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Optimal modal vibration suppression of a fluid-conveying pipe with a divergent mode

Yih-Hwang Lin[a,*], Rui-Cheng Huang[a], Chih-Liang Chu[b]

[a] *Department of Mechanical and Marine Engineering, National Taiwan Ocean University, Keelung 20224, Taiwan, Republic of China*

[b] *Department of Mechanical Engineering, Southern Taiwan University of Technology, Tainan, Taiwan 710, Republic of China*

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Abstract

This study deals with the divergence characteristics of pipes conveying fluid and explores the applicability of active modal vibration control for suppressing the associated excessive structural vibration. The Timoshenko beam theory is used to establish the system equation of motion. The analysis is based on the finite element method. Active modal control technique is developed in this work for pipes conveying fluid with a flow speed exceeding the critical one. Optimal independent modal space control (IMSC) is applied for the design. For pipes conveying super-critical flow speed, as considered in this work, the system's eigenvalues have both real and complex roots, which must be dealt with in a different way from what has been established in the literature. A weighting matrix with finite weights is applied for the control of complex modes, whereas a weighting matrix with an infinite weight is used for controlling the divergent mode, with roots being real. From this study, it is demonstrated that the control approach proposed in this work can ensure closed loop stability. The mode switching scheme of directing control to the mode which has higher modal response is found to be beneficial in reducing the overall structural vibration of the fluid-conveying pipe.

1. Introduction

Dynamic analysis of pipes conveying fluid has been an important subject both in industrial applications and academic research. Flutter instability was observed for cantilever pipes once the flow velocity exceeds the critical one [1,2]. When the critical flow velocity is exceeded, pipes

*Corresponding author. Tel: +886-224-622-192; fax: +886-224-620-836.
E-mail address: yhl2@mail.ntou.edu.tw (Y.-H. Lin).

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supported at both ends will buckle [3,4]. The subject is important in the design of mechanical components subjected to high flow disturbance, such as feed lines of rocket motors [5], piping systems [6], and nuclear reactor components [7]. A review article reported by Païdoussis and Li [8] shows that there has been hundreds of papers written on the subject.

Although the research efforts made on the dynamic analysis of fluid conveying pipes have been extensive, active vibration suppression of pipes conveying fluid has not been well studied. Tani and Sudani [9] applied a sub-optimal control law for vibration suppression of tubes conveying fluid using motor controlled tendons. The coupled mode control technique was used. Yau et al. [10] employed quantitative feedback theory to actively control the chaotic vibration of a constrained flexible fluid-conveying pipe. The piezoelectric actuators were used in their work. The analysis was conducted using a two-degree-of-freedom model. Sugiyama et al. [11] studied a vibration suppression technique by using an electronic valve to control the internal flowing fluid for a cantilever pipe with sub-critical flow speeds. The valve was used to adjust the speed of the flowing fluid through a feedback on–off control. To suppress the flutter instability of cantilever pipes subjected to high flow disturbance, Lin and Chu [12] presented an independent modal control technique in accordance with a new design method developed by Lin and Chu [13]. The control action was provided by a pair of surface mounted piezoelectric actuators. The technique has been shown to have advantages over the traditional coupled mode control scheme, in that it requires far less computer storage, demands considerably less computational effort, and allows a larger choice of control approaches to be used, including non-linear control [14]. Tsai and Lin [15] reported a further investigation on flutter control of cantilever pipes conveying fluid by using an adaptive approach. An instantaneous optimal control method was reported by Lin and Tsai [16] for non-linear vibration suppression of a cantilever pipe conveying fluid with flutter instability.

To the authors' knowledge, active modal control of pipes conveying fluid with divergence instability considered has not been reported in the literature. A new control formulation must be developed to address such a concern. In the following sections, a general finite element formulation with the modal control approach is presented. A detailed examination of the case of using one actuator for the control of one complex mode is addressed. A subsequent investigation of the control formulation for a divergent mode is then conducted. A numerical example is provided to illustrate the approach developed.

2. Model development

Fig. 1 shows a fixed–fixed pipe conveying fluid with two independent arms which are controlled by extension or contraction of the attached springs to create moments acting on the pipe. Two control inputs can be realized by using the present configuration. A simpler form of the control mechanism, which is capable of providing only one control input, has been reported by Lin and Trethewey [17] for active vibration control of a beam subjected to a moving load by using the coupled mode control formulation. The following equation of motion can be obtained with the use of finite element modelling technique:

$$[\mathbf{M}]\{\ddot{\mathbf{D}}(t)\} + [\mathbf{C}]\{\dot{\mathbf{D}}(t)\} + [\mathbf{K}]\{\mathbf{D}(t)\} = \lfloor \bar{\mathbf{N}} \rfloor_L^{\mathrm{T}} T_L(t) + \lfloor \bar{\mathbf{N}} \rfloor_R^{\mathrm{T}} T_R(t), \tag{1}$$

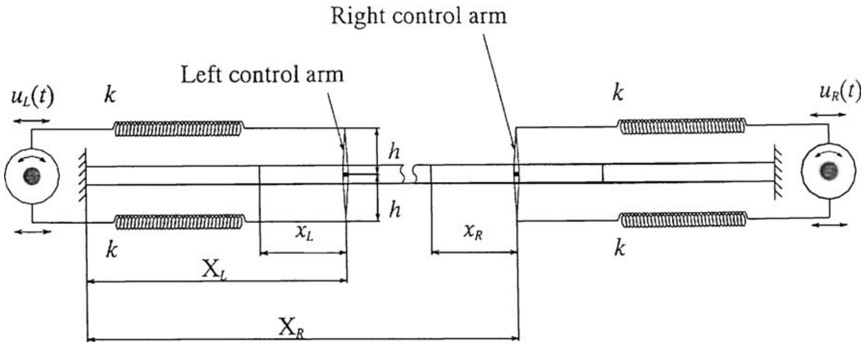

Fig. 1. Finite element model for the fluid-conveying pipe and the control mechanism.

where [**M**], [**C**], and [**K**] are the structural mass, damping, and stiffness matrices respectively. Contributions from both the pipe and the flowing fluid are accounted for. $\lfloor \bar{\mathbf{N}} \rfloor_L^{\mathrm{T}}$ and $\lfloor \bar{\mathbf{N}} \rfloor_R^{\mathrm{T}}$ are the transposition of the shape functions for rotation evaluated at the left and right control arms' positions, i.e., x_L and x_R, respectively, as shown in Fig. 1. $\{\mathbf{D}(t)\}$, $\{\dot{\mathbf{D}}(t)\}$, and $\{\ddot{\mathbf{D}}(t)\}$, denote displacement, velocity, and acceleration vectors respectively. The detailed description of the element entries within the structural matrices and the shape functions can be found in the work of Chu and Lin [18] where the dynamics of fluid-conveying Timoshenko pipes was analyzed. $T_L(t)$ and $T_R(t)$ are the left and right control moments respectively. They are created due to the extension or contraction of the corresponding springs and can be described as

$$T_L(t) = 2k_s h(u_L(t) - h\lfloor \bar{\mathbf{N}} \rfloor_L \{\mathbf{D}(t)\}),$$
$$T_R(t) = 2k_s h(u_R(t) - h\lfloor \bar{\mathbf{N}} \rfloor_R \{\mathbf{D}(t)\}), \tag{2}$$

where k_s denotes the control spring constant; h is the length of the control arm; $u_L(t)$ and $u_R(t)$ are the actively controlled extension or contraction of the left and right spring pairs respectively. The last term on the right side of Eq. (2) expresses the passive effect of the springs. Note that clockwise rotation is taken to be positive for both the finite element nodes of the pipe and control motors which produce the control inputs, $u_L(t)$ and $u_R(t)$.

