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Updating complex structures by a robust multilevel condensation approach

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Abstract

The method of updating proposed in this paper is based on condensed models and the frequency responses. A robust approach of condensation with two levels is presented. The first reduction basis denoted T_{01} reduces the size of the model to a few hundred of degrees of freedom. The vectors of the basis are sufficient to make the condensed model reliable and robust with respect to the perturbations of the model in the observed frequency band. The second reduction basis denoted T_{12} condenses the model to the measurement points. It is updated during the updating procedure. The formulation of the updating problem with residues on the inputs leads to a linear relation of the parameters to be identified. This method makes it possible to detect strong errors of mass and stiffness if the Ritz basist T_{01} is reliable. In practice, the reduced matrices evolve with the modifications of the model. One can update this basis during updating. A strategy of iterative regularization makes it possible to obtain the variations of the parameters. This method is validated on an industrial example. In order to test the robustness of this strategy, the quality of the readjusted model in comparison with the experimental results for various loading configurations of the structure is evaluated.
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1. Introduction

Finite element modeling is used to predict dynamic behaviour. Reliability is essential. Finite element (FE) simulations must be compared with direct measurements made on the body under study. Convergence of the calculated results with the experimental observations is achieved by

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correcting the mass, stiffness and damping matrices. These corrections are calculated with updating parameters using residues:

- residues on input [1];
- residues on output [2,3];
- residues on complex powers [4];
- residues based on eigensolutions [5];
- behaviour law error [6].

Applying forced responses makes it possible to account for the static contribution of high-frequency modes. The updated model should be usable in a static problem, but if updating is achieved solely with eigenmodes, the necessary static applications can be troublesome.

The interesting aspect is to compare structural behaviour at measured points with the corresponding behaviour at the same points in the condensed model. The degrees of freedom (d.o.f.) of the condensed model are physical points (sensors) on the structure.

This selection leads to a simple formulation of the updating problem applying residues on input giving a linear relationship with the parameters to identify. The method used enables updating strong mass and stiffness errors if the condensed model is reliable. In practice, the condensation matrices change as the model changes. It can be judicious to recalibrate them at the end of the updating process.

This leads to a two-level condensation:

- The first condensation T_{01}, reduces the size of the model to a few hundred d.o.f. The number of basis vectors of the T_{01} transformation is sufficient. The perturbed model, condensed by an initial transformation matrix T_{01}, should be reliable, accepting model perturbations in the frequency domain of the measurements.
- A second transformation, T_{12}, condenses the model obtained from the measured points. The T_{12} operation is recalibrated during the updating process. Recalibrating T_{01} may be necessary after updating if the distance between the initial model and the structure is considered to be too great.

2. Definition of the condensation bases

Updating procedures are based on iteration methods. For large-sized structures, large-dimension models cannot be used at each step due to the excessive calculation time.

One solution is to apply reanalysis techniques by reduced basis approach at each iteration.

2.1. Defining the model parameters

The first step in updating the model is the localization phase. This step defines the macro-elements of the model which contain the modelization errors. For each macroelement, the operator chooses the type or types of parameters P_i (Young modulus, volumetric mass, thickness, etc…) which are to be characterized.

The mass, damping and stiffness matrices of the macroelement number i are defined from the elementary matrices by

$$\mathbf{M}^{i}_{macro} = \sum_{e=1}^{nelem} \mathbf{M}^{elem}_{e}, \tag{1}$$

$$\mathbf{B}^{i}_{macro} = \sum_{e=1}^{nelem} \mathbf{B}^{elem}_{e}, \tag{2}$$

$$\mathbf{K}^{i}_{macro} = \sum_{e=1}^{nelem} \mathbf{K}^{elem}_{e}. \tag{3}$$

The impedance $\mathbf{L}^{i}_{macro}$ of macroelement number i is defined by

$$\mathbf{L}^{i}_{macro} = \mathbf{K}^{i}_{macro} + \mathrm{j}\omega\mathbf{B}^{i}_{macro} - \omega^{2}\mathbf{M}^{i}_{macro}. \tag{4}$$

In general, the updating parameters (P_i) intervene non-linearly in the correction matrices of the macroelements. The matrices can be factorized as

$$\mathbf{L}^{i}_{macro}(P_i) = \sum (P_i)^{\alpha}\mathbf{L}^{i\alpha}_{macro}. \tag{5}$$

A variation (ΔP_i) in parameter (P_i), gives a variation in impedance:

$$\Delta\mathbf{L}^{i}_{macro}(P_i) = \Delta P_i \sum \alpha (P_i)^{\alpha-1}\mathbf{L}^{i\alpha}_{macro}. \tag{6}$$

A dimensional coefficients of variation are used to avoid processing problems when searching for a parametric solution:

$$p_i = \frac{\Delta P_i}{P_i}, \tag{7}$$

so

$$\Delta\mathbf{L}^{i}_{macro}(p_i) = p_i \sum \alpha (P_i)^{\alpha}\mathbf{L}^{i\alpha}_{macro}. \tag{8}$$

An impedance correction (ΔL_0) of the model is defined by

$$\Delta\mathbf{L}_0 = \sum_{i=1}^{np} \Delta\mathbf{L}^{i}_{macro}. \tag{9}$$

2.2. Approached reanalysis on a reduced basis

The reduced basis $\mathbf{T}_{01}$ applied to perturbed problems must produce reliable condensed matrices.

