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Combinatorial optimal design of number and positions of actuators in actively controlled structures using genetic algorithms

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Abstract

In this paper, the optimal design of the numbers and positions of actuators in actively controlled structures is formulated as a three-level optimal design problem. Features of this design problem such as discreteness, multi-modality and hierarchical structure are discussed. A two-level genetic algorithm (TLGA) is proposed for solving this problem. The concept, principle and solution process of the TLGA are described. A case study is presented, in which a building is subjected to earthquake excitation and controlled by active tendon actuators. The results of this study show that: (1) the design problem for optimizing number and configuration of actuators simultaneously in actively controlled structures has the features of non-linearity, mixed-discreteness and multi-modality; (2) a three-level design model can give a reasonable description for this kind of design problem; (3) TLGA is an effective algorithm for solving the combinatorial optimization problem.

1. Introduction

Over the years, vibration control of structures (especially high-rise buildings, long-span bridges, and offshore structures) to strong earthquakes, winds or waves has drawn considerable interest of many researchers. Various classical and advanced control strategies have been proposed, and many control devices, passive as well as active, have been developed and applied to civil engineering structures. However, numerous technical concerns remain. Further research efforts are still required to thoroughly solve practical problems in the implementation of control systems

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before widespread applications of this novel technology become possible. These practical problems include control of non-linear structural systems, discrete-time nature of control environment, limited number of sensors and controllers, optimal configuration of control devices, robustness of control devices, and reliability issues, etc. The number of actuators applied in an actively controlled structure is generally limited by the constraints of control cost, placing space, etc. Furthermore, configuration of the limited number of actuators in the structure has significant effect on the reduction level of structural dynamic response because the control effect strongly depends upon the configuration of actuators in such a controlled structure. Thus, in order to get optimal control performance with satisfactory control cost, the actuator configuration problem needs to be further investigated.

There are two approaches in determining actuator configuration in controlled structures: the first is that the actuators are located based on system eigenvalues or controllability index [1], and the second, which is a more general, is that the actuators configuration is described as an optimization problem according to certain design criteria, and then solved by optimization techniques, the solution is the optimal placement of actuators [2]. Because the available positions for actuators in a structure are spatially discrete, the effect of adding or removing actuators on the overall structural dynamic responses is also discrete. Therefore, the optimal placement of actuators is a discrete non-linear optimization problem. This problem has been investigating by many researchers (e.g., [3–5]). But these researches were carried out based on an assumption: the number of sensors and actuators is determined in advance. In fact, the number of actuators has significant effects on the control performance of structural responses. Determination of the actuator number in a structure is affected by many factors such as total cost of control, type of actuator, controlled structures and so on. If the number and configuration of actuators in an actively controlled structure are considered simultaneously, the optimal design problem becomes more complicated. A multi-level genetic algorithm (MLGA) was developed to solve this type of problems under wind load [6] and the results were quite satisfactory.

In this paper, the combinatorial optimal design for optimizing the number of actuators and their configuration in actively controlled structures simultaneously is formulated as a three-level optimization problem. Then, the features of this design problem such as discreteness, non-linearity and multi-modality are discussed, and a three-level optimal design model is presented. In order to solve the optimization problem effectively, a simple two-level genetic algorithm (TLGA) is proposed. Both the three-level design model and the TLGA are verified through a case study in which an active controlled building is excited by a record of earthquake excitation.

2. Formulations and analysis

2.1. Analysis of the design problem

If the number of actuators and their configuration in structures are optimized simultaneously, the design problem becomes a three-level optimization problem (Fig. 1) with reference to the conventional design procedure. The first level implements the optimal control, the second level seeks the optimal configuration of actuators and the third level optimizes the number of actuators based on the control cost and control performance. Obviously, the three-level design problem is

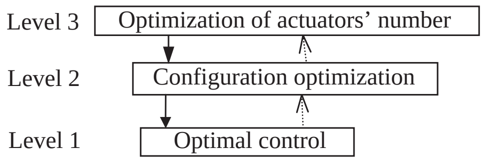

Fig. 1. Hierarchical structure of actuator number and configuration optimization.

