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Letter to the Editor

Non-linear response of cables subjected to periodic support excitation considering cable loosening

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1. Introduction

Many researchers have studied parametric vibrations in cables. Kovács et al. [1] pointed out that parametric instability of a cable employed as a tension member for a cable-stayed bridge or guy tower may occur under an axial periodic load due to the bending vibration of the tower or the girder of the bridge. He examined the parametric vibration of a string without sag and discussed using a vibration absorber to control the unstable vibration. Takahashi [2] calculated the instability boundaries of the principal regions of a flat-sag cable under an axial, sinusoidally time-varying load. Lilien and Pinto da Costa [3] developed non-dimensional analytical formulas that can be used to calculate the threshold amplitudes and limit cycle amplitudes in the stay cable. Perkins [4] described the modal interactions of in-plane/out-of-plane motions in the non-linear response of elastic cables subjected to parametric/external excitations. Many researchers [5–8] have reported on vibrations in bridge stay cables that are induced by periodic motions of the deck and/or tower was reported. Cable loosening may be observed in the non-linear parametric response of a cable since large amplitude vibration occurs. However, the effect of cable loosening in handling the non-linear parametric responses has been neglected in their analyses.

The authors described a method that can explicitly evaluate cable loosening and evaluated the effect of cable loosening on the non-linear responses of the cables [9]. Physical explanation of cable loosening and how it appears in real application are shown in Ref. [9]. This note examines the effect that loosening has on the non-linear parametric responses of flat cables that are subjected to periodic time-varying horizontal displacement/horizontal force at the support. The note also evaluates the regions of generated compressive forces and the effect of loosening on the non-linear parametric responses of the cables.

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2. Equation of motion

A horizontal cable with a uniform cross section hanging between two points is analyzed. If the profile is flat, so that the sag-to-span ratio is the order of 1:8 or less, the following equations of motion for non-linear vibration of a cable (neglecting the horizontal inertia force) are obtained [10]

$$m \frac{\partial^2 w}{\partial t^2} - (H + \Delta H) \frac{\partial^2 w}{\partial x^2} + (H + \Delta H) \frac{8f}{L^2} = mg, \tag{1}$$

$$\Delta H = \frac{EA}{L_E} \left\{ u(L) - u(0) + \frac{8f}{L^2} \int_0^L w \, \mathrm{d}x + \frac{1}{2} \int_0^L \left(\frac{\partial w}{\partial x} \right)^2 \mathrm{d}x \right\}, \tag{2}$$

where m is the mass per unit length, H is the initial horizontal tension induced by the dead load of the cable, ΔH is the additional horizontal tension, E is the Young's modulus, A is the cross-sectional area, $L_E = L\{1 + 8(f/L)^2\}$ is the length of the cable, L is the span of the cable, f is the sag of the cable, g is the gravitational acceleration, w is the deflection of the cable, u is the horizontal displacement of the cable, x is the span-wise co-ordinate, and t is the time.

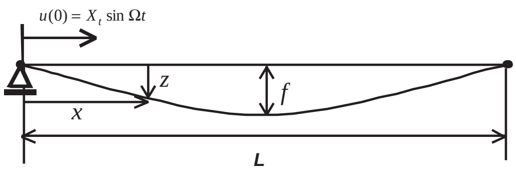
If the cable is subjected to horizontal support excitation displacement $u(0) = X_t \sin \Omega t$ at $x = 0$ and is fixed at $x = L$ as shown in Fig. 1(a), Eq. (2) is rewritten as

$$\Delta H = H_t \sin \Omega t + H_p, \tag{3}$$

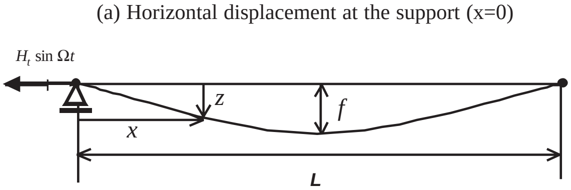
$$H_t = -\frac{EA}{L_E} X_t, \tag{4}$$

$$H_p = \frac{EA}{L_E} \left\{ \frac{8f}{L^2} \int_0^L w \, \mathrm{d}x + \frac{1}{2} \int_0^L \left(\frac{\partial w}{\partial x} \right)^2 \mathrm{d}x \right\}, \tag{5}$$

where H_t is the amplitude of the horizontal force due to excitation of the support, Ω is the circular frequency of the support excitation and H_p is the deflection-induced additional horizontal tension.