Recalibration of the $\mathbf{T}_{01}$ condensation matrices is costly in time and space for large-sized models. The $\mathbf{T}_{01}$ condensation matrix evolves during the updating process with

- $\mathbf{T}_{01}$: the condensation matrix of the initial model ($\mathbf{M}_0$, $\mathbf{B}_0$, and $\mathbf{K}_0$).
- $\mathbf{T}_{01}+\Delta\mathbf{T}_{01}$: the condensation matrix of the perturbed model ($\mathbf{M}_0+\Delta\mathbf{M}_0$, $\mathbf{B}_0+\Delta\mathbf{B}_0$ and $\mathbf{K}_0+\Delta\mathbf{K}_0$).

The following approximations are made:

$$(\mathbf{T}_{01} + \Delta\mathbf{T}_{01})^t * (\mathbf{M}_0 + \Delta\mathbf{M}_0)(\mathbf{T}_{01} + \Delta\mathbf{T}_{01}) \approx \mathbf{T}_{01}^t * (\mathbf{M}_0 + \Delta\mathbf{M}_0) * \mathbf{T}_{01}, \quad (10)$$

$$(\mathbf{T}_{01} + \Delta\mathbf{T}_{01})^t * (\mathbf{B}_0 + \Delta\mathbf{B}_0)(\mathbf{T}_{01} + \Delta\mathbf{T}_{01}) \approx \mathbf{T}_{01}^t * (\mathbf{B}_0 + \Delta\mathbf{B}_0) * \mathbf{T}_{01}, \quad (11)$$

$$(\mathbf{T}_{01} + \Delta\mathbf{T}_{01})^t * (\mathbf{K}_0 + \Delta\mathbf{K}_0)(\mathbf{T}_{01} + \Delta\mathbf{T}_{01}) \approx \mathbf{T}_{01}^t * (\mathbf{K}_0 + \Delta\mathbf{K}_0) * \mathbf{T}_{01}, \quad (12)$$

and the following terms are neglected:

$$\Delta\mathbf{T}_{01}^t * (\mathbf{M}_0 + \Delta\mathbf{M}_0) * (\mathbf{T}_{01} + \Delta\mathbf{T}_{01}) + \mathbf{T}_{01}^t * (\mathbf{M}_0 + \Delta\mathbf{M}_0) * \Delta\mathbf{T}_{01} \approx 0, \quad (13)$$

$$\Delta\mathbf{T}_{01}^t * (\mathbf{B}_0 + \Delta\mathbf{B}_0) * (\mathbf{T}_{01} + \Delta\mathbf{T}_{01}) + \mathbf{T}_{01}^t * (\mathbf{B}_0 + \Delta\mathbf{B}_0) * \Delta\mathbf{T}_{01} \approx 0, \quad (14)$$

$$\Delta\mathbf{T}_{01}^t * (\mathbf{K}_0 + \Delta\mathbf{K}_0) * (\mathbf{T}_{01} + \Delta\mathbf{T}_{01}) + \mathbf{T}_{01}^t * (\mathbf{K}_0 + \Delta\mathbf{K}_0) * \Delta\mathbf{T}_{01} \approx 0. \quad (15)$$

The expected perturbations of the model induce deformations. The reduced basis $\mathbf{T}_{01}$ must be able to represent

- the movement of the initial system with impedance $\mathbf{L}_0$;
- the movement of the system submitted to the given parametric modifications $\Delta\mathbf{L}$.

The nature and the localization of these modifications are known, but the amplitudes of their variations are not.

The behaviour of the initial model is represented by

$$\mathbf{L}_0 * \mathbf{y}_0 = \mathbf{f}_0, \quad (16)$$

and the behaviour of the condensed initial model by

$$\mathbf{L}_1 * \mathbf{y}_1 = \mathbf{f}_1, \quad (17)$$

with

$$\mathbf{T}_{10} = \mathbf{T}_{01}^t, \quad (18)$$

$$\mathbf{L}_1 = \mathbf{T}_{10} * \mathbf{L}_0 * \mathbf{T}_{01}, \quad (19)$$

$$\mathbf{y}_0 = \mathbf{T}_{01} * \mathbf{y}_1, \quad (20)$$

$$\mathbf{f}_1 = \mathbf{T}_{10} * \mathbf{f}_0. \quad (21)$$

Reanalysis of the modified system at step i of the parameter correction process is achieved on the reduced modified system:

$$\mathbf{L}_1^i = \mathbf{L}_1^{i-1} + \Delta\mathbf{L}_1^{i-1}, \quad (22)$$

where $\Delta\mathbf{L}_0^{i-1}$ is the impedance correction issuing from the updating step $(i-1)$ and $\Delta\mathbf{L}_1^{i-1} = \mathbf{T}_{10}\Delta\mathbf{L}_0^{i-1}\mathbf{T}_{01}$.

2.3. Adaptation of the condensation basis to changes

The behaviour of the modified model is represented by

$$[\mathbf{L}(\omega) + \Delta\mathbf{L}(\omega)]\mathbf{y}(\omega) = \mathbf{f}_e(\omega), \tag{23}$$

or by

$$\mathbf{L}(\omega)\mathbf{y}(\omega) = \mathbf{f}_e(\omega) - \Delta\mathbf{L}(\omega)\mathbf{y}(\omega), \tag{24}$$

this means that the modified model can be considered as an initial model loaded by error forces $\Delta\mathbf{L}(\omega)\mathbf{y}(\omega)$.