Independent modal space control technique is considered in this work. To proceed, the system equations must first be decoupled in modal space, where the modal equations are independent of each other. Eq. (1) is rearranged by decomposing the second order ordinary differential equations into first order ones:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \tag{3}$$

in which

$$\mathbf{x}(t) = \begin{pmatrix} \{\dot{\mathbf{D}}(t)\} \\ \{\mathbf{D}(t)\} \end{pmatrix},$$

$$\mathbf{A} = \begin{bmatrix} -[\mathbf{M}]^{-1}[\mathbf{C}] & -[\mathbf{M}]^{-1}([\mathbf{K}] + [\bar{\mathbf{k}}]) \\ [\mathbf{I}] & [\mathbf{0}] \end{bmatrix},$$

$$\mathbf{B} = 2k_s h \begin{pmatrix} [\mathbf{M}]^{-1}\lfloor \bar{\mathbf{N}} \rfloor_L^{\mathrm{T}} & [\mathbf{M}]^{-1}\lfloor \bar{\mathbf{N}} \rfloor_R^{\mathrm{T}} \\ \{\mathbf{0}\} & \{\mathbf{0}\} \end{pmatrix} \tag{4}$$

and

$$\mathbf{u}(t) = \lfloor u_L(t) \quad u_R(t) \rfloor^{\mathrm{T}}. \tag{5}$$

where

$$[\bar{\mathbf{k}}] = 2[\hat{\mathbf{N}}]k_s h^2, \tag{6}$$

in which

$$[\hat{\mathbf{N}}] = (\lfloor \bar{\mathbf{N}} \rfloor_L^{\mathrm{T}} \lfloor \bar{\mathbf{N}} \rfloor_L + \lfloor \bar{\mathbf{N}} \rfloor_R^{\mathrm{T}} \lfloor \bar{\mathbf{N}} \rfloor_R). \tag{7}$$

The right and left modal matrices in real quantities can be arranged as [19]

$$\mathbf{R} = \begin{bmatrix} \mathbf{a}_1 & \mathbf{b}_1 & \mathbf{a}_2 & \mathbf{b}_2 & \cdots & \mathbf{a}_n & \mathbf{b}_n \end{bmatrix},$$

$$\mathbf{L} = \begin{bmatrix} \mathbf{c}_1 & -\mathbf{d}_1 & \mathbf{c}_2 & -\mathbf{d}_2 & \cdots & \mathbf{c}_n & -\mathbf{d}_n \end{bmatrix}, \tag{8}$$

where the odd and even column entries represent the real and imaginary parts of the corresponding eigenvectors respectively. Note that the general descriptions as shown in Eq. (8) need to be modified for the pipe conveying fluid with a divergent mode. As the flow speed exceeds the critical one, the fundamental mode of the fixed–fixed pipe system considered here is divergent [8], i.e., the eigenvalues of the first mode do not appear as complex conjugate pairs, as opposed to the other modes. They are real numbers with the imaginary part being zero. The corresponding modal vectors are real, instead of complex. The divergence instability in this case is due to the centrifugal fluid force, which acts in the same manner as a compressive load. The effective stiffness of the system is diminished with increasing fluid flow speed. For sufficiently large flow speed, the restoring flexural force cannot resist the destabilizing centrifugal fluid force, which results in buckling or known as divergence. For a fixed–fixed pipe conveying fluid, the coriolis forces do no work [8]. The first two columns of the left and right modal matrices shown in Eq. (8) are replaced with the real modal vectors corresponding to the first two eigenvalues, the fundamental mode. The orthogonality condition leads to:

$$\mathbf{L}^{\mathrm{T}}\mathbf{R} = \begin{bmatrix} 2 & & & & & & \\ & 2 & & & zeros & & \\ & & 1 & & & & \\ & & & 1 & & & \\ & & & & \ddots & & \\ & zeros & & & & 1 & \\ & & & & & & 1 \end{bmatrix} \tag{9}$$

and

$$\mathbf{L}^{\mathrm{T}}\mathbf{A}\mathbf{R} = \Lambda, \tag{10}$$

in which

$$\Lambda = \begin{bmatrix} 2\sigma_{11} & 0 & & & & & \\ 0 & 2\sigma_{12} & & & & zeros & \\ & & \sigma_2 & \omega_2 & & & \\ & & -\omega_2 & \sigma_2 & & & \\ & & & & \ddots & & \\ & zeros & & & & \sigma_n & \omega_n \\ & & & & & -\omega_n & \sigma_n \end{bmatrix}, \tag{11}$$

where σ_{11} and σ_{12} denote the eigenvalues of the first mode, with imaginary parts being zero and σ_i and ω_i, $i = 2, 3, \ldots, n$, represent the real and imaginary parts of the ith mode respectively. Alternatively, the following equations can be used:

$$\Lambda_{11} = \begin{bmatrix} 2\sigma_{11} & 0 \\ 0 & 2\sigma_{12} \end{bmatrix}, \tag{12}$$

$$\Lambda_s = \begin{bmatrix} \sigma_s & \omega_s \\ -\omega_s & \sigma_s \end{bmatrix}, \qquad s = 2, 3, \ldots, n. \tag{13}$$

The physical state vector can be transformed to the modal co-ordinate by using the following transformation:

$$\mathbf{x}(t) = \mathbf{R}\mathbf{z}(t). \tag{14}$$

Substituting Eq. (14) into Eq. (3) and premultiplying by $\mathbf{L}^{\mathrm{T}}$ yields

$$\mathbf{L}^{\mathrm{T}}\mathbf{R}\dot{\mathbf{z}}(t) = \Lambda\mathbf{z}(t) + \mathbf{Z}(t), \tag{15}$$

where

$$\mathbf{Z}(t) = \mathbf{L}^{\mathrm{T}}\mathbf{B}\mathbf{u}(t) \tag{16}$$

are the modal control forces. Eq. (15) can be represented by n pairs of equation in the following form:

$$\begin{aligned} \dot{\mathbf{z}}_1(t) &= \Lambda_1\mathbf{z}_1(t) + \tfrac{1}{2}\mathbf{Z}_1(t) \\ \dot{\mathbf{z}}_s(t) &= \Lambda_s\mathbf{z}_s(t) + \mathbf{Z}_s(t), \qquad s = 2, 3, \ldots, n, \end{aligned} \tag{17}$$

where

$$\begin{aligned} \mathbf{z}_1(t) &= \lfloor z_1(t) \quad z_2(t) \rfloor^{\mathrm{T}}, \\ \Lambda_1 &= \begin{bmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{12} \end{bmatrix}, \\ \mathbf{Z}_1(t) &= \lfloor Z_1(t) \quad Z_2(t) \rfloor^{\mathrm{T}}, \\ \mathbf{z}_s(t) &= \lfloor z_{2s-1}(t) \quad z_{2s}(t) \rfloor^{\mathrm{T}}, \\ \mathbf{Z}_s(t) &= \lfloor Z_{2s-1}(t) \quad Z_{2s}(t) \rfloor^{\mathrm{T}}. \end{aligned} \tag{18}$$

Note that the factor, 1/2, preceeding the first modal force vector can be absorbed in the corresponding left eigenvectors because the eigenvectors can be arbitrarily normalized. This results in an identical equation format for the divergent mode and the rest complex modes. Optimal independent modal space control of a system can be realized by minimizing the cost function

$$J = \sum_{s=1}^{q} J_s, \tag{19}$$

in which J_s is the independent modal cost function for steady state system response, and q is the number of modes to be controlled. J_s is defined as

$$J_s = \int_0^\infty (\mathbf{z}_s^{\mathrm{T}}(t)\mathbf{z}_s(t) + \mathbf{Z}_s^{\mathrm{T}}(t)\mathbf{F}_s\mathbf{Z}_s(t))\,\mathrm{d}t, \qquad s = 1, 2, \ldots, q, \tag{20}$$

where $\mathbf{F}_s$ is the weighting matrix to be selected by the analyst. For this formulation, the optimal modal control forces can be obtained as [20]

$$\mathbf{Z}_s(t) = -\mathbf{F}_s^{-1}\mathbf{P}_s\mathbf{z}_s(t) = -\mathbf{K}_s\mathbf{z}_s(t), \qquad s = 1, 2, \ldots, q, \tag{21}$$

where $\mathbf{K}_s$ is the feedback gain matrix designed for this optimal controller and $\mathbf{P}_s$ is obtained by solving the matrix Riccati equation

$$\mathbf{P}_s\Lambda_s + \Lambda_s^{\mathrm{T}}\mathbf{P}_s - \mathbf{P}_s\mathbf{F}_s^{-1}\mathbf{P}_s + \mathbf{I} = 0, \qquad s = 1, 2, \ldots, q. \tag{22}$$

To control a complex mode, Meirovitch and Baruh [19] and Meirovitch and Ghosh [21] proposed the control of the two-dimensional modal displacement vector $\mathbf{z}_s$ by only one component of the modal force control vector. This is accomplished by choosing

$$\mathbf{F}_s^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & F_s^{-1} \end{bmatrix}. \tag{23}$$

This is equivalent to using an infinite cost weight for the first component of the complex modal force vector and that component will then become zero in the optimization process since its cost is infinite. The problem is that control spillover into the first component of the modal force vector for the complex mode controlled in the present case may lead to instability, depending on how the eigenvectors are normalized, as explored in detail by Lin and Chu [13] concerning modal control of general dynamic systems. For controlling a complex mode, a different form for the weighting matrix is proposed, which is given below

$$\mathbf{F}_s^{-1} = \begin{bmatrix} \bar{F}_s & 0 \\ 0 & \bar{F}_s \end{bmatrix}, \qquad s = 2, 3, \ldots, q, \tag{24}$$

where

$$\bar{F}_s \equiv F_s^{-1}, \qquad s = 2, 3, \ldots, q. \tag{25}$$

In the new design presented by Lin and Chu [13] for independent modal space control of general dynamic systems, it has been shown that the new design cannot lead to instability by using one actuator for the control of one stable complex mode, regardless of how the complex eigenvectors are normalized in the design process, whereas the previous reported approach may

