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Letter to the Editor

Dynamic response of moderately thick composite plates

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1. Introduction

One of the major concerns in designing composite plates is their susceptibility to dynamic loads. These dynamic loads can cause severe damage in composite plates, such as matrix cracking, fibre cracking and delaminating. An understanding of the behaviour of composite plates under dynamic loading is, therefore, essential.

Ghosh and Dey [1,2] have presented a formulation based on higher-order shear deformation theory using 4-noded element for the static and free vibration analysis of laminated composite plates. A formulation based on higher-order shear deformation theory using 9-noded Lagrangean element is given by Kant et al. [3,4], wherein the effects of consistent diagonal mass matrix, time step and finite element mesh are investigated and the results for multi-layer unsymmetric laminates are presented. Transient analysis of layered anisotropic plates is investigated by Reddy [5] using a shear deformable finite element based on first-order shear deformation theory in which parametric effects of time step, lamination schemes, etc. on the transient response are brought out.

Even though a good amount of literature [6–12] is available on dynamic analysis of laminated composites using different elements and shear deformation theories giving specific examples, a detailed study on the effects of various parameters on the dynamic behaviour as well as the frequency response of laminated composites is not available in the literature. Higher-order shear deformation theory is required to get acceptable results, especially in the case of thick plates. But, results for dynamic analysis of laminated plates, based on a higher-order shear deformation theory, are not available in literature. This paper presents some results of the dynamic analysis of laminated plates, using higher-order shear deformation theory, which can be used as benchmark solutions. A parametric study is also done and the results are presented in subsequent sections.

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2. Finite element formulation

A 4-noded non-conforming element with seven degrees of freedom per node, viz., u_0, v_0, w_0, $\partial w_0/\partial x$, $\partial w_0/\partial y$, θ_x and θ_y, based on higher-order shear deformation theory, is used for the analysis. This theory assumes a parabolic variation of transverse shear strains across the thickness, and satisfies the condition of zero transverse shear stresses at top and bottom surfaces of the plate. Use of the shear correction factor is not necessary in this case.

The governing equation of motion for a system subjected to external time-varying loads is expressed as

$$[M]\{\ddot{d}(t)\} + [C]\{\dot{d}(t)\} + [K]\{d(t)\} = \{R(t)\}. \tag{1}$$

Detailed formulation for the development of stiffness matrix, $[K]$, is given by Ghosh and Dey [1]. The mass matrix, $[M]$, is assembled from element mass matrices which are developed by a modified scheme of lumping, called HRZ lumping scheme, producing a diagonal mass matrix. Various steps involved in the development of mass matrix are given by Ghosh and Dey [2]. The effect of damping in the system is taken into account by means of damping matrices. It is difficult to determine the element damping parameters for general finite element assemblages, as the damping parameters are frequency dependent. Hence, in general, damping matrix, $[C]$, is not assembled from the element damping matrices. Damping is generally expressed in terms of damping ratios rather than by means of an explicit damping matrix. Transient analysis, which involves the determination of time-wise history of displacements and stresses in a structure under the action of external time-varying loads, is done by adopting mode superposition method. Accordingly,

$$[\bar{C}] = [\Phi]^{\mathrm{T}}[C][\Phi] \begin{bmatrix} 2\xi_1\omega_1 & & & & \\ & 2\xi_2\omega_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & 2\xi_n\omega_n \end{bmatrix}$$

since $[C]/[C_{cr}] = [\xi]$, the damping ratio, $[C_{cr}] = [2M_{ii}\omega_i]$, $[\Phi]^{\mathrm{T}}[M][\Phi] = [I]$, the unit matrix and $[\Phi]$, the modal matrix. The equilibrium equation corresponding to the generalized displacements may be expressed as

$$\{\ddot{X}_i(t)\} + 2\omega_i\xi_i\{\dot{X}_i(t)\} + \omega_i^2\{X_i(t)\} = \{\bar{R}_i(t)\}. \tag{2}$$

Eq. (2) can be easily solved by any of the direct integration methods and Wilson-θ time integration technique [13] is used in the present study. Subspace iteration technique [13] is used to extract the eigenvalues and eigenvectors.

3. Transient analysis

The analysis of composite plates is done by using 2×2 and 3×3 Gauss integration for the evaluation of stiffness and mass terms respectively employing 16×16 mesh division for full plate.

