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Book review

Finite Element Methods for Structures with Large Stochastic Variations by I. Elishakoff and Y.J. Ren, Oxford University Press, Oxford, 2003, pp. ix + 260, price £45, ISBN 0 19 852631 8

This book contains 260pp. It has seven chapters, a prologue and an epilogue, 11 appendices and a bibliography with over a thousand references. An author and subject indices are also included. The style of the book is typical of that of the senior author in the sense that everywhere in the book shows that the authors read widely and the senior author collaborated his research work with many international workers. In the prologue it contains an interesting or arguably somewhat stretched but brief introduction to the history of the finite element method (FEM).

It is mentioned in the prologue that the authors prefer the term, *finite element method for stochastic problems* (*FEMSP*) instead of *stochastic finite element method* (*SFEM*). This reviewer thinks that while FEMSP is an improvement over SFEM it is too general since it can include temporally stochastic loadings or spatially stochastic problems or both. Therefore, a more appropriate term for the problems dealt with in the above book would be *finite element method for spatially stochastic problems* (*FEMSSP*).

Returning to the individual chapters of the book, Chapter 1 is concerned with the fundamentals of the FEM. The formulations included are for beam bending and plane stress/strain analyses. The formulations are of the displacement type and the treatments here are very elementary. Readers looking for more advanced topics will be disappointed.

Chapter 2 presents a brief review of the FEM for stochastic structures. The techniques included in this chapter are: finite element formulation (FEF) by the perturbation technique, FEF by series expansion, FEF by homogeneous chaos, improved first order perturbation FEF, and numerical results for a two-bar truss with stochastic Young's moduli. This chapter concludes with illustrations of various simple examples. Results and discussion of the examples are also presented.

Chapters 3–5 present several new approaches which are not perturbative FEM for stochastic structures. In Chapter 3, considerable length is devoted to the FEM based on the exact inverse of the stiffness matrices of a bar and beam by Fuch's method. It should be pointed out that for a linear uniform bar and beam their element stiffness matrices can be obtained explicitly, in the sense that no numerical matrix inversion and numerical integration are required. These explicit stiffness matrices can lead to exact displacement solutions. For example, the stiffness matrix of a uniform bar has been obtained explicitly by selecting linear polynomials for the shape functions. Similarly, the stiffness matrix of a uniform Bernoulli–Euler beam has been obtained explicitly by selecting cubic polynomial shape functions and exact solution can be obtained accordingly. In this chapter the so-called new formulation of the finite element stiffness matrix is presented. The

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conventional or *old* (as in contrast to *new* applied by the authors) formulation results in element stiffness matrix of order 8 while the new formulation presented gives the element stiffness matrix of order 4. Naturally, the imposition of displacement constraints is different from the conventional one. While the three step uniform beam is fine for illustration purpose it may be noted that the stresses at nodes 2 and 3 are discontinuous. This is due to the fact that in the book the displacement formulation-based FEM is used. The stiffnesses contain parameters assumed to possess the Pearson type II distribution.

Chapter 4 deals with the exact solutions of stochastic shear and Bernoulli–Euler beams as benchmark problems. The comment made on Chapter 3 above concerning the exact solutions applies here. The detailed derivation of exact solutions for stochastic Bernoulli–Euler beams is presented in this chapter. It includes, for example, the deterministic beam under a random load treated as a random field, and beam with random flexibility under a random field load.

The variational principle-based FEM for stochastic beams is presented in Chapter 5. The stochastic versions of Bubnov–Galerkin and Rayleigh–Ritz methods as well as the FEM were constructed by the application of variational principles. The shear beam and beam with various boundary conditions under uniform pressure were considered. As this reviewer pointed out in 2000 [1], and noted by the authors in Section 5.1 that these variational principles have no physical implication or meaning. This non-physical basis is still considered by this reviewer to be a major shortcoming of the procedures presented for large stochastic variations in the above book.

Chapter 6 is different from Chapters 3–5 in that here the element-level flexibility based FEM for stochastic structures is emphasized. The mean and covariance of the displacement are obtained in terms of the mean and covariance of flexibility. The plane stress/strain of a square and rectangle with Pearson type II probability density function (PDF) are included. It is understood that the examples concerning the rectangle applying a mesh of 3×5 (mentioned in p. 154) are for illustration of the approaches presented. However, it has two major problems and should not be disregarded. First, even for deterministic structures it cannot give accurate displacement solutions and therefore, any conclusion drawn from the displacement solution may not be correct. In the stochastic situations, the coarse mesh can lead to even more inaccuracy. The reason that the comparison of results in the book is good is due to the fact that the results for the comparison were all based on the structure discretized by the same mesh. Second, it is likely that terms associated with the inner forces cannot be assumed to have zero contribution. In turn, it is equally likely that Eq. (6.43) or results obtained by this latter equation are incorrect.

In Chapter 7, structural uncertainties are modelled by both the stochastic and interval methods to quantify the uncertainties in response quantities. Results using these approaches were obtained for a shear frame. A comparison is also made.

The above book contains many typographical errors. These include those in the body of the text, the references, author and subject indices. For brevity, the following examples are given. First, in p. 118, three lines below Eq. (5.5), it was mentioned that the proof of the variational principles ... is given in Appendix F. But in the table of contents and the body of the text, Appendix F includes the proof of the boundary conditions in Eq. (5.32). In this same page, there is a missing term in the denominator of the first integral in Eq. (5.7). Second, examples of typographical errors in the author index are: (1) Sankar, T.S. 211, ... should read Sankar, T.S. 212, ... (2) Ma, F. 224 should read Ma, F. 225, (3) Lin, Y.K. 29, 201, 222 should read Lin, Y.K. 29, 202, 223, and (4) To, C.W.S. 239 should read To, C.W.S. 241. In the latter page with references by this reviewer, two of them have three typographical mistakes. This reviewer will just give his corrected first reference which should read "To, C.**W**.S. (2001). On computational stochastic structural dynamics applying finite elements, Arch. Comput. Meth. **Eng.** ..." The two bolds or highlights are the corrections in this reference indicated by the present reviewer.

To conclude, the level of FEM adopted in the book is very elementary in the sense that it is at the junior or senior undergraduate or first year graduate level depending of which school one attends. The structures dealt with are the most basic and simple ones, namely, bars, beams, and plane stress/strain two-dimensional structures. The problems presented in the book are entirely of static type. Those who work on applications of elementary FEM and simple static stochastic structures with large stochastic variation will likely find the book a satisfactory and an informative one.

On the other hand, those who work on application of elementary and advanced level of FEM dealing with plates and shells or combination of them, with large stochastic variations and dynamic loadings will have to look elsewhere. In this respect, it might be relevant to mention that FEM approaches for linear dynamic problems including beam structures [1], and linear and non-linear shell structures [2,3] with large spatially stochastic variations have appeared in the literature.

References

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[2] C.W.S. To, On computational stochastic structural dynamics applying finite elements, *Archives of Computational Methods in Engineering* 8 (1) (2001) 3–40.

[3] C.W.S. To, Nonlinear random responses of shell structures with spatial uncertainties, *Proceedings of Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, Montreal, Canada, September 29–October 2, 2002, DETC2002/CIE-34472, pp. 1–11.

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