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Propagation characteristic of Rayleigh waves in orthotropic fluid-saturated porous media

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Abstract

In this paper, the influence of anisotropy of the solid skeleton on the propagation characteristic of Rayleigh waves in orthotropic fluid-saturated porous media is studied from a more general sense based on Biot's theory. Firstly, the governing equations for orthotropic fluid-saturated porous media are derived. Then the three-dimensional complex characteristic equations for Rayleigh waves are deduced and their existence conditions are given. Based on the characteristic equations, the Rayleigh wave speeds along arbitrary directions and particle traces in arbitrary sagittal planes are numerically calculated. The effects of anisotropy of the solid skeleton on the propagation characteristic of Rayleigh waves are analyzed in detail. The results show that the Rayleigh waves display different characteristic in orthotropic fluid-saturated porous media from that they have exhibited in isotropic or transversely isotropic fluid-saturated porous media.

1. Introduction

In pure solid half-space, the discovery of Rayleigh waves is tightly connected with the earthquake spectrum analysis [1]. Owing to the slower attenuation of the energy than that of the body waves and the characteristic that it propagates along the surface, Rayleigh waves cause destructive vibration to the structures. Since most of the geological material can be treated as some kind of fluid-saturated porous media and the actual media shows anisotropy, clarification of the characteristic of Rayleigh waves in anisotropic fluid-saturated porous media has significant practical meaning in many fields such as earthquake engineering, soil dynamics, geophysics and hydrology.

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However, due to the non-linearity, it is difficult to get exact solutions of the characteristic equations of Rayleigh waves in anisotropic media. Especially for fluid-saturated porous media, the existence of the fluid phase in governing equations increases the order of the characteristic equations and makes the problem more complicated. As a result, though much research [2,3] has been carried out on the propagation of Rayleigh waves in anisotropic solids, less has been done on the Rayleigh waves in anisotropic fluid-saturated porous media. Deresiewicz [4] first researched the Rayleigh waves in porous media based on Biot's theory. Johns [5] studied the Rayleigh waves in isotropic saturated soil, but only one kind of longitudinal waves was considered in his characteristic equations and thus the equations obtained by him are not complete. Tajuddin [6] considered two kinds of the longitudinal waves and got the characteristic equations for isotropic fluid-saturated porous media. Hirai [7] analyzed the Rayleigh waves in isotropic layered fluid-saturated porous media by finite element methods, and the effects of the dynamic permeability on the propagation characteristic of Rayleigh waves are discussed. Sharma and Gogna [8] studied the Rayleigh waves in transversely isotropic porous media but the dissipation is ignored. Liu and de Boer [9] studied the Rayleigh waves in isotropic fluid-saturated porous media using mixture theory. Kumar [10] analyzed the influence of the heterogeneous base on the propagation of Rayleigh waves in isotropic fluid-saturated porous layer. Liu and Liu [11,12] discussed the fluid viscous effects on the propagation characteristic of Rayleigh waves in transversely isotropic fluid-saturated porous media. From the discussion above we can see that the researches about Rayleigh waves in fluid-saturated porous media are mainly focused on the isotropic and transversely isotropic ones. However, according to the research results of mutli-scale analysis, geological materials with random cracks/joints distribution, such as rocks, can be classified as some kinds of transversely isotropic or orthotropic materials after statistical treatment [13]. By now, as per the knowledge of authors, the works about the Rayleigh waves in the orthotropic fluid-saturated porous media have not been available in the literatures. Obviously, orthotropy of the solid skeleton has what effects on the propagation of Rayleigh waves is an important problem that should be clarified. Here we try to discuss the effect of anisotropy of the solid skeleton on the propagation characteristic of Rayleigh waves and show more features of Rayleigh waves in anisotropic fluid-saturated porous media.

In the present paper, the characteristic analysis of Rayleigh waves in orthotropic fluid-saturated porous media is carried out in the next two sections. The governing equations in orthotropic fluid-saturated porous media are obtained based on Biot's theory [14] (Section 2). The three-dimensional complex characteristic equations of Rayleigh waves are derived and their existence conditions are given (Section 3). In Section 4, the Rayleigh wave speeds along arbitrary directions and the particle traces in arbitrary sagittal planes are numerically calculated. The effects of anisotropy of the solid skeleton on the propagation characteristic of Rayleigh waves are discussed in the final section and the conclusion is given.

2. Characteristic analysis

Following Biot [14], ignoring the dispersion caused by the fluid, the equations of motion for anisotropic fluid-saturated porous media can be written in terms of Cartesian co-ordinates as

$$\tau_{ij,j} = \rho \ddot{u}_i + \rho_f \ddot{w}_i, \tag{1a}$$

$$-p_{,i} = \rho_f \ddot{u}_i + \frac{\rho_f}{\phi}\ddot{w}_i, \tag{1b}$$

where τ_{ij} are the stress components of the solid skeleton, p the pore pressure in the fluid, $w_i = \phi(U_i - u_i)$ the displacement components of the fluid relative to the solid skeleton in which u_i and U_i are the displacement components of the solid skeleton and the saturated pore fluid, respectively, ϕ the porosity, $\rho = (1-\phi)\rho_s + \phi\rho_f$ the composite density in which ρ_s and ρ_f are the densities of the solid skeleton and the pore fluid. Here $i, j = 1, 2, 3$ are corresponding to x, y, z.