The effects of various parameters like damping ratio, width-to-thickness ratio, material anisotropy, fibre orientation, number of layers and aspect ratio on the dynamic response is studied by analyzing plates having the following geometry and material properties. $a = b = 25\,\text{cm},\ E_2 = 2.1 \times 10^6\,\text{N/cm}^2,\ E_1/E_2 = 25,\ \nu_{12} = 0.25,\ G_{12} = G_{13} = 0.5E_2,\ G_{23} = 0.2E_2,$ $\rho = 8 \times 10^{-6}\,\text{N s}^2/\text{cm}^4$. In all cases, the plates are assumed to be simply supported on all edges. A suddenly applied non-dimensional uniform load, $Q = qb^4/(E_2h^4) = 10$, is considered for the analysis. In all cases a time step of 5 μs is used.

(i) *Effect of damping*: To study the effect of damping, a square simply supported 4-layer cross-ply (0/90/90/0) laminate with $b/h = 10$ and different percentages of damping is considered and the results are presented in Fig. 1. The amplitude of vibration is found to decay fast with increase in percentage damping. It is seen that for low percentages of damping (1%) the amplitude decays almost linearly with respect to time. As the percentage of damping increases, the amplitude of vibration decays exponentially. It is also observed that it takes certain time to respond and hence the increase in damping ratio does not reduce the response proportionately during the initial period of time. The value of logarithmic decrement is found to agree with the assumed damping ratio in all the three cases, which confirms the correctness of the procedure.

(ii) *Effect of width-to-thickness ratio*: The central displacement response for 4-layer symmetric cross-ply laminate with different width-to-thickness ratios is presented in Fig. 2. From this figure, it is seen that the amplitude of vibration decreases at a very fast rate with increase in plate width-to-thickness ratio in the thick plate range ($b/h < 20$), the decrease is at a very small rate in the thin plate range ($b/h > 20$). This implies that the effect of transverse shear is significant for $b/h \leqslant 20$. Moreover, the load applied being non-dimensional, thick plates are subjected to heavier loads compared to thin plates. Similar variation is observed for normal stresses also because the stresses are evaluated from displacements. The variation of transverse shear stress (τ_{yz}) for different b/h

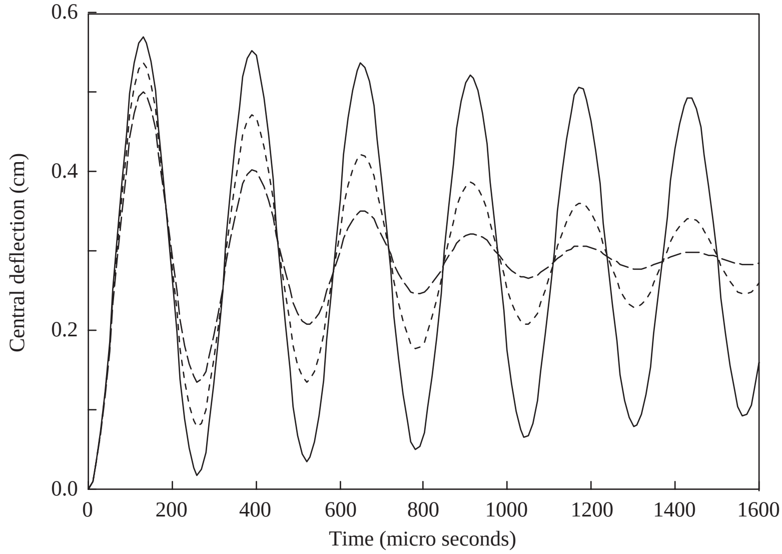

Fig. 1. Effect of damping on central displacement response of a square simply supported plate (0/90/90/0, $b/h = 10$): —— 1% damping, - - - 5% damping, – – – 10% damping.