(a) Horizontal displacement at the support (x=0)



(b) Horizontal force at the support (x=0)

Fig. 1. Geometry of a cable: (a) horizontal displacement at the support ($x = 0$); (b) horizontal force at the support ($x = 0$).

In the present analysis, the horizontal support displacement $u(0)$ is assumed to be smaller than deflection w.

If a cable is subjected to the horizontal support excitation force $H(0) = \overline{H_t} \sin \Omega t$ as shown in Fig. 1(b), Eq. (4) should be $H_t = \overline{H_t}$. Therefore, Eq. (3) is valid for both the excitation displacement and the excitation force.

By considering the effects of flexural rigidity and damping, the calculation is able to stably handle cable loosening, as proposed in the previous paper [9]. Eq. (1) is rewritten as follows:

$$m\frac{\partial^2 w}{\partial t^2} + c\frac{\partial w}{\partial t} + EI\frac{\partial^4 w}{\partial x^4} - (H + H_t \sin \Omega t + H_p)\frac{\partial^2 w}{\partial x^2} + (H + H_t \sin \Omega t + H_p)\frac{8f}{L^2} = mg, \quad (6)$$

where c is the damping coefficient and I is the geometrical moment of inertia.

By making Eqs. (6), (4) and (5) non-dimensional by the span of the cable L, the initial horizontal tension H and the first natural circular frequency ω_0 of the string, the following equations are obtained:

$$\frac{\partial^2 \bar{w}}{\partial \tau^2} + 2h\bar{\omega}_1\frac{\partial \bar{w}}{\partial \tau} + \frac{k^2\delta}{\pi^2}\frac{\partial^4 \bar{w}}{\partial \bar{x}^4} - (1 + h_t \sin \omega\tau + h_p)\frac{1}{\pi^2}\frac{\partial^2 \bar{w}}{\partial \bar{x}^2} + (1 + h_t \sin \omega\tau + h_p)\frac{8\gamma}{\pi^2} = \frac{8\gamma}{\pi^2}, \quad (7)$$

$$h_t = -\frac{k^2}{1 + 8\gamma^2}\overline{X_t}, \quad (8)$$

$$h_p = \frac{\lambda^2}{64}\cdot\left\{8\int_0^1 \frac{1}{\gamma}\bar{w}\,\mathrm{d}\bar{x} + \frac{1}{2}\int_0^1\left(\frac{1}{\gamma}\frac{\partial \bar{w}}{\partial \bar{x}}\right)^2 \mathrm{d}\bar{x}\right\}, \quad (9)$$

where $\bar{w} = w/L$ is the non-dimensional displacement in the z direction, $\bar{x} = x/L$ is the non-dimensional co-ordinate in the x direction, $\tau = \omega_0 t$ is the non-dimensional time, $\omega_0 = \sqrt{(H/m)\,(\pi/L)^2}$ is the first natural circular frequency of a taut string that has no sag [10], $\bar{\omega}_1 = \omega_1/\omega_0$ is the first non-dimensional natural circular frequency of the cable, ω_1 is the first natural circular frequency of the cable, h is the damping constant, $\omega = \Omega/\omega_0$ is the non-dimensional circular frequency of the support excitation, $h_t = H_t/H$ is the non-dimensional amplitude of the horizontal support excitation force, $h_p = H_p/H$ is the non-dimensional additional horizontal tension, $\overline{X_t} = X_t/L$ is the non-dimensional amplitude of the horizontal support excitation displacement, $\gamma = f/L$ is the sag-to-span ratio of the cable, $k^2 = EA/H$ is the ratio of the elongation stiffness to the horizontal tension of the cable (the square of the ratio of the longitudinal wave propagation velocity to the transversal wave propagation velocity), $\delta = (EI/L^2)(1/EA)$ is the ratio of flexural rigidity to elongation stiffness [9], and $\lambda^2 = 64\,k^2\gamma^2/(1 + 8\gamma^2)$ is an Irvine parameter [10].

