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Critical states of thin ellipsoidal shells in simple and compound rotations

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Abstract

The similarity of modes of static buckling of spinning elastic ellipsoid shells and their elastic resonant precession vibrations in compound rotation is investigated by theoretical simulation through the use of immovable, slewing and rotating reference frames. It is established that the elongated shells lose their equilibrium stability more readily in simple rotation, while the flattened ones are more easily involved in resonant regimes of vibrations in compound rotations.

An elaborate approach may be used for numerical simulation of the critical states of thin-walled elastic rotors of engines of aircrafts during their translational motions and attitude manoeuvres.

1. Introduction

It is common knowledge in the theory of the equilibrium of elastic rectilinear rotating shafts, carrying rigid discs, that the centrifugal inertia forces generated by spinning can provoke system stability loss. In these cases the axial line of the shaft buckles and assumes the shape of a plane curve spinning together with the shaft [1]. In as much as in the considered case the inertia forces moving the shaft off balance depend on the magnitudes of the shaft displacements, they are positional ones. For this reason their inclusion into the constitutional equations causes change of the structure of their remaining members. As this takes place, it turns out that the equation of its critical equilibrium is identical in form to the characteristic equation of its free vibrations, whereas the critical value of the rotation velocity equals the first frequency of free vibrations of the non-rotating elastic system. Like simplified models, where the rotors are substituted by absolutely rigid bodies and critical states of the system are considered to result from the elastic pliability of the

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shaft, they are usually used in investigations of the phenomena of static and dynamic loss of stability of turbine structures. One would expect that the analogous bifurcational phenomena may also occur in stationary rotation of thin-walled axisymmetrical shells of turbine rotors around an immovable axis.

However, the character of mechanical behaviour of a thin-walled elastic rotor becomes much more complicated in its compound rotation, when the rotor is installed on a flying apparatus performing attitude manoeuvres and the rotor axis performs additional forced slewing with small angular velocity in some plane. In this case, the precession elastic vibrations of the rotor are excited as a result of superposition and interaction of different kinds of rotation. In the inertial reference system, the vibrations manifest themselves as a stationary state deformed symmetrically relative to the plane, containing the axes of the rotor spinning and slewing. Initiation of critical states of a system performing compound rotation can be associated with precession resonances realized at some (critical) values of the system rotation velocity [2,3].

Despite the apparent distinctions between the considered phenomena, they have essential resemblances in the character of their proceeding, with the only difference that it is more convenient to observe them in different co-ordinate systems. Thus, the static loss of the elastic thin-wall rotor stability in simple rotation is realized in the form of its stationary buckling in the co-ordinate system spinning together with the rotor. For this reason, in the inertial reference frame it is manifested in the form of a rotating buckle.

In compound rotation, there is the inverse picture. In the co-ordinate system connected with the rotor, its vibrations show themselves as buckles moving (processing) in the direction opposite to the direction of the rotor spinning, which is known as inverse regular precession. In this connection they appear as immovable buckles relative to the inertial reference frame. So it is possible to achieve some similarities in the problem formulations and techniques of their solution choosing a rotating co-ordinate system for the description of the first phenomenon and a slewing reference frame for analysis of the second one.

In this connection, the simultaneous solution of these problems, establishment of the possibilities of initiation of static and dynamic critical states of the shell rotors in simple and compound rotations, and determination of what type of these instabilities comes earlier are of interest.

Some issues of the dynamics of the rotating plate and shell systems were considered in Refs. [4–9]. The precession vibrations of elastic discs and conic shells in compound rotation were analyzed in Ref. [10].

2. Equations of dynamic equilibrium of a shell element

Let a thin-walled axisymmetric ellipsoidal shell be fixed by one of its circular bases to a rigid carrier. The carrier rotates together with the co-ordinate system $Oxyz$ relative to the axis of symmetry Oz (Fig. 1) with the constant by module angular velocity ω. The second contour of the shell is free from constraints and forces. Introduce curvilinear orthogonal co-ordinate system $ox^1x^2x^3$ in the middle surface of the shell. Its co-ordinate line x^1 lies in a generatrix section, x^2 is circumferentially directed, and x^3 is oriented by an internal normal to the shell surface.

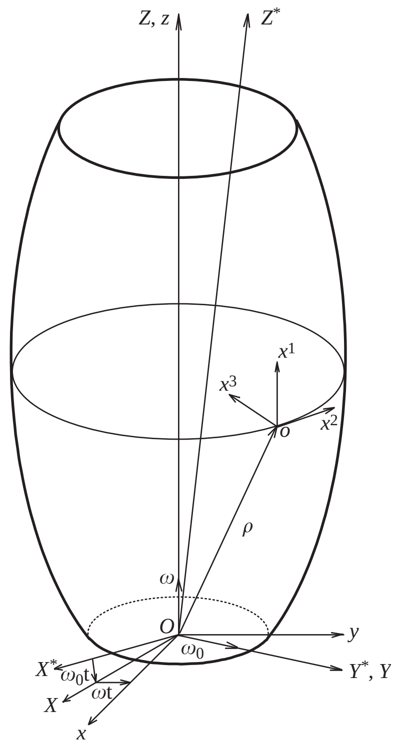

Fig. 1. Shell design scheme.

Note that the constituent equations of the shell dynamics in compound rotation are adduced in Ref. [10]. Here their concise survey used for formulation of the problem of the shell spinning stability will be outlined.

Taking into account that, at initiation of critical phenomena in a rotating shell, it is prestressed by centrifugal inertia forces, one uses the general form of geometrically non-linear equations of dynamic equilibrium of its element [3]. In co-ordinate system $Oxyz$ they appear as

$$\nabla_\alpha \mathbf{T}^\alpha + \mathbf{p} = 0, \qquad \nabla_\alpha \mathbf{M}^\alpha + (\mathbf{e}^\alpha \times \mathbf{T}^\alpha)\sqrt{a_{11}a_{22} - a_{12}a_{21}} = 0 \quad (\alpha = 1, 2). \tag{1}$$

Here $\mathbf{T}^\alpha$ is the vector of internal forces in the shell, $\mathbf{M}^\alpha$ the vector of internal moments, ∇_α the symbol of the covariant derivative, $\mathbf{p}$ the vector of intensity of external distributed forces; and a_{ij} the coefficients of the first quadratic form of the shell midsurface.

