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Letter to the Editor

A note on the effect of compressibility on the propagation of free waves in sandwich plates with heavy fluid loading

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Analysis of wave propagation in an infinitely long plate, which separates a half-space occupied by a fluid from a half-space occupied by a vacuum, is the classic problem in stationary structural acoustics. A tremendous number of publications on this subject is available in the literature (see, for example, a survey in Ref. [1]). In most of the publications, the classic Kirchhoff plate model and the standard model of a compressible fluid (an acoustic medium) are used. However, modern naval structures are often made of sandwich plates and their dynamics in many cases may adequately be described within the framework of the model of an incompressible fluid. This brief note is aimed at comparison of the behaviour of a fluid-loaded sandwich plate in the cases, when the fluid's compressibility is both ignored and taken into account.

A plate of sandwich composition is considered, which consists of two symmetrical relatively thin, stiff skin plies and a thick, soft, core ply. All plies are assumed to be isotropic and the following non-dimensional parameters are introduced to describe the internal structure of a sandwich plate: $\varepsilon = h_{skin}/h_{core}$ as a thickness parameter, $\delta = \rho_{core}/\rho_{skin}$ as a density parameter and $\gamma = E_{core}/E_{skin}$ as a stiffness parameter. In the framework of the theory given in Refs. [2–4], the deformation of a sandwich plate is governed by two independent variables: a displacement of the mid-surface of a plate w (which is the same for all plies) and a shear angle between skin plies θ. As is shown in Ref. [4], this theory reliably describes the first and second branches of the dispersion curves. The plate is loaded by an acoustic medium, which occupies the lower half-space ($z<0$) and equations of motions of a plate (a sandwich beam) are

$$\frac{1}{12}\left(2+\frac{\gamma}{\varepsilon^3}\right)\frac{\partial^4 w}{\partial x^4}-\frac{(1-\nu)}{2}\left(1+\frac{1}{\varepsilon}\right)^2\varepsilon\gamma\left(\frac{\partial\theta}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)+\left(2+\frac{\delta}{\varepsilon}\right)\left(\frac{h}{c}\right)^2\frac{\partial^2 w}{\partial t^2}$$
$$-\frac{1}{12}\left(2+\frac{\delta}{\varepsilon^3}\right)\left(\frac{h}{c}\right)^2\frac{\partial^4 w}{\partial x^2\partial t^2}=p(x,0,t),$$

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$$-\frac{1}{2}\frac{\partial^2\theta}{\partial x^2}+\frac{(1-\nu)}{2}\varepsilon\gamma\left(\theta+\frac{\partial w}{\partial x}\right)+\frac{1}{2}\left(\frac{h}{c}\right)^2\frac{\partial^2\theta}{\partial t^2}=0. \tag{1}$$

This system of linear differential equations is written in non-dimensional form in the variables $w=w_{dim}/h, x=x_{dim}/h, p=p_{dim}/\rho c^2, c=\sqrt{E/\rho(1-\nu^2)}$, $h=h_{skin}$, $\rho=\rho_{skin}$, $E=E_{skin}$, t is dimensional time.

A non-dimensional velocity potential is introduced as $\varphi=\varphi_{dim}/hc$ and it obeys the wave equation ($z=z_{dim}/h$)

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial z^2}-\frac{h^2}{c_{fl}^2}\frac{\partial^2\varphi}{\partial t^2}=0. \tag{2}$$

The non-dimensional acoustic pressure is defined as

$$p(x,z,t)=-\frac{\rho_{fl}}{\rho}\frac{h}{c}\frac{\partial\varphi(x,z,t)}{\partial t}. \tag{3}$$

The continuity condition at the fluid–structure interface $z=0$ is formulated as

$$\frac{\partial\varphi}{\partial z}=\frac{h}{c}\frac{\partial w}{\partial t}. \tag{4}$$

A solution of these equations is sought in the form

$$\begin{aligned} w(x,t)&=A_w\exp(\mathrm{i}Kx-\mathrm{i}\omega t),\\ \theta(x,t)&=A_\theta\exp(\mathrm{i}Kx-\mathrm{i}\omega t),\\ \varphi(x,z,t)&=A_w\exp(\Gamma z+\mathrm{i}Kx-\mathrm{i}\omega t). \end{aligned} \tag{5}$$

Here the following non-dimensional parameters are introduced:

$$\Gamma=\sqrt{K^2-\tilde{c}^2\Omega^2},\quad K=kh,\quad \Omega=\omega h/c,\quad \tilde{\rho}=\rho/\rho_{fl},\quad \tilde{c}=c/c_{fl}.$$