The constitutive equations for anisotropic fluid-saturated porous media may be stated as [14]

$$\begin{aligned} \tau_{ij} &= A_{ijkl}e_{kl} + M_{ij}\varsigma, \\ p &= M_{ij}e_{ij} + M\varsigma, \\ e_{ij} &= \tfrac{1}{2}(u_{i,j} + u_{j,i}), \\ \varsigma &= -w_{i,i}, \end{aligned} \tag{2}$$

where e_{ij} are the strain components of the solid skeleton, ς the volumetric strain of the fluid. A_{ijkl} ($A_{ijkl} = A_{jikl} = A_{klij}$, the number of independent variables is 21), M_{ij} ($M_{ij} = M_{ji}$, the number of independent variables is 6) and M are parameters for the anisotropic solid skeleton and pore fluids, the independent variables are 28. For orthotropic fluid-saturated porous media, the stress–strain relationship can be expressed as

$$\begin{Bmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_{zz} \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \\ p \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 & 0 & 0 & M_{11} \\ A_{12} & A_{22} & A_{23} & 0 & 0 & 0 & M_{22} \\ A_{13} & A_{23} & A_{33} & 0 & 0 & 0 & M_{33} \\ 0 & 0 & 0 & 2A_{44} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2A_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2A_{66} & 0 \\ M_{11} & M_{22} & M_{33} & 0 & 0 & 0 & M \end{bmatrix} \begin{Bmatrix} e_{xx} \\ e_{yy} \\ e_{zz} \\ e_{yz} \\ e_{zx} \\ e_{xy} \\ \varsigma \end{Bmatrix}, \tag{3}$$

where A_{11}, A_{12}, A_{13}, A_{22}, A_{23}, A_{33}, A_{44}, A_{55}, A_{66}, M_{11}, M_{22}, M_{33}, and M are 13 independent elastic parameters.

Substituting Eq. (3) into Eq. (1), the governing equations for orthotropic fluid-saturated porous media can be expressed in the following form:

$$\begin{aligned} &A_{11}\frac{\partial^2 u_x}{\partial x^2} + A_{66}\frac{\partial^2 u_x}{\partial y^2} + A_{55}\frac{\partial^2 u_x}{\partial z^2} + (A_{12} + A_{66})\frac{\partial^2 u_y}{\partial x \partial y} + (A_{13} + A_{55})\frac{\partial^2 u_z}{\partial x \partial z} \\ &\quad - M_{11}\left(\frac{\partial^2 w_x}{\partial x^2} + \frac{\partial^2 w_y}{\partial x \partial y} + \frac{\partial^2 w_z}{\partial x \partial z}\right) = \rho \ddot{u}_x + \rho_f \ddot{w}_x, \end{aligned} \tag{4a}$$

$$\begin{aligned} &(A_{12} + A_{66})\frac{\partial^2 u_x}{\partial x \partial y} + A_{66}\frac{\partial^2 u_y}{\partial x^2} + A_{22}\frac{\partial^2 u_y}{\partial y^2} + A_{44}\frac{\partial^2 u_y}{\partial z^2} + (A_{23} + A_{44})\frac{\partial^2 u_z}{\partial y \partial z} \\ &\quad - M_{22}\left(\frac{\partial^2 w_x}{\partial x \partial y} + \frac{\partial^2 w_y}{\partial y^2} + \frac{\partial^2 w_z}{\partial y \partial z}\right) = \rho \ddot{u}_y + \rho_f \ddot{w}_y, \end{aligned} \tag{4b}$$

$$
\begin{aligned}
&(A_{13}+A_{55})\frac{\partial^2 u_x}{\partial x\partial z}+(A_{23}+A_{44})\frac{\partial^2 u_y}{\partial y\partial z}+A_{55}\frac{\partial^2 u_z}{\partial x^2}+A_{44}\frac{\partial^2 u_z}{\partial y^2}+A_{33}\frac{\partial^2 u_z}{\partial z^2}\\
&\quad -M_{33}\left(\frac{\partial^2 w_x}{\partial x\partial z}+\frac{\partial^2 w_y}{\partial y\partial z}+\frac{\partial^2 w_z}{\partial z^2}\right)=\rho\ddot{u}_z+\rho_f\ddot{w}_z,
\end{aligned}
\tag{4c}
$$

$$
\begin{aligned}
&M_{11}\frac{\partial^2 u_x}{\partial x^2}+M_{22}\frac{\partial^2 u_y}{\partial x\partial y}+M_{33}\frac{\partial^2 u_z}{\partial x\partial z}-M\left(\frac{\partial^2 w_x}{\partial x^2}+\frac{\partial^2 w_y}{\partial x\partial y}+\frac{\partial^2 w_z}{\partial x\partial z}\right)\\
&\quad =-\rho_f\ddot{u}_x-\frac{\rho_f}{\phi}\ddot{w}_x,
\end{aligned}
\tag{4d}
$$

$$
\begin{aligned}
&M_{11}\frac{\partial^2 u_x}{\partial x\partial y}+M_{22}\frac{\partial^2 u_y}{\partial y^2}+M_{33}\frac{\partial^2 u_z}{\partial y\partial z}-M\left(\frac{\partial^2 w_x}{\partial x\partial y}+\frac{\partial^2 w_y}{\partial y^2}+\frac{\partial^2 w_z}{\partial y\partial z}\right)\\
&\quad =-\rho_f\ddot{u}_y-\frac{\rho_f}{\phi}\ddot{w}_y
\end{aligned}
\tag{4e}
$$

$$
\begin{aligned}
&M_{11}\frac{\partial^2 u_x}{\partial x\partial z}+M_{22}\frac{\partial^2 u_y}{\partial y\partial z}+M_{33}\frac{\partial^2 u_z}{\partial z^2}-M\left(\frac{\partial^2 w_x}{\partial x\partial z}+\frac{\partial^2 w_y}{\partial y\partial z}+\frac{\partial^2 w_z}{\partial z^2}\right)\\
&\quad =-\rho_f\ddot{u}_z-\frac{\rho_f}{\phi}\ddot{w}_z.
\end{aligned}
\tag{4f}
$$