Using the relations between contravariant components of the functions of internal forces T^{ij} and moments M^{ij} and covariant components of the functions of strains ε_{ij} and curvature increments μ_{ij},

$$\begin{aligned} T^{ij} &= \mathbf{E}h\varepsilon_{\alpha\beta}[va^{ij}a^{\alpha\beta} + (1-v)a^{i\alpha}a^{j\beta}]/(1-v^2), \\ M^{ij} &= \mathbf{E}h^3\mu_{\alpha\beta}[va^{ij}a^{\alpha\beta} + (1-v)a^{i\alpha}a^{j\beta}]/12(1-v^2), \end{aligned} \tag{2}$$

expressing these functions via covariant components u_1, u_2, u_3 of the displacement vector $\mathbf{u}$ and the angles ϑ_i of the cross-section rotation,

$$\varepsilon_{ij} = (\mathbf{e}_i\partial\mathbf{u}/\partial x^j + \mathbf{e}_j\partial\mathbf{u}/\partial x^i + \vartheta_i\vartheta_j)/2, \qquad \vartheta_i = (\partial\mathbf{u}/\partial x^i)\mathbf{e}_3,$$
$$\mu_{ij} = (1/c^{ik}\mathbf{e}^k\partial\mathbf{\Omega}/\partial x^j + 1/c^{jk}\mathbf{e}^k\partial\mathbf{\Omega}/\partial x^i)/2, \quad \mathbf{\Omega} = c^{ij}\vartheta_i\mathbf{e}_j \quad (i,j,k = 1,2), \tag{3}$$

and taking into account change of the parameters b_i^j, b_{ij} of the second quadratic form at the process of the shell deformation, one gains the equations of its dynamic equilibrium. Here, c^{ij} are the components of the discriminant tensor of the surface.

3. Forces of inertia of compound and simple rotation

In the considered case only the inertia forces play the role of active forces applied to the shell. To calculate them one uses the equality

$$\mathbf{p} = -\gamma h\mathbf{a}, \tag{4}$$

where γ is the shell material density, h its thickness and $\mathbf{a}$ the vector of the shell element absolute acceleration.

Inasmuch as the inertia forces of a simple rotation constitute a particular case of the inertia forces of the compound rotation, the more general case will be one's special concern. Assume the shell compound rotation to be conducted by forced in-plane slewing its axis Oz with the constant angular velocity ω_0. Let $\omega \perp \omega_0$.

The feature of the technique of calculation of the vectors $\mathbf{a}$ and $\mathbf{p}$ lies in the fact that it is more convenient to define them in the rotating co-ordinate system $Oxyz$ and thereupon to apply them in Eqs. (1), using the local co-ordinate system $ox^1x^2x^3$. To perform this transition, introduce additional co-ordinate systems (Fig. 1): $OX^*Y^*Z^*$ is the inertial co-ordinate system with its origin at the centre of the supporting contour of the shell with its axis OY^* collinear with the ω_0 vector; $OXYZ$ the slewing co-ordinate system whose immovable axis OY coincides with the OY^*-axis and axis OZ is in line with the Oz-axis.

The $\mathbf{a}$ vector is calculated by the formula

$$\mathbf{a} = \mathbf{a}_e + \mathbf{a}_r + \mathbf{a}_c. \tag{5}$$

To define the vectors of the bulk ($\mathbf{a}_e$), relative ($\mathbf{a}_r$) and Coriolis ($\mathbf{a}_c$) accelerations the following formulae are used [11]:

$$\mathbf{a}_e = \boldsymbol{\varepsilon} \times \boldsymbol{\rho} + (\boldsymbol{\omega}_0 + \boldsymbol{\omega}) \times [(\boldsymbol{\omega}_0 + \boldsymbol{\omega}) \times \boldsymbol{\rho}], \quad \mathbf{a}_r = \dot{\mathrm{d}}^2\boldsymbol{\rho}/\mathrm{d}t^2, \quad \mathbf{a}_c = 2(\boldsymbol{\omega}_0 + \boldsymbol{\omega}) \times (\mathrm{d}\boldsymbol{\rho}/\mathrm{d}t). \tag{6}$$

Here $\boldsymbol{\omega}_0 + \boldsymbol{\omega}$, $\boldsymbol{\varepsilon} = \boldsymbol{\omega}_0 \times \boldsymbol{\omega}$ are correspondingly the vectors of absolute angular velocity and angular acceleration of the spinning co-ordinate system $Oxyz$; $\boldsymbol{\rho}$ is the radius vector of the shell element in this system.

Assuming $\omega \gg \omega_0$, fulfilling vector procedures (6) and neglecting the value ω_0^2, the contravariant components of the acceleration vectors (6) are found to be

$$a_e^1 = -\omega^2 r \sin\varphi/\sqrt{a_{11}} + 2\omega_0\omega r \sin(\omega t + x^2)\cos\varphi/\sqrt{a_{11}}$$
$$- \omega^2 u_1 \sin^2\varphi/a_{11} + \omega^2 u_3 \sin\varphi\cos\varphi/\sqrt{a_{11}},$$

$$\begin{aligned}
a_e^2 &= -\omega^2 u_2/a_{22}, \\
a_e^3 &= \omega^2 r\cos\varphi + 2\omega_0\omega r\sin(\omega t + x^2)\sin\varphi + \omega^2(u_1\sin\varphi/\sqrt{a_{11}} - u_3\cos\varphi)\cos\varphi, \\
a_c^1 &= -2\omega\dot{u}_2\sin\varphi/\sqrt{a_{11}a_{22}}, \\
a_c^2 &= 2\omega\dot{u}_1\sin\varphi/\sqrt{a_{11}a_{22}} - 2\omega\dot{u}_3\cos\varphi/\sqrt{a_{22}}, \quad a_c^3 = 2\omega\dot{u}_2\cos\varphi/\sqrt{a_{22}}, \\
a_r^1 &= \ddot{u}_{11}/a_{11}, \quad a_r^2 = \ddot{u}_2/a_{22}, \quad a_r^3 = \ddot{u}_3. \qquad (7)
\end{aligned}$$

Here r is the distance from the considered element to the rotation axis, φ is the angle between the tangent to the shell generatrix and the rotation axis. With the allowance made for Eqs. (4) and (7), the contravariant components of the vector of intensity of inertia forces acting on the shell in compound rotation are found:

$$\begin{aligned}
p^1 &= -\gamma h(-\omega^2 r\sin\varphi/\sqrt{a_{11}} + 2\omega_0\omega r\sin(\omega t + x^2)\cos\varphi/\sqrt{a_{11}} - \omega^2 u_1\sin^2\varphi/a_{11} \\
&\quad + \omega^2 u_3\sin\varphi\cos\varphi/\sqrt{a_{11}} - 2\omega\dot{u}_2\sin\varphi/\sqrt{a_{11}a_{22}} + \ddot{u}_{11}/a_{11}), \\
p^2 &= -\gamma h(2\omega\sin\varphi\cdot\dot{u}_1/\sqrt{a_{11}a_{22}} - 2\omega\cos\varphi\dot{u}_3/\sqrt{a_{22}} + \ddot{u}_2/a_{22} - \omega^2 u_2/a_{22}), \\
p^3 &= -\gamma h(\omega^2 r\cos\varphi + 2\omega_0\omega r\sin(\omega t + x^2)\sin\varphi \\
&\quad - \omega^2(u_1\sin\varphi/\sqrt{a_{11}} - u_3\cos\varphi)\cos\varphi + 2\omega\dot{u}_2\cos\varphi/\sqrt{a_{22}} + \ddot{u}_3). \qquad (8)
\end{aligned}$$

They include three types of forces. The first one involves ω^2, does not depend on time and plays the role of active static forces. The second force type incorporates the multiplier $\sin(\omega t + x^2)$, depends on the phase function $\omega t + x^2$ and plays the role of active dynamic forces. The forces of the third type contain the velocities $\dot{u}_1$, $\dot{u}_2$, $\dot{u}_3$ and accelerations $\ddot{u}_1$, $\ddot{u}_2$, $\ddot{u}_3$ and arise only at the dynamic deforming of the shell.

