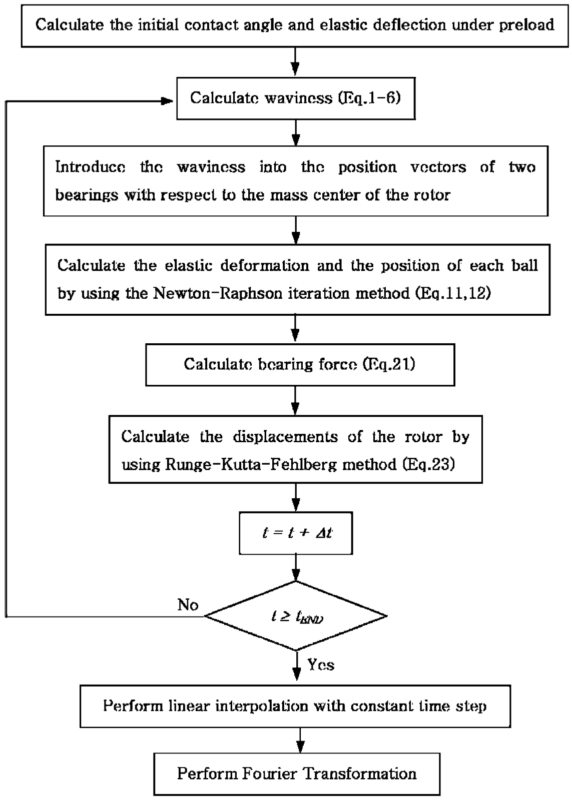

Fig. 4. Numerical procedures to calculate the ball bearing vibration due to the effect of waviness.

The natural frequencies of the analysis model are calculated at the rotating speed of 10000 rpm, and they are 657, 745, 2361, and 3073 Hz, corresponding to the backward rocking, forward rocking, radial and axial modes, respectively. Also, the effect of the numerical damping is almost negligible and it only shifts the natural frequencies by 0.1 Hz (0.01%), 0.1 Hz (0.01%), 1.9 Hz (0.12%), and 7.3 Hz (0.3%), corresponding to the backward rocking, forward rocking, radial and axial modes, respectively.

3.2. Bearing vibration due to the centrifugal force and gyroscopic moment of ball

The bearing contact force due to the waviness of the rolling elements produces not only the principal frequencies but also their harmonics and sideband frequencies due to the non-linear load–deflection characteristics. Table 3 shows the axial and radial vibration frequencies with the variation of waviness order presented by prior researchers [1,3]. Table 4 shows the sideband frequencies due to the waviness interaction between the rolling elements by prior researchers [1], and Table 5 shows the radial sideband frequencies additionally generated by the interaction between the waviness to produce the axial and radial vibration by prior researchers [6]. Based on Tables 3–5, principal frequencies and sideband frequencies are represented as the linear

Table 3
Principal vibration frequencies ($i = 1$) and their harmonics ($i > 1$) due to the waviness of the rolling elements ($i \geqslant 1$: intger, Z: number of ball, f: rotating frequency of inner race, f_c: cage rotating frequency, f_b: ball spinning frequency)

Type of waviness	Waviness order	Principal frequencies and their harmonics (Hz)	Type of motion
Outer race	$l = iZ$	iZf_c	Axial
	$l = iZ \pm 1$	iZf_c	Radial
Inner race	$l = 1$	f	Radial
	$l = iZ$	$iZ(f - f_c)$	Axial
	$l = iZ \pm 1$	$iZ(f - f_c) \pm f$	Radial
Ball	Oversize	f_c	Radial
	$l = 2i$	$2if_b$	Axial
	$l = 2i$	$2if_b \pm f_c$	Radial

Table 4
Sideband frequencies due to the waviness interaction between the rolling elements [1]

Interacting surfaces	Sideband frequencies (Hz)	
	Axial vibration	Radial vibration
Outer race Ball oversize	$iZf_c \pm jf_c$	$iZf_c \pm jf_c$
Outer race Ball	$iZf_c \pm j2f_b$	$iZf_c \pm j2f_b$
Outer race Inner race	$iZf_c \pm jf$	$iZf_c \pm if$
Inner race Ball oversize	$iZ(f - f_c) + jf_c$	$iZ(f - f_c) \pm f \pm jf_c$
Inner race Ball	$iZ(f - f_c) \pm 2jf_b$	$iZ(f - f_c) \pm f \pm 2jf_b$

combination of the number of balls, rotating frequency of inner race and cage, and ball spinning frequency.