According to Simon and Paul [15], the material coefficients are related to the elastic parameters of the solid skeleton C_{ijkl}, the bulk modulus of solid grains K_s, and the bulk modulus of fluids K_f by

$$
\begin{aligned}
A_{11}&=C_{11}+\alpha_1^2 M,\quad A_{12}=C_{12}+\alpha_1\alpha_2 M,\quad A_{13}=C_{13}+\alpha_1\alpha_3 M,\\
A_{22}&=C_{22}+\alpha_2^2 M,\quad A_{23}=C_{23}+\alpha_2\alpha_3 M,\quad A_{33}=C_{33}+\alpha_3^2 M,\\
A_{44}&=C_{44},\quad A_{55}=C_{55},\quad A_{66}=C_{66},\\
M_{11}&=-M\alpha_1,\quad M_{22}=-M\alpha_2,\quad M_{33}=-M\alpha_3,\\
M&=\left(\frac{\phi}{K_f}+\frac{1-\phi}{K_s}-\frac{C_{11}+C_{22}+C_{33}+2C_{12}+2C_{13}+2C_{23}}{9\,K_s^2}\right)^{-1},\\
\alpha_1&=1-\frac{C_{11}+C_{12}+C_{13}}{3\,K_s},\quad \alpha_2=1-\frac{C_{12}+C_{22}+C_{23}}{3\,K_s},\quad \alpha_3=1-\frac{C_{13}+C_{23}+C_{33}}{3\,K_s}.
\end{aligned}
$$

It is clear that when $C_{11}=C_{22}$, $C_{13}=C_{23}$, $C_{44}=C_{55}$ and $C_{66}=(C_{11}-C_{12})/2$, Eq. (4) degenerates to the governing equations for the transversely isotropic fluid-saturated porous media [11]. When $C_{11}=C_{22}=C_{33}$, $C_{12}=C_{13}=C_{23}$, $C_{44}=C_{55}=C_{66}$ and $C_{44}=(C_{11}=C_{12})/2$, Eq. (4) becomes the governing equations for isotropic situation. Thus, the characteristic equations for Rayleigh waves in isotropic and transversely isotropic fluid-saturated porous media can be obtained through appropriate degeneration of the equations for orthotropic fluid-saturated porous media.

3. Characteristic equations of Rayleigh waves

Assuming the wave motion is caused by a pulse at the infinite distance (for example, earthquake), the frequency is positive. Establish Cartesian co-ordinates with plane xoy the free surface of the half-plane and positive z-axis pointing inside the media. We seek the solutions of Eq. (4) that represent the Rayleigh waves traveling in a half-space with symmetry about z-axis in the form

$$[u_i, w_i] = [A_i, B_i]\exp\mathrm{i}k(l_1x + l_2y + rz - ct), \tag{5}$$

where A_i and B_i are amplitudes of displacement components, i the imaginary unit, l_i the direction cosines, r a complex attenuation coefficient whose imaginary part should be positive corresponding to the half-space $z>0$, $k=\omega/c$ the wave number, in which c is the Rayleigh wave speed and ω the frequency.

Substitution of Eq. (5) into Eq. (4) leads to simultaneous equations

$$[d_{kl}]\{A_x, A_y, A_z, B_x, B_y, B_z\}^{\mathrm{T}} = 0, \tag{6}$$

where the elements d_{kl} of the 6×6 complex matrix are given as

$$d_{11} = \rho\omega^2 - (A_{11}l_1^2 + A_{66}l_2^2 + A_{55}r^2)k^2, \quad d_{12} = d_{21} = -(A_{12}+A_{66})l_1l_2k^2,$$
$$d_{13} = d_{31} = -(A_{13}+A_{55})l_1rk^2, \quad d_{14} = d_{41} = \rho_f\omega^2 + M_{11}l_1^2k^2,$$
$$d_{15} = d_{51} = M_{11}l_1l_2k^2, \quad d_{16} = d_{61} = M_{11}l_1rk^2,$$
$$d_{22} = \rho\omega^2 - (A_{66}l_1^2 + A_{22}l_2^2 + A_{44}r^2)k^2, \quad d_{23} = d_{32} = -(A_{23}+A_{44})l_2rk^2,$$
$$d_{24} = d_{42} = M_{22}l_1l_2k^2, \quad d_{25} = d_{52} = \rho_f\omega^2 + M_{22}l_2^2k^2,$$
$$d_{26} = d_{62} = M_{22}l_2rk^2, \quad d_{33} = \rho\omega^2 - (A_{55}l_1^2 + A_{44}l_2^2 + A_{33}r^2)k^2,$$
$$d_{34} = d_{43} = M_{33}l_1rk^2, \quad d_{35} = d_{53} = M_{33}l_2rk^2,$$
$$d_{36} = d_{63} = \rho_f\omega^2 + M_{33}r^2k^2, \quad d_{44} = \rho_f\omega^2/\phi - Ml_1^2k^2,$$
$$d_{45} = d_{54} = -Ml_1l_2k^2, \quad d_{46} = d_{64} = -Ml_1rk^2,$$
$$d_{55} = \rho_f\omega^2/\phi - Ml_2^2k^2, \quad d_{56} = d_{65} = -Ml_2rk^2,$$
$$d_{66} = \rho_f\omega^2/\phi - Mr^2k^2.$$

In order to have a non-trivial solution of Eq. (6), we have $\mathrm{Det}[d_{kl}] = 0$, i.e.,

$$a_1r^8 + a_2r^6 + a_3r^4 + a_4r^2 + a_5 = 0. \tag{7}$$

The coefficients a_m ($m = 1, \ldots, 5$) are very miscellaneous, the explicit forms are not given here.