In simple rotation the forces of the second and third types are not generated, so in this case

$$\begin{aligned}
p^1 &= -\gamma h[\omega^2 r\sin\varphi/\sqrt{a_{11}} - \omega^2 u_1\sin^2\varphi/a_{11} + \omega^2 u_3\sin\varphi\cos\varphi/\sqrt{a_{11}}], \\
p^2 &= 0, \qquad p^3 = -\gamma h[\omega^2 r\cos\varphi + \omega^2(u_1\sin\varphi/\sqrt{a_{11}} - u_3\cos\varphi)\cos\varphi]. \qquad (9)
\end{aligned}$$

Expressions (8) and (9) are used for simulation of the inertia forces acting on a shell at the precritical and critical states.

4. The equations of critical states of a shell in simple and compound rotation

The equations of shell dynamics in compound rotation follow from relationships (1)–(3) transformed with allowance made for Eqs. (4)–(8). In this case it is necessary to take into account the change of geometry of the shell at its loading and to use the values $r + \Delta r$, $\varphi + \Delta\vartheta_1^*$ instead of r and φ in the equalities containing the large value ω^2. Then the equations of the force group of system (1) assume the form

$$
\begin{aligned}
&\partial T^{11}/\partial x^1 + \partial T^{12}/\partial x^2 + (2\Gamma^1_{11} + \Gamma^2_{21})T^{11} + \Gamma^1_{22}T^{22} - b^1_1 T^{13} \\
&\quad = \gamma h[-\omega^2(r+\Delta r)\sin(\varphi + \Delta\vartheta^*_1)/\sqrt{a_{11}} + 2\omega_0\omega r \sin(\omega t + x^2)\cos\varphi/\sqrt{a_{11}} \\
&\quad - 2\omega \sin\varphi \dot{u}_2/\sqrt{a_{11}a_{22}} + \ddot{u}_{11}/a_{11}], \\
&\partial T^{12}/\partial x^1 + \partial T^{22}/\partial x^2 + (3\Gamma^2_{12} + \Gamma^1_{11})T^{12} - b^2_2 T^{23} \\
&\quad = \gamma h[\omega^2 r \cos\varphi \cos(\pi/2 + \Delta\vartheta^*_2)\sqrt{a_{22}} + 2\omega \sin\varphi \dot{u}_1/\sqrt{a_{11}a_{22}} \\
&\qquad - 2\omega \cos\varphi \dot{u}_3/\sqrt{a_{22}} + \ddot{u}_2/a_{22} - \omega^2 u_2/a_{22}], \\
&\partial T^{13}/\partial x^1 + \partial T^{23}/\partial x^2 + (\Gamma^2_{12} + \Gamma^1_{11})T^{13} + b_{11}T^{11} + b_{22}T^{22} \\
&\quad = \gamma h[\omega^2(r+\Delta r)\cos(\varphi + \Delta\vartheta^*_1) + 2\omega_0\omega r \sin(\omega t + x^2)\sin\varphi \\
&\qquad + 2\omega\cos\varphi \dot{u}_2/\sqrt{a_{22}} + \ddot{u}_3].
\end{aligned}
\tag{10}
$$

Here Γ^i_{jk} are the Christoffel symbols.

Note that existence of the multipliers $\sin(\omega t + x^2)$ in the right of these equations is associated with the type of inertia forces acting on the shell, in as much as they are harmonic relative to x^2 and t and run in the circumferential direction with the angular velocity ω initiating the shell precession vibrations with frequency ω. At simulation of the precession vibrations excited by these forces, it is believed that they are small because $\omega \gg \omega_0$. Owing to this assumption, firstly the state of simple rotation with the velocity ω is separated and the shell stress–strain state is calculated. Thereupon, the precession vibrations of the shell in compound rotation are analyzed with the help of the motion equations linearized in the vicinity of the first state. They stem from Eqs. (10):

$$
\begin{aligned}
&\partial\Delta T^{11}/\partial x^1 + \partial\Delta T^{12}/\partial x^2 + (2\Gamma^1_{11} + \Gamma^2_{21})\Delta T^{11} + \Gamma^1_{22}\Delta T^{22} - b^1_1\Delta T^{13} \\
&\quad - \gamma h[-\omega^2 \sin\varphi \Delta r/\sqrt{a_{11}} - \omega^2 r\cos\varphi\Delta\vartheta^*_1/\sqrt{a_{11}} - 2\omega\sin\varphi\Delta\dot{u}_2/\sqrt{a_{11}a_{22}} + \Delta\ddot{u}_1/a_{11}] \\
&\qquad = 2\gamma h\omega_0\omega r\sin(\omega t + x^2)\cos\varphi/\sqrt{a_{11}}, \\
&\partial\Delta T^{12}/\partial x^1 + \partial\Delta T^{22}/\partial x^2 + (3\Gamma^2_{12} + \Gamma^1_{11})\Delta T^{12} - b^2_2\Delta T^{23} \\
&\quad - \gamma h[-\omega^2 r\cos\varphi\Delta\vartheta^*_2/\sqrt{a_{22}} + 2\omega\sin\varphi\Delta\dot{u}_1/\sqrt{a_{11}a_{22}} \\
&\quad - 2\omega\cos\varphi\Delta\dot{u}_3/\sqrt{a_{22}} + \Delta\ddot{u}_2/a_{22} - \omega^2\Delta u_2/a_{22}] = 0, \\
&\partial\Delta T^{13}/\partial x^1 + \partial\Delta T^{23}/\partial x^2 + (\Gamma^2_{12} + \Gamma^1_{11})\Delta T^{13} + b_{11}\Delta T^{11} + \Delta b_{11}T^{11} + b_{22}\Delta T^{22} + \Delta b_{22}T^{22} \\
&\quad - \gamma h[\omega^2\cos\varphi\Delta r - \omega^2 r\sin\varphi\Delta\vartheta^*_1 + 2\omega\cos\varphi\Delta\dot{u}_2/\sqrt{a_{22}} + \Delta\ddot{u}_3] \\
&\qquad = 2\gamma h\omega_0\omega r\sin\varphi\sin(\omega t + x^2).
\end{aligned}
\tag{11}
$$