3. Numerical analysis method

Eqs. (7)–(9) are non-linear equations of the motion of a flat horizontal cable. Additional horizontal tensions are constant at every point on a cable because the horizontal inertial force of the cable is neglected, based on the assumption of a flat-sag cable. This means that the total non-dimensional horizontal tension is given by $(1 + h_t \sin \omega\tau + h_p)$. In a non-linear vibration analysis that considers cable loosening, the value of the total horizontal tension is set to zero when it becomes less than zero. That is

$$1 + h_t \sin \omega\tau + h_p = 0 \quad (1 + h_t \sin \omega\tau + h_p < 0). \tag{10}$$

In order to solve Eqs. (7)–(9) while evaluating Eq. (10), the numerical method should be used. The explicit formula of the finite difference method [11] is also employed in this study. The definite integrals in Eq. (9) are calculated using Simpson's 1/3 formula. The time interval for the numerical analysis should be defined so as to satisfy the stability condition of the scheme that is used.

4. Numerical method and results

4.1. Numerical conditions

In this note, the parametric excitation is the horizontal support excitation displacement $\overline{X_t}$ or the horizontal support excitation force h_t.. The circular frequency ω of the parametric excitation is assumed to be ω_1 or $2\omega_1$. If $\omega = \omega_1$ is used, the parametric excitation corresponds to the parametric responses of the second unstable region, while if $\omega = 2\omega_1$ is used, it corresponds to that of the principal unstable region.

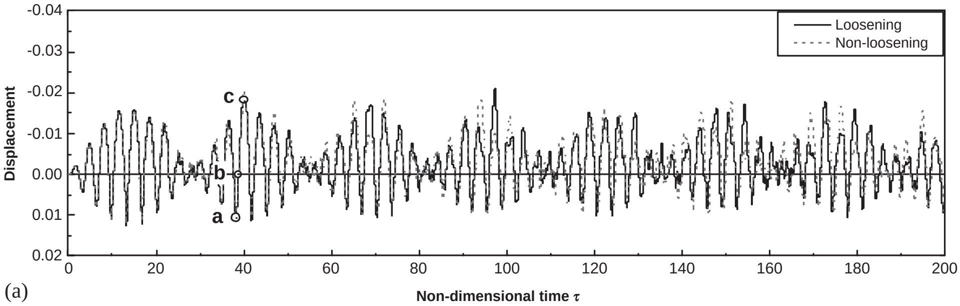
The number of divisions of a cable is 100. In other words, $\Delta x = 0.01$. In order to satisfy the stability conditions, the time interval must be less than 2.5×10^{-5}. 1.0×10^{-5} is used here. The parameters δ and h, needed to solve the divergence problem, have the same values as the previous paper [9], i.e., $\delta = 10^{-7}$ and $h = 0.001$. For the case in which compressive forces do not appear, the time histories produced by the finite difference method and a Galerkin method coincide very well in terms of both the displacement and the horizontal tension of the parametric response.

4.2. Non-linear parametric responses of the second unstable region

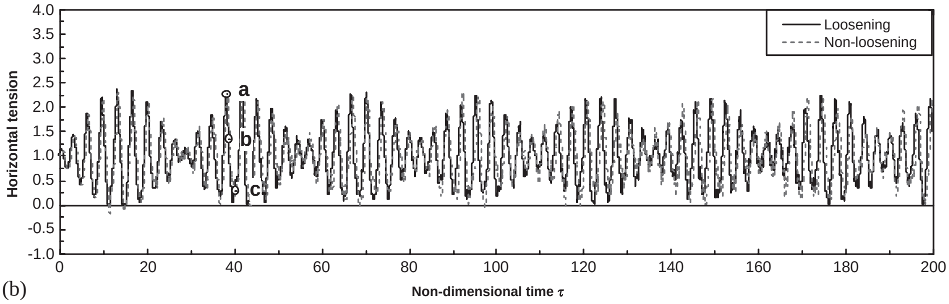
Fig. 2 shows the non-linear parametric responses of the second unstable region ($\omega = \omega_1$) when the cable is subjected to compressive forces. Fig. 2(a) and (b) are time histories of the center point of the cable and the total horizontal tension, respectively. The corresponding space shapes and the maximum displacement during nonlinear vibration of the cable are shown in Fig. 2(c) and (d). Notations a, b and c correspond to the maximum, zero, and minimum displacements at the center point of the cable.