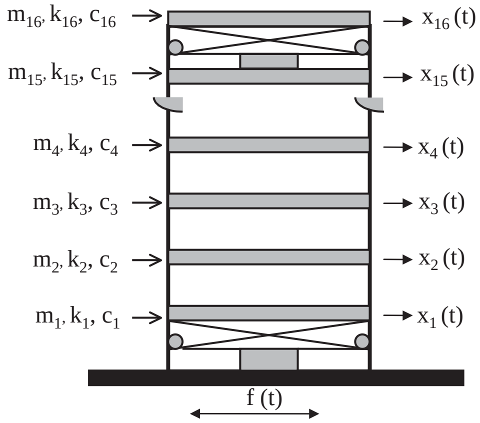

Fig. 2. A 16-storey building with active tendon actuators.

Table 1
Structural parameters of the example building

Floor	Mass (kN)	Stiffness (10^8 N/m)	Damping (kN s/m)
1	6723	2.56	27.0
2–13	5684	2.56	27.0
14–16	5559	1.74	27.0

much more complex than the optimization problem in which only the configuration of actuators is optimized. This can be seen from the following analysis:

(1) *Actuator configuration is optimized while the number of actuators is determined in advance*: In order to analyze the characteristics of actuator placement problem with certain number of actuators, a 16-storey building subjected to earthquake excitation (Fig. 2) is considered here. The structural parameters of this building are listed in Table 1. Two actuators are supposed to be installed to control the structural dynamic responses, and the positions of the two actuators are to be determined. The time history of Tianjin earthquake excitation (Fig. 3) recorded in 1976 is used as earthquake excitation for this building, and the linear quadratic regulator (LQR) control law is adopted in control implementation. It can be concluded from the results that the actuator configuration problem has the following features:

- Actuator positions in a structure are discrete, and their values are integer. Thus, the design space of this problem is not continuous but a set of discrete points.

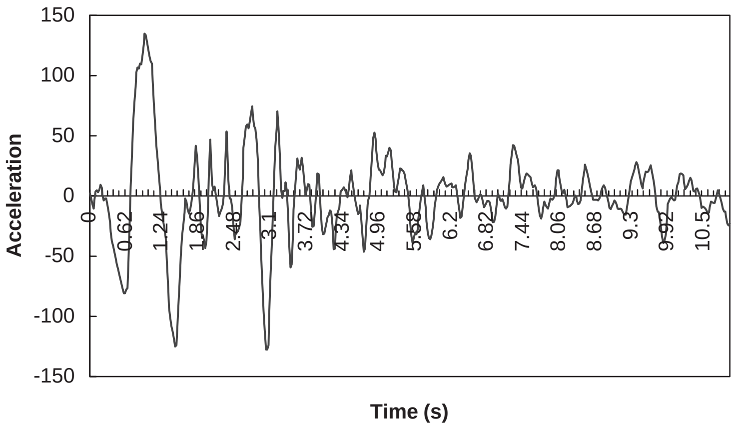

Fig. 3. Tianjin earthquake record.

- With reference to the conventional design procedure, it is a two-level design problem, the first level implements the optimal control and the second level optimizes the configuration of actuators.
- The objective function (maximum displacement of the building with different possible positions of actuators) is discrete, non-linear and multi-modal.

(2) *Actuator numbers and their configurations are optimized simultaneously*: If the actuator numbers and their configurations are optimized simultaneously, the design becomes a three-level procedure as shown in Fig. 1. For a complex multi-level optimization problem, two important solving techniques are decomposition and co-ordination. By decomposition, the problem is divided into sub-problems, and the coupling between the sub-problems prevents the direct solution of the overall problem. In general, decomposition models can be divided into two classes: process oriented decomposition and system oriented decomposition [7]. For the combinatorial optimization problem considered in this paper, process oriented decomposition is employed, where the determination of actuator numbers, positions and calculation of optimal control force are decomposed into different subsequent steps.

Co-ordination is a scheme of revising sub-problem optimization so that the final solution is that of the original problem. The interconnection of sub-problems may take many forms, but one of the most common forms is the hierarchical form in which a second-level unit co-ordinates the units on the level below. The goal of the second level is to co-ordinate the action of the first-level units so that the solution of the original problem is obtained.