Table 6 shows the principal vibration frequencies due to the various waviness of the rolling element by prior researchers [3], and by this proposed method with or without considering the centrifugal force and moment of ball. To validate the accuracy of this research, the vibration frequencies due to the effect of the waviness of the ball bearing without considering the centrifugal force and gyroscopic moment of the ball are compared with those of the prior researchers as shown in the third and fourth columns of Table 6. It shows that the proposed model exactly matches those of prior researchers, and that the centrifugal force and gyroscopic moment of the ball change the principal frequencies in this model significantly as shown in the fifth column of

Table 5
Radial sideband frequencies due to the waviness interactions between the waviness of axial and radial vibration [6]

Interacting surfaces	Waviness order	Sideband frequencies (Hz)
Outer race	$l = iZ$	$iZf_c \pm 2jf_b \pm f_c$
Ball	$l = 2i$	
Outer race	$l = iZ$	$iZf_c \pm jZ(f - f_c) \pm f$
Inner race	$l = iZ \pm 1$	
Outer race	$l = iZ \pm 1$	$iZf_c \pm jZ(f - f_c)$
Inner race	$l = iZ$	
Inner race	$l = iZ$	$iZ(f - f_c) \pm jf_c$
Ball oversize	1	
Inner race	$l = iZ$	$iZ(f - f_c) \pm 2jf_b \pm f_c$
Ball	$l = 2i$	

Table 6
Comparisons of principal vibration frequencies due to the waviness of the rolling elements

Type	Waviness order	Principal frequencies (Hz)			Type of motion
		Prior model [3]	Proposed model without F_{cj} and M_{Gj}	Proposed model with F_{cj} and M_{Gj}	
Cage rotating frequency	f_c	71.63	72.06	76.72	
Ball spinning frequency	f_b	460.30	461.11	500.91	
Outer race	15	1146.10	1146.52	1227.60	Radial
	16	1146.10	1146.10	1227.60	Axial
	17	1146.10	1146.10	1227.60	Radial
Inner race	15	1353.90	1650.47	1272.40	Radial
	16	1520.57	1517.15	1439.10	Axial
	17	1687.24	1680.81	1605.73	Radial
Ball		848.97	850.07	925.09	Radial
	2	920.60	922.23	1001.82	Axial
		992.23	994.39	1078.54	Radial

Table 6. The centrifugal force and gyroscopic moment of a ball increase the contact angle of the inner race, and they decrease the contact angle of the outer race and angle ζ, so that the cage rotating frequency and ball spinning frequency increase as shown in Eqs. (16) and (17). Therefore, the principal vibration frequencies due to ball waviness ($2f_b \pm f_c$ and $2f_b$) and outer race waviness (Zf_c) become bigger than those in the case without considering the centrifugal force and gyroscopic moment of the ball, and the principal vibration frequencies due to inner race waviness

$(Z(f - f_c) \pm f$ and $Z(f - f_c))$ become smaller than those in the case without considering the centrifugal force and gyroscopic moment of the ball. The centrifugal force and gyroscopic moment of a ball also affect the sideband frequencies, because they are represented as the linear combination of the cage and ball spinning frequencies as shown in Tables 4 and 5. This research shows that the centrifugal force and gyroscopic moment of a ball play an important role in determining the bearing vibration frequencies.

3.3. Bearing vibration due to the waviness interaction of two ball bearings constrained by a rotor

Prior researchers investigated the bearing vibration in a rotor supported by a single ball bearing or by two ball bearings with the same waviness [1,6]. In most cases, however, a rotor is supported by two or more ball bearings with different waviness. Even if it is supported by two identical ball bearings, unsymmetrical load distribution may result in different waviness in each bearing.