Fig. 3 shows the time histories and space shapes when the amplitude of parametric excitation is twice amplitude shown in Fig. 2. The compressive forces do not increase very much from the value shown in Fig. 2(b) although the negative displacements are obvious. This can be interpreted as meaning that the cable maintains space shapes that do not easily generate the compressive forces shown in Figs. 2(c)and 3(c).

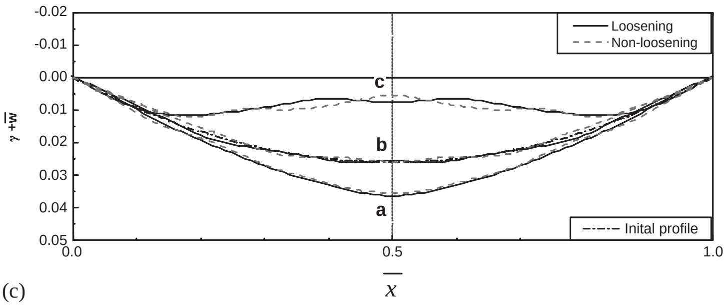
Fig. 4 shows the relationship between the maximum displacement response and the amplitude of the horizontal tension h_t due to excitation of the support. The corresponding amplitude of support excitation displacement $\overline{X_t}$ is also shown in Fig. 4. Loosening begins to appear when h_t is greater than 0.1. Loosening affects the negative maximum response, while the positive maximum response is scarcely affected.



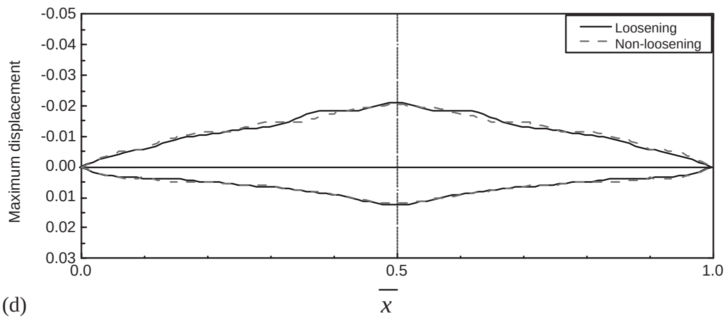
(a)



(b)



(c)



(d)

Fig. 2. Time histories in the second unstable region of (a) displacement at the center point, (b) total horizontal tension, (c) space shapes and (d) maximum displacement of a cable for $\gamma = 0.026$, $k^2 = 900$, $h_t = 0.2$ ($\bar{X}_t = 2.24 \times 10^{-4}$), $\delta = 10^{-7}$ and $h = 0.001$.