In the expression of these forces, $\mathbf{y}(\omega)$ is not known. It is assumed that the displacement vector $\mathbf{y}(\omega)$ is decomposed on the truncated basis of the eigenmodes $\mathbf{Y}_{md}$ enriched by the static residues $\mathbf{Y}_{rs}$ [7,8].

This gives

$$\Delta\mathbf{L}(\omega)\mathbf{y}(\omega) = \Delta\mathbf{L}(\omega)[\mathbf{Y}_{md}\mathbf{Y}_{rs}]c(\omega) = \sum_{i=1}^{np} \Delta\mathbf{L}^{i}_{macro}(\omega)[\mathbf{Y}_{md}\mathbf{Y}_{rs}]c(\omega), \tag{25}$$

and by deduction, the error forces $\mathbf{F}$ can be represented together as

$$\mathbf{F} = \lfloor\ \Delta\mathbf{L}^{1}_{macro}[\mathbf{Y}_{md}\mathbf{Y}_{rs}] \quad \Delta\mathbf{L}^{2}_{macro}[\mathbf{Y}_{md}\mathbf{Y}_{rs}] \quad \ldots \quad \ldots \quad \Delta\mathbf{L}^{i}_{macro}[\mathbf{Y}_{md}\mathbf{Y}_{rs}]\ \rfloor, \tag{26}$$

or as

$$\mathbf{F} = \lfloor\ \Delta\mathbf{K}^{1}_{macro}[\mathbf{Y}_{md}\mathbf{Y}_{rs}] \quad \Delta\mathbf{M}^{1}_{macro}[\mathbf{Y}_{md}\mathbf{Y}_{rs}] \quad \ldots \quad \Delta\mathbf{K}^{i}_{macro}[\mathbf{Y}_{md}\mathbf{Y}_{rs}] \quad \Delta\mathbf{M}^{i}_{macro}[\mathbf{Y}_{md}\mathbf{Y}_{rs}] \quad \ldots\ \rfloor. \tag{27}$$

This matrix $\mathbf{F}$ is decomposed into singular values to extract the independent vectors $\mathbf{F}_{fe}(\mathbf{F}_{fe} = DVS(\mathbf{F}))$ representing all possible error forces.

The condensation basis $\mathbf{T}_{01}$ is formed by uniting

- a modal sub-basis $\mathbf{Y}_{md}$;
- a static residues basis $\mathbf{Y}_{rs}$ able to represent the static behaviour of the structure:

$$\mathbf{Y}_{rs} = \mathbf{K}_0^{-1}\mathbf{f}_e. \tag{28}$$

- a static residues basis $\mathbf{Y}_{fe}$ able to approximate the dynamic behaviour of the structure resulting from changes in the parameters:

$$\mathbf{Y}_{fe} = \mathbf{Y}_{md}\mathbf{\Lambda}_{md}^{-1}\mathbf{Y}^{t}_{md}\mathbf{F}_{fe}. \tag{29}$$

This gives

$$\mathbf{T}_{01} = [\,\mathbf{Y}_{md} \quad \mathbf{Y}_{rs} \quad \mathbf{Y}_{fe}\,]. \tag{30}$$

2.4. Multiple reductions to achieve updating

The updating process requires an adaptation of the size of the model to the size of the data observed on the structure.

Two steps are necessary for large-sized models.

The first step is condensation reduction to obtain a reduced model with a level 1 impedance $\mathbf{L}_1$ with a few hundred d.o.f. on mixed co-ordinates

$$\mathbf{L}_1 = \mathbf{T}_{10}\mathbf{L}_0\mathbf{T}_{01}. \tag{31}$$

The level 1 reduction basis $\mathbf{T}_{10}$ can represent the displacements of the corrected mode authorizing recalibration of the model at this level of reduction. The level 1 model is recalibrated and reanalyzed at each iteration of the process. The eigenmodes and the static residues obtained are used to construct the new level 2 reduction basis $\mathbf{T}_{12}$.

The second reduction produces a reduced level 2 model with d.o.f. based on physical co-ordinates:

$$\mathbf{L}_2 = \mathbf{T}_{21}\mathbf{L}_1\mathbf{T}_{12} = \mathbf{T}_{21}\mathbf{T}_{10}\mathbf{L}_0\mathbf{T}_{01}\mathbf{T}_{12}, \tag{32}$$

the dimension of the condensation basis $\mathbf{T}_{12}$ depends on the number of sensors placed on the structure. In general, there are fewer sensors than static residues necessary to obtain a robust condensation basis. This necessitates a recalibration of the reduction basis $\mathbf{T}_{12}$ at each iteration of the updating process.

3. Updating by condensation

3.1. Presentation

The method consists of updating using residues on input in the following manner:

- the transfer functions measured by "c" sensors placed on the structure are used for updating;
- the finite element model of the structure is condensed on c d.o.f. corresponding to the sensors' d.o.f.;
- the dampings introduced into the model are modal dampings;
- in the procedure used, condensation is achieved at two levels:
  - Level 1: by a condensation matrix $\mathbf{T}_{10}$ bringing the initial model (size of approximately 100 000 d.o.f.) down to a intermediate model (size of approximately 1000 d.o.f.).
  - Level 2: by a transformation $\mathbf{T}_{21}$ which brings the intermediate model down to final model (size approximately 20–100 d.o.f.: this is the number of sensors on the structure).
- During updating, the level 2 condensed model is recalibrated from the level 1 condensed model considered to be reliable and robust. Here the robustness is defined by

  The aptitude of the condensed model to the level 1 to represent parametric modifications of the initial model not condensed.