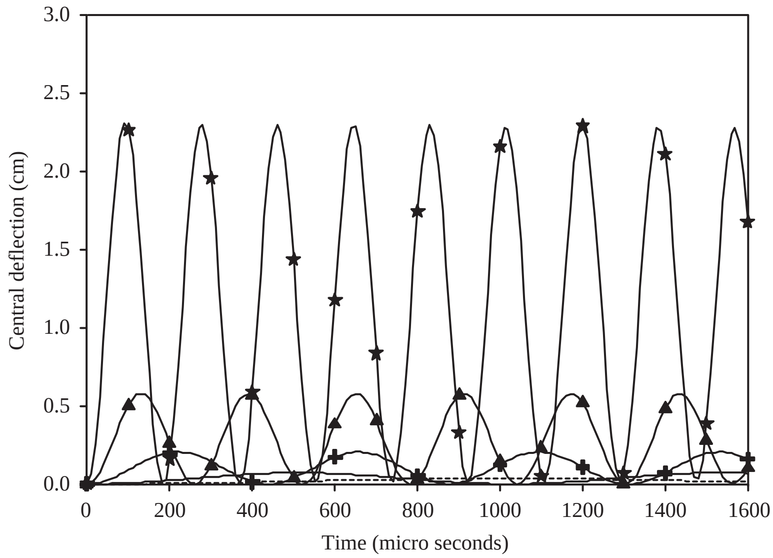

Fig. 2. Effect of width-to-thickness ratio on central displacement response of a square simply supported plate (0/90/90/0): —★— $b/h = 5$, —▲— $b/h = 10$, —+— $b/h = 20$, —— $b/h = 50$, - - - $b/h = 100$.

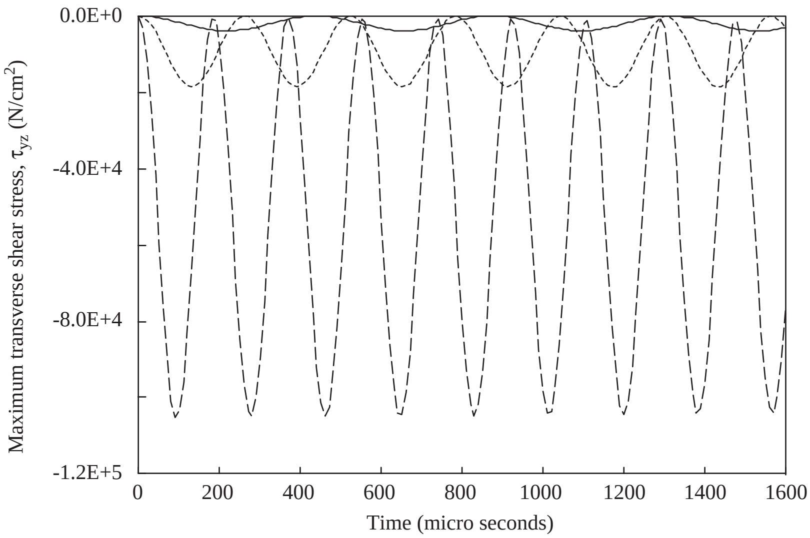

Fig. 3. Effect of width-to-thickness ratio on the transverse shear stress (τ_{yz}) response of a square simply supported plate (0/90/90/0): – – – $b/h = 5$, - - - $b/h = 10$, —— $b/h = 20$.

ratios is given in Fig. 3 and the behaviour is same as that of deflection. Similar behaviour is observed in the case of angle-ply laminates also.

(iii) *Effect of material anisotropy*: The displacement history for a 3-layer cross-ply (0/90/0) laminate with $b/h = 10$ is shown in Fig. 4. The increase in the value of E_1/E_2 ratio from 5 to 40,