Figs. 5–7 show the frequency spectra of the radial force, axial force and moment at each bearing in the case that the left bearing has the ball waviness of order 2 and the right bearing has the inner race waviness of order 16. In following figures, the characters C, B, I, O, *, (i,j) and $\boxed{(i,j)}$ denote the cage rotating frequency, the principal frequencies due to ball waviness, inner race waviness and

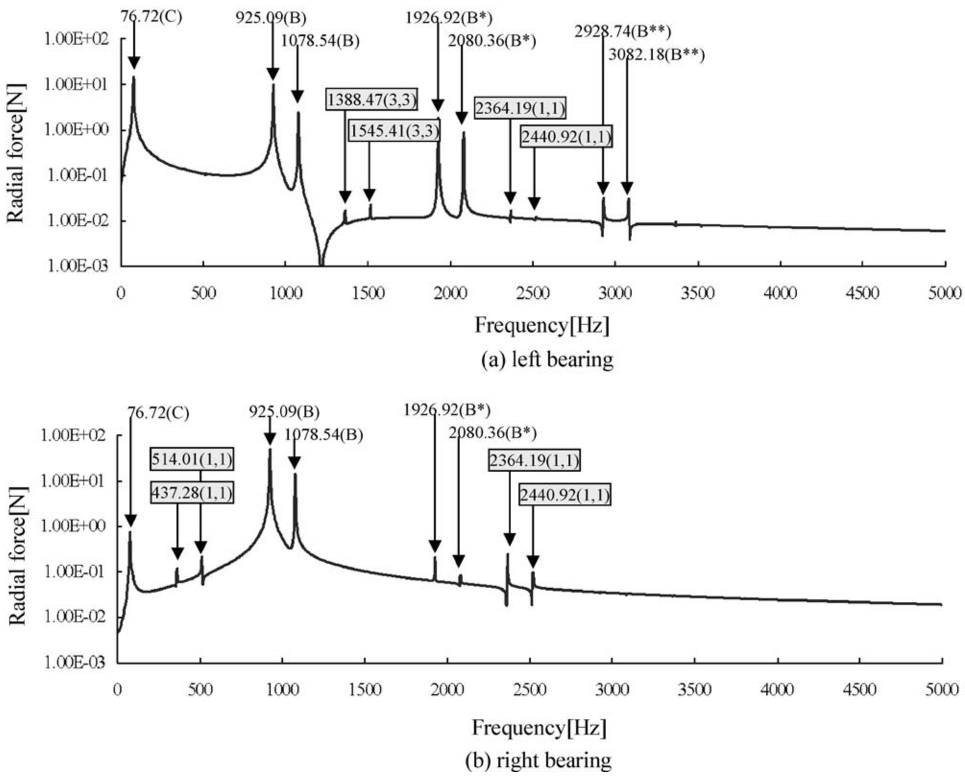

Fig. 5. Frequency spectra of the radial force at each ball bearing in the case that the left bearing has the ball waviness of order 2 and the right bearing has the inner race waviness of order 16.