- A $\mathbf{T}_{10}$ recalibration may be necessary at the end of the updating process;
- Use of residues on input has a dual interest:
  - The problem to be solved is linear in relation to the local stiffness, mass, or damping perturbations.
  - The transfer functions can be used without identifying modes, so errors introduced by the usual modal extraction procedures can be eliminated. In this application, the measures used resulted from a set of recordings made over several days which necessarily contained

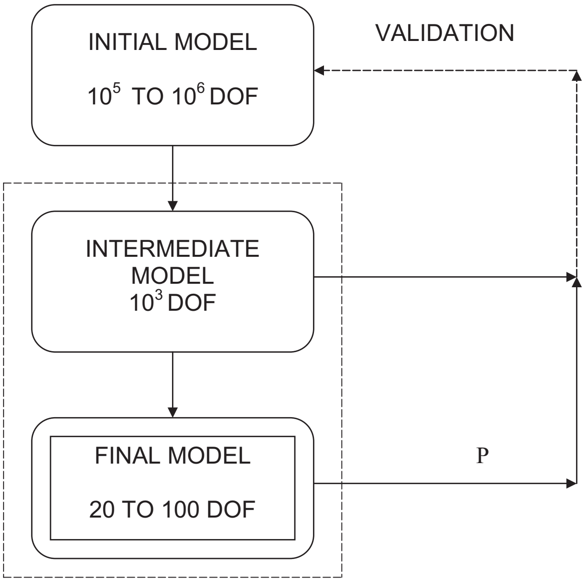

Fig. 1. The totality of the method.

eigenfrequency variations between the different measurements. The transfer functions used were reconstituted from the modal identification.

- The drawback of this updating method is that it is sensitive to measurement errors. This sensitivity is directly related to the processing of the matrix to invert. This processing depends on the position of the sensors and the unknown errors, and as such it cannot be predicted from a raw model, so it may be necessary to replace the sensors after the first attempt at updating. To avoid this problem, an overly abundant number of sensors are used so that the sensors that produce a good processing of the matrix to invert can be selected out. The totality of the method is illustrated by Fig. 1.

3.2. Method principle

There is a condensed model with c physical d.o.f.:

$$\mathbf{L}_c * \mathbf{y}^{(m)} = \mathbf{f}^{(m)}. \tag{33}$$

For a given excitation, the movement recorded on the structure can be represented by

$$(\mathbf{L}_c + \Delta\mathbf{L})\mathbf{y}^{(s)} = \mathbf{f}^{(s)}, \tag{34}$$

so that if

$$\mathbf{f}^{(s)} = \mathbf{f}^{(m)}, \tag{35}$$

then

$$\Delta\mathbf{L} * \mathbf{y}^{(s)} = \mathbf{L}_c(\mathbf{y}^{(m)} - \mathbf{y}^{(s)}), \tag{36}$$

if the modifiable zones are defined beforehand so

$$\Delta\mathbf{L} = \sum_{i=1} p_i \Delta\mathbf{L}_i, \tag{37}$$

the problem is linear

$$\mathbf{S}*\mathbf{p} = \mathbf{L}_c(\mathbf{y}^{(m)} - \mathbf{y}^{(s)}). \tag{38}$$

To limit amplifying measurement errors in the product $\mathbf{L}_c(\mathbf{y}^{(m)} - \mathbf{y}^{(s)})$, the problem is best presented as

$$\mathbf{A}*\mathbf{p} = \mathbf{y}^{(m)} - \mathbf{y}^{(s)}. \tag{39}$$

3.3. Introduction of damping parameters

Damping is not introduced into the finite element model.

In a body, the principal sources of damping are localized at the joints between components, and it is difficult to have a linear damping model in the joints.

The simplest solution is to introduce modal dampings into the model using the generalized damping matrix ß diagonal

$$\boldsymbol{\beta} = \mathbf{Y}_m^t \mathbf{B}_m \mathbf{Y}_m \Rightarrow \mathbf{B}_m = \mathbf{Y}_m^{t(-1)} \boldsymbol{\beta} \mathbf{Y}_m^{(-1)} = \mathbf{M}_m \mathbf{Y}_m \boldsymbol{\beta} \mathbf{Y}_m^t \mathbf{M}_m, \tag{40}$$

where $\mathbf{Y}_m$ is the truncated modal matrix of the conservative model.

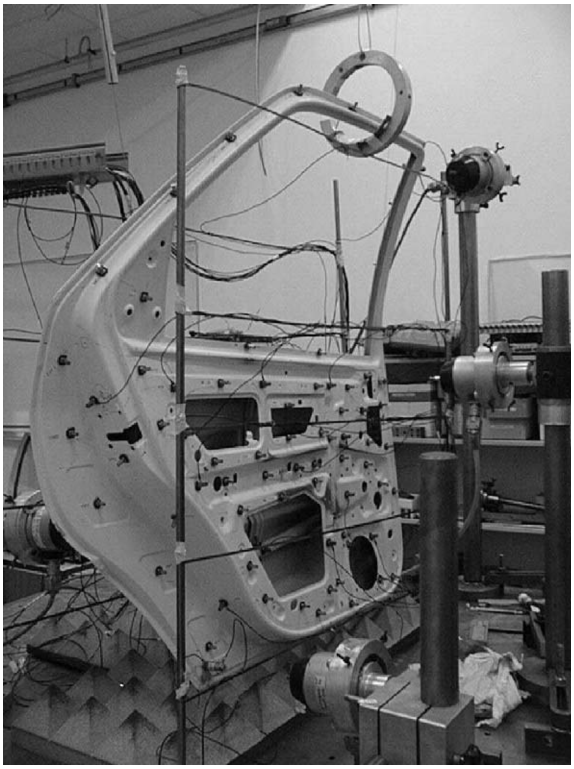

Fig. 2. Experimentation with a free–free door without loading.