The relationships among the design variables, objective functions and constraint functions in a multi-level optimization problem can be described by a problem matrix [8,7]: an entry v in position, i, j of this matrix indicate that the function i depends on the jth sub-vector of variables. The block-angular problem matrix of the three-level optimization problem considered in this study is shown in Fig. 4, which is of a structure of coupling variables. It can be described as the following non-linear programming problem:Find N, $\mathbf{P}$ and $\mathbf{U}(t)$ such that

$$\begin{aligned} \min \quad & \Phi = f(N, \mathbf{P}, \mathbf{U}(t)) \\ \text{s.t.} \quad & \mathbf{G}(N, \mathbf{P}, \mathbf{U}(t)) \leqslant 0, \end{aligned} \tag{1}$$

	level 3	level 2	level 1
variables:	N	**P**	**U** (t)
objective function	v	v	v
global constraints	v		
local constraints	v	v	
	v		v

Fig. 4. Block-angular problem matrix with coupling variables.

where Φ is objective function to be minimized and $\mathbf{G}$ represents constraint function. N, $\mathbf{P}$ and $\mathbf{U}(t)$ are design variables in the third, second and first level, respectively. In this paper, N in level 3 represents the number of actuators, variable set $\mathbf{P}$ in level 2 represents the positions of actuators, and variable set $\mathbf{U}(t)$ in level 1 means the control forces optimized through LQR strategy. Satisfying the optimality conditions for the first-, second- and third-level sub-problems in this case guarantees that the conditions for the original problem are also satisfied.

2.2. Formulation of the optimization problem

(1) *Sub-problem in level 3: optimal number of actuators*: This sub-problem aims to get the optimal number of actuators. Design variable in this level is the number of actuators, and the objective function may vary according to design purposes. In general, the objective function should include the factors such as cost, control target, etc. In this sub-problem, the cost of control system and the dynamic response of the building (maximum displacement), are to be minimized, and so, this sub-problem is described as

$$\begin{aligned} \min \quad & \Phi = \alpha_1 O(N, \mathbf{P}) + \alpha_2 \Omega(N) \\ \text{s.t.} \quad & \Omega(N) \leqslant [\Omega_{max}], \\ & O(N, \mathbf{P}) \leqslant [O_{max}], \\ & [N_{min}] \leqslant N \leqslant [N_{max}], \end{aligned} \tag{2}$$

where N is the design variable which represents the number of actuators in this level and it is also the global design variable in the optimization problem. Ω represents the control cost which is a function of N, and O is the structural displacement response which is a function of N and the actuator configuration $\mathbf{P}$. $[\Omega_{max}]$, $[O_{max}]$ and $[N_{max}]$ are allowable maximum values of the corresponding parameters, and $[N_{min}]$ is the allowable minimum number of actuators. α_1 and α_2 are two weighting coefficients.

Generally, the maximum floor displacement decreases with increase of the number of actuators, this can be seen from Fig. 5 which shows the variation of the top floor displacement of the building with the number of actuators. However, with the increase of the actuator numbers, the control cost also increases. For some kinds of actuators such as ATMD, the control cost grows sharply. Functions representing the control cost may be in various forms, and different forms generate different results. Currently, there is no representative form for the function. For convenience of analysis, this study assumes that the control cost is a parabolic function of the

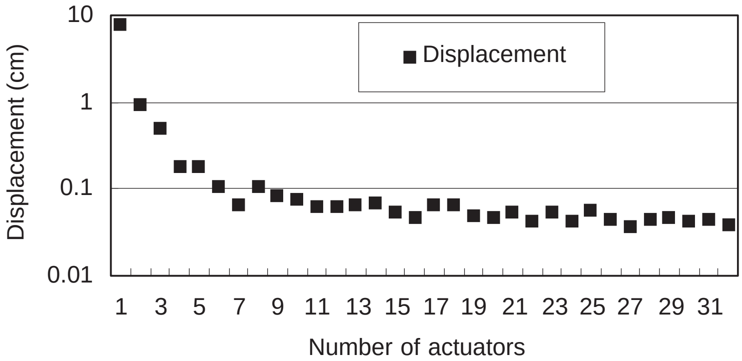

Fig. 5. Optimal top floor displacement with increase of actuators.