On the left sides of these equations there are summands containing the multipliers Δr, $\Delta\vartheta^*_1$, $\Delta\vartheta^*_2$, Δb_{11}, Δb_{22}. To calculate them

$$
\begin{aligned}
&\Delta r = \Delta u_1 \sin\varphi/\sqrt{a_{11}} - \Delta u_3\cos\varphi, \qquad \Delta\vartheta^*_1 = (-\partial\Delta u_3/\partial x^1 - b^1_1\Delta u_1)/\sqrt{a_{11}}, \\
&\Delta\vartheta^*_2 = (-\partial\Delta u_3/\partial x^2 - b^2_2\Delta u_2)/\sqrt{a_{22}}, \qquad \Delta b_{11} = -\Delta\mu_{11}, \qquad \Delta b_{22} = -\Delta\mu_{22}, \\
&\Delta\mu_{11} = \partial\Delta\vartheta^*_1\sqrt{a_{11}}/\partial x^1 - \Delta\vartheta^*_1\sqrt{a_{11}}\Gamma^1_{11}, \qquad \Delta\mu_{22} = \partial\Delta\vartheta^*_2\sqrt{a_{22}}/\partial x^2 - \Delta\vartheta^*_1\sqrt{a_{11}}\Gamma^1_{22}
\end{aligned}
\tag{12}
$$

are used. In as much as the forces containing these multiplier change at the shell deformation, they are positional ones. Taking into consideration that these forces cause variation of the structure of the left hand side of Eq. (11) and since it has ω^2 coefficients, the system's critical states can be achieved with increased ω, when system (11) left-hand member operator degenerates.

The summands with the multipliers $\Delta\dot{u}_1$, $\Delta\dot{u}_2$, $\Delta\dot{u}_3$ in Eq. (11) characterize the gyroscopic-type forces. Their total power equals zero, but their presence leads to a change of the values of free vibration frequencies and splitting their multiple values, as well as to a change of free vibration modes, excluding the possibility of shell vibrations with steady waves.

The expressions on the right of Eqs. (11) play the role of active forces. Owing to their explicit dependence on the function $\sin(\omega t + x^2)$, they generate harmonic waves propagating (processing) in the circumferential direction. Eqs. (11) and (12) are not homogeneous, so the states can be achieved with a change of ω and coefficients of the left-hand parts, when the calculated values of the precession vibration amplitudes begin to enlarge infinitely. The states are known as precession resonances.

Theoretical simulation of the shell stability loss in the simple rotation is performed on the basis of relationships (11) simplified with allowance made for the equality $\omega_0 = 0$ and the fact that the shell does not vibrate. In so doing, it becomes possible to consider the static deformation of the shell in the rotating co-ordinate system $Oxyz$ and if so one has instead of Eq. (7)

$$\begin{aligned}
&a_e^1 = -\omega^2 r \sin\varphi/\sqrt{a_{11}} - \omega^2 u_1 \sin^2\varphi/a_{11} + \omega^2 u_3 \sin\varphi\cos\varphi/\sqrt{a_{11}},\\
&a_e^2 = 0, \quad a_e^3 = \omega^2 r\cos\varphi + \omega^2(u_1\sin\varphi/\sqrt{a_{11}} - u_3\cos\varphi)\cos\varphi,\\
&a_c^1 = 0, \quad a_c^2 = 0, \quad a_c^3 = 0, \qquad a_r^1 = 0, \quad a_r^2 = 0, \quad a_r^3 = 0.
\end{aligned} \tag{13}$$

Substituting equalities (13) into (4) and then into Eq. (1) and linearizing them in the vicinity of the state of simple rotation with the velocity ω with allowance made for Eq. (12), one has the homogeneous system of neutral equilibrium of the shell in some perturbed state:

$$\begin{aligned}
&\partial\Delta T^{11}/\partial x^1 + \partial\Delta T^{12}/\partial x^2 + (2\Gamma_{11}^1 + \Gamma_{21}^2)\Delta T^{11} + \Gamma_{22}^1\Delta T^{22} - b_1^1\Delta T^{13}\\
&\quad - \gamma h[\omega^2 r\cos\varphi(\partial\Delta u_3/\partial x^1 + b_1^1\Delta u_1)/a_{11} - \omega^2\Delta u_1\sin^2\varphi/a_{11} + \omega^2\Delta u_3\sin\varphi\cos\varphi/\sqrt{a_{11}}] = 0,\\
&\partial\Delta T^{12}/\partial x^1 + \partial\Delta T^{22}/\partial x^2 + (3\Gamma_{12}^2 + \Gamma_{11}^1)\Delta T^{12} - b_2^2\Delta T^{23}\\
&\quad - \gamma h[\omega^2 r\cos\varphi(\partial\Delta u_3/\partial x^2 + b_2^2\Delta u_2)/a_{22} - \omega^2\Delta u_2/a_{22}] = 0,\\
&\partial\Delta T^{13}/\partial x^1 + \partial\Delta T^{23}/\partial x^2 + (\Gamma_{12}^2 + \Gamma_{11}^1)\Delta T^{13} + b_{11}\Delta T^{11} + \Delta b_{11}T^{11}\\
&\quad + b_{22}\Delta T^{22} + \Delta b_{22}T^{22} - \gamma h[\omega^2 r\sin\varphi(\partial\Delta u_3/\partial x^1 + b_1^1\Delta u_1)/\sqrt{a_{11}}\\
&\quad + \omega^2\Delta u_1\sin\varphi\cos\varphi/\sqrt{a_{11}} - \omega^2\Delta u_3\cos^2\varphi] = 0.
\end{aligned} \tag{14}$$

The ω values characterized by non-trivial solutions of homogeneous system (14) are critical and the appropriate solutions conform to the modes of stability loss. As a rule, the practical interest represents only the lowest critical value of ω.

The coefficients of Eqs. (11) and (14) are determined by the functions a_{ij}, b_{ij}, Γ_{ij}^k. To calculate them, it is necessary to preset the equation of the shell midsurface. Inasmuch as the Oz-axis of the considered ellipsoidal shell is the axis of its symmetry, the equations can be represented in the form

$$x = (b/a)\sqrt{a^2 - (x^1 - a\sqrt{4b^2 - d^2}/2b)^2}\cos x^2,$$
$$y = (b/a)\sqrt{a^2 - (x^1 - a\sqrt{4b^2 - d^2}/2b)^2}\sin x^2, \qquad z = x^1. \tag{15}$$

Here a, b are the ellipsoid semi-axes and d is the diameter of the clamped contour circle of the shell.