Since the velocity potential cannot grow exponentially when $z\to-\infty$ and it should not describe waves coming from infinity, the following conditions are imposed:

$$\mathrm{Re}\,\Gamma\equiv\mathrm{Re}\lfloor\sqrt{K^2-\tilde{c}^2\Omega^2}\rfloor>0, \tag{6a}$$

$$\text{if }\ \mathrm{Re}\,\Gamma=0,\quad\text{then }\ \mathrm{Im}[\Gamma]<0. \tag{6b}$$

Hereafter, the consideration is restricted by the waves, which propagate or decay in the positive direction of the x-axis and, therefore, the following conditions are imposed:

$$\mathrm{Im}\,K>0, \tag{6c}$$

$$\text{if }\ \mathrm{Im}\,K=0,\quad\text{then }\ \mathrm{Re}\,K>0. \tag{6d}$$

The dispersion equation for a sandwich plate with heavy fluid loading in the non-dimensional form is

$$\left[\frac{1}{12}\left(2+\frac{\gamma}{\varepsilon^3}\right)K^4-\left(2+\frac{\delta}{\varepsilon}\right)\Omega^2-\frac{1}{12}\left(2+\frac{\delta}{\varepsilon^3}\right)K^2\Omega^2+\frac{1-v}{2}\left(1+\frac{1}{\varepsilon}\right)^2\varepsilon\gamma K^2-\frac{\Omega^2}{\tilde{\rho}\sqrt{K^2-\tilde{c}^2\Omega^2}}\right]$$
$$\times\left[K^2-\Omega^2+(1-v)\varepsilon\gamma\right]-\tfrac{1}{2}(1-v)^2\left(1+\frac{1}{\varepsilon}\right)^2\varepsilon^2\gamma^2K^2=0. \tag{7}$$

In fact, it does not present serious difficulties to compute all roots (purely real, purely imaginary or complex) of this dispersion equation. However, this task becomes truly elementary when some algebraic transformations are performed to obtain a dispersion equation in the polynomial form. This equation is of the 14th order and it is formulated as

$$(p_1p_3+p_4)^2\Gamma^2-p_2^2p_3^2=0, \tag{8}$$

$$p_1=\frac{1}{12}\left(2+\frac{\gamma}{\varepsilon^3}\right)K^4-\left(2+\frac{\delta}{\varepsilon}\right)\Omega^2-\frac{1}{12}\left(2+\frac{\delta}{\varepsilon^3}\right)K^2\Omega^2+\frac{1-v}{2}\left(1+\frac{1}{\varepsilon}\right)^2\varepsilon\gamma K^2,$$
$$p_2=-\frac{\Omega^2}{\tilde{\rho}},\quad p_3=K^2-\Omega^2+(1-v)\varepsilon\gamma,\quad p_4=-\tfrac{1}{2}(1-v)^2\left(1+\frac{1}{\varepsilon}\right)^2\varepsilon^2\gamma^2K^2.$$

Each root of the polynomial equation (8) is easily found by the use of any standard software and should be checked whether it originates from Eq. (7) and whether all conditions (6) are held.

Vibrations of a plate with steel skin plies in water ($\tilde{\rho}=7.81$) are considered and the dispersion equation (7) is solved for several values of the compressibility parameter $\tilde{c}$, which varies from $\tilde{c}=0$ (an incompressible fluid) to $\tilde{c}=3.7$ (the actual ratio of the sound speed in steel to the sound speed in water). The parameters of a sandwich plate composition are $\varepsilon=0.05$, $\delta=0.1$, $\gamma=0.01$.

Consider the case of an incompressible fluid. As discussed in Ref. [4], the dispersion equation (7) at any frequency has a purely real root of relatively large magnitude, which (similar to Kirchhoff theory) describes propagation of the dominantly flexural wave in a plate both with and without fluid loading. The dependence of Re[K] and the dependence of the real part of the relevant 'fluid' wave number Re[Γ] on Ω are shown in Fig. 1a and b. The dependence of this root on frequency has been explored, for example, in Ref. [5] for a Kirchhoff plate vibrating in water. The influence of the compressibility on this root is not analyzed here any further and only the behaviour of the small magnitude roots, which are related to the dominantly shear wave in a sandwich plate is discussed.