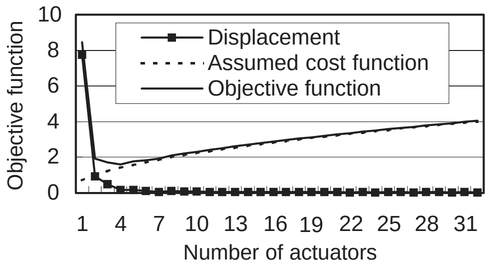

Fig. 6. Objective function curve in level 3.

number of actuators N, $\sqrt{2N}$. Fig. 6 shows variation of the objective function Φ (Eq. (2)) with the actuator numbers, which is obtained based on Eq. (2), Fig. 5 and the cost function with coefficients $\alpha_1 = 99.5$ and $\alpha_2 = 0.5$. In terms of these simulation conditions, it can be seen from Fig. 6 that there must be a reasonable number of actuators which makes the value of the objective function be a minimum.

(2) *Sub-problem in level 2: optimal configuration of actuators*: In the second level, configuration of N actuators obtained in level 3 is optimized. Objective function in this sub-problem is the top floor displacement response (relative to ground) of the building. Design variables are the position of actuators. Each actuator represents a design variable $p_i (i = 1, 2, \ldots, N)$; the value of p_i denotes that there is an actuator on the p_ith floor. The mathematical formulation of this sub-problem is described as

$$\begin{aligned} \min_{N,\mathbf{P}} \quad & O(N,\mathbf{P}) = \min_{N,\mathbf{P}} \max_{\substack{1 \leqslant j \leqslant q \\ 1 \leqslant l \leqslant h}} |x_j(t_l)| \\ & \mathbf{P} = \{p_1, p_2, \ldots, p_N\}^{\mathrm{T}} \\ \text{s.t.} \quad & O(N,\mathbf{P}) \leqslant [G] \text{ and } 1 \leqslant p_i \leqslant n, \quad i = 1, 2, \ldots, N, \end{aligned} \tag{3}$$

where $x_j(t_l)$ is the top floor displacement with the jth actuator permutation at time t_l, h is the total number of time intervals in a time history simulation, q is the total number of possible placements of actuators in the building, $[G]$ is the allowable maximum displacement response, n is the number of building floors.

(3) *Sub-problem of level 1: implementation of optimal control*: Formulation of the sub-problem in the first level is an optimal control problem. For an actively controlled building with n degrees of freedom, which is subjected to external loads $\mathbf{F}(t)$ and counteracted by control forces $\mathbf{U}(t)$, the governing equation of motion for the building can be expressed as [9,10]

$$\mathbf{M}\ddot{X}(t) + \mathbf{C}\dot{X}(t) + \mathbf{K}\mathbf{X}(t) = \mathbf{D}\mathbf{U}(t) + \mathbf{F}(t), \tag{4}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the $n \times n$ mass, damping and stiffness matrix, respectively. $\mathbf{X}(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}}$ is an $n \times 1$ displacement vector with $x_j(t)$ being the deformation of the jth floor. $\mathbf{U}(t)$ is an $N \times 1$ control force vector consisting of N control forces, and is the function of state-space $\mathbf{Z}(t)$ and time t; $\mathbf{F}(t)$ is an earthquake ground excitation force vector, and $\mathbf{D}$ is an $n \times N$ matrix denoting the location of N actuators. Represented in a state-space form, the second order differential equation (4) is rewritten as a first order differential equation

$$\dot{Z}(t) = \mathbf{A}\mathbf{Z}(t) + \mathbf{B}\mathbf{U}(t) + \mathbf{E}, \tag{5}$$

where $Z(t) = [X(t), \dot{X}(t)]^{\mathrm{T}}$ is a $2n \times 1$ state vector.

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \text{ is a } 2n\times 2n \text{ system matrix,}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ M^{-1}D \end{bmatrix} \text{ is a } 2n \times N \text{ control matrix,}$$

$$\mathbf{E} = \begin{bmatrix} 0 \\ M^{-1}F(t) \end{bmatrix} \text{ is a } 2n \times n \text{ load matrix.}$$