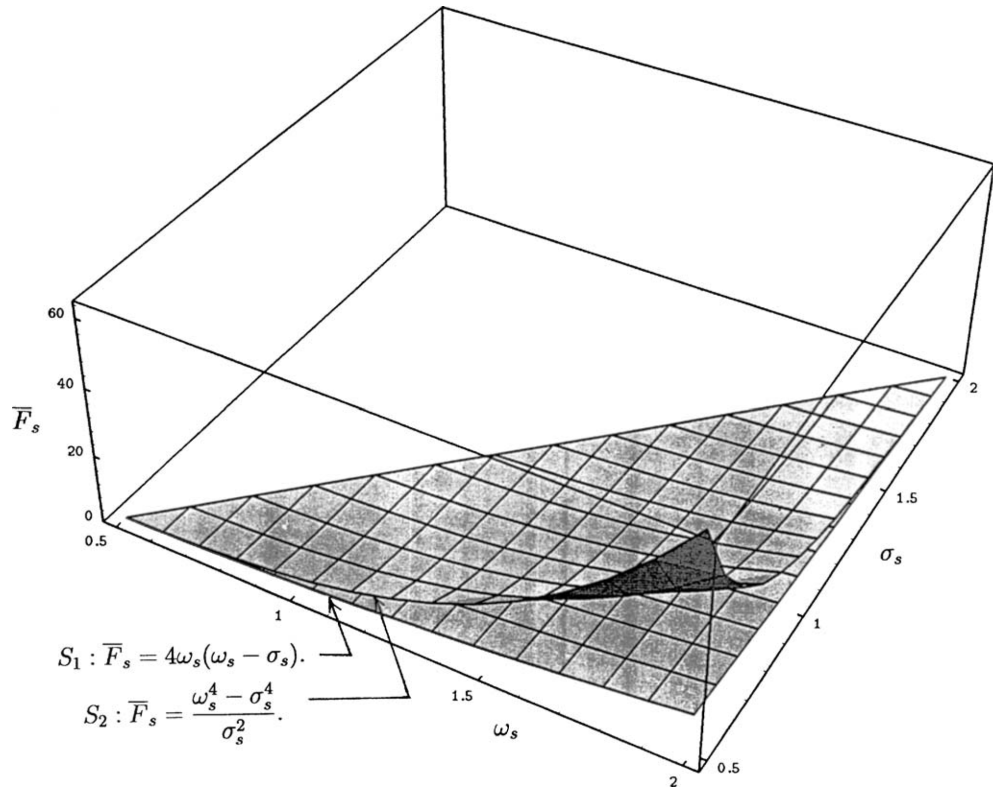

Fig. 2. A typical stability map for controlling a complex mode using one actuator.

lead to closed loop instability depending on the normalization procedure of the complex eigenvectors. However, a complete stability analysis of the control formulation has not been established. Stability criteria will be derived in this work to examine the stability property of the controlled system with a typical stability map shown to illustrate the concept. This will make clear the applicable regions and limitations of controlling one complex mode using one actuator for independent modal space control of general dynamic systems.

When the approach as suggested by Lin and Chu [13] is applied, the closed loop eigenvalues for the complex mode controlled can be shown as

$$\lambda_s = \sigma_s - \frac{\sigma_s + \sqrt{\sigma_s^2 + \bar{F}_s}}{2} \pm \frac{1}{2}\sqrt{(\sigma_s + \sqrt{\sigma_s^2 + \bar{F}_s})^2 - 4\omega_s^2}. \tag{26}$$

Unlike the approach proposed by the pioneer developers of the IMSC technique, the present formulation leads to closed loop eigenvalues which are independent of how the complex eigenvectors are normalized. It is apparent from inspection of Eq. (26) that for a stable complex mode, the control can never destabilize the system and always adds to improve the system performance. However, for an unstable complex mode, the stability characteristics of the closed loop mode needs to be established. From Eq. (26), it can be shown that if the last term in the right side turns out to be imaginary or zero, the closed loop mode is stable since the resultant of the first

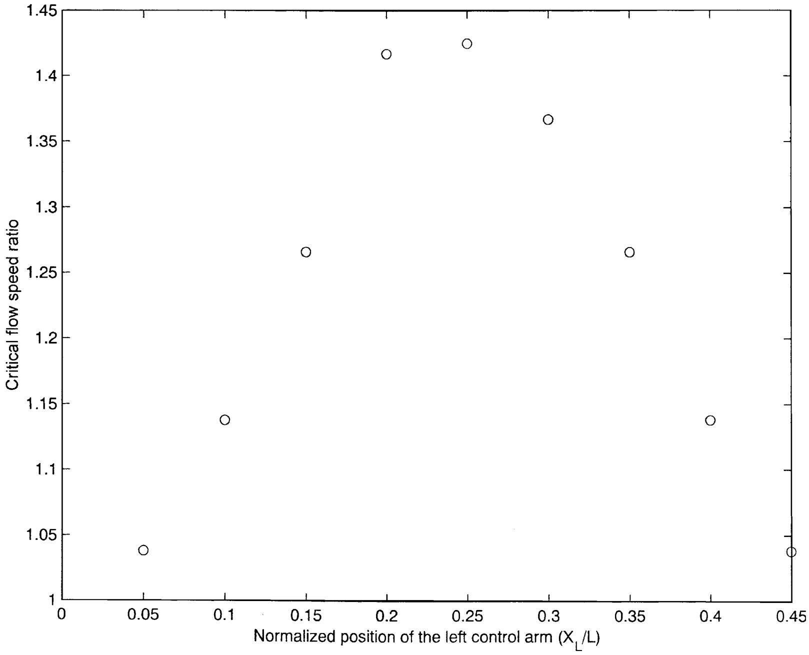

Fig. 3. Effect of the positions of the control arms on the critical flow speed.

two terms are always negative, knowing that $\bar{F}_s$ is positive in the optimal control formulation. This implies

$$(\sigma_s + \sqrt{\sigma_s^2 + \bar{F}_s})^2 - 4\omega_s^2 \leqslant 0, \tag{27}$$

thus

$$\sigma_s + \sqrt{\sigma_s^2 + \bar{F}_s} \leqslant 2\omega_s, \tag{28}$$

which leads to the determination of $\bar{F}_s$ for stability:

$$\bar{F}_s \leqslant 4\omega_s(\omega_s - \sigma_s). \tag{29}$$

If the last term in the right side of Eq. (26) turns out to be a non-zero real number, then stability of the controlled complex mode can be assured if the closed loop eigenvalues are negative real numbers. This condition requires that

$$\sigma_s - \frac{\sigma_s + \sqrt{\sigma_s^2 + \bar{F}_s}}{2} < -\frac{1}{2}\sqrt{(\sigma_s + \sqrt{\sigma_s^2 + \bar{F}_s})^2 - 4\omega_s^2}. \tag{30}$$

Solving for $\bar{F}_s$ yields the following criterion for stability when controlling an unstable complex mode:

$$\bar{F}_s < \frac{\omega_s^4 - \sigma_s^4}{\sigma_s^2}. \tag{31}$$

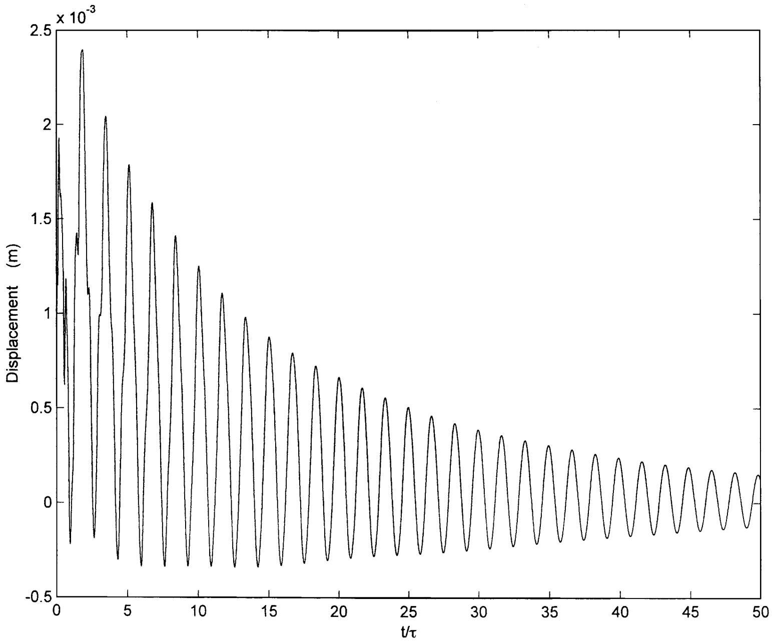

Fig. 4. Central displacement response of the fluid-conveying pipe with the first two modes aimed for control.