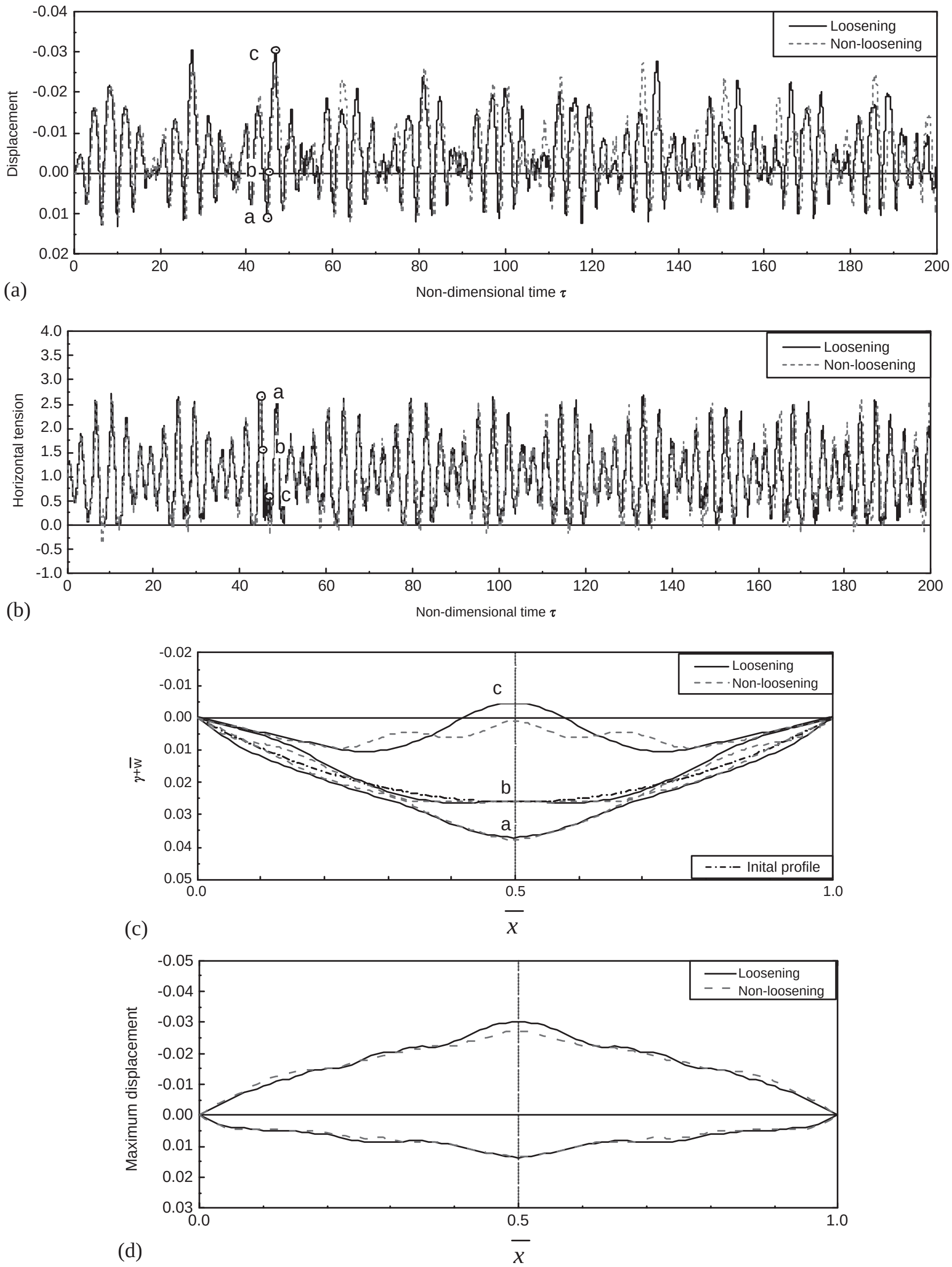


Fig. 3. Time histories in the second unstable region of (a) displacement at the center point, (b) total horizontal tension, (c) space shapes and (d) maximum displacement of a cable for $\gamma = 0.026$, $k^2 = 900$, $h_t = 0.4$ ($\bar{X}_t = 4.48 \times 10^{-4}$), $\delta = 10^{-7}$ and $h = 0.001$.

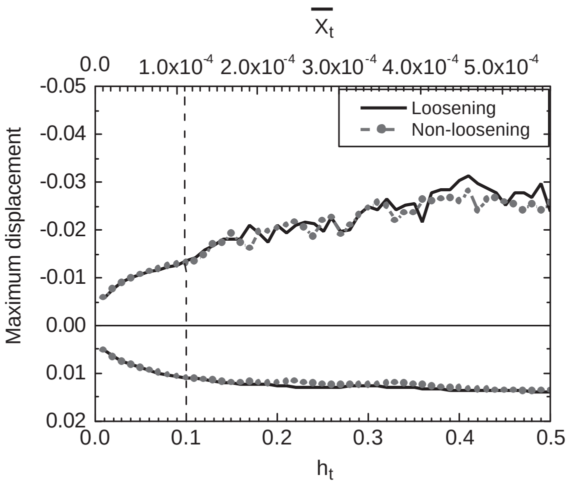


Fig. 4. Relationship between the maximum displacement and the amplitude of the support excitation in the second unstable region for $\gamma = 0.026$ and $k^2 = 900$.

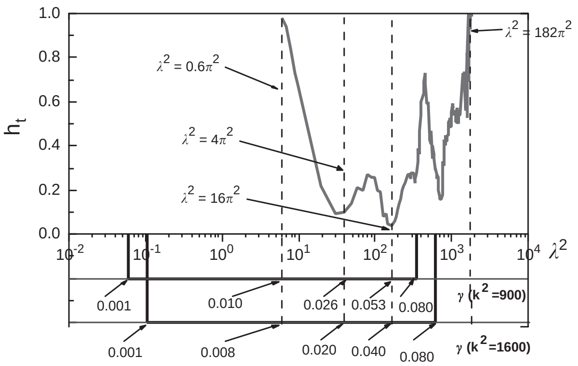


Fig. 5. Relationship between the amplitude of the excitation in which the compressive force appear and Irvine parameter in the second unstable region.