The control system described by the state equation (5) is a linear and time-invariant system. The current optimal active control forces can be determined subjected to the condition that the quadratic objective function (performance index) $J(t)$ is minimized. Thus, the first-level optimization problem can be described as

$$\begin{aligned} \min \quad & J(N, \mathbf{P}, \mathbf{U}(t)) = 0.5 \int_0^{tf} [\mathbf{Z}(t)^{\mathrm{T}}\mathbf{Q}\mathbf{Z}(t) + \mathbf{U}(t)^{\mathrm{T}}\mathbf{R}\mathbf{U}(t)]\,\mathrm{d}t \\ \text{s.t.} \quad & \mathbf{U}(t) \leqslant [\mathbf{U}_{max}], \\ & \text{and } \mathbf{M}\ddot{X}(t) + \mathbf{C}\dot{X}(t) + \mathbf{K}\mathbf{X}(t) = \mathbf{D}\mathbf{U}(t) + \mathbf{F}(t). \end{aligned} \tag{6}$$

In Eq. (6), $\mathbf{Q}$ is a $2n \times 2n$ positive semi-definite weighting matrix. $\mathbf{R}$ is an $N \times N$ symmetric positive definite weighting matrix for the input control forces. $[\mathbf{U}_{max}]$ is a vector of maximum control force of actuators. By applying the optimal control theory and assuming that the control force vector $\mathbf{U}(t)$ may be generated by feedback of the state vector $\mathbf{X}(t)$ and $\dot{X}(t)$ alone, the optimal control force is

$$\mathbf{U}^*(t) = -\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{W}\mathbf{Z}(t), \tag{7}$$

where $\mathbf{W}$ is the Riccati matrix that can be obtained by solving the Riccati matrix equation

$$\mathbf{W}\mathbf{A} + \mathbf{A}^{\mathrm{T}}\mathbf{W} - \mathbf{W}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{W} + \mathbf{Q} = 0. \tag{8}$$

Substituting Eq. (7) into Eq. (4) yields the following equation of motion:

$$\mathbf{M}\ddot{X}(t) + (\mathbf{C} + \Delta\mathbf{C})\dot{X}(t) + (\mathbf{K} + \Delta\mathbf{K})\mathbf{X}(t) = \mathbf{F}(t), \tag{9}$$

where $\Delta\mathbf{C}$ and $\Delta\mathbf{K}$ represent the damping gain and stiffness gain, respectively. The solution of Eq. (9), which is the response of the structure, can be obtained by solving Eq. (9) using the Newmark method or other methods [11].

3. Solution procedures and TLGA

Based on the formulation presented above, the solution procedure of the three-level optimization problem is:

(1) Determination of the value scope of actuator numbers according to the design demands and constraints. The third-level optimization sub-problem is implemented first, and the initial value of the number of actuators, N_0, is obtained.
(2) The sub-problem in level 2 is solved based on the feed-forward message N_0, then following is the first-level sub-problem. For the sub-problem in level 1, it is solved according to optimal control algorithm and may be included in the second-level sub-problem. Solutions of the sub-problems in levels 2 and 1 are the optimal configuration of the N_0 actuators and the maximum floor displacement under the optimal configuration.
(3) The results obtained in level 2 are fed back to level 3, and then the second solving loop starts. The next value of the number of actuators, N_1, is obtained based on the redesign rules and optimization design algorithm. The process is repeated until the final optimum has been reached.

It is difficult for the above solution procedure to solve the three-level optimization problem efficiently. First, the sub-problem in level 1 is an optimization problem with continuous objective function and design space, but in levels 2 and 3, the optimization problems have features of discrete design variables, non-continuous design space, non-linear and multi-modal objective functions. Thus, conventional optimization methods cannot solve the problem effectively. Second, because the optimization problem has a hierarchical structure, there is no representative method that can solve this problem efficiently. Additionally, computational cost could be very high if the optimal method is not selected properly. In fact, solving optimization problem with features of discreteness, non-linearity and multi-modality is still an important and difficult research branch in the field of optimization theory and practice. In the following section, a TLGA is proposed for solving the three-level optimization problem considered in this study.

3.1. Two-level genetic algorithm

(1) *Genetic algorithms*: GA is a kind of heuristic random search technique based on the concept of natural selection and natural genetics of population, and so they are of "population-based" method of searching large combinatorial design spaces to find the optimum combination of design variables. Detailed discussions on the mechanisms of GA can be found in Refs. [12,13]. As an

algorithm, GA is different from the conventional optimization methods encountered in engineering optimization problems in the following ways:

- GA works with a coding set of variables and not with the variables themselves.
- GA searches from a population of points rather than by improving a single point, GA can reduce the possible chance of being trapped into a local optimum.
- GA uses objective function information without any gradient information.
- GA uses probabilistic transition rules whereas traditional methods use gradient information.