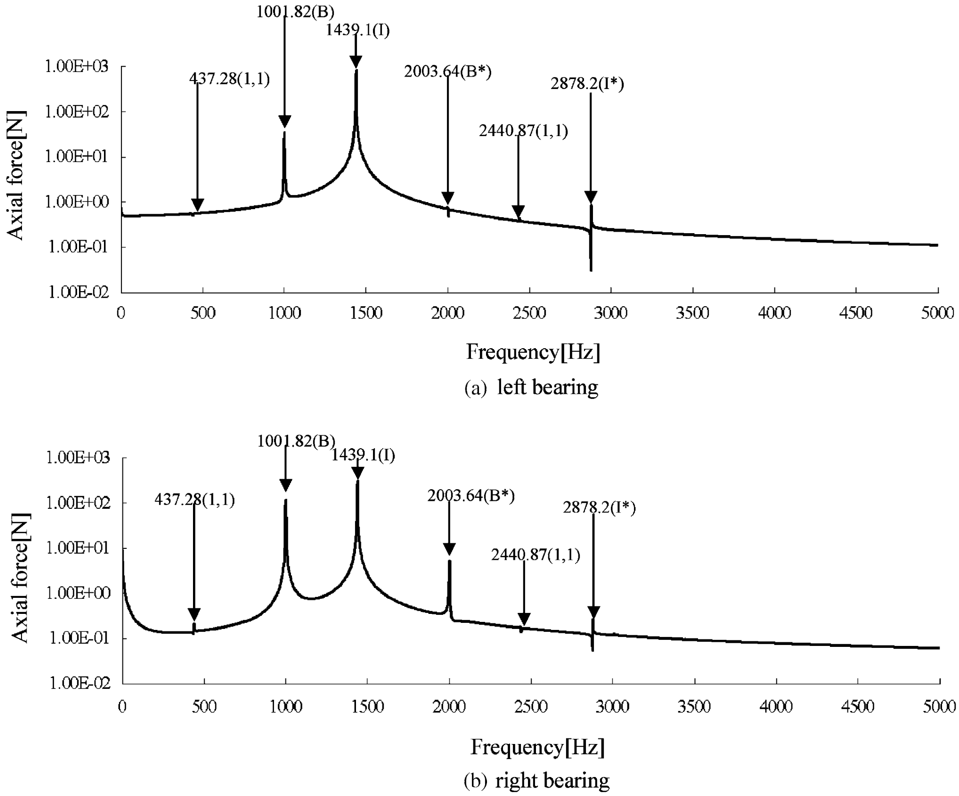

Fig. 6. Frequency spectra of the axial force at each ball bearing in the case that the left bearing has the ball waviness of order 2 and the right bearing has the inner race waviness of order 16.

outer race waviness, their harmonics, and the sideband frequencies in Tables 4 and 5, respectively. In Fig. 5, the radial force has the principal frequencies due to the ball waviness of order 2 and their harmonics, because the inner race waviness of order 16 does not generate the radial vibration. It also has the sideband frequencies in Table 5, which result from the waviness interaction between the ball waviness of the left bearing to generate radial vibration and the inner race waviness of the right bearing to generate the axial vibration. In Fig. 6, the axial force has the principal frequencies due to the ball waviness of order 2 and the inner race waviness of order 16, and their harmonics. They also have the sideband frequencies in Table 4, which result from the waviness interaction between the ball waviness of the left bearing and inner race waviness of the right bearing. Fig. 7 shows the frequency spectra of the moment, which has the same frequency composition as the radial force, because the radial force in each bearing produces the moment. Figs. 5–7 show that the principal frequencies and their harmonics in the radial force, axial force and moment resulting from the waviness of one ball bearing, are transferred to the other bearing through the rotor. They also show that the sideband frequencies due to the load–deflection non-linearity may be produced through the rotor not only from the waviness interaction between the rolling elements of a ball bearing, but also from the waviness interaction between the rolling elements of two ball bearings constrained by a rotor.