In Fig. 2a and c, the dependence of the real parts of the wave numbers Re[K] and Re[Γ] on the frequency parameter Ω is presented. In Fig. 2b, a similar graph is shown for Im[K]. Two different regimes of wave motions are observed in these graphs. They are separated by the cut-on frequency of the dominantly shear wave, which is easily found from Eq. (7) as K vanishes, $K=0$:

$$\Omega_c=\sqrt{(1-v)\varepsilon\gamma}. \tag{9}$$

This threshold frequency does not depend on fluid's parameters and in this case it is $\Omega=0.0187$. In the 'low-frequency' range ($\Omega<0.0187$), the dispersion equation (7) has four roots, which satisfy all conditions (11). All of them are complex conjugate (see curves 1–4 in Fig. 2a–b). There also exists a purely real one (see Fig. 1), which is not discussed hereafter. As is seen from Fig. 2c, all

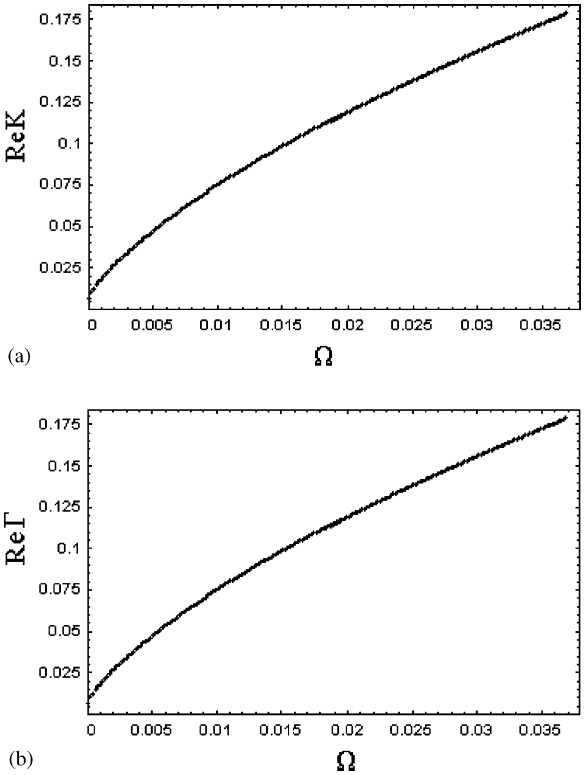

Fig. 1. For a dominantly flexural wave in a plate loaded by an incompressible fluid, the frequency parameter Ω versus (a) the real part of wave number K; (b) the real part of wave number Γ.

fluid wave numbers Γ have positive real parts and therefore wave motion exponentially decays with the growth in a distance from a vibrating plate. In the 'high-frequency' range ($\Omega > 0.0187$) there are two complex conjugate roots (curves 3, 4) and a purely real root (curve 5), which describes propagation of the dominantly shear wave in a plate with heavy fluid loading [3]. Thus, the number of roots has reduced by one as a cut-on frequency is passed. There is only one propagating wave (curve 5) instead of two evanescent waves (curves 1, 2), which are characterized by complex conjugate roots,.

In the case of a sandwich plate loaded by water, the dispersion curves similar to those, presented in Fig. 2 are displayed in Fig. 3a–c. The same notations are used. As is seen, in the low-frequency range, the behaviour of roots is the same as in the previous case. However, unlike the previous case there is no propagating shear wave at $\Omega > 0.0187$, since in this frequency region, condition (6a) for this wave is violated. Thus, there exists only propagating flexural wave at $\Omega > 0.0187$, see Fig. 1.

To clarify the matter, consider a weakly compressible fluid, $\tilde{c} = 0.629$. The results of computations are presented in Fig. 4a–c. Admissible dispersion curves 1–4 below the cut-on frequency are located similarly to the both previous cases. However, above the cut-on frequency, the fluid's compressibility introduces the qualitatively new effect, which has not yet been observed