GA has been proved to be a versatile and effective approach for solving optimization problems such as combinatorial and discrete optimization problems [14], mixed-discrete non-linear optimization problems [15]. Nevertheless, there are many cases in which the simple GA does not perform particularly well, and various modified GAs and hybrid GAs have been proposed. For example, Miller et al. [16] presented local improvement operators in pure GA for NP-hard optimization problems. Jenkins [17] presented the methods of enhancing GA with control adaptation and described the development of a combinatorial space reduction heuristic. When GA is used to solve the three-level optimization problem under consideration, a direct and simple way is to treat the second- and third-level sub-problems as a single optimization problem, in which all variables are coded in an individual (a chromosome) and a sharing objective function is selected. It is clear that this way cannot solve the problem effectively because it cannot handle the interactions between levels and/or sub-problems in the three-level structure. However, it will be shown that another approach, TLGA, can meet the needs and solve the problem effectively.

(2) *Two-level genetic algorithm*: Fig. 7 shows the structure of TLGA. In level GA1, there is a GA module (GA_{11}), but there are many modules (GA_{2i}) in level GA2. Each module corresponds to a sub-problem, or in other words each module solves a sub-problem. Because of the coupling among sub-problems in the same level, and the interactions between modules in adjacent levels, these modules are not independent. A module in upper level acts not only as a solver for the corresponding sub-problem, but also as a co-ordinator or controller for the implementation of the modules in the lower level. Each module is a relative independent GA module; it has all genetic operators such as population, selection, crossover, mutation, etc. But the implementation of each module is affected by its adjacent modules because it needs message from these modules. The implementation of TLGA and the interactions between modules will be described in detail through the following case study.

When TLGA is applied to solve the optimization problem considered in this study, module GA_{11} in level GA1 is used to solve the sub-problem of optimal actuator numbers, and the module GA_{21} in level GA2 is used to solve the optimal placement problem of actuators in the building. Thus, only one module in the lower level GA is needed. The sub-problem of optimal control force in the first level (Fig. 1) is solved using the optimal control algorithm and is included in the

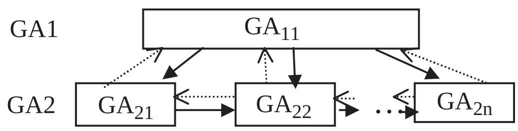

Fig. 7. Structure of the TLGA.

Table 2
Coding and parameters of TLGA

	GA_{11}	GA_{21}
Variables	N	$P_1, P_2, P_3, \ldots, P_N$
Range of value	$[1, N_{max}]$, $N_{max} = 32$	$[1, n]$, $n = 16$
Coding method	Binary	Binary
Binary bits of each variable	5	4
Example of chromosome	10011	0110 0010 0111 1110 1010 0101 1100 0111
Size of population	$P_{op1} = 20$	$P_{op2} = 20 + N \times 2$
Fitness function	$F_{it1} = 10 - \Phi$	$F_{it2} = 1.0–O(N, \mathbf{P})$
Value of genetic parameters	$P_{c1} = 0.7$, $P_{m1} = 0.055$, $G_{en1} = 20$	$P_{c2} = 0.6$, $P_{\mathrm{m}2} = 0.05$, $G_{en2} = 40$
Optimization solution	4	Floor: 9, 14, 15 and 16
Optimization value of objective function	1.59	0.1785 (cm)

configuration optimization sub-problem. The interactions between modules GA_{11} and GA_{21} include:

- The number of actuators, N, obtained in module GA_{11} is sent to module GA_{21}, which determines the number of variables in module GA_{21} according to the problem formulation in Section 2. Thus, the chromosome length of individuals, population size and parameters of genetic operators in module GA_{21} will vary with actuator number N.
- Module GA_{21} feeds the following information back to module GA_{11} in the upper level: (1) maximum displacement response when the N actuators are located at the optimal positions, and (2) terminal and convergent parameters in module GA_{21} such as final evolutionary generation, errors, optimal placement of the N actuators, etc. The feedback information is used for the next computing loop by module GA_{11}. Optimal control sub-problem in level 1 is solved in module GA_{21} when evaluation of each individual in GA_{21} is made.