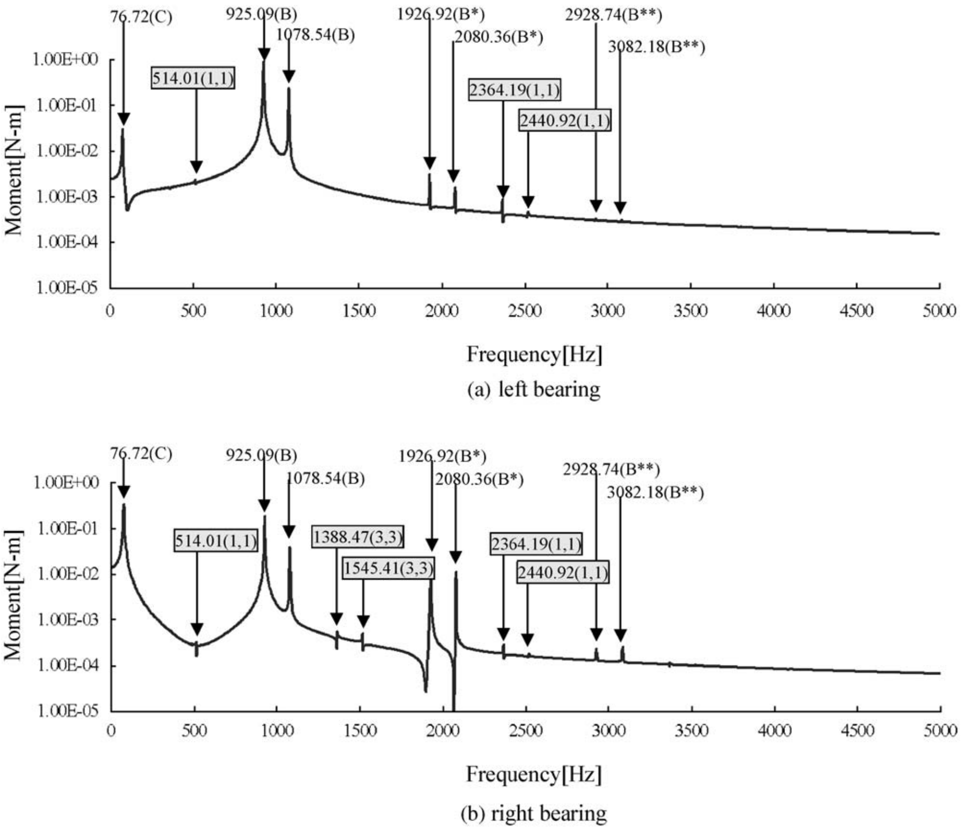

Fig. 7. Frequency spectra of the moment at each ball bearing in the case that the left bearing has the ball waviness of order 2 and the right bearing has the inner race waviness of order 16.

Fig. 8 shows the frequency spectra of the displacements of the rotor due to the application of the bearing force and moment in the case that the left bearing has the ball waviness of order 2 and the right bearing has the inner race waviness of order 16. In Figs. 8(a) and (c), the radial and angular displacement has the principal frequencies due to the ball waviness of order 2, their harmonics and the sideband frequencies, which are similar to those of Figs. 5 and 7. However, in Fig. 8(b), the axial displacement does not have the harmonics and sideband frequencies but only principal frequencies, because the axial forces in the pair of ball bearings cancel each other. Resonance is observed in the forward rocking and radial vibration modes (745 and 2361 Hz) due to the excitation of bearing frequencies.

Fig. 9 shows the frequency spectra of the radial force, axial force and moment of the left bearing in the case that the left bearing has the inner race waviness of order 15 and the right bearing has the outer race waviness of order 16. The inner race waviness of order 15 generates the radial vibration, and the outer race waviness of order 16 generates the axial vibration. In Figs. 9(a) and (c), the radial force and moment have the principal frequency and its harmonics due to the inner race waviness of order 15. Even though the inner race waviness of order 15 at the left