With the use of Eq. (15) the following functions are determined:

$$a_{ij} = \frac{\partial x}{\partial x^i}\frac{\partial x}{\partial x^j} + \frac{\partial y}{\partial x^i}\frac{\partial y}{\partial x^j} + \frac{\partial z}{\partial x^i}\frac{\partial z}{\partial x^j} \quad (i,j = 1,2),$$
$$b_{ij} = \frac{1}{\sqrt{a}}\left[\left(\frac{\partial y}{\partial x^1}\frac{\partial z}{\partial x^2} - \frac{\partial z}{\partial x^1}\frac{\partial y}{\partial x^2}\right)\frac{\partial^2 x}{\partial x^i \partial x^j} + \left(\frac{\partial z}{\partial x^1}\frac{\partial x}{\partial x^2} - \frac{\partial x}{\partial x^1}\frac{\partial z}{\partial x^2}\right)\frac{\partial^2 y}{\partial x^i \partial x^j}\right.$$
$$\left. + \left(\frac{\partial x}{\partial x^1}\frac{\partial y}{\partial x^2} - \frac{\partial y}{\partial x^1}\cdot\frac{\partial x}{\partial x^2}\right)\frac{\partial^2 z}{\partial x^i \partial x^j}\right] \quad (i,j, = 1,2),$$
$$\Gamma_{k,ij} = \frac{1}{2}\left(\frac{\partial a_{ik}}{\partial x^j} + \frac{\partial a_{jk}}{\partial x^i} - \frac{\partial a_{ij}}{\partial x^k}\right), \qquad \Gamma^k_{ij} = a^{kl}\Gamma_{l,ij} \quad (i,j,k,l = 1,2). \tag{16}$$

Underline that the functions $T^{11}(x^1, x^2)$, $T^{22}(x^1, x^2)$ appearing in Eqs. (11) and (14) play a role of known coefficients. They are found from the preliminary solution of the problem of the shell stress–strain state in simple rotation with the velocity ω.

5. Technique of numerical simulation

For construction of solutions of the non-homogeneous equation system (11) let us proceed from the point that their right sides contain the functions with the phase co-ordinate $\omega t + x^2$ describing the running forces. Owing to this, it is possible to represent the required functions in the form of harmonic waves running in the circumferential direction. As this takes place, the functions which are even relative to the co-ordinate $\omega t + x^2$ are substituted by the expressions with the multiplier $\cos(\omega t + x^2)$ and the uneven functions have the multiplier $\sin(\omega t + x^2)$:

$$\Delta u_1(x^1, x^2, t) = u_{(1)}(x^1)\sin(\omega t + x^2), \quad \Delta u_2(x^1, x^2, t) = u_{(2)}(x^1)\cos(\omega t + x^2),$$
$$\Delta u_3(x^1, x^2, t) = u_{(3)}(x^1)\sin(\omega t + x^2), \quad \Delta\vartheta_1(x^1, x^2, t) = \vartheta_{(1)}(x^1)\sin(\omega t + x^2),$$
$$\Delta\varepsilon_{11}(x^1, x^2, t) = \varepsilon_{(11)}(x^1)\sin(\omega t + x^2), \quad \Delta\varepsilon_{12}(x^1, x^2, t) = \varepsilon_{(12)}(x^1)\cos(\omega t + x^2),$$
$$\Delta\mu_{11}(x^1, x^2, t) = \mu_{(11)}(x^1)\sin(\omega t + x^2), \quad \Delta T^{13}(x^1, x^2, t) = T^{(13)}(x^1)\sin(\omega t + x^2). \tag{17}$$

The rest of the required variables in Eq. (11) are expressed through functions (17).

After substitution of Eq. (17) into Eq. (11) and linearized equalities (2), (3) and cancelling by $\cos(\omega t + x^2)$, the phase co-ordinate $\omega t + x^2$ and the derivatives with respect to x^2, t are excluded and a non-homogeneous system of ordinary differential equations of the eighth order with respect to the required functions $u_{(1)}$, $u_{(2)}$, $u_{(3)}$, $\vartheta_{(1)}$, $\varepsilon_{(11)}$, $\varepsilon_{(12)}$, $\mu_{(11)}$, $T^{(13)}$ with the independent function x^1 is constructed. Note that this substitution is equivalent to investigation of the dynamics of the spinning shell in the unspinning (but slewing with the velocity ω_0) co-ordinate system $OXYZ$. This equation system can be presented in general form as

$$\mathrm{d}\mathbf{y}/\mathrm{d}x = \mathbf{A}_1(x)\mathbf{y} + \mathbf{f}(x). \tag{18}$$

Here $\mathbf{y} = \mathbf{y}(x)$ is the required eight-dimensional vector function

$$\mathbf{y} = (u_{(1)}, u_{(3)}, \vartheta_{(1)}, \varepsilon_{(11)}, \mu_{(11)}, T^{(13)}, u_{(2)}, \varepsilon_{(12)})^{\mathrm{T}}, \tag{19}$$

where $x \equiv x^1$ is the independent variable changing in the segment $0 \leqslant x \leqslant l$; $\mathbf{A}_1(x)$ is the coefficient matrix of eighth order which is determined by the shell theory equations and correlations presetting the coefficients of the first and second quadratic forms; and $\mathbf{f}(x)$ is the preset vector of the right sides.

The solution of system (18) has to satisfy the boundary conditions

$$\mathbf{By}(0) = 0, \qquad \mathbf{Dy}(l) = 0, \tag{20}$$

where the constant matrices $\mathbf{B}$ and $\mathbf{D}$ have dimension 4×8.

The $\mathbf{y}(x)$ vector function is constructed through the use of the transfer matrix method in the form of superposition of the particular solution $\mathbf{y}_0(x)$ of non-homogeneous system (18) and general solution $Y(x)\mathbf{C}$ of the appropriate homogeneous system; the $\mathbf{C}$ vector is found from the condition of satisfying equalities (20). The ω values in the coefficients of the matrix $\mathbf{A}_1(x)$ for which the matrix resulting from conditions (20) and serving for the $\mathbf{C}$ vector determination degenerates are resonant.

The problem of the shell stability at simple rotation is solved on the basis of Eq. (14) via determination of the ω eigenvalues, for which this system has non-trivial eigensolutions. Among these solutions, there are the modes possessing cyclic symmetry relative to the Oz-axis and containing multipliers in the form of harmonics $\sin nx^2$, $\cos nx^2$ with different values of n. The least energy-consuming modes for the considered shells turned out to be the modes with the harmonic number $n = 1$. For this reason the replacement is used:

$$\begin{aligned}
&\Delta u_1(x^1, x^2) = u_{(1)}(x^1)\sin x^2, \quad \Delta u_2(x^1, x^2) = u_{(2)}(x^1)\cos x^2,\\
&\Delta u_3(x^1, x^2) = u_{(3)}(x^1)\sin x^2, \quad \Delta\vartheta_1(x^1, x^2) = \vartheta_{(1)}(x^1)\sin x^2,\\
&\Delta\varepsilon_{11}(x^1, x^2) = \varepsilon_{(11)}(x^1)\sin x^2, \quad \Delta\varepsilon_{12}(x^1, x^2) = \varepsilon_{(12)}(x^1)\cos x^2,\\
&\Delta\mu_{11}(x^1, x^2) = \mu_{(11)}(x^1)\sin x^2, \quad \Delta T^{13}(x^1, x^2) = T^{(13)}(x^1)\sin x^2.
\end{aligned} \tag{21}$$

After substitution of Eq. (21) into Eq. (14) and linearized equalities (2), (3) and cancelling $\cos x^2$, the independent variable x^2 is excluded and a system of homogeneous ordinary differential equations of the eighth order relative to the functions $u_{(1)}(x^1)$, $u_{(2)}(x^1)$, $u_{(3)}(x^1)$, $\Delta\vartheta_{(1)}(x^1)$, $\Delta\varepsilon_{(11)}(x^1)$, $\Delta\varepsilon_{(12)}(x^1)$, $\Delta\mu_{(11)}(x^1)$, $\Delta T^{(13)}(x^1)$ is constructed. Its non-trivial solutions are found with the help of the transfer matrix method. The ω values for which the determinant of the matrix derived from conditions (20) takes the zero value are critical.