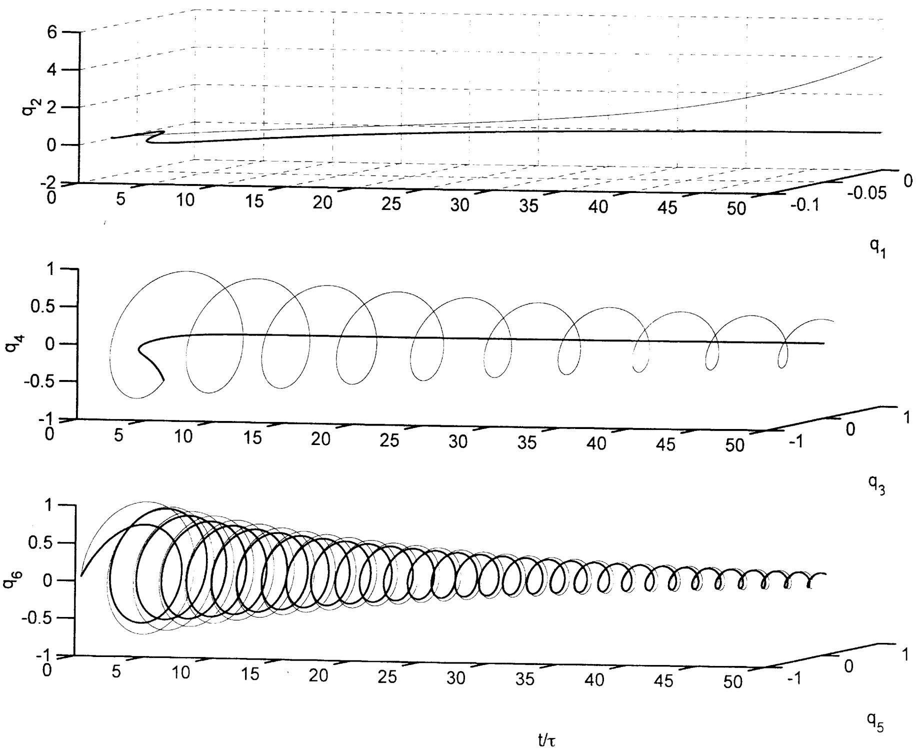

Fig. 5. The first three modal responses of the fluid-conveying pipe with the first two modes aimed for control. Top: the first mode; middle: the second mode; bottom: the third mode. Thick line: controlled; thin line: uncontrolled.

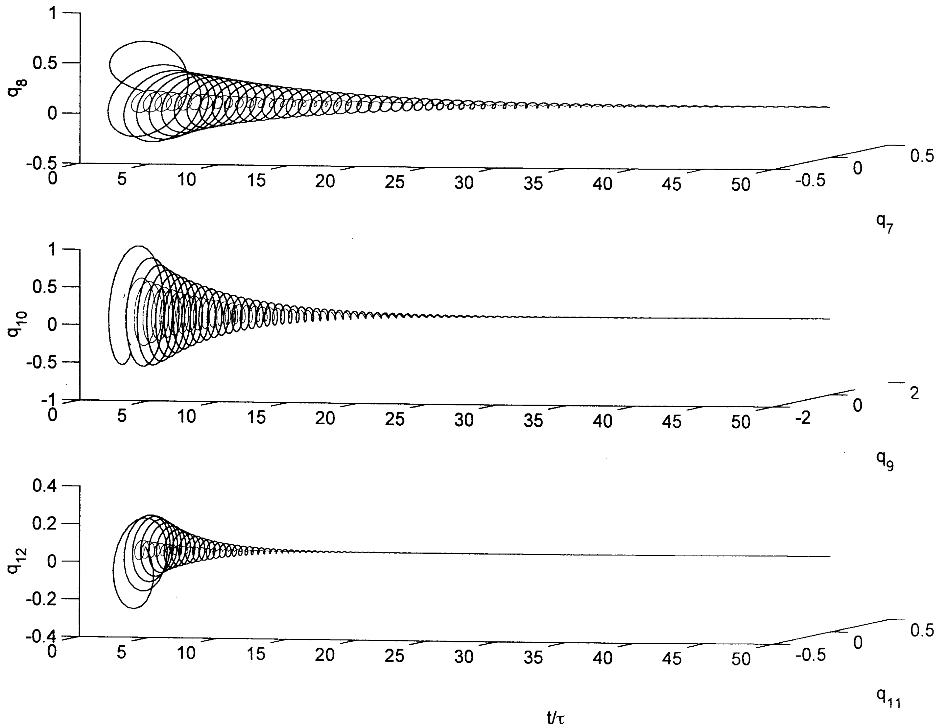

Fig. 6. Modal responses of the fourth to the sixth modes of the fluid-conveying pipe with the first two modes aimed for control. Top: the fourth mode; middle: the fifth mode; bottom: the sixth mode. Thick line: controlled; thin line: uncontrolled.

A typical stability map for controlling an unstable complex mode is illustrated in Fig. 2. The stability characteristics can be summarized as follows:

1. For the region below S_1, the closed loop eigenvalues lie in the left side of the complex plane, hence the controlled complex mode is underdamped.
2. On surface S_1, the closed loop eigenvalues are repeated negative real numbers, hence the controlled complex mode is critically damped. This feature is utilized when controlling complex modes of the fluid-conveying pipe discussed in the subsequent section.
3. For the region between surfaces S_1 and S_2, the closed loop eigenvalues are negative real numbers, hence the controlled complex mode is overdamped.
4. On surface S_2, the closed loop eigenvalues have a root being zero, that is, a root is at the origin of the complex plane. Thus, surface S_2 is the divergence boundary for the system considered.
5. For the region above S_2, the closed loop eigenvalues contain a positive real root, hence the controlled complex mode is divergent.

From observation of Eqs. (29) and (31), any positive $\bar{F}_s$ will make the system unstable when $\sigma_s \geqslant \omega_s$. In this situation, it is impossible to control an unstable complex mode using one actuator.

For the system considered in this work concerning a fixed–fixed fluid-conveying pipe with flow speed above the critical one, the fundamental mode exhibits divergence instability. The

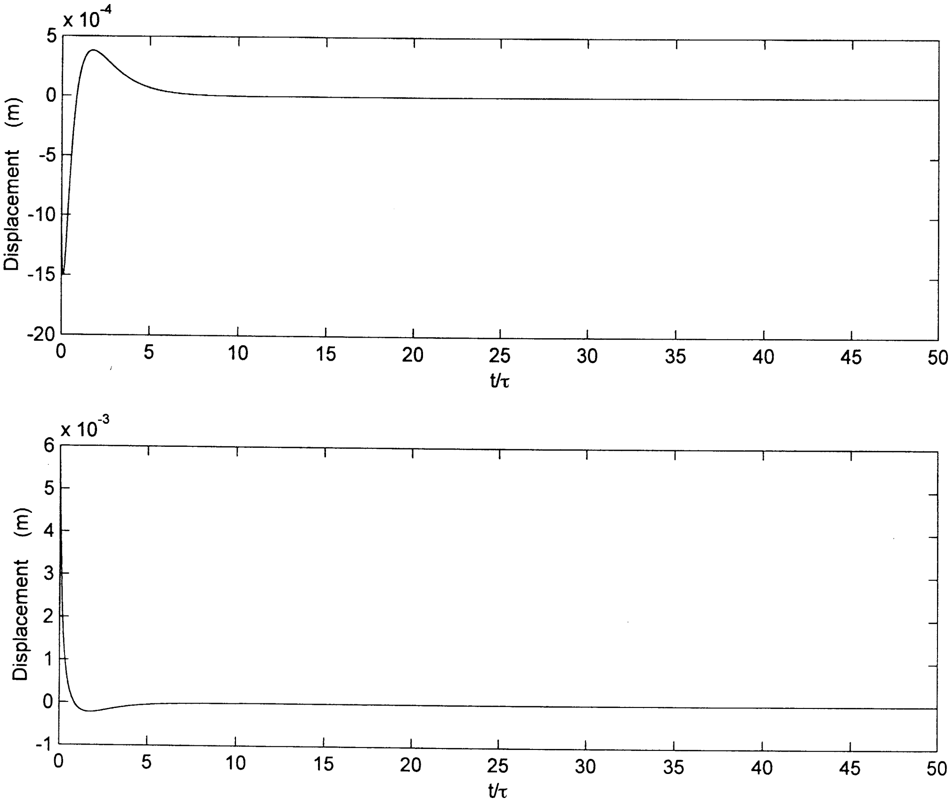

Fig. 7. Active extension/contraction of the control springs for the fluid-conveying pipe with the first two modes aimed for control. Top: left actuator; bottom: right actuator.