In isotropic or transversely isotropic situations (Eq. (7) degenerates to four order or six order, respectively), the solutions of Eq. (7) is pure imaginary [11,12]. But in the orthotropic porous media, the roots of Eq. (7) are in general complex. Assuming $r = \xi + \mathrm{i}\eta$, Eq. (5) can be rewritten as

$$[u_i, w_i] = [A_i, B_i]\exp(-k_r^Iz)\exp(\mathrm{i}k_r^Rz)\exp\mathrm{i}k(l_1x + l_2y - ct), \tag{8}$$

with $k_r = k(\xi + i\eta) = k_r^R + ik_r^I$. From Eq. (8) it can be obtained that the disturbance direction of Rayleigh waves inclines to the free surface with the angle of $\alpha = \cos^{-1}\xi$. Moreover, it is apparent that

$$|k_r^R|^2 - |k_r^{\rm I}|^2 = k^2(\xi^2 - \eta^2), \quad |k_r^R||k_r^{\rm I}|\cos\beta = \tfrac{1}{2}k^2\xi\eta. \tag{9}$$

As shown in Fig. 1, though the dispersion caused by the fluid viscosity is ignored, the phase plane (the plane vertical to the vector k_r^R) and the amplitude plane (the plane vertical to the vector k_r^I) are not parallel to each other any more. The maximal attenuation is not along the direction of the wave propagation, but along the direction of the vector k_r^I. The Rayleigh wave in orthotropic fluid-saturated porous media is a non-homogenous wave.

The condition to be Rayleigh waves requires the imaginary part of r being positive in Eq. (8). Assume $r^{(n)} = \xi^{(n)} + i\eta^{(n)}$ $(n = 1, \ldots, 4)$ be the four of the roots of Eq. (7) with positive imaginary parts. If $\Delta_r^{(n)}$ are normalized solutions of Eq. (6), the general solutions of Eq. (6) must be of the form

$$\begin{aligned}&[A_x^{(n)}, A_y^{(n)}, A_z^{(n)}, B_x^{(n)}, B_y^{(n)}, B_z^{(n)}]\\ &\quad = [\Delta_1^{(n)}, \Delta_2^{(n)}, \Delta_3^{(n)}, \Delta_4^{(n)}, \Delta_5^{(n)}, \Delta_6^{(n)}]K_r^{(n)}f_r^{(n)},\end{aligned} \tag{10}$$

where $f_r^{(n)}(n = 1, \ldots, 4)$ are arbitrary constants, $K_r^{(n)} = [\sum_{l=1}^{6}\left|\Delta_l^{(n)}\right|^2]^{-1/2}$,

$$\Delta_1^{(n)} = -\begin{vmatrix} d_{16}^{(n)} & d_{12}^{(n)} & d_{13}^{(n)} & d_{14}^{(n)} & d_{15}^{(n)} \\ d_{26}^{(n)} & d_{22}^{(n)} & d_{23}^{(n)} & d_{24}^{(n)} & d_{25}^{(n)} \\ d_{36}^{(n)} & d_{32}^{(n)} & d_{33}^{(n)} & d_{34}^{(n)} & d_{35}^{(n)} \\ d_{46}^{(n)} & d_{42}^{(n)} & d_{43}^{(n)} & d_{44}^{(n)} & d_{45}^{(n)} \\ d_{56}^{(n)} & d_{52}^{(n)} & d_{53}^{(n)} & d_{54}^{(n)} & d_{55}^{(n)} \end{vmatrix}, \quad \Delta_2^{(n)} = -\begin{vmatrix} d_{11}^{(n)} & d_{16}^{(n)} & d_{13}^{(n)} & d_{14}^{(n)} & d_{15}^{(n)} \\ d_{21}^{(n)} & d_{26}^{(n)} & d_{23}^{(n)} & d_{24}^{(n)} & d_{25}^{(n)} \\ d_{31}^{(n)} & d_{36}^{(n)} & d_{33}^{(n)} & d_{34}^{(n)} & d_{35}^{(n)} \\ d_{41}^{(n)} & d_{46}^{(n)} & d_{43}^{(n)} & d_{44}^{(n)} & d_{45}^{(n)} \\ d_{51}^{(n)} & d_{56}^{(n)} & d_{53}^{(n)} & d_{54}^{(n)} & d_{55}^{(n)} \end{vmatrix},$$

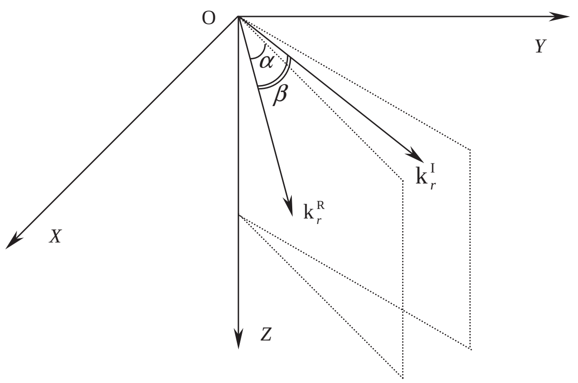

Fig. 1. Complex wave vector of Rayleigh waves in orthotropic fluid-saturated porous media.

$$\Delta_3^{(n)} = -\begin{vmatrix} d_{11}^{(n)} & d_{12}^{(n)} & d_{16}^{(n)} & d_{14}^{(n)} & d_{15}^{(n)} \\ d_{21}^{(n)} & d_{22}^{(n)} & d_{26}^{(n)} & d_{24}^{(n)} & d_{25}^{(n)} \\ d_{31}^{(n)} & d_{32}^{(n)} & d_{36}^{(n)} & d_{34}^{(n)} & d_{35}^{(n)} \\ d_{41}^{(n)} & d_{42}^{(n)} & d_{46}^{(n)} & d_{44}^{(n)} & d_{45}^{(n)} \\ d_{51}^{(n)} & d_{52}^{(n)} & d_{56}^{(n)} & d_{54}^{(n)} & d_{55}^{(n)} \end{vmatrix}, \quad \Delta_4^{(n)} = -\begin{vmatrix} d_{11}^{(n)} & d_{12}^{(n)} & d_{13}^{(n)} & d_{16}^{(n)} & d_{15}^{(n)} \\ d_{21}^{(n)} & d_{22}^{(n)} & d_{23}^{(n)} & d_{26}^{(n)} & d_{25}^{(n)} \\ d_{31}^{(n)} & d_{32}^{(n)} & d_{33}^{(n)} & d_{36}^{(n)} & d_{35}^{(n)} \\ d_{41}^{(n)} & d_{42}^{(n)} & d_{43}^{(n)} & d_{46}^{(n)} & d_{45}^{(n)} \\ d_{51}^{(n)} & d_{52}^{(n)} & d_{53}^{(n)} & d_{56}^{(n)} & d_{55}^{(n)} \end{vmatrix},$$