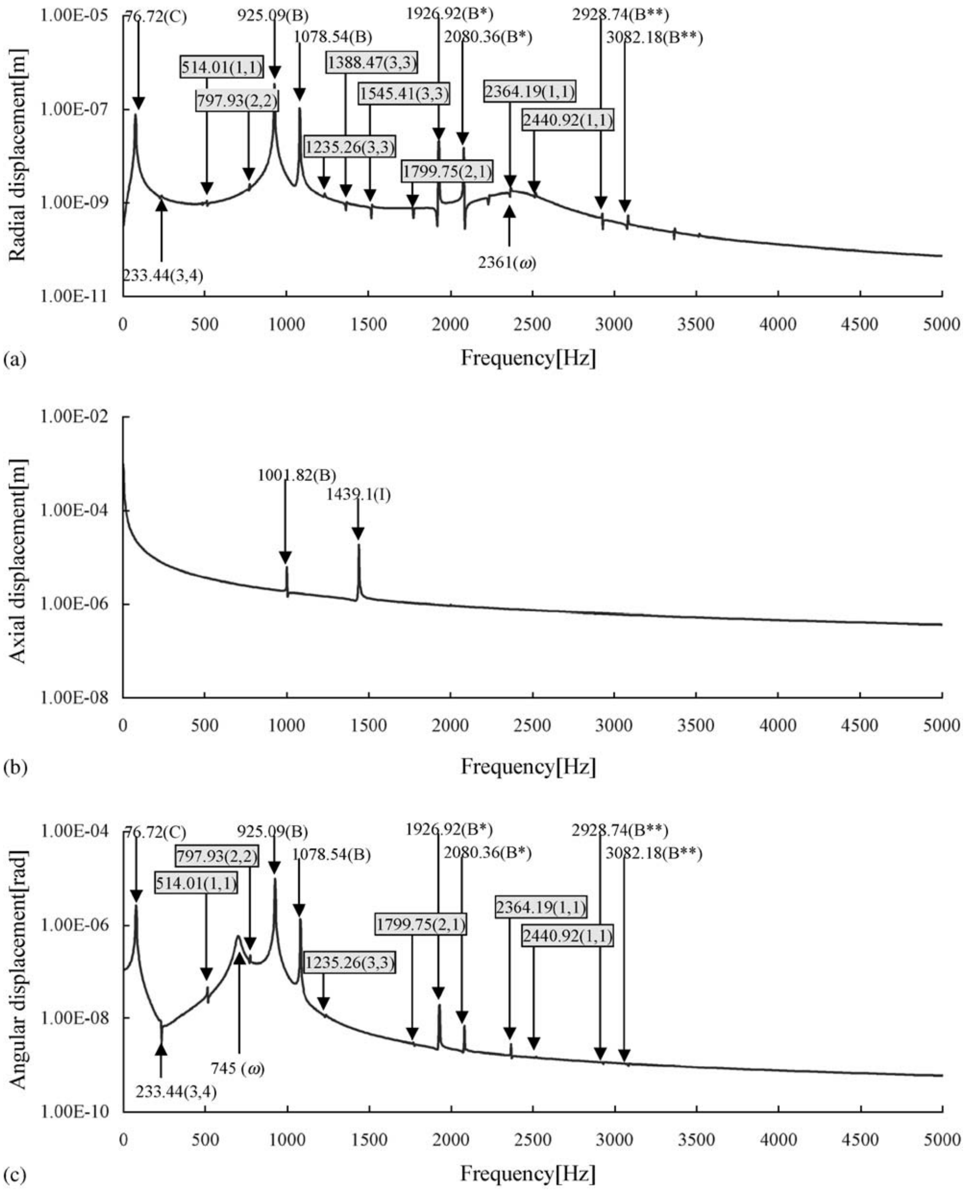

Fig. 8. Frequency spectra of the displacements of the rotor due to the application of the bearing force and moment in the case that the left bearing has ball waviness of order 2 and the right bearing has inner race waviness of order 16.

bearing only generates the radial force, the axial force as shown in Fig. 9(b) is generated at the left bearing through the waviness interaction with the right bearing, which has the outer race waviness of order 16. However, there exists only the axial force in the right bearing, as shown in Fig. 10, without the radial force and moment through the waviness interaction of the left bearing with the inner race waviness of order 15. It can be explained that the axial force is much bigger than the