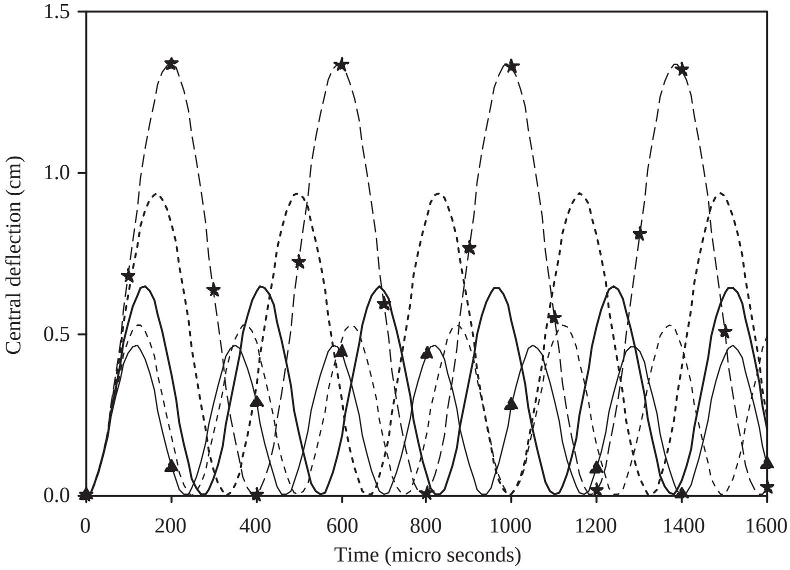

Fig. 4. Effect of material anisotropy on the central displacement response of a square simply supported plate (0/90/0, $b/h = 10$): —★— $E_1/E_2 = 5$, - - - $E_1/E_2 = 10$, —— $E_1/E_2 = 20$, – – – $E_1/E_2 = 30$, —▲— $E_1/E_2 = 40$.

keeping E_2 the same, causes a reduction in the displacement response. This is expected because increase in E_1 means increase in stiffness of the plate. Frequency of vibration also increases as E_1/E_2 increases, because of the increase in stiffness. A similar behaviour is observed in stress response also.

(iv) *Effect of fibre orientation*: The dynamic response of a 4-layer laminate with the angle of orientation varying from 0° to 45° is given in Fig. 5 for symmetric $(\alpha/-\alpha/-\alpha/\alpha)$ arrangement of layers with $b/h = 20$. A similar behaviour is observed for antisymmetric $(\alpha/-\alpha/\alpha/-\alpha)$ arrangement also. In both cases, it is seen that the displacement decreases with increase in fibre orientation angle, the minimum being for 45°. This is because of the change in stiffness caused by change in fibre orientation. It is also observed that the peak response occurs at different times, for different fibre orientation angles. This is due to the change in frequency, following a change in stiffness. Moreover, the displacement is less for antisymmetric arrangement than symmetric arrangement. It is observed that the stresses are also minimum for $\alpha = 45°$.

(v) *Effect of number of layers*: Cross-ply and angle-ply laminates of width-to-thickness ratio 10, with symmetric and antisymmetric arrangement, are analyzed to study the effect of number of layers. The displacement response for antisymmetric cross-ply laminates is shown in Fig. 6. Other results are not presented for the sake of brevity. From the figure, it is seen that the amplitude of vibration decreases with increase in number of layers. Behaviour of 2-layered plate is significantly different from that of a multi-layered plate. Difference between the amplitudes of 4-layer plate and 6-layer plate is very small, the decrease being negligible beyond 6 layers. This is because the plate behaves more or less like an orthotropic plate beyond 6-layers. It is concluded that number of layers has no appreciable effect on the amplitude of vibration for a multi-layered plate.

(vi) *Effect of aspect ratio*: The effect of aspect ratio on the dynamic response of a 3-layer cross-ply (0/90/0) laminate with $b/h = 10$ is studied by varying a/b ratio, keeping the value of b