eigenvalues corresponding to this divergent mode are real, as opposed to the other modes which are complex. One actuator is proposed to control this mode. The relationship between the modal control force and the physical control input for the fundamental mode can be shown as

$$\begin{pmatrix} Z_1(t) \\ Z_2(t) \end{pmatrix} = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} \bar{u}(t), \tag{32}$$

in which $\bar{u}(t)$ represents the actual control force; η_1 and η_2 are not unique due to non-unique normalized eigenvectors. When the weighting matrix in the form as shown in Eq. (24) is applied, the closed loop system matrix with control spillover between the complex modal force vectors considered can be written as

$$A_1^* = \begin{bmatrix} \sigma_{11} - \frac{\eta_1^2}{\eta_1^2+\eta_2^2}\bar{F}_1\bar{P}_1 & -\frac{\eta_1\eta_2}{\eta_1^2+\eta_2^2}\bar{F}_1\bar{P}_1 \\ -\frac{\eta_1\eta_2}{\eta_1^2+\eta_2^2}\bar{F}_1\bar{P}_1 & \sigma_{12} - \frac{\eta_2^2}{\eta_1^2+\eta_2^2}\bar{F}_1\bar{P}_1 \end{bmatrix}, \tag{33}$$

where $\bar{F}_1$ and $\bar{P}_1$ denote an element of the weighting matrix and the corresponding solution for the elements of the Riccati matrix respectively. Note that the present formulation yields a diagonal Riccati matrix with identical diagonal elements [13]. The eigenvalues, the closed loop poles, solved

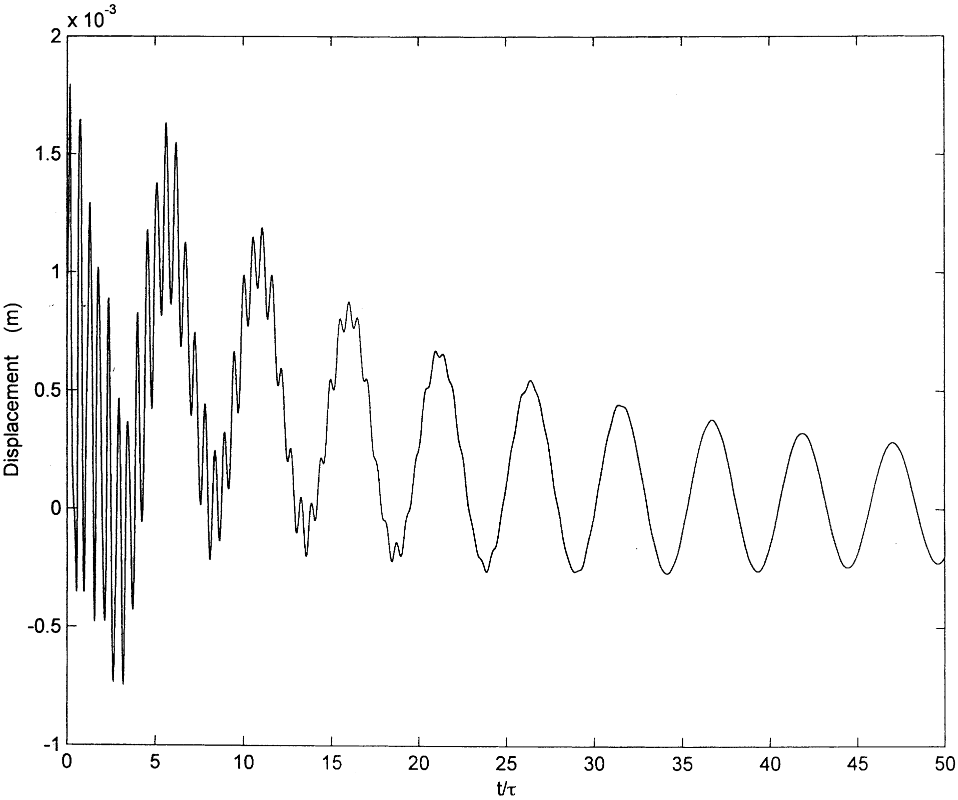

Fig. 8. Central displacement response of the fluid-conveying pipe with the first mode and the third mode aimed for control.

from Eq. (33) are very complicated and are dependent of how the eigenvectors are normalized. This feature is highly undesirable because the way to normalize the eigenvectors is arbitrary. This situation is similar to the case when the form as shown in Eq. (23) is used to control a complex mode. For the system considered here, the eigenvalues of the divergent mode appear to be $\sigma_{11} < 0$ and $\sigma_{12} > 0$. It is natural to aim the control for the positive eigenvalue which leads to divergent response. To realize this, a weighting matrix in the form of Eq. (23) is examined here, where an infinite weight is used for the first modal force. In the optimization process, the control for the first modal force will then turn out to be zero, indicating the first eigenvalue, which is negative, is left uncontrolled by the intention of system design. However, because control spillover exists between the modal forces, the system behavior must be analyzed taking into account such an effect. It turns out the Riccati matrix equation for this divergent mode can be solved in closed form and is

$$\mathbf{P}_1 = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2\sigma_{11}} & 0 \\ 0 & \frac{\sigma_{12}+\sqrt{\sigma_{12}^2+\bar{F}_1}}{\bar{F}_1} \end{bmatrix}, \tag{34}$$

in which

$$\bar{F}_1 \equiv F_1^{-1}. \tag{35}$$

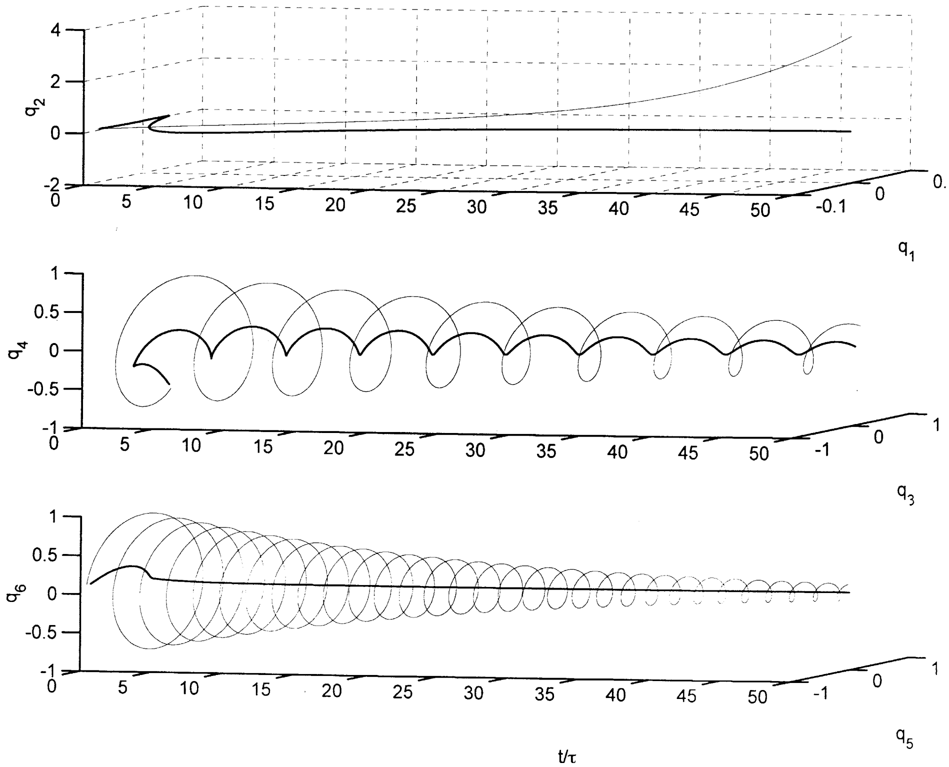

Fig. 9. The first three modal responses of the fluid-conveying pipe with the first mode and the third mode aimed for control. Top: the first mode; middle: the second mode; bottom: the third mode. Thick line: controlled; thin line: uncontrolled.