The relationship between the minimal amplitude of parametric excitation h_t that generates compressive forces in the cable and the Irvine parameter λ^2 is shown in Fig. 5. For the reference, the sag-to-span ratio γ is shown for the cases where $k^2 = 900$ and 1600. The compressive force appears easily in the particular magnitude of parameter λ^2. The amplitude of the parametric excitation h_t has the local minimum near $\lambda^2 = 4\pi^2$ which corresponds to the modal transition region from the first to the second symmetric vibration (see Fig. 5). Next, it has a minimum near $\lambda^2 = 16\pi^2$. This sag-to-span ratio γ also corresponds to the mode transition region from the second to the third symmetric vibration. In this way, cables with a sag-to-span ratio γ corresponding to the natural mode transition region can be easily compressed. This conclusion

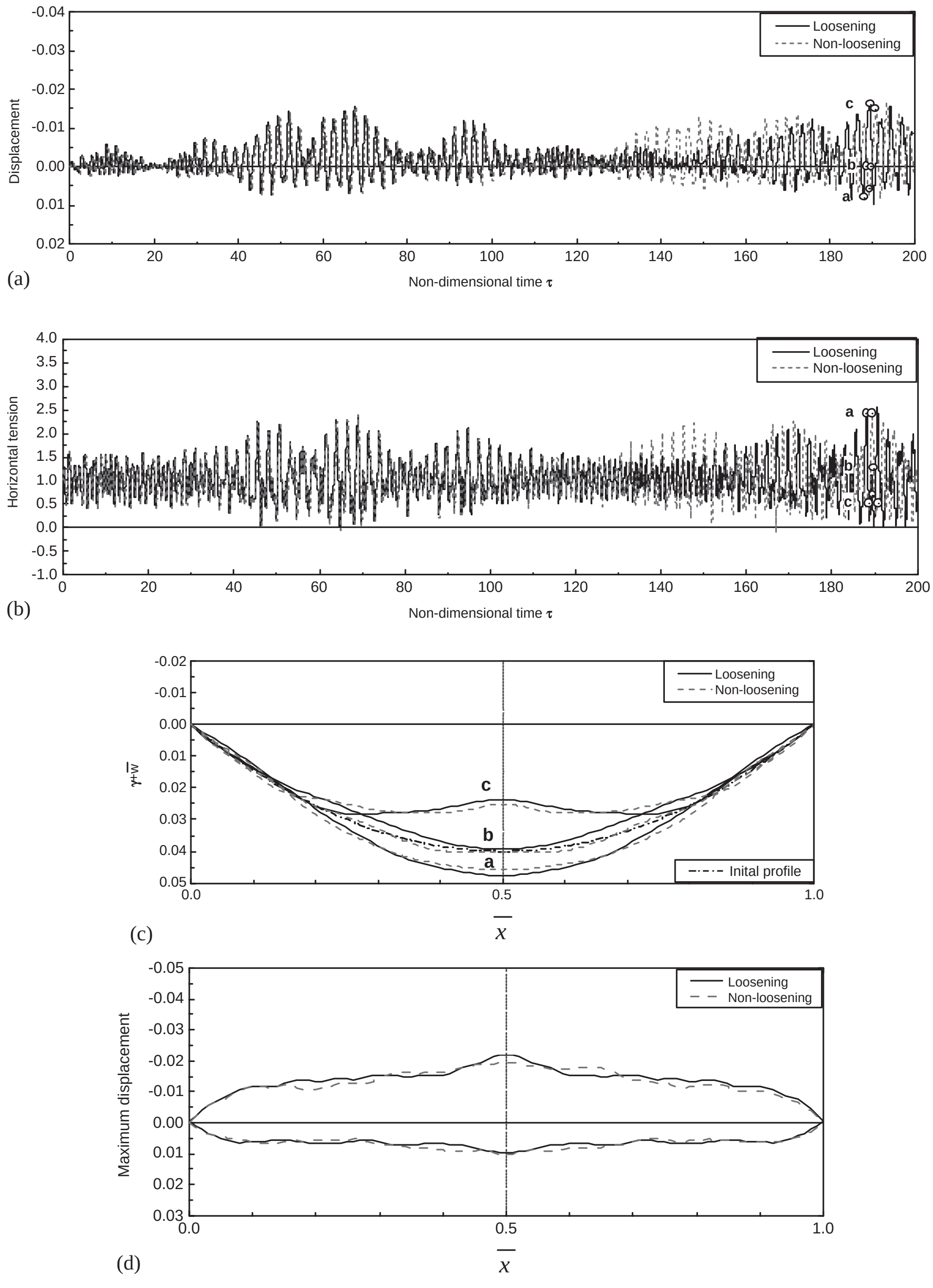


Fig. 6. Time histories in the principal unstable region of (a) displacement at the center point, (b) total horizontal tension, (c) space shapes and (d) maximum displacement of a cable for $\gamma = 0.040$, $k^2 = 900$, $h_t = 0.3$ ($\bar{X}_t = 3.38 \times 10^{-4}$), $\delta = 10^{-7}$ and $h = 0.001$.