$$\Delta_5^{(n)} = -\begin{vmatrix} d_{11}^{(n)} & d_{12}^{(n)} & d_{13}^{(n)} & d_{14}^{(n)} & d_{16}^{(n)} \\ d_{21}^{(n)} & d_{22}^{(n)} & d_{23}^{(n)} & d_{24}^{(n)} & d_{26}^{(n)} \\ d_{31}^{(n)} & d_{32}^{(n)} & d_{33}^{(n)} & d_{34}^{(n)} & d_{36}^{(n)} \\ d_{41}^{(n)} & d_{42}^{(n)} & d_{43}^{(n)} & d_{44}^{(n)} & d_{46}^{(n)} \\ d_{51}^{(n)} & d_{52}^{(n)} & d_{53}^{(n)} & d_{54}^{(n)} & d_{56}^{(n)} \end{vmatrix}, \quad \Delta_6^{(n)} = \begin{vmatrix} d_{11}^{(n)} & d_{12}^{(n)} & d_{13}^{(n)} & d_{14}^{(n)} & d_{15}^{(n)} \\ d_{21}^{(n)} & d_{22}^{(n)} & d_{23}^{(n)} & d_{24}^{(n)} & d_{25}^{(n)} \\ d_{31}^{(n)} & d_{32}^{(n)} & d_{33}^{(n)} & d_{34}^{(n)} & d_{35}^{(n)} \\ d_{41}^{(n)} & d_{42}^{(n)} & d_{43}^{(n)} & d_{44}^{(n)} & d_{45}^{(n)} \\ d_{51}^{(n)} & d_{52}^{(n)} & d_{53}^{(n)} & d_{54}^{(n)} & d_{55}^{(n)} \end{vmatrix},$$

in which $d_{kl}^{(n)}$ can be got by substitution of $r^{(n)}$ for r in the elements d_{kl} of Eq. (6).

Then the displacements due to the Rayleigh wave propagation in the porous media are

$$[u_x, u_y, u_z, w_x, w_y, w_z] = \sum_{n=1}^{4} f_r^{(n)} K_r^{(n)} [\Delta_1^{(n)}, \Delta_2^{(n)}, \Delta_3^{(n)}, \Delta_4^{(n)}, \Delta_5^{(n)}, \Delta_6^{(n)}] \times \exp(-k\eta^{(n)} z) \exp \mathrm{i}k(l_1 x + l_2 y + \xi^{(n)} z - ct). \tag{11}$$

The free boundary conditions are

$$\tau_{zz} = \tau_{xz} = \tau_{yz} = p = 0 \quad \text{at } z = 0. \tag{12}$$

By making use of Eqs. (3) and (5), the boundary conditions can be expressed as

$$[H_{kn}]\{f_r^{(1)}, f_r^{(2)}, f_r^{(3)}, f_r^{(4)}\}^{\mathrm{T}} = 0 \quad (k, n = 1, \ldots, 4), \tag{13}$$

where

$$\begin{aligned} H_{1n} &= A_{13} l_1 \Delta_1^{(n)} + A_{23} l_2 \Delta_2^{(n)} + A_{33} r^{(n)} \Delta_3^{(n)} \\ &\quad - M_{33} l_1 \Delta_4^{(n)} - M_{33} l_2 \Delta_5^{(n)} - M_{33} r^{(n)} \Delta_6^{(n)}, \\ H_{2n} &= A_{44} l_2 \Delta_3^{(n)} + A_{44} r^{(n)} \Delta_2^{(n)}, \\ H_{3n} &= A_{55} l_1 \Delta_3^{(n)} + A_{55} r^{(n)} \Delta_1^{(n)}, \\ H_{4n} &= M_{11} l_1 \Delta_1^{(n)} + M_{22} l_2 \Delta_2^{(n)} + M_{33} r^{(n)} \Delta_3^{(n)} \\ &\quad - M l_1 \Delta_4^{(n)} - M l_2 \Delta_5^{(n)} - M r^{(n)} \Delta_6^{(n)}, \end{aligned}$$

In order to get the non-trivial solution of $f_r^{(n)}$, the determinant of matrix $[H_{kl}]$ must be equal to zero, so the characteristic equation of Rayleigh waves in the orthotropic fluid-saturated porous media is

$$\mathrm{Det}\, H = \begin{vmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{21} & H_{22} & H_{23} & H_{24} \\ H_{31} & H_{32} & H_{33} & H_{34} \\ H_{41} & H_{42} & H_{43} & H_{44} \end{vmatrix}. \tag{14}$$

In general, $\mathrm{Det}\, H$ is complex. Eq. (14) can be rewritten as

$$\mathrm{Re}[\mathrm{Det}\, H] = 0, \tag{15a}$$

$$\mathrm{Im}[\mathrm{Det}\, H] = 0. \tag{15b}$$

Eq. (15) are functions of Rayleigh wave speed c. For a given frequency, the speed c satisfying Eqs. (15a) and (15b) simultaneously is the solution that we need.