In order to construct a mode of stability loss, one of the required components of vector $\mathbf{C}$ is preset to be arbitrary and the other seven components are determined from system (20). After this the mode of buckling is calculated.

The particular solutions of the homogeneous system of ordinary differential equations are constructed through the use of the Runge–Kutta method with application of the orthogonalization procedure.

6. Testing the investigation techniques

For the purpose of testing the selected approach, the simpler problems of simple and compound rotations of thin-walled shells were considered. Thus the verification of the technique of theoretical analysis of thin-wall shell buckling under conditions of simple rotation was performed with the help of the example of stability analyses of an elongated cylindrical shell and a tube beam with equivalent parameters of stiffness and inertia characteristics. The radius of the shell external surface constitutes $R_1 = 25.5$ mm, the radius of its internal surface $R_2 = 24.5$ mm, its thickness $h = 1$ mm, the material elasticity modulus $E = 2.1 \times 10^{11}$ Pa, the Poisson ratio $\nu = 0.3$, and the density $\gamma = 7.8 \times 10^3$ kg/m^3. The shell length is chosen to be $l = 0.5$, 1 and 2 m. Two types of boundary conditions were considered: for the first one the left end of the cylindrical shell (tubular beam) was clamped and the right end was free from constraints; for the second type of boundary conditions both the ends of each elastic system were clamped.

Calculations of the shell critical states were performed on the basis of the technique outlined in Section 5. The shell midsurface equations are represented in the form

$$x = R\cos x^2, \quad y = R\sin x^2, \quad z = x^1.$$

It was found via use of formulae (16):

$$\begin{aligned} &a_{11} = 1, \quad a_{22} = R^2, \quad a_{12} = a_{21} = 0, \quad a^{11} = 1, \quad a^{22} = R^{-2}, \\ &b_{11} = 0, \quad b_{22} = -R, \quad b_1^1 = 0, \quad b_2^2 = -R^{-1}. \end{aligned} \tag{22}$$

These values were substituted into Eqs. (14), whereupon the problem of static stability loss was solved for different values of angular velocity ω with the step $\Delta\omega = 25$ s^{-1}. In the vicinity of the critical velocity, the increment $\Delta\omega$ was diminished till 1 s^{-1}. In doing so, the segment $0 \leqslant x^1 \leqslant l$ was divided into 3200 integration steps and 80 points of orthogonalization were preset.

As an alternative, the stability of the elongated spinning cylindrical shell was simulated with the use of a beam theory. It was considered that an equivalent tube beam was rotating around the OZ-axis with the angular velocity ω. Its equilibrium in a perturbed state is described by uncoupled ordinary differential equations

$$EI\mathrm{d}^4u/\mathrm{d}z^4 = q_x, \quad EI\mathrm{d}^4v/\mathrm{d}z^4 = q_y, \tag{23}$$

where u, v are the beam displacements along the axes Ox and Oy, correspondingly, z the longitudinal co-ordinate, EI the bending stiffness; and q_x, q_y the intensities of external loads.

For the considered disturbed case, one has

$$q_x = \gamma F\omega^2 u, \quad q_y = \gamma F\omega^2 v, \tag{24}$$

where γ is the beam material density and F its cross-sectional area.

Substituting Eq. (24) into Eq. (23), one gains homogeneous equations of equilibrium stability of the spinning tube

$$EI\mathrm{d}^4u/\mathrm{d}z^4 - \gamma F\omega^2 u = 0, \quad EI\mathrm{d}^4v/\mathrm{d}z^4 - \gamma F\omega^2 v = 0. \tag{25}$$

The values ω_{cr} for which Eqs. (25) have non-trivial solutions are critical and the corresponding non-trivial solutions represent the modes of stability loss. They were found via solving Eqs. (25) with prescribed boundary conditions using the transfer matrix method. In doing so, the particular solutions were constructed by joint application of the Runge–Kutta method and the method of orthogonalization.

Emphasizing that these equations coincide in form with the characteristic equations of free vibrations of non-rotating beams and the critical value ω_{cr} of the velocity, ω is equal to the first frequency of free vibrations which turns out to be multiple. If the beam spins, its free vibrations in the two mutually perpendicular planes proved to be coupled through gyroscopic interaction. In this connection, the equations of free vibrations take the form

$$EI\partial^4 u/\partial z^4 + \gamma F(-\omega^2 u - 2\omega\dot{v} + \partial^2 u/\partial t^2) = 0,$$
$$EI\partial^4 v/\partial z^4 + \gamma F(-\omega^2 v + 2\omega\dot{u} + \partial^2 v/\partial t^2) = 0. \quad (26)$$

This system's solutions testify that the vibration mode of the rotating beam can be precessive only and, generally, the frequencies of precession vibrations do not coincide with the multiple frequencies of a non-rotating beam but are the result of its splitting into two different values, one of which is equal to the angular velocity of precession to the direction of rotation and the other corresponds to the opposite direction.

The results of calculations of critical velocities for the cylindrical shells and equivalent tubular beams are listed in Table 1.

Close agreement between the critical velocities of the thin-walled rotating tubes found through the use of shell theory and beam theory is a testimony of plausibility of the applied approach.

For testing the technique of investigation of the shell precession vibrations in compound rotation, a gyroscopic moment $\mathbf{M}_g$ acting on an inertially equivalent rigid body was also calculated.

It is an integral measure of a mechanical system dynamical behaviour in compound rotation. Its display for a rotating axisymmetrical solid body whose axis performs additional compulsory slewing consists in generation of the body support reactions making a force couple with the moment

$$\mathbf{M}_g = I_z\omega \times \omega_0. \quad (27)$$

Here I_z is the body inertia moment relative to the rotation axis.