It contains the eigenmodes present in the frequency band of the measurements. The modes are normalized in relation to the mass matrix

$$\mathbf{Y}_m^t \mathbf{M}_m \mathbf{Y}_m = \mathbf{I}_n \quad \text{with } n < c. \tag{41}$$

The structure's damping matrix is thus given in the form

$$\mathbf{B}_s = \sum_i \boldsymbol{\beta}_i \mathbf{M}_m \mathbf{Y}_{mi} \mathbf{Y}_{mi}^t \mathbf{M}_m, \tag{42}$$

then condensed by transformations $\mathbf{T}_{10}$ and $\mathbf{T}_{21}$.

The $\boldsymbol{\beta}_i$ are the damping parameters.

For neighbouring frequencies, extradiagonal terms can be introduced into the generalized damping matrix.

3.4. Development of the optimization problem

Assuming that the forces applied to the structure during the trial are observed correctly, i.e. such that: $\mathbf{f}(s) = \mathbf{f}(m)$, we have

$$(\Delta \mathbf{K}_c + \mathrm{j}\omega \Delta B_c - \omega^2 \Delta M_c) y_s \approx (K_c^{(m)} + \mathrm{j}\omega B_c^{(m)} - \omega^2 M_c^{(m)})(y^{(m)} - y^{(s)}), \tag{43}$$

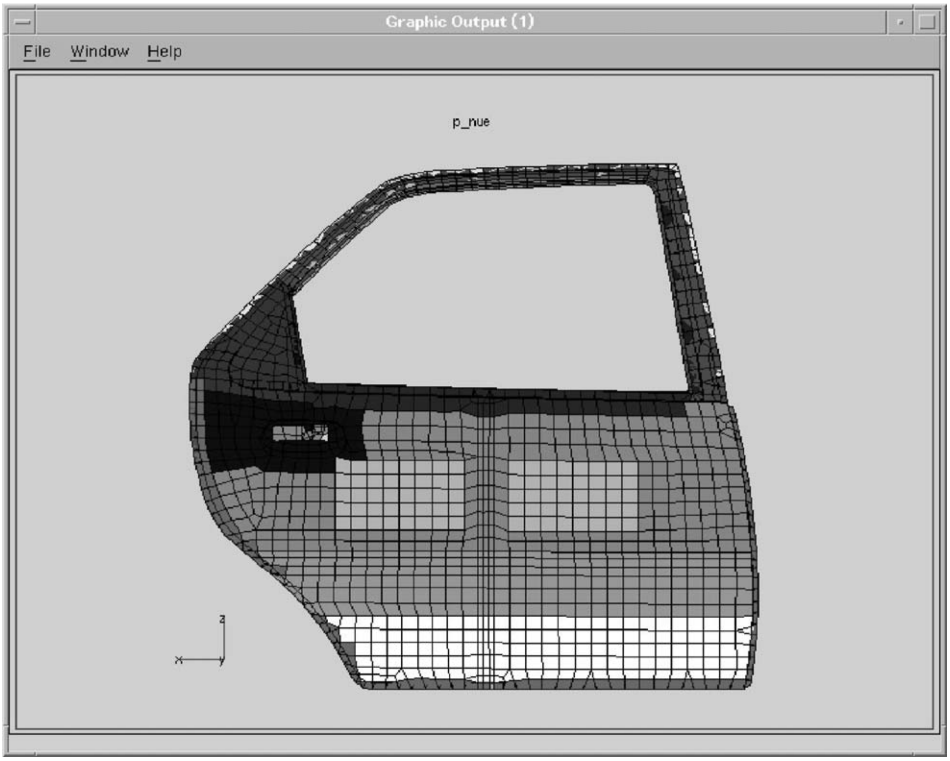

Fig. 3. Definition of the zones of the updating parameters on the outer side of the door.

or

$$\Delta \mathbf{Z}_c * \mathbf{y}_s \cong \mathbf{Z}_c^{(m)}(\mathbf{y}^{(m)} - \mathbf{y}^{(s)}), \tag{44}$$

with $\Delta \mathbf{Z}_c$ the global correction matrix at frequency ω condensed twice, $\mathbf{Z}_c^{(m)}$ the dynamic stiffness matrix at frequency ω condensed twice:

$$\Delta \mathbf{Z}_c = \sum_{i=1}^{q_k} k_i \mathbf{K}_{ic}^{(m)} + \mathrm{j}\omega \sum_{l=1}^{q_b} b_l \mathbf{B}_{lc}^{(m)} - \omega^2 \sum_{j=1}^{q_m} m_j \mathbf{M}_{jc}^{(m)}, \tag{45}$$

with

$$\mathbf{K}_{ic}^{(m)} = \mathbf{T}_{21}\mathbf{T}_{10}\mathbf{K}_i^{(m)}\mathbf{T}_{01}\mathbf{T}_{12}, \tag{46}$$

$$\mathbf{B}_{lc}^{(m)} = \mathbf{T}_{21}\mathbf{T}_{10}\mathbf{B}_l^{(m)}\mathbf{T}_{01}\mathbf{T}_{12}, \tag{47}$$

$$\mathbf{M}_{jc}^{(m)} = \mathbf{T}_{21}\mathbf{T}_{10}\mathbf{M}_j^{(m)}\mathbf{T}_{01}\mathbf{T}_{12}. \tag{48}$$