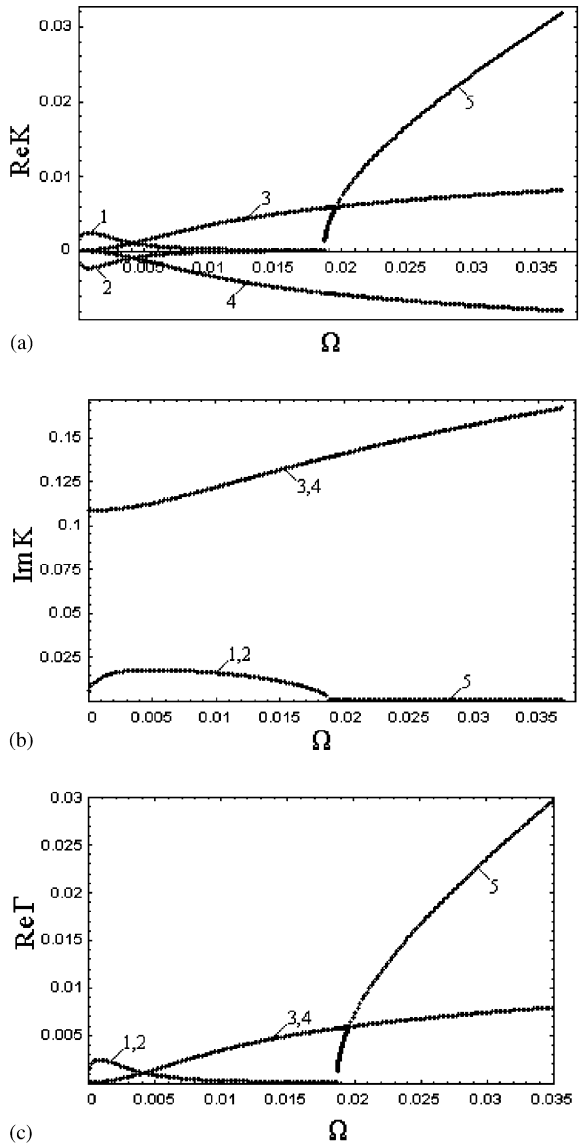

Fig. 2. For a dominantly shear wave in a plate loaded by an incompressible fluid, the frequency parameter Ω versus (a) the real part of wave number K; (b) the imaginary part of wave number K; (c) the real part of wave number Γ.

in the previous case. As follows from condition (6a), for a wave, which decays into fluid, the following inequality should hold: $K \geqslant \tilde{c}\Omega$. The threshold frequency is easily found from Eq. (8), when $K = \tilde{c}\Omega$. It is given by the elementary formula

$$\Omega_G = \sqrt{\frac{(1-v)\varepsilon\gamma}{1-\tilde{c}^2}}. \tag{10}$$

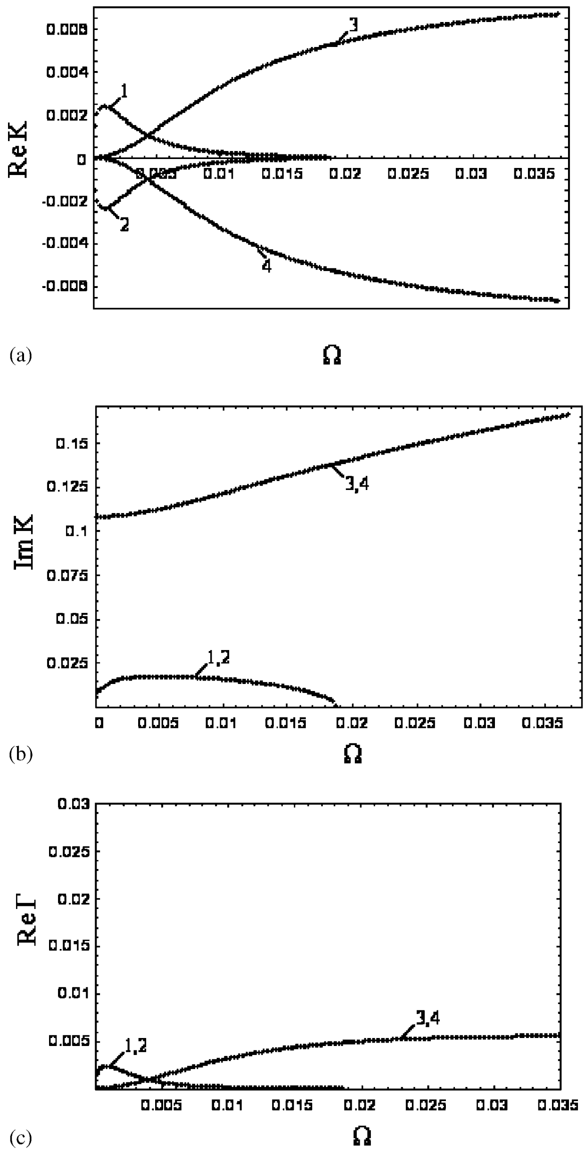

Fig. 3. For a dominantly shear wave in a plate loaded by water, the frequency parameter Ω versus (a) the real part of wave number K; (b) the imaginary part of wave number K; (c) the real part of wave number Γ.