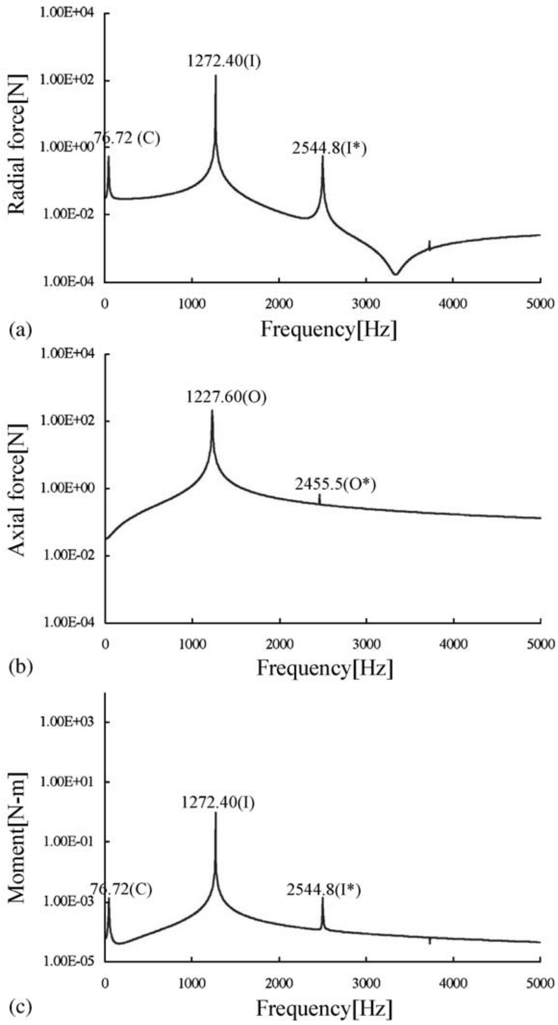

Fig. 9. Frequency spectra of the radial force, axial force and moment of the left bearing in the case that the left bearing has the inner race waviness of order 15 and the right bearing has the outer race waviness of order 16.

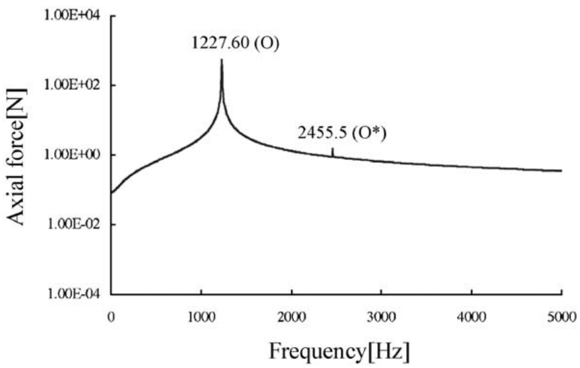

Fig. 10. Frequency spectra of the axial force of the right bearing in the case that the left bearing has the inner race waviness of order 15 and the right bearing has the outer race waviness of order 16.