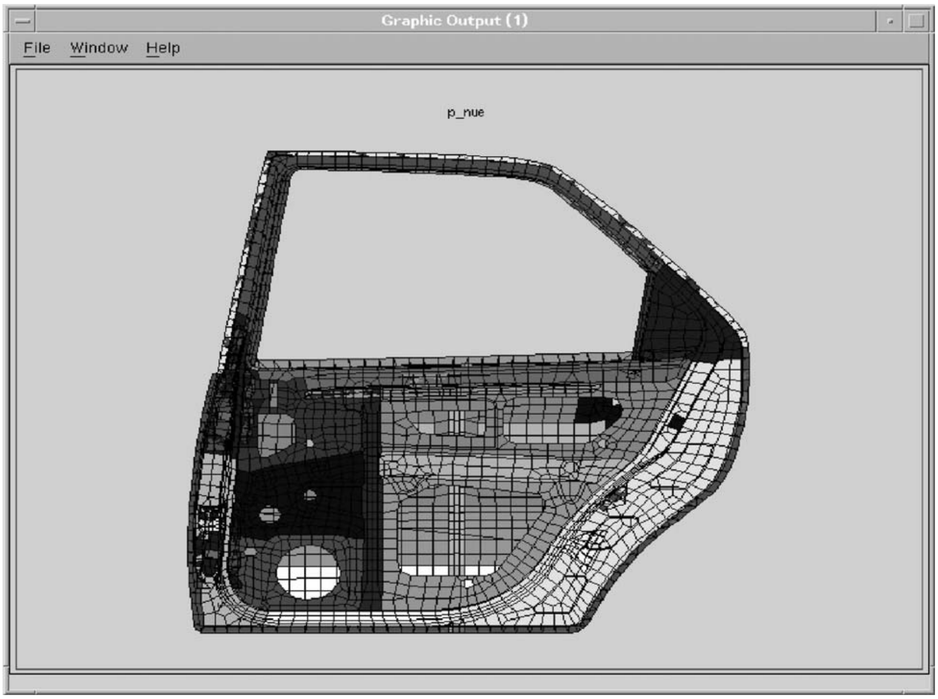

Fig. 4. Definition of the zones of the updating parameters on the inner side of the door.

Grouping the equations written by all the excitation frequencies ω, gives

$$\begin{bmatrix} \cdots & \mathbf{K}_{ic}^{(m)}(\omega_1)\mathbf{y}_1^{(s)} & \cdots & -\omega_1^2\mathbf{M}_{jc}^{(m)}(\omega_1)\mathbf{y}_1^{(s)} & \cdots \\ & \vdots & & \vdots & \\ \cdots & \mathbf{K}_{ic}^{(m)}(\omega_h)\mathbf{y}_h^{(s)} & \cdots & -\omega_h^2\mathbf{M}_{jc}^{(m)}(\omega_h)\mathbf{y}_h^{(s)} & \cdots \\ & \vdots & & \vdots & \\ \cdots & \mathbf{K}_{ic}^{(m)}(\omega_{ne})\mathbf{y}_{ne}^{(s)} & \cdots & -\omega_{ne}^2\mathbf{M}_{jc}^{(m)}(\omega_{ne})\mathbf{y}_{ne}^{(s)} & \cdots \end{bmatrix} \begin{bmatrix} \vdots \\ k_i \\ \vdots \\ b_l \\ \vdots \\ m_j \end{bmatrix} \approx \begin{bmatrix} \mathbf{Z}_c^{(m)}(\omega_1)(\mathbf{y}_1^{(m)} - \mathbf{y}_1^{(s)}) \\ \vdots \\ \mathbf{Z}_c^{(m)}(\omega_h)(\mathbf{y}_h^{(m)} - \mathbf{y}_h^{(s)}) \\ \vdots \\ \mathbf{Z}_c^{(m)}(\omega_{ne})(\mathbf{y}_{ne}^{(m)} - \mathbf{y}_{ne}^{(s)}) \end{bmatrix}, \quad (49)$$

with

$$\mathbf{y}_h^{(s)} = \mathbf{y}^{(s)}(\omega_h), \quad (50)$$

$$\mathbf{y}_h^{(m)} = \mathbf{y}^{(m)}(\omega_h), \quad (51)$$

or in a modified and simplified form:

$$\mathbf{S}*\Delta\mathbf{p} \approx \mathbf{b}, \quad (52)$$

where b is the vector constructed by the subvectors $[\mathbf{y}^{(m)} - \mathbf{y}^{(s)}]$.

An updating iteration is performed due to the approximations introduced by the two successive condensations.