The designed modal control forces are then

$$\begin{pmatrix} Z_1(t) \\ Z_2(t) \end{pmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\bar{F}_1 P_{22} \end{bmatrix} \begin{pmatrix} z_1(t) \\ z_2(t) \end{pmatrix}. \tag{36}$$

The designed physical control input can be obtained from Eqs. (32) and (36)

$$\bar{u}(t) = -\frac{\bar{F}_1 P_{22}}{\eta_2} z_2(t). \tag{37}$$

The actual modal control forces, as opposed to those designed as shown in Eq. (36), are determined by substituting Eq. (37) into Eq. (32)

$$\begin{pmatrix} Z_1(t) \\ Z_2(t) \end{pmatrix} = \begin{bmatrix} 0 & -\frac{\eta_1}{\eta_2}\bar{F}_1 P_{22} \\ 0 & -\bar{F}_1 P_{22} \end{bmatrix} \begin{pmatrix} z_1(t) \\ z_2(t) \end{pmatrix}. \tag{38}$$

The closed loop system equations for the fundamental mode then become

$$\dot{\mathbf{z}}_1 = A_1^* \mathbf{z}_1, \tag{39}$$

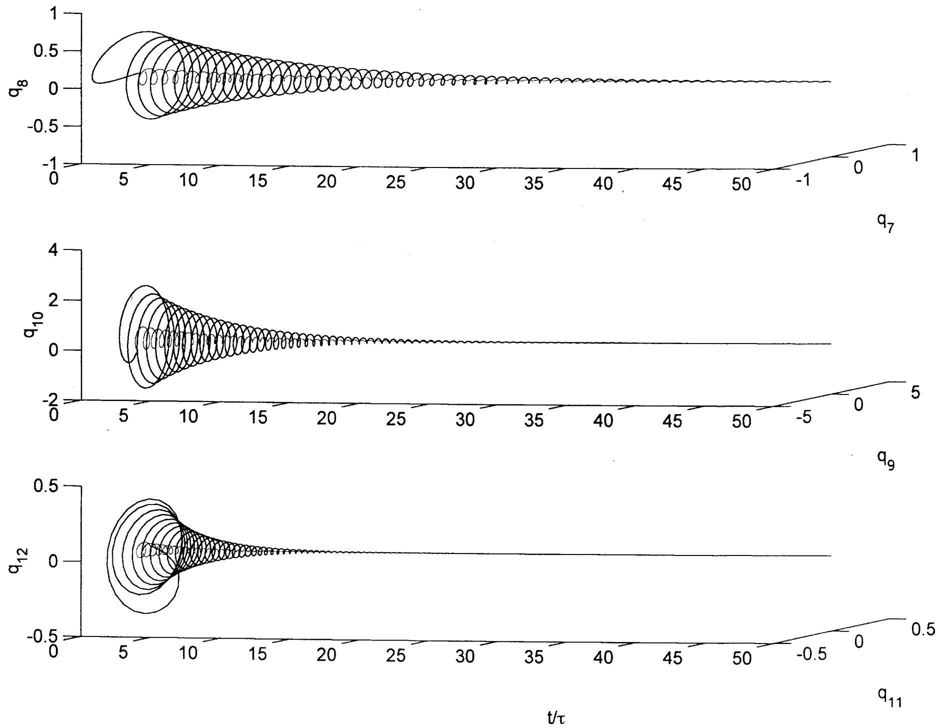

Fig. 10. Modal responses of the fourth to the sixth modes of the fluid-conveying pipe with the first mode and the third mode aimed for control. Top: the fourth mode; middle: the fifth mode; bottom: the sixth mode. Thick line: controlled; thin line: uncontrolled.

where

$$A_1^* = \begin{bmatrix} \sigma_{11} & -\frac{\eta_1}{\eta_2}\bar{F}_1 P_{22} \\ 0 & \sigma_{12} - \bar{F}_1 P_{22} \end{bmatrix}. \tag{40}$$

The closed loop eigenvalues can then be obtained as

$$\lambda_{11} = \sigma_{11}, \tag{41}$$

$$\lambda_{12} = -\sqrt{\bar{F}_1 + \sigma_{12}^2}. \tag{42}$$

Note that the first eigenvalue, which is negative, remains unchanged after control. With the use of any weighting factor, which is a positive number by design, the second eigenvalue is always negative disregarding how positive the open loop eigenvalue is. This demonstrates that the approach proposed here can assure closed loop stability for the divergent mode. It is interesting to note that a weighting matrix in a form as shown in Eq. (23) suffers a serious stability problem when used for controlling a complex mode [13], whereas it can be successfully applied for controlling a divergent mode with closed loop stability guaranteed.

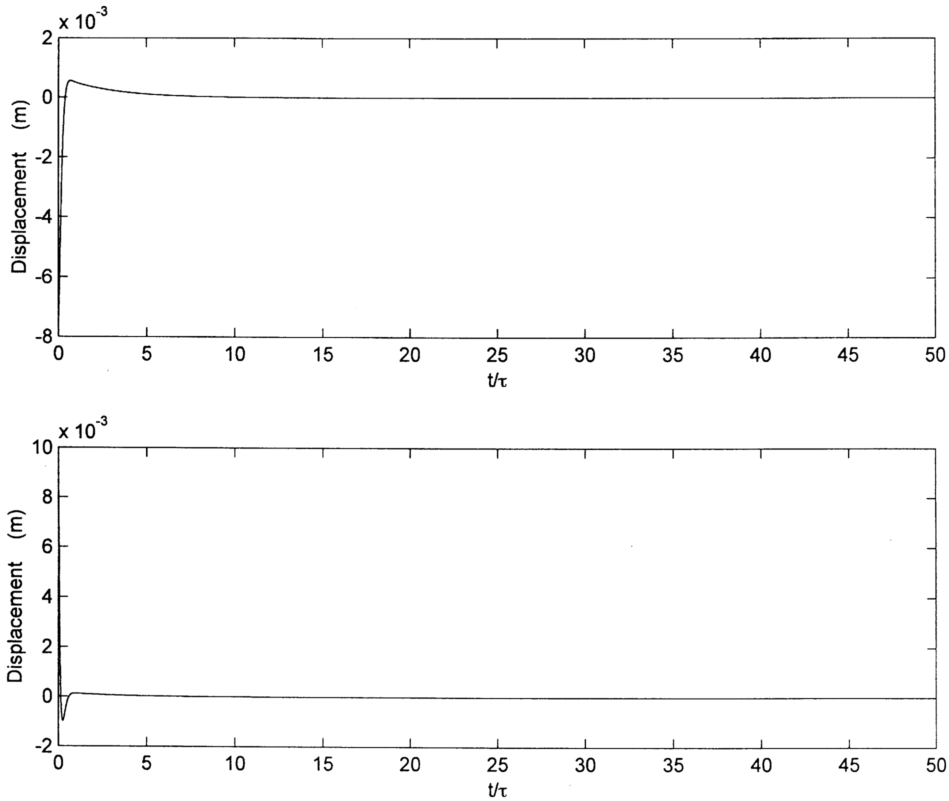

Fig. 11. Active extension/contraction of the control springs for the fluid-conveying pipe with the first mode and the third mode aimed for control. Top: left actuator; bottom: right actuator.

3. Numerical results

The pipe parameters used for the simulation study are: Young's modulus, $E = 6.89(10^{10})\ \mathrm{N/m^2}$; length, $L = 0.5585$ m; mass per unit length, $\rho A = 0.342$ kg/m; outside diameter, $d_o = 0.0254$ m; inside diameter, $d_i = 0.0221$ m. In this study, a total of 12 elements is used for the numerical analysis. Rayleigh damping is considered for the pipe. The pipe modal dampings for the first mode and the sixth mode are taken to be one percent and two percents of the critical damping respectively, which results in the modal dampings for the modes between these two specified frequencies lying below those just specified [22]. The mass per unit length of the fluid, μ_f, is 0.0855 kg/m. The length of the control arms, h, is 0.07 m. The control spring constant, k_s, is $900EI/L^3$.

Fig. 3 shows the effect of control arms' positions on the critical flow speed ratio, defined as the ratio of the critical flow speed with and without the passive effect of the control mechanism considered. The right control arm is considered to be symmetrically positioned with respect to the left one. As can be seen from Fig. 3, the favorable locations of the control arms are located at 1/4 and 3/4 of the pipe length for the left and the right control arms respectively. This configuration is

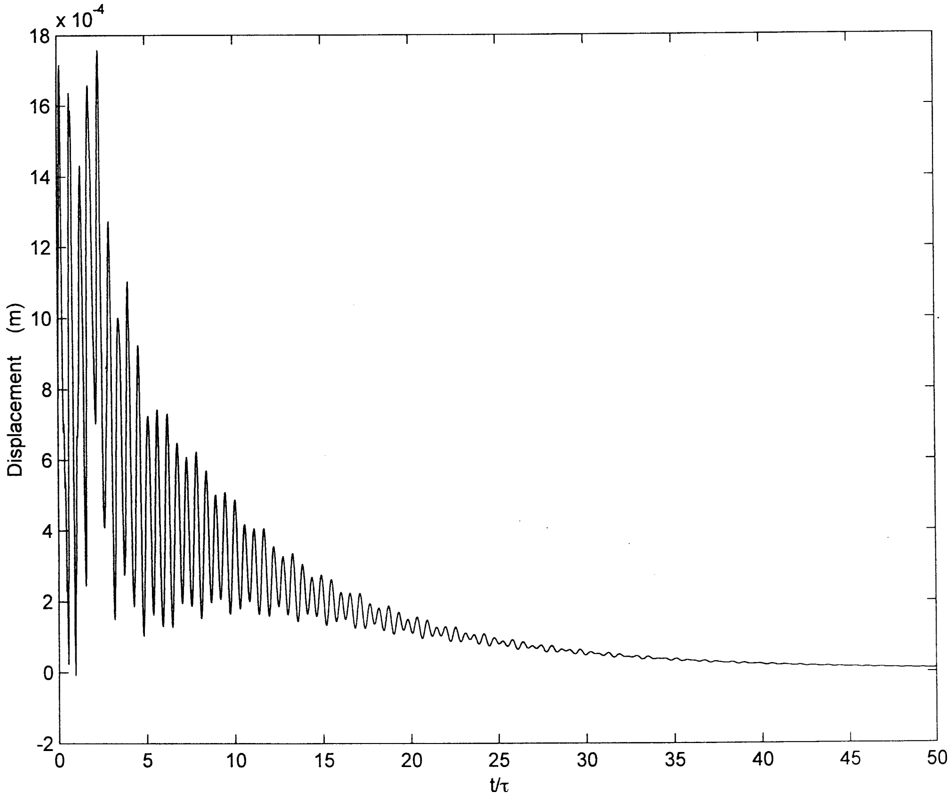

Fig. 12. Central displacement response of the fluid-conveying pipe with the mode switching control enabled.

utilized subsequently for active control of the fluid-conveying pipe. Note that the modal control inputs are designed first when applying IMSC and are independent on the positions of the control arms. However, in the process of synthesizing the actual control inputs from the modal control inputs, positions of the control arms play an important role. The use of the positions of the control arms as determined above results in smaller actual control inputs as compared to those when other positions are used, which is evident from discussions presented in the previous section.