Moreover, the displacements of the media are given by

$$\begin{aligned}[u_x, u_y, u_z, w_x, w_y, w_z] &= \mathrm{Re}[\sum_{n=1}^{4} f_r^{(n)} K_r^{(n)} [\Delta_1^{(n)}, \Delta_2^{(n)}, \Delta_3^{(n)}, \Delta_4^{(n)}, \Delta_5^{(n)}, \Delta_6^{(n)}] \\ &\quad \times \exp(-k\eta^{(n)} z) \exp \mathrm{i}k(l_1 x + l_2 y + \xi^{(n)} z - ct)] \\ &= \sum_{n=1}^{4} [|\Gamma_i^{(n)}| \cos[k(l_1 x + l_2 y + \xi^{(n)} z - ct) + \theta_i^{(n)}], \\ &\quad |\Pi_i^{(n)}| \cos[k(l_1 x + l_2 y + \xi^{(n)} z - ct) + \varphi_i^{(n)}]], \end{aligned} \tag{16}$$

where

$$[\Gamma_x^{(n)}, \Gamma_y^{(n)}, \Gamma_z^{(n)}, \Pi_x^{(n)}, \Pi_y^{(n)}, \Pi_z^{(n)}] = f_r^{(n)} K_r^{(n)} [\Delta_1^{(n)}, \ldots, \Delta_6^{(n)}] \exp(-k\eta^{(n)} z),$$

$\theta_i^{(n)}$ and $\varphi_i^{(n)}$ are the corresponding polar angles of $\Gamma_i^{(n)}$ and $\Pi_i^{(n)}$, respectively.

3.1. Special case

Pure solids can be treated as the porous media in the case $\phi = 0$, that is, $\rho_f = 0$, $A_{ijkl} = C_{ijkl}$, $\rho = \rho_s$, $M_{ij} = M = 0$. Thus, Eq. (6) degenerate to

$$[d'_{kl}]\{A'_x, A'_y, A'_z\}^{\mathrm{T}} = 0 \quad (k, l = 1, 2, 3), \tag{17}$$

where

$$\begin{aligned} d'_{11} &= \rho_s \omega^2 - (C_{11} l_1^2 + C_{66} l_2^2 + C_{55} r'^2) k^2, & d'_{12} &= d'_{21} = -(C_{12} + C_{66}) l_1 l_2 k^2, \\ d'_{13} &= d'_{31} = -(C_{13} + C_{55}) l_1 r' k^2, & d'_{22} &= \rho_s \omega^2 - (C_{66} l_1^2 + C_{22} l_2^2 + C_{44} r'^2) k^2, \\ d'_{23} &= d'_{32} = -(C_{23} + C_{44}) l_2 r' k^2, & d'_{33} &= \rho_s \omega^2 - (C_{55} l_1^2 + C_{44} l_2^2 + C_{33} r'^2) k^2. \end{aligned}$$

Consequently, we have

$$a'_1 r'^6 + a'_2 r'^4 + a'_3 r'^2 + a'_4 = 0. \tag{18}$$

Following the same routine, we can get the three roots with positive imaginary parts $\boldsymbol{r}'^{(n)}$ and corresponding normalized solutions of Eq. (17) with the form

$$[A_x'^{(n)}, A_y'^{(n)}, A_z'^{(n)}] = [\Delta_1'^{(n)}, \Delta_2'^{(n)}, \Delta_3'^{(n)}] K_r'^{(n)} f_r'^{(n)}, \tag{19}$$

where $f_r^{\prime(n)}(n=1,2,3)$ are arbitrary constants, $K_r^{\prime(n)} = [\sum_{l=1}^{3} |\Delta_l^{\prime(n)}|]^{-1/2}$,

$$\Delta_1^{\prime(n)} = -\begin{vmatrix} d_{13}^{\prime(n)} & d_{12}^{\prime(n)} \\ d_{23}^{\prime(n)} & d_{22}^{\prime(n)} \end{vmatrix}, \quad \Delta_2^{\prime(n)} = -\begin{vmatrix} d_{11}^{\prime(n)} & d_{13}^{\prime(n)} \\ d_{12}^{\prime(n)} & d_{23}^{\prime(n)} \end{vmatrix}, \quad \Delta_3^{\prime(n)} = \begin{vmatrix} d_{11}^{\prime(n)} & d_{12}^{\prime(n)} \\ d_{12}^{\prime(n)} & d_{22}^{\prime(n)} \end{vmatrix},$$

in which $d_{kl}^{\prime(n)}$ can be got by substitution of $r^{\prime(n)}$ for r' in the elements d'_{kl} of Eq. (17).Then the displacements due to the Rayleigh wave propagation in the pure solids are

$$[u_x, u_y, u_z] = \sum_{n=1}^{3} f_r^{\prime(n)} K_r^{\prime(n)} [\Delta_1^{\prime(n)}, \Delta_2^{\prime(n)}, \Delta_3^{\prime(n)}] \exp(-k\eta^{\prime(n)} z) \times \exp \mathrm{i}k(l_1 x + l_2 y + \xi^{\prime(n)} z - ct). \tag{20}$$

The free boundary conditions are

$$\tau_{zz} = \tau_{xz} = \tau_{yz} = 0, \quad z = 0. \tag{21}$$

By making use of Eq. (13), the boundary conditions can be expressed as

$$[H'_{kn}]\{f_r^{\prime(1)}, f_r^{\prime(2)}, f_r^{\prime(3)}\}^{\mathrm{T}} = 0 \quad (k, n = 1, 2, 3), \tag{22}$$

where

$$\begin{aligned} H'_{1n} &= C_{13} l_1 \Delta_1^{\prime(n)} + C_{23} l_2 \Delta_2^{\prime(n)} + C_{33} r^{\prime(n)} \Delta_3^{\prime(n)}, \\ H'_{2n} &= C_{44} l_2 \Delta_3^{\prime(n)} + C_{44} r^{\prime(n)} \Delta_2^{\prime(n)}, \\ H'_{3n} &= C_{55} l_1 \Delta_3^{\prime(n)} + C_{55} r^{\prime(n)} \Delta_1^{\prime(n)}. \end{aligned} \tag{23}$$

Thus, the characteristic equation of Rayleigh waves in the orthotropic media is

$$\mathrm{Det}\, H' = \begin{vmatrix} H'_{11} & H'_{12} & H'_{13} \\ H'_{21} & H'_{22} & H'_{23} \\ H'_{31} & H'_{32} & H'_{33} \end{vmatrix}. \tag{24}$$

Eq. (24) is just the expansion expression of the solution given by Ref. [2] for orthotropic solids. This shows that the characteristic equations for Rayleigh waves in pure solids are particular cases of fluid-saturated porous media and can be obtained directly from the corresponding characteristic equations for Rayleigh waves in fluid-saturated porous media through appropriate degeneration.