Table 2 shows the coding, parameters and other information about the application of TLGA to the combinatorial optimization problem under consideration. Due to discreteness and small search space of the variables, binary-coding method is adopted for the encoding of chromosome in levels GA1 and GA2. Parameters of genetic operators in module GA_{11} are expressed as probability of crossover P_{c1}, probability of mutation P_{m1}, size of population P_{op1}, generation of evolution G_{en1}. Correspondingly, parameters in module GA_{21} are expressed as P_{c2}, P_{m2}, P_{op2} and G_{en2}, respectively.

4. Numerical example

The example building described in Section 2 is used in this case study to verify the effectiveness of the proposed hierarchical model and the TLGA for the combinatorial optimization problem considered in this study. The parameters of genetic operators in the two levels are selected and showed in Table 2. Fitness functions in modules GA_{11} and GA_{21} are

$$GA_{11} : F_{it1} = \beta_1 - [\alpha_1 O(N, \mathbf{P}) + \alpha_2 \Omega(N)], \tag{10}$$

where the coefficients $\alpha_1 = 99.5$ and $\alpha_2 = 0.5$. Control cost function is simply assumed as a parabola function:

$$\Omega(N) = \sqrt{2N}, \tag{11}$$

$$\mathrm{GA}_{21} : F_{it2} = \beta_2 - O(N, \mathbf{P}), \tag{12}$$

where β_1 and β_2 are two coefficients that transfer the minimum problems into the maximum optimization problems. The population size in module GA_{11} is set to a value of 20. But the size of the population in module GA_{21} is expressed as a function of the number of actuators, N, because more variables should correspond to a relatively large population to obtain the global optimization solution. The function is expressed as

$$P_{op2} = 20 + N \times 2. \tag{13}$$

By using the software developed by the authors for the proposed TLGA and the optimization model established in this paper, the results of the numerical example are obtained and listed in Table 2. The optimal design result of sub-problem in level 3 (solved by module GA_{11}), optimal number of actuators in the building, is 4 and the corresponding objective value is 1.592714. Comparing this result to that shown in Fig. 6 which is obtained with the same simulation parameters as those in the case study, it can be seen that the two results are the same. Thus, it is concluded that the result is reliable.

The solution of the sub-problem in level 2 (solved by GA_{21}) is that the four actuators are placed at floors 9, 14, 15 and 16, respectively, when the maximum top floor displacement, which is obtained from the analysis of time history response under different placements of four actuators, achieves a minimum of 0.1785 cm. Fig. 8 shows the evolution process of module GA_{11} and Fig. 9 shows that in module GA_{21}. The variation of fitness function with the increase of generation is also shown in Figs. 8 and 9. From Fig. 8 it can be seen that the maximum fitness in each generation reaches the maximum value quickly, and that the variation of average fitness is scaled down with the increase of evolution generation. That is to say, the objective function gets to the minimum quickly. Fig. 9 indicates the changes of the top floor displacement and maximum fitness with evolutionary process in module GA_{21}. From this figure it can be seen that the top floor displacement achieves the minimum value quickly with the increase of genetic generation.

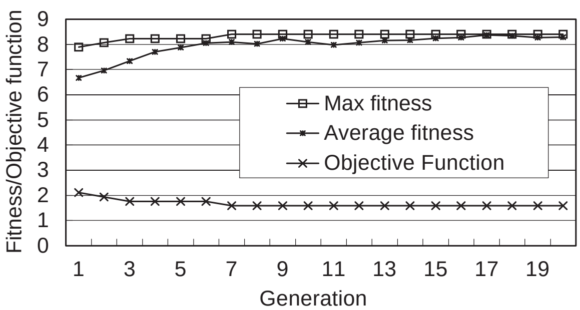

Fig. 8. Variation of objective function and fitness with evolutionary generation in module GA_{11}.