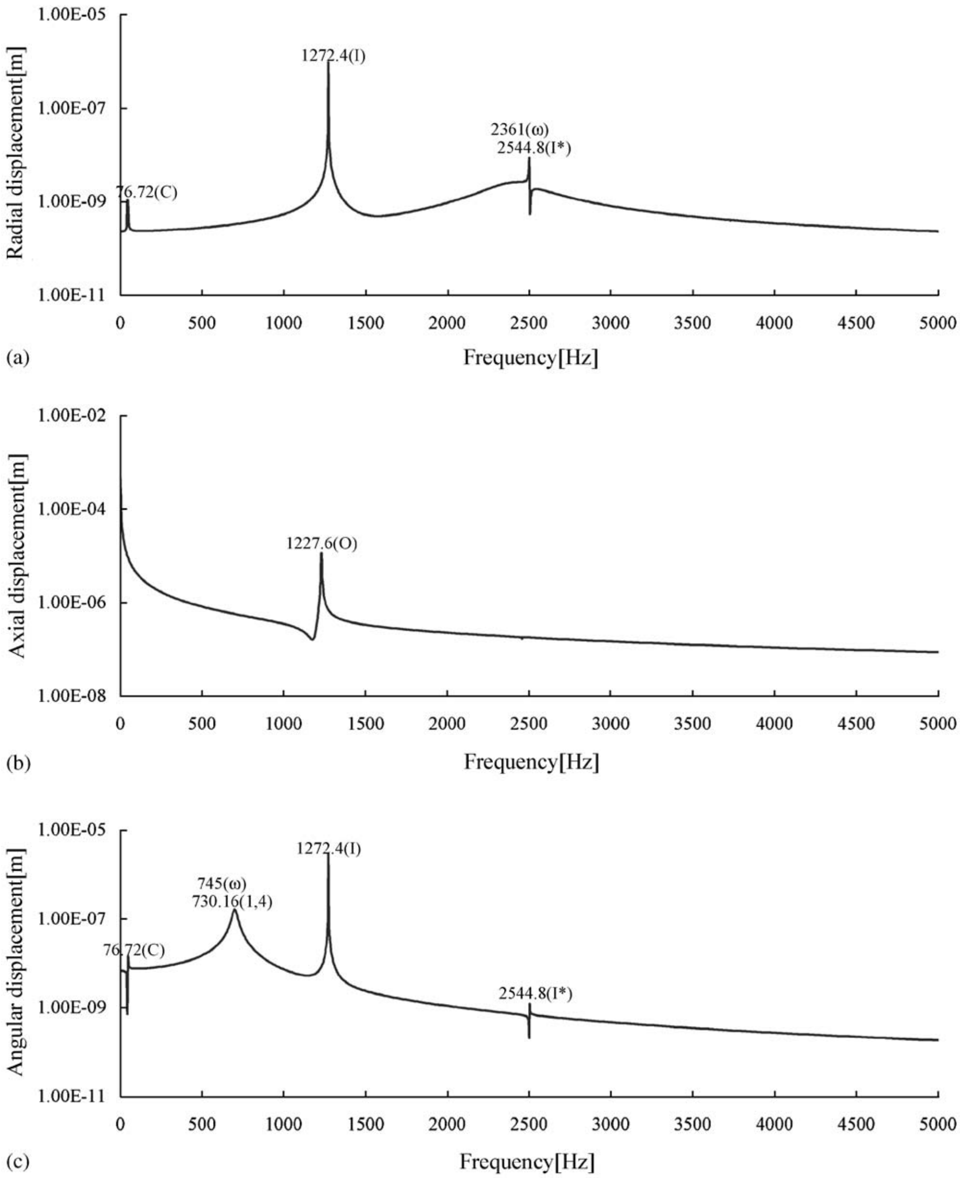

Fig. 11. Frequency spectra of the displacements of the rotor due to the application of the bearing force and moment in the case that the left bearing has the inner race waviness of order 15 and the right bearing has the outer race waviness of order 16.

radial force in this axially preloaded ball bearing, so that the waviness interaction is transferred from the axial force to the radial force, not in the opposite direction. Also, sideband frequencies are not observed in this case. It can be explained that the inner race waviness at the left bearing does not directly contact the outer race waviness at the right bearing, so that the effect of the waviness interaction between the two ball bearings is almost negligible.

Fig. 11 shows the frequency spectra of the displacements of the rotor due to the application of the bearing force and moment in the case that the left bearing has the inner race waviness of order 15 and the right bearing has the outer race waviness of order 16. In Figs. 11(a) and (c), the radial and angular displacement has the principal frequency due to the inner race waviness of order 15, and its harmonics, which are similar to those of Fig. 9. However, in Fig. 11(b), the axial displacement does not have the harmonics but only the principal frequency, because the axial forces in a pair of ball bearings cancel each other. Resonance is observed in the forward rocking and radial vibration modes (745 and 2361 Hz) due to the excitation of bearing frequencies.

4. Conclusions

1. This research presents an analytical model to investigate vibration due to ball bearing waviness in a rotating system supported by two or more ball bearings, taking account of the centrifugal force and gyroscopic moment of the ball.
2. The centrifugal force and gyroscopic moment of the ball play an important role in determining the bearing vibration frequencies, i.e., the principal frequencies, their harmonics and the sideband frequencies resulting from the waviness of rolling elements of ball bearing.
3. In a rotor supported by two or more ball bearings, the principal frequencies and their harmonics of the radial force, axial force and moment resulting from the waviness of one ball bearing is transferred to those of the other bearing through the rotor.
4. In a rotor supported by two or more ball bearings, the sideband frequencies may be produced not only from the waviness interaction between the rolling elements of one ball bearing, but also from the waviness interaction between the rolling elements of two or more ball bearings constrained by a rotor.

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