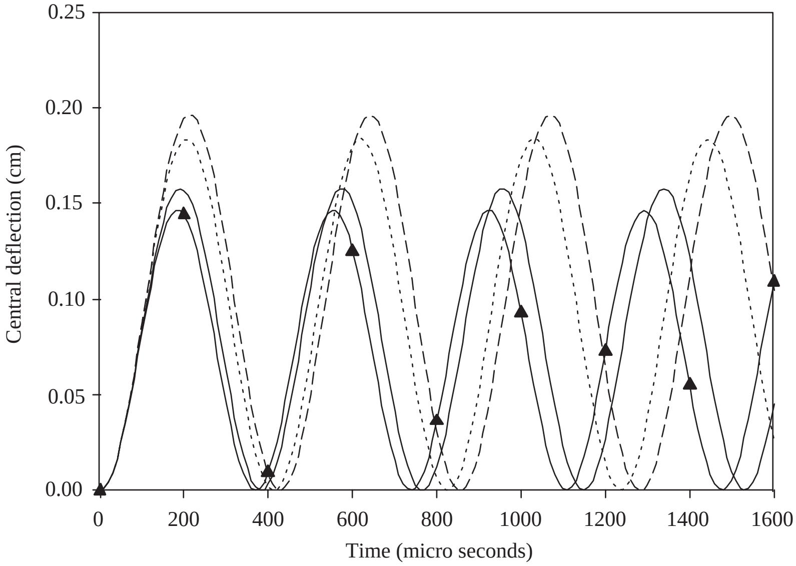

Fig. 5. Effect of fibre orientation angle on the central displacement response of a square simply supported plate ($\alpha/-\alpha/-\alpha/\alpha$, $b/h=20$): ––– $\alpha=0°$, - - - $\alpha=15°$, —— $\alpha=30°$, —▲— $\alpha=45°$.

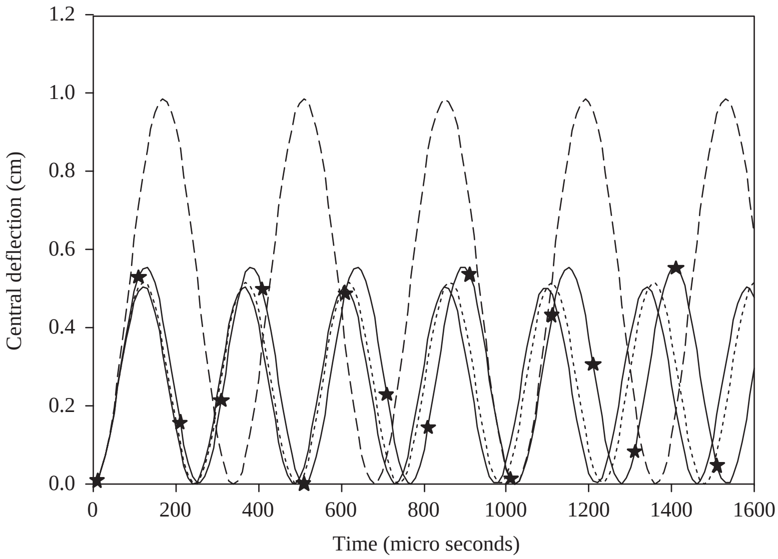

Fig. 6. Effect of number of layers on the central displacement response of a square simply supported antisymmetric laminate ($b/h=10$): ––– $N=2$, —★— $N=4$, - - - $N=6$, —— $N=8$.

constant in all cases. The displacement history corresponding to various a/b ratios is given in Fig. 7. It is seen that the displacement increases with increase in aspect ratio. This is because the behaviour of the plate gradually changes to that of an one-dimensional structure with increase in aspect ratio.

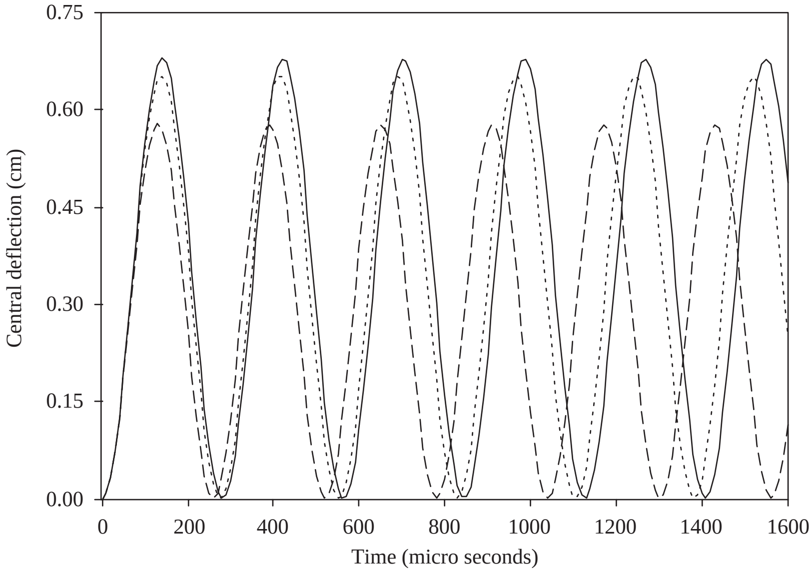

Fig. 7. Effect of aspect ratio on central displacement response of a square simply supported plate (0/90/0, $b/h = 10$): – – – $a/b = 1$, - - - $a/b = 1.4$, —— $a/b = 1.8$.