At the same time, the elastic vibrations of a real thin-walled rotor excited by compound rotation are accompanied by the generation of a system of distributed edge elastic bending

Table 1
Critical values of angular velocity for cylindrical shell and tubular beams

Types of boundary conditions	Length l (m)	The first critical angular velocity, ω_{cr} (rad s^{-1})	
		Cylindric shell	Tubular beam
End $x = 0$ is clamped,	0.5	1287	1290
end $x = \ell$ is free	1.0	329	323
	2.0	82	81
End $x = 0$ is clamped,	0.5	7943	8292
end $x = \ell$ is clamped	1.0	1986	2072
	2.0	511	518

moments, torques, longitudinal and shear forces applied to the solid spinning carrier and producing a total resultant elastic moment $\mathbf{M}_e$. It is useful to compare the moments $\mathbf{M}_g$ and $\mathbf{M}_e$ which should approximately coincide in the preresonant zones of the ω value varying.

7. Investigation results

The outlined techniques were used for investigation of critical states of axisymmetrical ellipsoidal shells in simple and compound rotations. The equations of midsurface geometry of the shells were prescribed in form (15). Based on these correlations, geometrical parameters (16) were computed.

The investigations were performed in the limits $0 \leqslant \omega \leqslant 2500\ \mathrm{s}^{-1}$, and the ω_0 value was assumed to be $1\ \mathrm{s}^{-1}$. The results of investigations are given in Table 2. Recall the designations used. Here a and b are the ellipsoid semi-axes along the axes Oz and Ox (Oy), respectively, cases $a > b$ and $a < b$ correspond to elongated and flattened ellipsoids, if $a = b$ the ellipsoid is spherical; l is the distance between the planes of the ellipsoid circular contours; R is the radius of the shell free contour; ω_{cr} is the critical velocity value for the shell buckling in simple rotation; ω_{res} is the velocity value of the shell resonant precession in compound rotation. In all the cases, the diameter of the shell clamping at the left end equals $d = 0.008$ m and the shell thickness $h = 0.002$ m.

It can be seen that the type of the shell critical state set on originally in simple or compound rotations depends on the ratio between diameter $2R$ and distance l defining its dimensions. Thus, the static stability loss of simple rotation precedes the resonant precession for rather elongated ellipsoids without larger ratios $2R/l$ (cases 3–9 in Table 2, $2R/l \leqslant 2.17$) and the resonant vibration

Table 2
Critical and resonant values of angular velocity for ellipsoid shells

Case number	Type of shell shape	a (m)	b (m)	R (m)	l (m)	$2R/l$	ω_{cr} (rad s^{-1})	ω_{res} (rad s^{-1})
1	Elongated ellipsoids ($a > b$)	0.4	0.2	0.132	0.10	2.64	2533	1147
2		0.4	0.2	0.143	0.12	2.38	1043	925
3		0.4	0.2	0.152	0.14	2.17	666	776
4		0.4	0.2	0.160	0.16	2.00	483	673
5		0.4	0.2	0.167	0.18	1.86	375	596
6		0.4	0.2	0.173	0.20	1.73	303	538
7		0.4	0.2	0.199	0.40	0.995	89	358
8		0.4	0.2	0.173	0.60	0.58	49	—
9		0.4	0.2	0.132	0.70	0.38	40	—
10	Hemisphere ($a = b$)	0.4	0.4	0.4	0.39996	2.00	95	94
11		0.2	0.2	0.2	0.19998	2.00	415	423
12	Flattened ellipsoids ($a < b$)	0.2	0.4	0.265	0.05	10.6	—	627
13		0.2	0.4	0.346	0.10	6.92	—	242
14		0.2	0.4	0.399	0.20	3.99	—	134

state in compound rotation precedes the critical static state in simple rotation for the elongated and flattened shells with ratios $2R/l \geqslant 2.38$ (cases 1, 2, 12–14). For the hemispherical shells, the critical states of spinning and resonant regimes of compound rotation preceed nearly simultaneously (cases 10 and 11). Note that in cases 8 and 9 the resonant states were not achieved at all and in cases 12–14 the shells do not buckle statically in the adopted diapason of the angular velocity ω varying. Note, that during calculation, it was difficult to attain convergence of numerical results. One of the reasons for this feature is connected with the rigidity of the constituent differential equations caused by availability of large multipliers ω^2 and small thickness h in numerators and denominators of their coefficients. The second reason is fast varying of the functions of physical components of internal forces $T_{(13)}(x^1)$, $T_{(11)}(x^1)$ in simple spinning, which are constructed at investigation of simple rotation buckling and, besides, are included into coefficients of linearized equations (11) and (14). To overcome the numerical difficulties at application of the Runge–Kutta method for determination of the particular solutions, the number of segments Δx^1, to which the diapason $0 \leqslant x^1 \leqslant l$ was divided, was assumed to be 64,000 and the number of orthogonalization points was selected to equal 80.

Convergence of numerical calculations also depends on the fields of distribution of internal forces in the shell prestressed by simple rotation. As the fields have zones with high gradients of these forces, linearized differential equations (14) have coefficients changing rapidly in the x^1 co-ordinate. Fig. 2 shows the functions of physical components $T_{(11)}$, $T_{(13)}$ in the generatrix section at the precritical state of simple rotation ($\omega = 1147$ rad/s) for case 1 in Table 2. They possess sharp inclinations in boundary segments of the section, which contribute to deterioration of calculation convergence.

Fig. 3a illustrates the mode of shell buckling at simple spinning (bold lines) in plane yOz of the rotating co-ordinate system $Oxyz$ for case 9 in Table 2. For the selected orientation of the reference frame, the buckling shell displaces symmetrically relative to plane yOz, experiencing the largest deflections in this plane.

The mode of the shell generatrix precession vibrations in compound rotation has a similar shape, with one difference that it is stationary in the slewing co-ordinate system $OXYZ$ (Fig. 3b for case 14 in Table 2). The mode shape is symmetrical relative to plane YOZ.

Fig. 4 shows the typical modes of the free boundary circle buckling relative to different co-ordinate systems. In co-ordinate system $Oxyz$, spinning together with the shell, the buckled

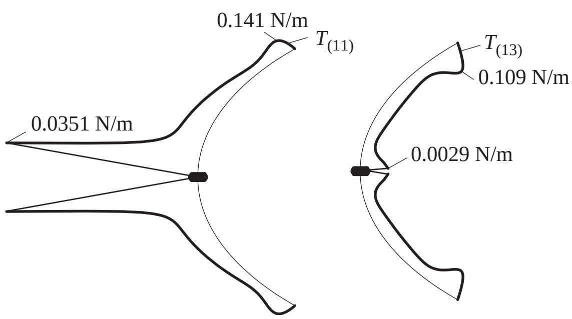

Fig. 2. Distribution of shell internal forces $T_{(11)}$ and $T_{(13)}$ at a precritical state of simple rotation.