Table 1
Parameters for solid skeleton and pore fluids (porosity $\Phi = 0.2$)

Solid skeleton											Pore fluid	
C_{11} (GPa)	C_{12} (GPa)	C_{13} (GPa)	C_{22} (GPa)	C_{23} (GPa)	C_{33} (GPa)	C_{44} (GPa)	C_{55} (GPa)	C_{66} (GPa)	K_s (GPa)	ρ_s (kg/m^3)	K_f (GPa)	ρ_f (kg/m^3)
9.57	1.19	2.33	9.3	2	8.32	3	3.2	3.8	35	2600	2	1000

4. Numerical results

In order to illustrate the effects of the orthotropic solid skeleton on the propagation characteristic of Rayleigh waves, variation of Rayleigh wave speeds with its propagation directions and the displacement field are calculated by using Eqs. (15) and (16), respectively. The material parameters for the orthotropic fluid-saturated porous medium are given in Table 1. For an assumed value of the speed c, from Eq. (7) we can obtain four values of $r^{(n)}$. Substitute $r^{(n)}$ into Eq. (15) and whether the Det H equals to zero is investigated. Here interval dichotomy is used to determine a root in a given range of the independent variable with a specified small relative error. When the speed c, satisfying Eq. (15) are determined, the corresponding displacement components are derived according to Eq. (16).

Fig. 2 shows the variation of Rayleigh wave speeds on the free surface of the half-space. Owing to anisotropy of the solid skeleton, the Rayleigh wave speed is no longer invariable as it shows in the isotropic or transversely isotropic fluid-saturated porous media (here transverse isotropy means the horizontal transverse isotropy). The Rayleigh wave speed changes with the variation of the propagation directions. But along a certain direction, it propagates with a certain speed, for example, the speed along the direction from the point (0, 0, 0) to the point (1, 1, 0) is 0.925 km/s.

Fig. 3 gives the particle traces at different depth in different sagittal planes (the plane normal to both the bounding surface and the wave front) with $A_r = [\sum_{n=1}^{3} |f_r^{(n)}|^2]^{-1/2} K_r^{(n)}$ for transversely isotropic fluid-saturated porous media. From plane xoz to plane yoz, every 15° a sagittal plane is selected. It can be seen that the particle traces are elliptical and along with the increase of the depth, the amplitudes decrease exponentially. At the same layer, the amplitudes of the particle traces are not changed and the shapes are identical. It can also be seen that in the transversely isotropic fluid-saturated porous media, every particle moves in its own sagittal plane.

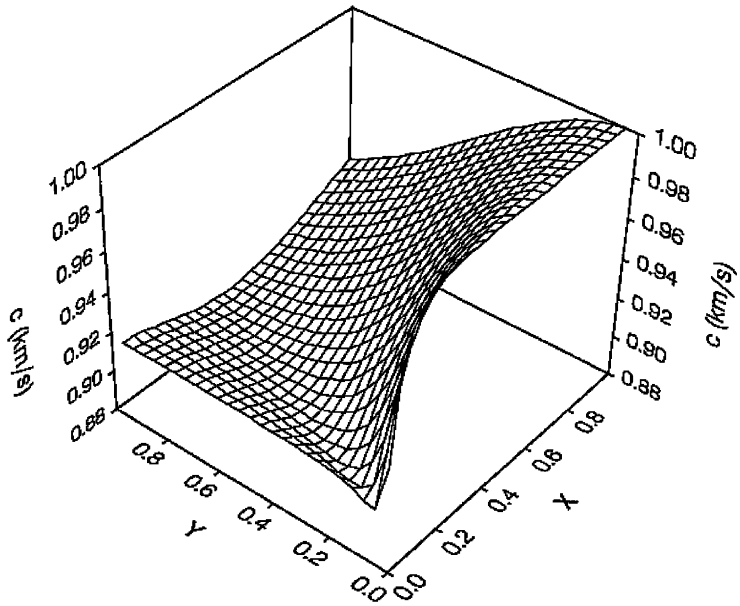

Fig. 2. Variation of Rayleigh wave speeds on the free surface.

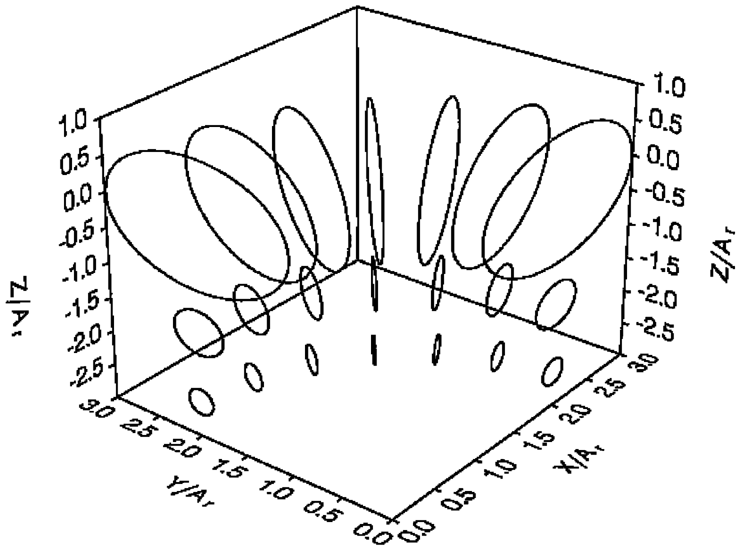

Fig. 3. Particle traces in different sagittal planes for transversely isotropic fluid-saturated porous media when fluid viscosity is omitted.