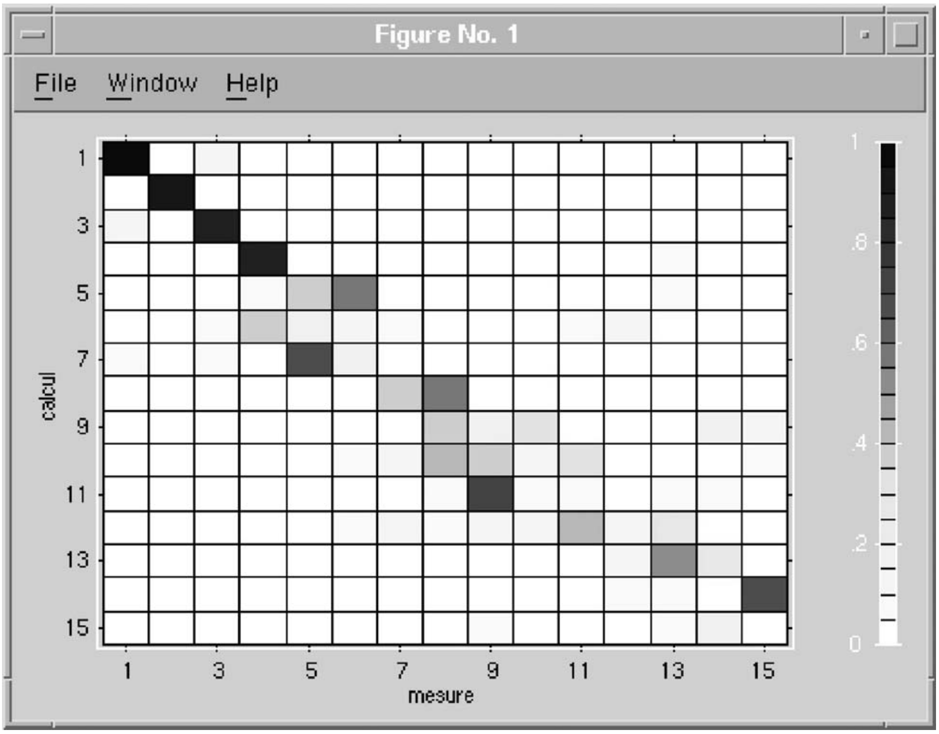

Fig. 5. Mac matrix between identified and calculated eigenvectors (without loading).

4. Results and conclusions

To access the validity of a model updating process, the effect of the process has to be assessed for different boundary conditions.

To do this, we used the experimental results obtained with a free-free configuration of an unloaded door was used to identify the zones and values of the parameters to update.

This first experimental phase is presented in Fig. 2.

The following tasks were performed on the finite element model to find the values of the parameters associated with each zone:

- Definition of the zones and the parameters used for updating.
- Condensation of the initial models of the free–free door without loading. The size of the intermediate model is 320 d.o.f. and the final model is 80 d.o.f.
- Determination of the variations in the parameters, solely using data coming from the free–free door without loading.

The second phase consisted in calculations on the finite element model as presented in Figs. 3 and 4.

The updated model was validated as follows

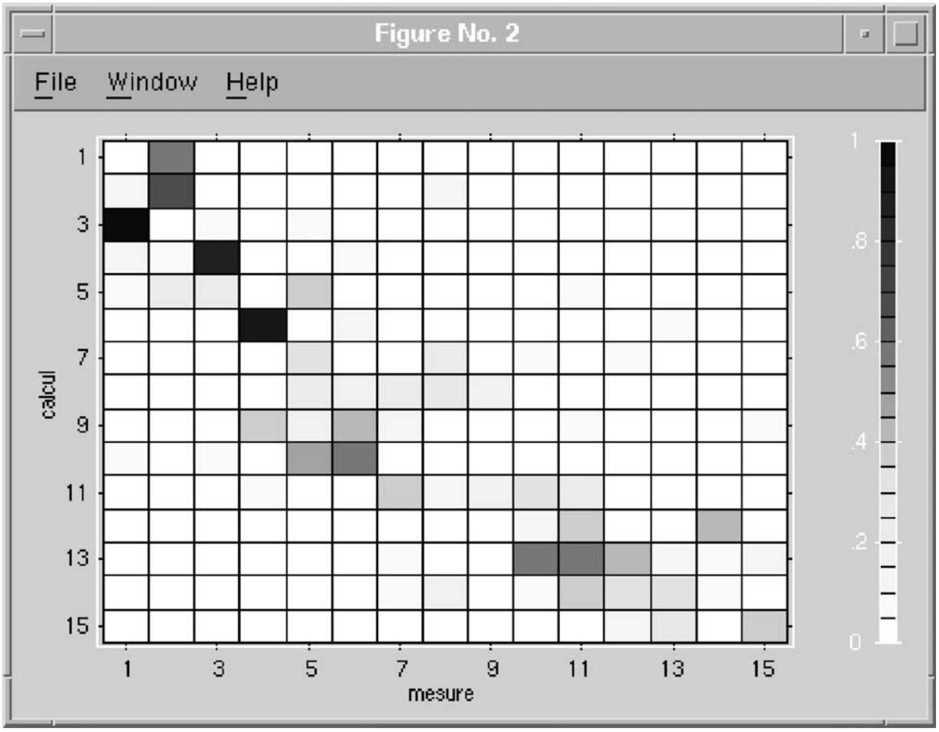

Fig. 6. Mac matrix between identified and calculated eigenvectors (with loading).

- The updating parameters found in the complete free–free door model without loading were introduced and compared with the measured experimentation data obtained under the same conditions.
- The boundary conditions were changed by loading the anchorage points on the door. The initial model of the door after loading was compared with the experimental data and with the updated model using the parameters obtained in the free–free model without loading.

The Mac matrix between the first fifteen modes identified and calculated from the initial finite elements model for the free–free door without loading is presented in Fig. 5.

The Mac matrix between the first 15 modes identified and calculated from the initial finite elements model for the free–free door with loading is presented in Fig. 6.

The Mac matrix between the first 15 modes identified and calculated from the complete updated model for the free–free door without loading is presented in Fig. 7.