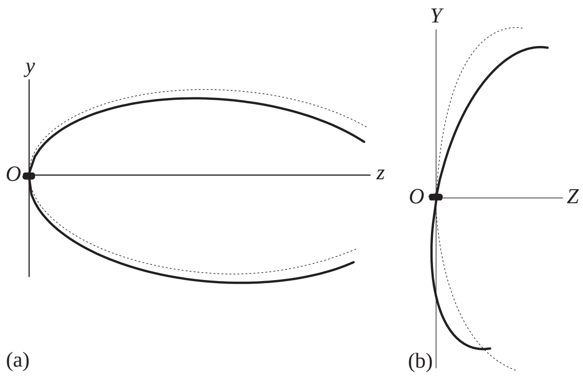

Fig. 3. Modes of shell stability loss in simple spinning (a) and shell preresonant vibrations in compound rotation (b).

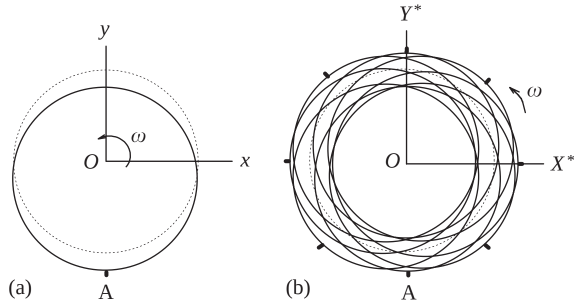

Fig. 4. Modes of shell buckling in spinning (a) and immovable (b) co-ordinate systems.

boundary circle shows up as an immovable displaced curve (Fig. 4a, the bold line), whereas in the inertial co-ordinate system $OX^*Y^*Z^*$, it is seen as a rotating displaced curve (Fig. 4b). The A label permits one to follow change of the section orientation relative to the chosen reference frame. Note that in both cases the A label is oriented radially.

The opposite pattern occurs for the mode of preresonant vibrations of the shells in compound rotation, because it has different manifestations in the spinning ($Oxyz$) and slewing ($OXYZ$) co-ordinate systems. Fig. 5a illustrates translational motion of the shell edge around the O centre relative to the $Oxyz$ co-ordinate system. In doing so, every point of the boundary circle describes a circular path clockwise with angular velocity ω similar to the closed curve swept through by the A label also clockwise, therewith its directions being in parallel (as it is called, the inverse regular precession). At the same time the $Oxyz$ co-ordinate system spins counterclockwise with respect to the $OXYZ$ co-ordinate system. So the resultant motion of the boundary line in the $OXYZ$ reference system is exhibited in the circle displaced by the distance $u_{(3)}(l)$ along the OY-axis and rotating around its own centre counterclockwise with the angular velocity ω (Fig. 5b). As this takes place, the A label is directed radially. The dashed circles in Figs. 4 and 5 define the contour lines of the shells in the undeformed states.

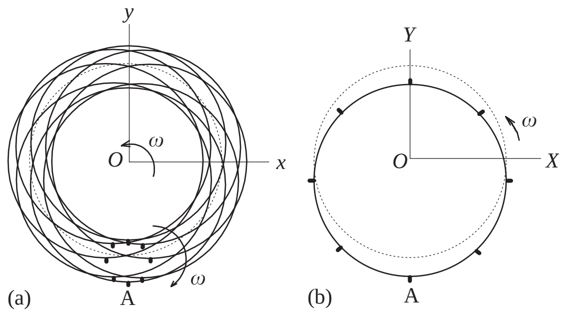

Fig. 5. Modes of shell precession vibrations in spinning (a) and slewing (b) co-ordinate systems.

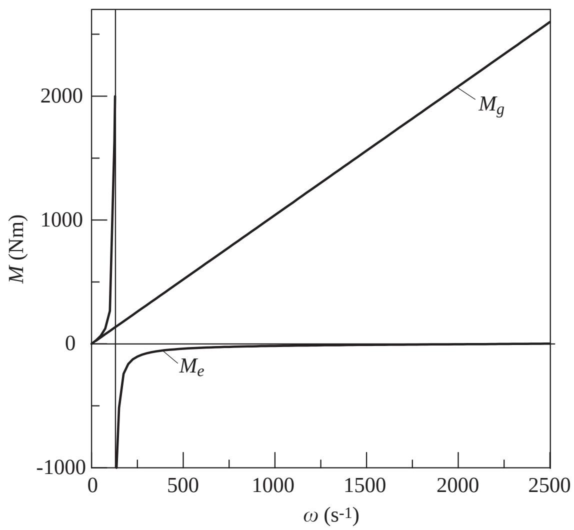

Fig. 6. Values of elastic (M_e) and gyroscopic (M_g) moments in the ellipsoid shell.

Fig. 6, which is typical for all the considered cases of compound rotation, shows the functions of moduli of the elastic (M_e) and gyroscopic (M_g) moments for case 14 in Table 2. For the M_e calculation the formula used was

$$M_e = \frac{\pi d}{2}\left(-T^*_{(13)}\frac{d}{2}\sin\alpha - T^*_{(11)}\frac{d}{2}\cos\alpha + M^*_{(11)}\right).$$

Here $T^*_{(13)}(0)$, $T^*_{(11)}(0)$, $M^*_{(11)}(0)$ are the amplitude values of physical components of the internal shear force, longitudinal force and bending moment at $x^1 = 0$; α is the angle between the Oz-axis and the tangent to the shell generatrix.

As $\omega \perp \omega_0$ in the considered cases, the equality $M_g = I_z\omega\omega_0$ is valid. The inertia moment I_z of ellipsoid shells relative to the Oz-axis is determined by

$$I_z = 2\pi h\gamma \int_0^1 f^3(z)\sqrt{1 + [f'(z)]^2}\,\mathrm{d}z.$$

Referring to Fig. 6, the moments moduli M_e and M_g practically coincide in the preresonant zone and differ essentially in the neighbourhood of the resonance and in the postresonant zones.

In closing, one can point out one further peculiarity of the dynamical behaviour of the studied elastic system. It is known [7,9] that rotation of an axisymmetric shell entails splitting its multiple frequencies and the emergence of frequency flairs in the vicinity of each of them. One out of the pair relates to the inverse regular precession, and the other is associated with the direct regular precession. Their values are moving apart with the ω enlargement. In the case of compound rotation, the inertia forces move opposite to the spinning direction generating the inverse regular precession, so the resonant vibrations, corresponding to the second frequency of the direct precession, cannot be realized.

The imposed mode of inverse precession in compound rotation also impedes the static stability loss in cases when the critical angular velocity ω_{cr} of simple spinning is lower than the resonant velocity ω_{res} of compound rotation.

8. Conclusions

The results of computer simulation of statics and dynamics of rotating ellipsoid shells make it possible to draw the following inferences:

1. The simple rotation of ellipsoid shells can be accompanied by their static buckling.
2. The compound rotation of ellipsoid shells is attended with their vibrations in the mode of inverse regular precession, which can acquire resonant character.
3. The static stability loss of simple rotation precedes the resonant precession for rather elongated ellipsoid shells and the resonance effect in compound rotation precedes the critical state in simple rotation for flattened shells.
4. The modes of deformation of the ellipsoid shells at critical states of simple and compound rotations have some similarities observed in rotating, slewing and immovable reference frames.

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