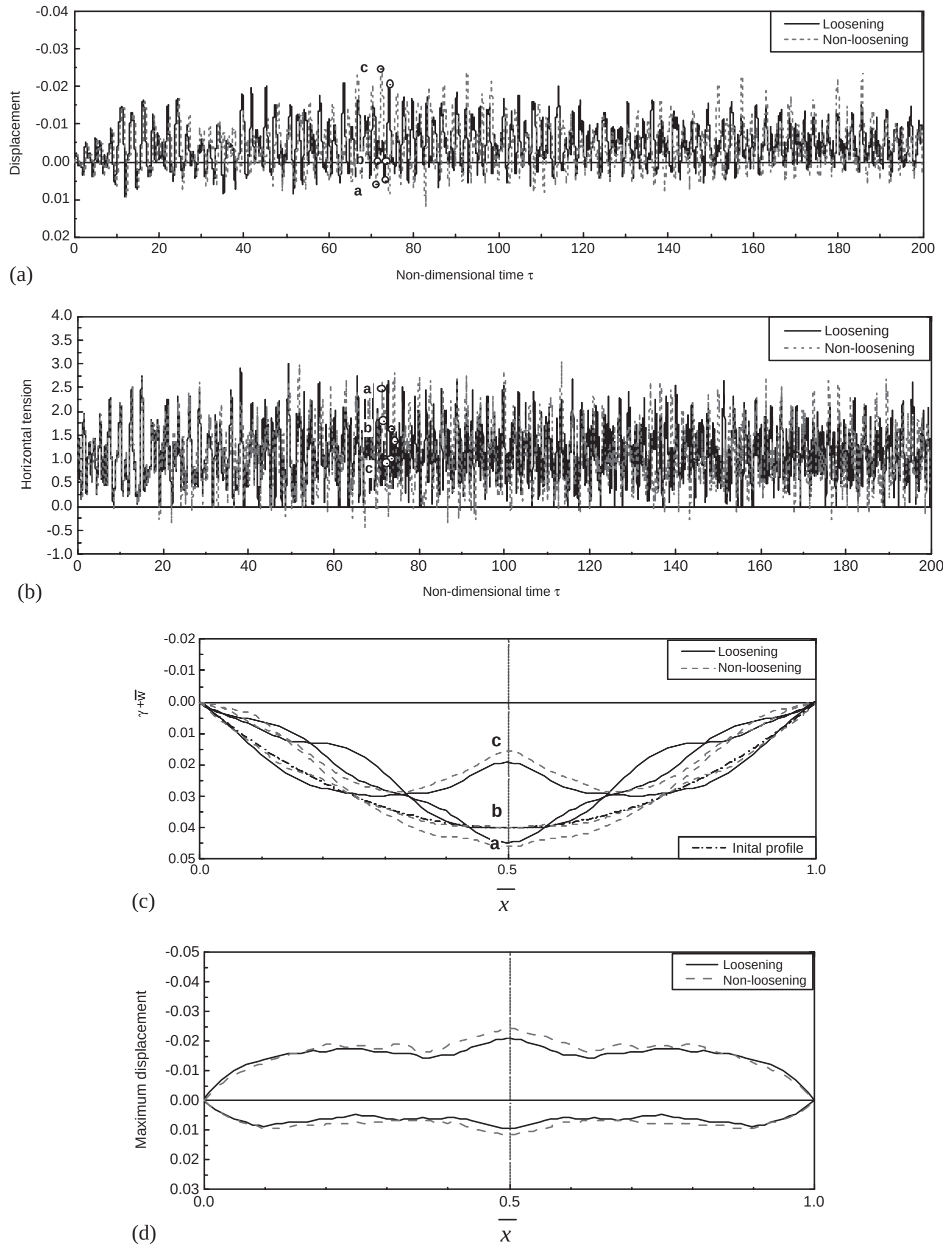


Fig. 7. Time histories in the principal unstable region of (a) displacement at the center point, (b) total horizontal tension, (c) space shapes and (d) maximum displacement of a cable for $\gamma = 0.040$, $k^2 = 900$, $h_t = 0.5$ ($\bar{X}_t = 5.63 \times 10^{-4}$), $\delta = 10^{-7}$ and $h = 0.001$.

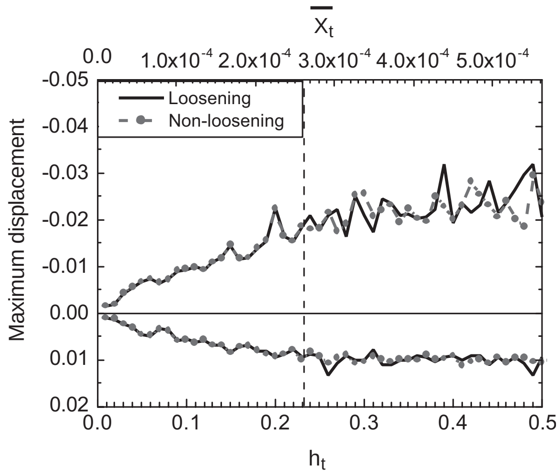


Fig. 8. Relationship between the maximum displacement and the amplitude of the support excitation in the principal unstable region for $\gamma = 0.040$ and $k^2 = 900$.

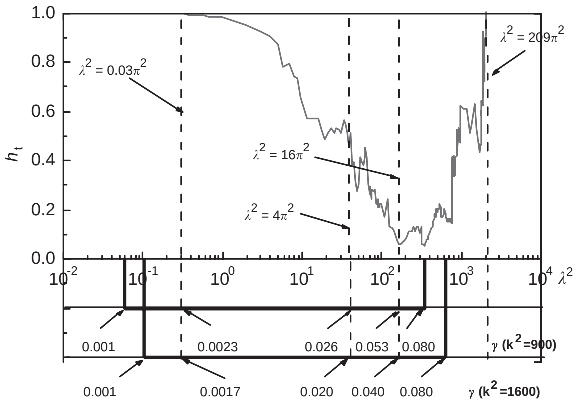


Fig. 9. Relationship between the amplitude of the excitation in which the compressive force appears and Irvine parameter in the principal unstable region.

was also reached for the non-linear response of the cables subjected to the vertical sinusoidally time-varying loading [9].

4.3. Non-linear parametric responses of the principal unstable region

Figs. 6 and 7 show the non-linear parametric responses of the principal unstable region ($\omega = 2\omega_1$) with compressive force in the cable. Fig. 8 shows the relationship between the maximum displacement response and the amplitude of the parametric excitation h_t due to excitation of the support. Loosening begins to appear when h_t is greater than 0.23. Loosening affects the negative maximum response, while the positive maximum response is scarcely affected. The results are similar to those obtained for the second unstable regions.

The relationship between the minimal amplitude of parametric excitation h_t that generates compressive forces in the cable and the Irvine parameter λ^2 is shown in Fig. 9. Comparing Fig. 9 with Fig. 5, it can be seen that the principal unstable region of the range that is generated by loosening is larger than the second unstable region.

5. Concluding remarks

This note examined the effect of cable loosening on the non-linear parametric vibrations of cables subjected to periodic support excitation. The main findings were as follows:

1. Loosening appears easily in cables with a particular sag-to-span ratio. The region in the time history of the cable in which the compressive force appears is narrow.
2. Loosening affects the negative maximum response, while the positive maximum response is scarcely affected.
3. The principal unstable region of the range that is generated by cable loosening is larger than the second unstable region.

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