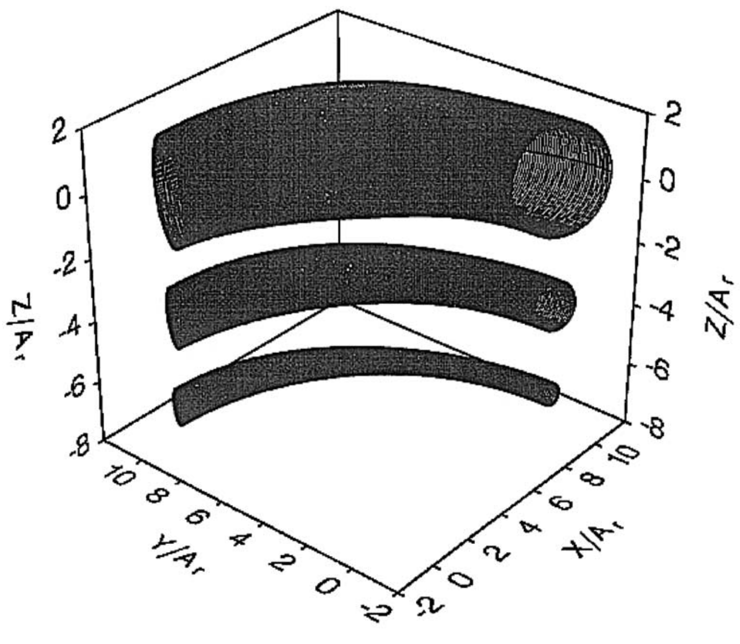

Fig. 4. Particle trace in different sagittal planes for orthotropic fluid-saturated porous media when fluid viscosity is omitted.

Fig. 4 describes the particle traces at different depth in different sagittal planes with $A_r = [\sum_{n=1}^{4} |f_r^{(n)}|^2]^{-1/2} K_r^{(n)}$ for orthotropic fluid-saturated porous media. Comparison between Figs. 3 and 4 shows that with the same situation in the isotropic or transversely isotropic fluid-saturated

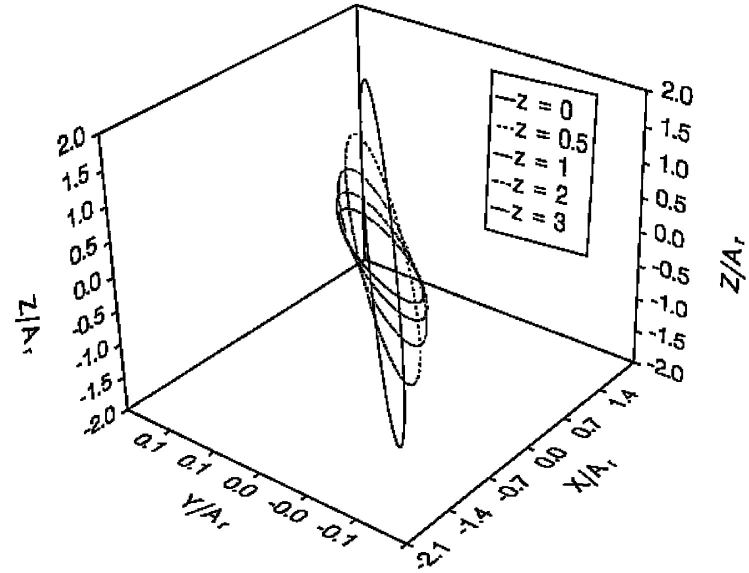

Fig. 5. Particle traces in 30° sagittal plane.

porous media, the particle traces in orthotropic fluid-saturated porous media are elliptical and the sizes of ellipses decrease exponentially with the increase of the depth. But owing to the orthotropic characteristic of the media, even on the same layer, the sizes of the ellipses change with the variation of the propagation directions which can be seen clearly in Fig. 4.

For orthotropic fluid-saturated porous media, the complex attenuation exponent r is no more pure imaginary as it is in the isotropic or transversely isotropic porous media. From Eq. (8) it can be seen that because the real part of complex attenuation exponent r is not equal to zero, the disturbance directions of Rayleigh waves in orthotropic fluid-saturated porous media may not be parallel to the free surface as they do in the isotropic or transversely isotropic ones and the motions of the particles also show different characteristic. The particle motions exhibit a sinusoidal variation with z-axis. The main axis of the elliptical trace is not vertical to the free surface any longer. The particles do not move in their own sagittal planes but do some out-of-plane motions, which is different from that they generally do in the isotropic or transversely isotropic ones. In order to see clearly the variations of the particle motions with the variation of the depth, the particle traces in the 30° sagittal plane are given in Fig. 5. It can be seen obviously that the particle motion planes incline to the sagittal plane.

5. Conclusion

Summarizing the results above, we assert that, due to anisotropy of the solid skeleton, the Rayleigh wave in orthotropic fluid-saturated porous media exhibits special characteristic. Its speed and particle traces show direction dependence. Though the dispersion caused by the fluid viscosity is omitted, the phase plane and the amplitude plane are not parallel to each other as they do in the isotropic or transversely isotropic fluid-saturated porous media. It is a non-homogenous wave and the maximal attenuation is not along the propagation directions. In orthotropic fluid-saturated porous media, the main axis of the elliptical traces of the particles is not vertical to the free surface but displays a sinusoidal variation with the depth. Obviously, in orthotropic fluid-saturated porous media, Rayleigh waves are not the classical Rayleigh waves in general sense. Anisotropy of the solid skeleton has great influence on the propagation characteristic of Rayleigh waves in fluid-saturated porous media.

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