The Mac matrix between the first 15 modes identified and calculated from the complete updated model for the free–free door with loading is presented in Fig. 8. These modes were obtained by introducing the values of the parameters identified in the free-free configuration without loading for each zone.

In conclusion, an acceptable model of the door was obtained in the free–free configuration valid for the first 10 modes without loading and for the first 8 modes with loading. The basis used for

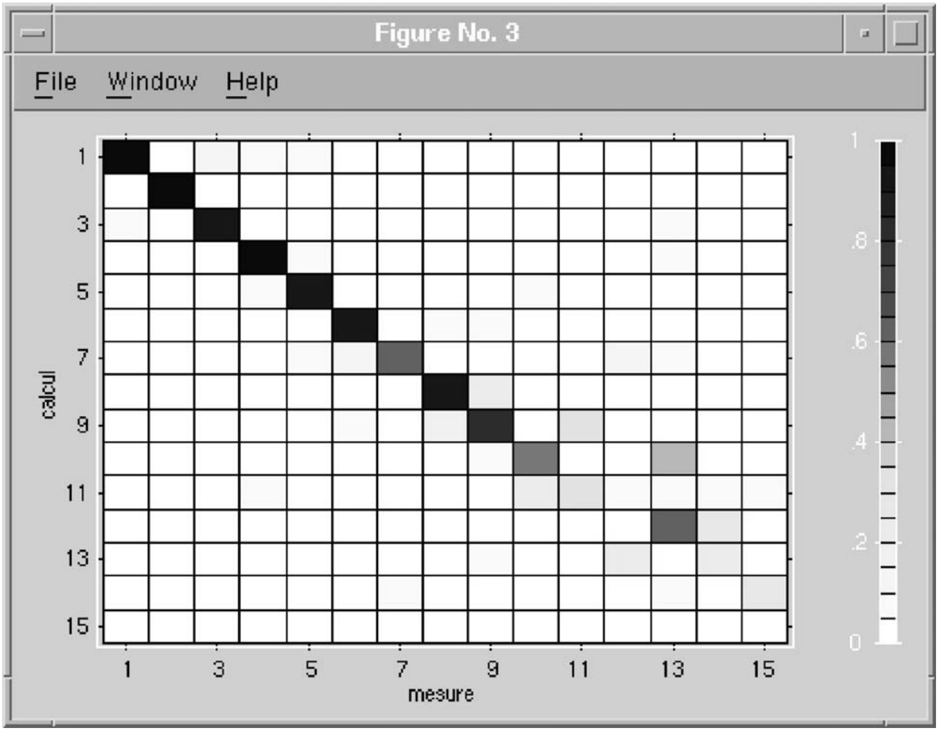

Fig. 7. Mac matrix between identified and calculated eigenvectors for the door without loading using the model from step no. 1.

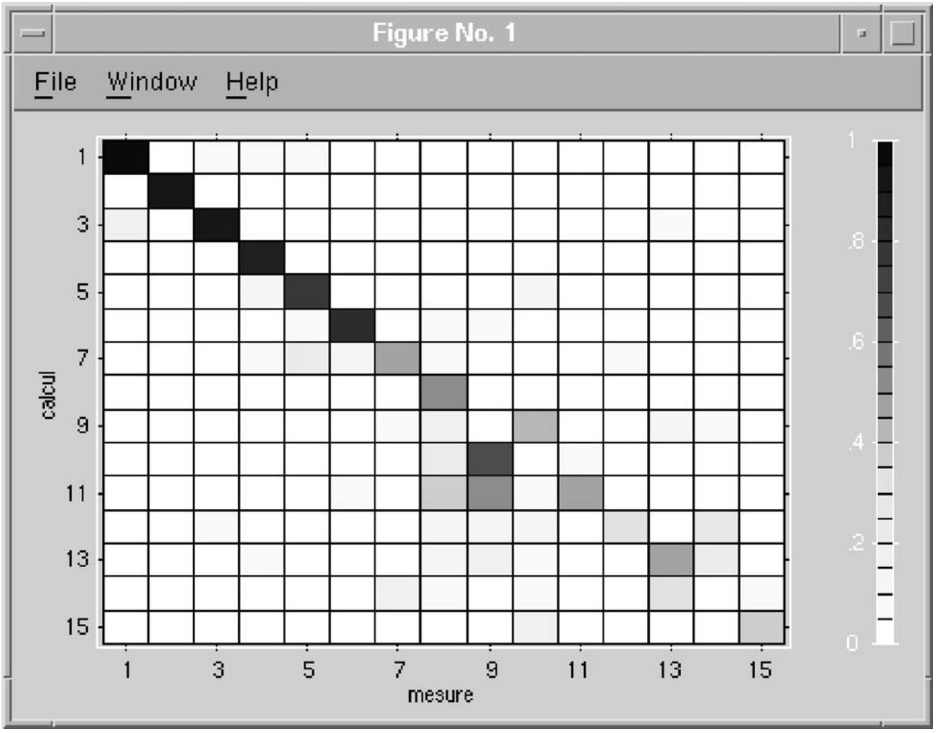

Fig. 8. Mac matrix between identified and recalculated modes using the complete model in the free–free configuration with loading.

the reanalysis of the modified structural behaviour may be composed of a variety of displacement fields, including free interface, blocked or mixed modes of the nominal model. Furthermore, this approach is particularly valuable in the context of stochastic updating and optimization procedures where the cost of an exact analysis is prohibitive.

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