4. Frequency response

To the best of the authors' knowledge, no published work on the frequency response of laminated composite plates is available. To fill this gap, an attempt has been made to carry out some investigations on the behaviour of laminated plates subjected to harmonic loading. Two-layer cross-ply (0/90) and angle-ply (45/−45) laminates of width-to-thickness ratio 5 and 100 are considered. The geometry and properties of the material of the plate are same as those used for the transient analysis. The plate is subjected to a uniform load of 10 N/cm^2 varying harmonically with respect to time. The response curve for the cross-ply laminate with different percentages of damping and 15 cycles of loading is shown in Fig. 8 for $b/h = 5$. Similar curves are obtained for angle-ply laminates as well as thin plates. From the figure, it is clear that as the percentage of damping increases, the resonant response decreases. It is also observed that the fibre orientation angle and b/h ratio does not have any influence on the response curve. The convergence study on the number of cycles of loading required to get a steady state response at resonance for the 2-layer laminates (0/90 and 45/−45) is shown in Fig. 9 for $b/h = 5$ and different percentages of damping. From the figure it is evident that the number of cycles of loading required decreases with increase in the percentage of damping. In other words, with a damping ratio of 0.02, 35 cycles of loading is required to get a converged steady state resonant response and with a damping ratio of 0.05, 20 cycles of loading is found to be sufficient. In order to study the effect of b/h ratio on the magnification factor at resonance, a 4-layer symmetric cross-ply (0/90/90/0) laminate is considered. The variation of dynamic load factor at resonance is given in Fig. 10, from which it is evident that there is only a reduction of 3% in the resonant response as the width-to-thickness ratio increases from 5 to 20. The response is almost constant beyond $b/h = 20$.

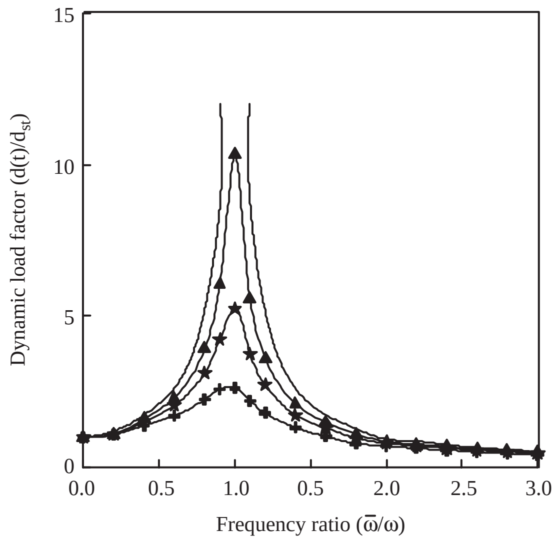

Fig. 8. Response curve of a 2-layer square simply supported laminate (0/90, $b/h = 5$): —— $\xi = 0\%$, —▲— $\xi = 5\%$, —★— $\xi = 10\%$, —+— $\xi = 20\%$.

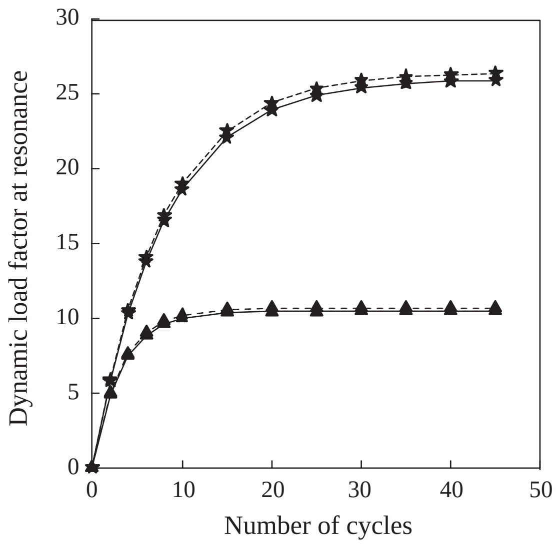

Fig. 9. Effect of number of cycles of loading on steady state response of a 2-layer square simply supported laminate ($b/h = 5$): —★— 0/90, $\xi = 2\%$, —▲— 0/90, $\xi = 5\%$, ---★--- 45/−45, $\xi = 2\%$, --▲--- 45/−45, $\xi = 5\%$.