The dynamic displacement response at the pipe center due to a unit impact load at its mid-span is shown in Fig. 4 with control of the first two modes being considered. The constant fluid flowing velocity is $1.0021v_{cr}$, where v_{cr} is the critical flow speed to buckle the pipe, with the passive effect of the control mechanism considered. The abscissa refers to a normalized time scale, in which τ is the time required for a fluid particle to travel across the pipe span. The pipe will buckle if left uncontrolled due to a super-critical flow speed. As can be seen in Fig. 4, the pipe vibration can be suppressed. The pipe response no longer is divergent and is now dominated by the third mode, which is left uncontrolled in the control system design. Figs. 5 and 6 show the first to the third and the fourth to the sixth modal responses respectively. As can be seen, vibrations of the first two modes are nicely suppressed, whereas the third mode is not affected much due to control spillover.

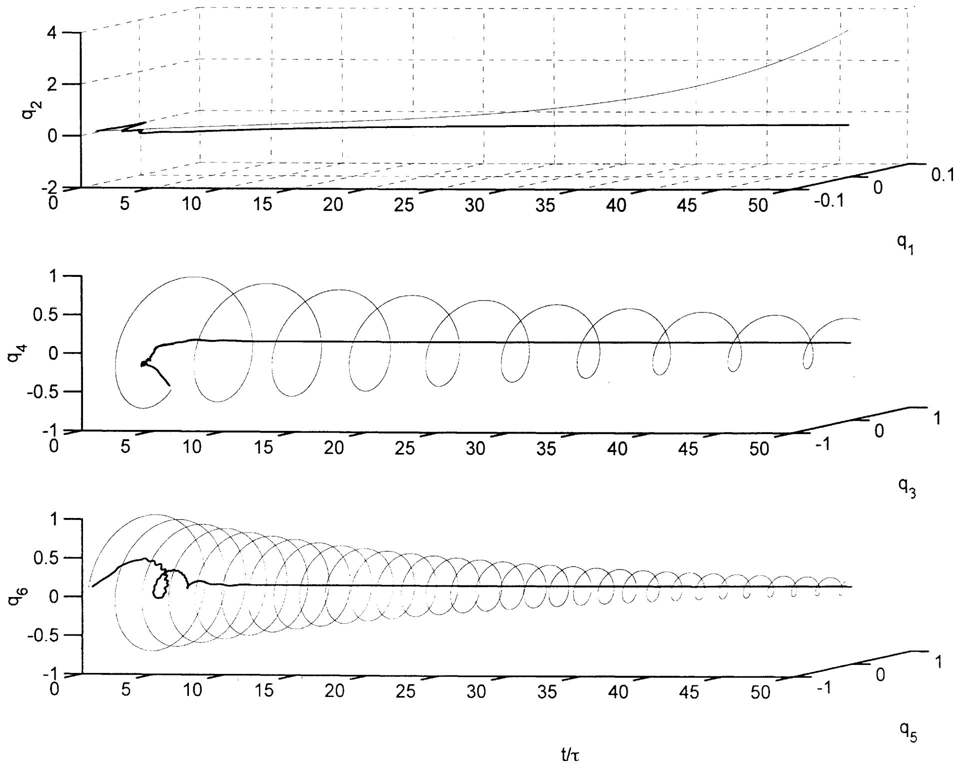

Fig. 13. The first three modal responses of the fluid-conveying pipe with the mode switching control enabled. Top: the first mode; middle: the second mode; bottom: the third mode. Thick line: controlled; thin line: uncontrolled.

The control spillover has a detrimental effect for the fourth to the sixth modal responses. However, higher modal response contribute less to the overall physical response due to their inherently higher rigidities and damping effects [23]. The uncontrolled first modal response can be found to grow monotonously, as predicted by linear theory. The post-buckling response of this mode can only be predicted accurately by a non-linear theory, which is beyond the scope of this work. Note that the IMSC technique can never destabilize the modes uncontrolled due to control spillover from the modes controlled because the modes are controlled independently. In the present case, the modal control forces only consist of modal states of the controlled modes, the first two modes in this case. The corresponding required control inputs, the extension or contraction of the control springs, of the left and right control mechanisms are shown in Fig. 7. The physical control inputs $\mathbf{u(t)}$, as computed from Eq. (16), are dependent on the controlled modal states only, and hence control spillover to all other uncontrolled modes cannot induce instability. The control spillover in such a case acts as disturbance, which is independent of the modal states of the uncontrolled modes.

Fig. 8 illustrates the mid-pipe response when the first and the third modes are aimed for control. The divergent behavior of the fundamental mode can be successfully controlled. The controlled pipe response can be seen to be dominated by the second mode. Figs. 9 and 10 show the first to the third and the fourth to the sixth modal responses respectively. Vibrations of the first and the third

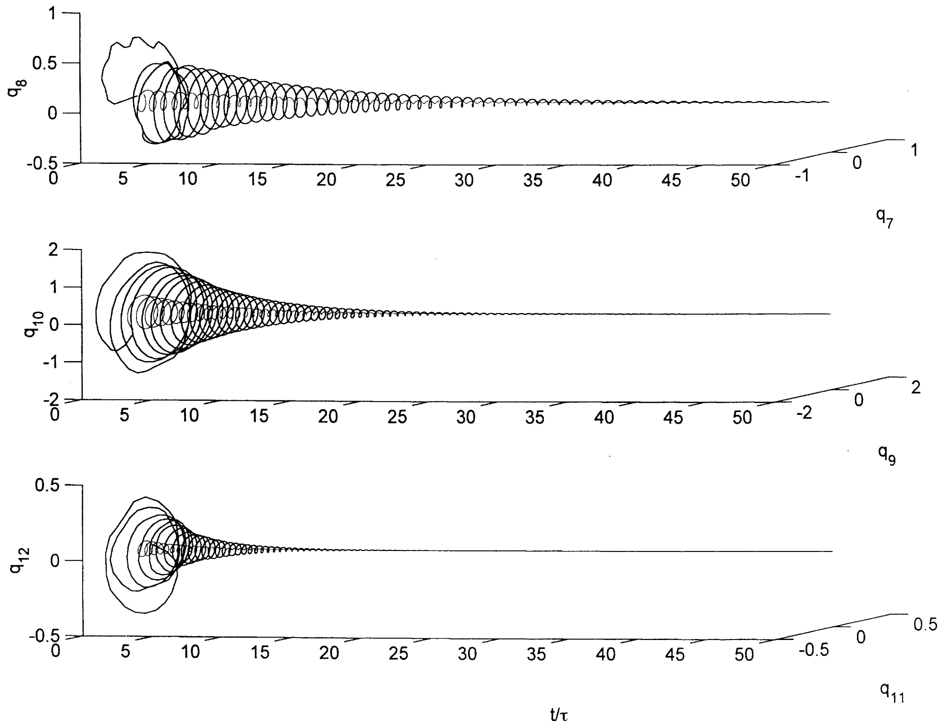

Fig. 14. Modal responses of the fourth to the sixth modes of the fluid-conveying pipe with the mode switching control enabled. Top: the fourth mode; middle: the fifth mode; bottom: the sixth mode. Thick line: controlled; thin line: uncontrolled.

modes are well suppressed. Control spillover seems to have a favorable effect on the second mode, contrary to the previous case. The effect of control spillover is not always detrimental. It depends on both the controlled and uncontrolled modal characteristics and the actuator locations. Similar to the previous case, the control spillover worsens the fourth to the sixth modal responses. The corresponding required control inputs are shown in Fig. 11.