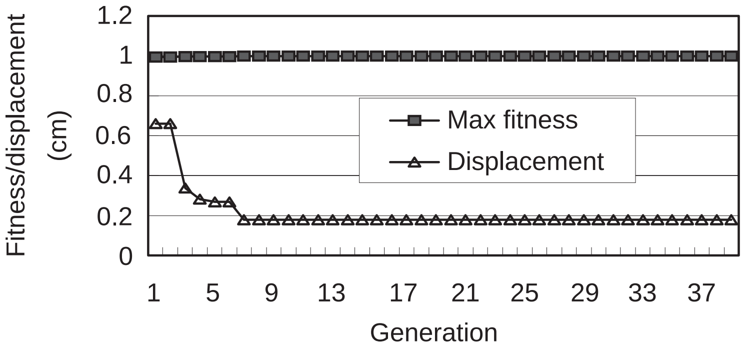

Fig. 9. Variation of fitness and top floor displacement with evolutionary generation in module GA_{21}.

Table 3
Number and optimal placement of actuators and the corresponding displacement responses

No. of actuators	Optimal placement	Displacement (cm)
1	10	7.7573
2	8 15	0.9212
3	9 15 16	0.4955
4	9 14 15 16	0.1785
5	3 11 15 14 16	0.1856
6	8 10 14 15 16 16	0.1067
7	8 9 14 15 16 16 16	0.0664
8	4 8 13 13 14 15 16 16	0.1066
9	2 8 13 14 15 16 16 16 16	0.0833
10	1 4 7 12 14 15 16 16 16 16	0.0759

Table 3 shows the optimal placement of different number of actuators as well as the corresponding top floor displacement response. These results are obtained by module GA_{21} in this paper.

5. Discussion

GA parameters have significant effects on the algorithm performance such as convergence and its speed. Discussions and studies on GA parameters can be found in Refs. [18,13,19,20]. However, the knowledge about the proper selection of GA parameters is still only fragmentary and has a rather empirical background. In this numerical simulation, parameters of GA operators in each GA level (or each module) are set constants during evolution process based on several times of adjustments with the exception of population size in GA_{21}, and the optimal solutions are obtained and then verified to be correct.

Among the genetic operator parameters, the population size is one of the most important choices faced by any user of GAs, since it has strong influence on the GA simulation cost and the two important issues in the evolution process: population diversity and selective pressure. Sometimes it may be critical in many applications. If the population size is too small and the search space is large, the GA may converge quickly, and converge to local optimum with high possibilities. On the other hand, if the population size is too large, the GA may waste computational resources. Furthermore, if the population size is too large and the search space is small, the global optimal solution may be included in the first generation. It might be the best way for setting population size to let it self-tune according to the GA actual needs, i.e., at different stages of the search process different sizes of the population might be optimal. In this study, the search space of module GA_{21} varies with the number of actuators determined in the upper level GA1: the more the number of actuators, the larger the search space. Thus, keeping the population size of module GA_{21}, P_{op2}, as a constant is not reasonable. After several trials, satisfactory results are obtained by setting P_{op2}, as a function of design variables in module GA_{21} (Eq. (13)). Also, because the search spaces in modules GA_{11} and GA_{21} are small, quick convergence is obtained as shown in Figs. 8 and 9. Changing genetic parameters will change the convergence of the algorithm.

Due to the complexity and differences of TLGA from single GA, the study on genetic operator parameters in different GA levels and interactions of parameters among all GA levels becomes more important. In order to examine the performance of the TLGA, a single-level GA is developed to solve the problem considered in the numerical example. Comparing the implementing process of the TLGA to that of the single-level GA, it is found that the single-level GA takes less time than TLGA in the implementing process. The main reason is that less computing is required in the single-level GA. However, it is found that more attentions need to be paid to the operation design of the single-level GA to make it possible for solving such a problem.

To verify the optimum results, an optimal search method, the uniform design method [21] is used to check the optimal actuator locations for given actuator numbers. The results from the uniform design method match exactly with those from the TLGA method. It is noted that other optimal search methods can be used to solve the problem where the configuration of actuators is being optimized. The TLGA method, however, is used to solve the combinational optimization problem, where both the number and the configuration of actuators are optimized simultaneously.

6. Conclusions

From this study it can be concluded that: (1) the combinatorial optimization problem, optimizing the number and configuration of actuators in actively controlled structures simultaneously, is a three-level optimal design problem. This design problem has features of mixed-discrete design variables, and non-linear and multi-modal objective functions. (2) The proposed three-level design model and the formulation in each level give a reasonable description for the combinatorial optimization problem. (3) The TLGA presented in this paper is verified as an effective algorithm for solving this kind of optimal design problem.

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