5. Conclusions

Based on the studies conducted, the following conclusions are drawn. With increase in damping ratio, the amplitude of vibration decays exponentially. The effect of transverse shear is significant for width-to-thickness ratio less than or equal to 20 in the case of composite plates. The increase in material anisotropy increases the stiffness of the plate. The stiffness of the plate being the same in

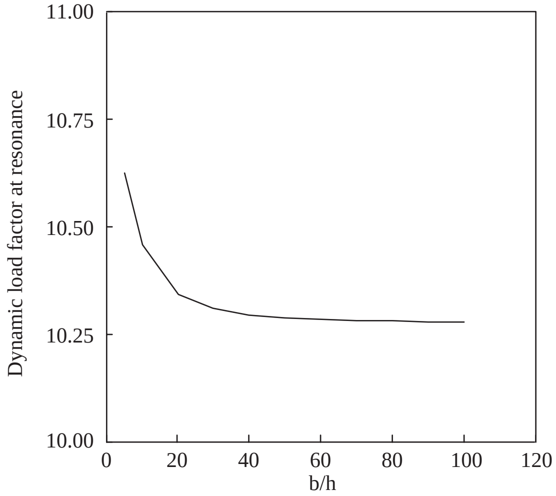

Fig. 10. Effect of width-to-thickness ratio on dynamic load factor at resonance for a square simply supported laminate (0/90/90/0): $\xi = 5\%$.

both directions, the response of the plate is minimum for a fibre orientation angle of 45°. The effect of number of layers on the transient response is negligible in multi-layered plates. The increase in aspect ratio changes the behaviour of the plate similar to that of an one-dimensional structure. The resonant response is not influenced by the width-to-thickness ratio and fibre orientation angle, but depends only on the percentage of damping.

References

[1] A.K. Ghosh, S.S. Dey, A simple finite element for the analysis of laminated plates, *Computers and Structures* 44 (1992) 585–596.

[2] A.K. Ghosh, S.S. Dey, Free vibration of laminated composite plates—a simple finite element based on higher-order theory, *Computers and Structures* 52 (1994) 397–404.

[3] T. Kant, Mallikarjuna, A higher-order theory for free vibration of unsymmetrically laminated composite and sandwich plates—finite element evaluation, *Computers and Structures* 32 (1989) 1125–1132.

[4] T. Kant, J.H. Varaiya, C.P. Arora, Finite element transient analysis of composite and sandwich plates based on a refined theory and implicit time integration schemes, *Computers and Structures* 36 (1990) 401–420.

[5] J.N. Reddy, Dynamic (transient) analysis of layered anisotropic composite material plates, *International Journal for Numerical Methods in Engineering* 19 (1983) 237–255.

[6] A.H. Sheikh, S. Haldar, D. Sengupta, Vibration of plates in different situations using a high-precision shear deformable element, *Journal of Sound and Vibration* 253 (2002) 329–345.

[7] D.J. Gorman, R. Singhal, Free vibration analysis of cantilever plates with step discontinuities in properties by the method of superposition, *Journal of Sound and Vibration* 253 (2002) 631–652.

[8] E. Pan, P.R. Heyliger, Free vibrations of simply supported and multilayered magneto-electro-elastic plates, *Journal of Sound and Vibration* 252 (2002) 429–442.

[9] A.H. Sheikh, A.S. Haldarb, D. Sengupta, A high precision shear deformable element for the analysis of laminated composite plates of different shapes, *Composite Structures* 55 (2002) 329–336.

[10] T.S. Lok, Q.H. Cheng, Bending and forced vibration response of a clamped orthotropic thick plate and sandwich panel, *Journal of Sound and Vibration* 245 (2001) 63–78.
[11] A.A. Khdeir, Free vibration and buckling of symmetric cross-ply laminated plates by an exact method, *Journal of Sound and Vibration* 126 (1988) 447–461.
[12] A.K. Noor, Free vibration of multilayered composite plates, *American Institute of Aeronautics and Astronautics Journal* 11 (1973) 1038–1039.
[13] K.J. Bathe, *Finite Element Procedures*, Prentice-Hall of India, New Delhi, 1996.