For $\tilde{c}=0$, i.e., in the case of an incompressible fluid, $\Omega_G=\Omega_C$ and the wave, which propagates in a plate, automatically decays in a fluid volume. However, if $\tilde{c}\neq 0$ and $\Omega_C<\Omega<\Omega_G$, then $\mathrm{Re}\lfloor\sqrt{K^2-\tilde{c}^2\Omega^2}\rfloor<0$. Thus, the amplitude of a wave grows exponentially as $z\to-\infty$ and it cannot exist in a plate with fluid loading. As is shown in Fig. 4a, the branch of dispersion curve, which presents the purely propagating dominantly shear wave, does not emerge from the co-ordinate axis. The length of a gap (Ω_C,Ω_G) progressively grows with the growth in the compressibility parameter $\tilde{c}$ and its upper boundary becomes infinite when $\tilde{c}\geqslant 1$. This is why this

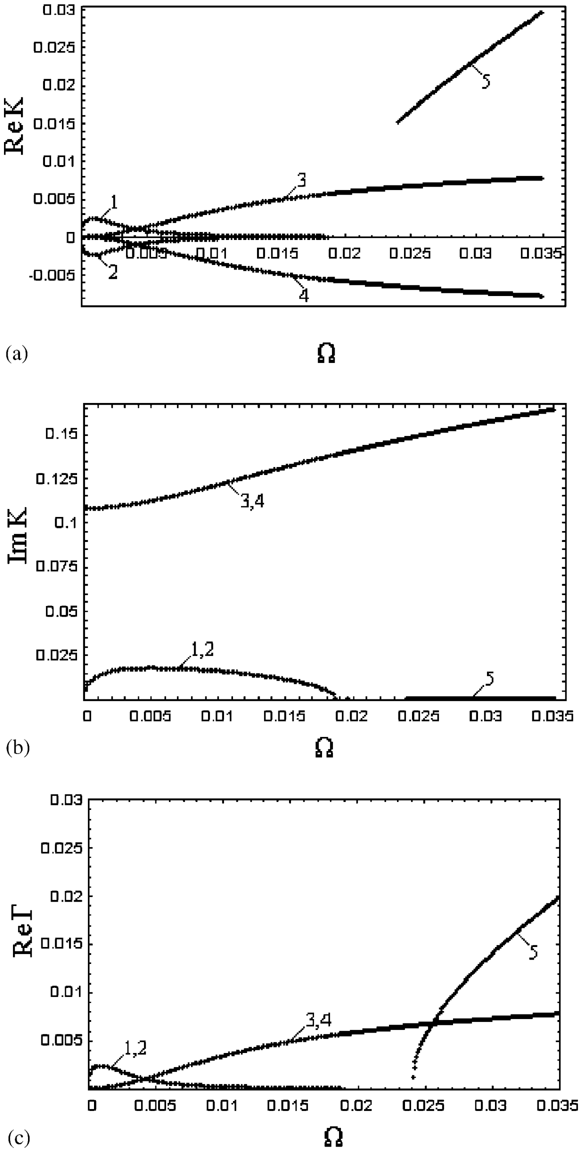

Fig. 4. For a dominantly shear wave in a plate loaded by a weakly compressible fluid, the frequency parameter Ω versus (a) the real part of wave number K; (b) the imaginary part of wave number K; (c) the real part of wave number Γ.

branch has disappeared in the case of a sandwich plate loaded by water. A dependence of the magnitude of a cut-on frequency parameter on $\tilde{c}$ is illustrated in Fig. 5.

In conclusion, it should be pointed out that the sound speed in modern composite materials may vary in a broad range and in practical design it is possible to face both regimes, discussed in this note.

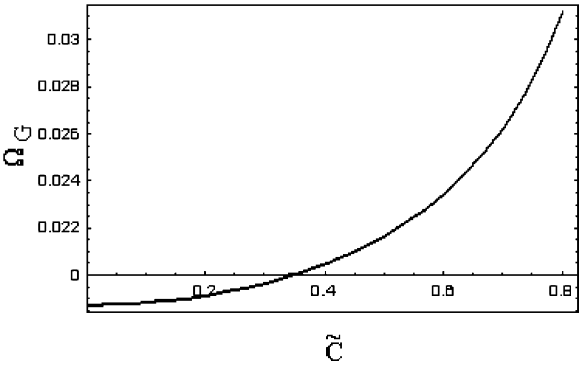

Fig. 5. The cut-on frequency parameter versus sound speed ratio.

Acknowledgements

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