From the above analyses, it shows that the third mode dominates the controlled pipe response when the first two modes are controlled, whereas the controlled pipe response is dominated by the second mode if the first and third modes are aimed for control. Note that the fundamental mode is unstable and must always be controlled. To further suppress the pipe vibration, an approach of switching the control target is examined. This is done by comparing the second and third modal responses at each control step and the control input is switched to control the mode which has a higher modal response. Fig. 12 shows the central pipe response with the switching scheme implemented. As can be seen, the controlled pipe vibration can be further suppressed, as compared to the previous two analysis cases. As shown in Fig. 13, vibrations of the first three modes are nicely suppressed. Control spillover effect for the fourth to the sixth modes is similar to the previous cases, as illustrated in Fig. 14. Fig. 15 shows the corresponding required control inputs. The switching pattern is illustrated in Fig. 16. Unlike the other two cases, the control inputs appear to be larger and consist of alternating spikes rather than smoothly

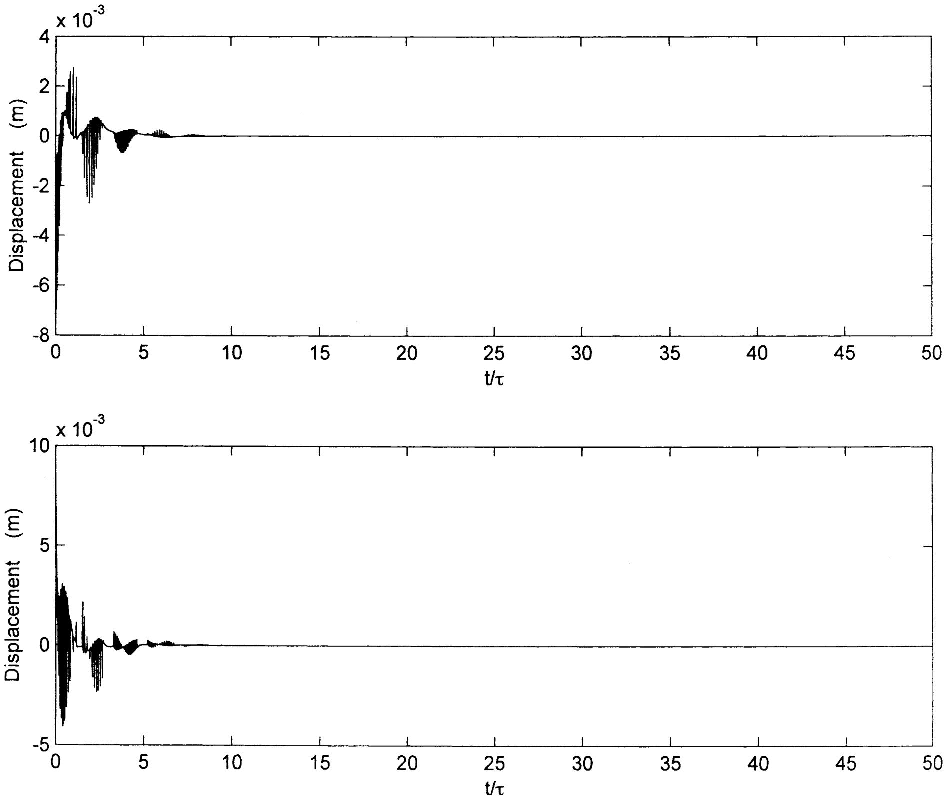

Fig. 15. Active extension/contraction of the control springs for the fluid-conveying pipe with the mode switching control enabled. Top: left actuator; bottom: right actuator.

changing curves. The designer should be aware of this effect during implementation of the control system.

4. Conclusions

Optimal independent modal space control for a fixed–fixed pipe conveying fluid with a supercritical flow speed has been investigated by using a general finite element formulation. The uncontrolled system has a divergence instability in the fundamental mode. The complete stability behavior of a controlled system with the use of one actuator for the control of one complex mode has been determined in this work. For a system having a divergent mode, such as the case as considered in this study, it has been demonstrated that one actuator is sufficient to control the unstable mode with stability of the closed system guaranteed. The Riccati matrix equations are notoriously stiff and often cause numerical convergence problems when coupled mode control technique is used. In this study, the closed form solution for the Riccati matrix equations has been derived, which is virtually impossible for a large dynamic system using the coupled mode control approach. Numerical analysis results show that by switching the control effort to the mode that

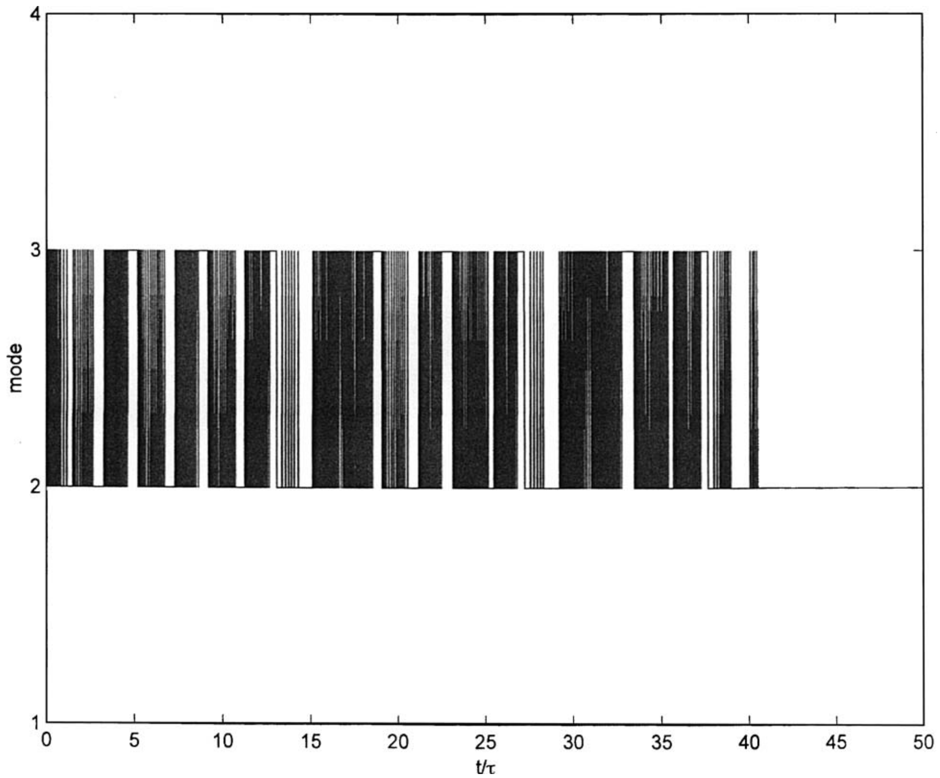

Fig. 16. Mode switching in the control process of the fluid-conveying pipe.

has a higher modal response, the overall pipe vibration can be further suppressed. However, more control inputs and fast acting actuators are required to meet the control requirement. In any case, the divergence instability of the fluid-conveying pipe can be successfully stabilized and excessive structural vibration is suppressed by using the modal control approach presented.

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References

[1] R.W. Gregory, M.P. Païdoussis, Unstable oscillations of tubular cantilevers conveying fluid, parts 1 and 2, Proceedings of the Royal Society of London, Series A 293 (1966) 512–542.
[2] M.P. Païdoussis, Dynamics of tubular cantilevers conveying fluid, Journal of Mechanical Engineering Science 12 (1970) 85–103.
[3] G.W. Housner, Bending vibrations of a pipe line containing flowing fluid, Journal of Applied Mechanics 19 (1952) 205–208.

[4] P.J. Holmes, Pipes supported at both ends cannot flutter, Journal of Applied Mechanics 45 (1978) 619–622.
[5] R.D. Blevins, Flow-induced Vibrations, Van Nostrand Reinhold, New York, 1977.
[6] A. Shulemovich, Flow-induced vibrations caused by roughness in pipes conveying fluid, Journal of Applied Mechanics 53 (1986) 181–186.
[7] T.M. Mulcahy, M.W. Wambsganss, Flow-induced vibration of nuclear reactor components, Shock and Vibration Digest 8 (1976) 318–332.
[8] M.P. Païdoussis, G.X. Li, Pipes conveying fluid: a model dynamical problem, Journal of Fluids and Structures 7 (1993) 137–204 doi: 10.1006/jfls.1993.1011.
[9] J. Tani, Y. Sudani, Active flutter suppression of a tube conveying fluid, The First European Conference on Smart Structures and Materials, Glasgow, 1992, pp. 333–336.
[10] C.-H. Yau, A.K. Bajaj, O.D.I. Nwokah, Active control of chaotic vibration in a constrained flexible pipe conveying fluid, Journal of Fluids and Structures 9 (1995) 99–122 doi: 10.1006/jfls.1995.1005.
[11] Y. Sugiyama, T. Katayama, E. Kanki, K. Nishino, B. Akesson, Stabilization of cantilevered flexible structures by means of an internal flowing fluid, Journal of Fluids and Structures 10 (1996) 653–661 doi: 10.1006/jfls.1996.0043.
[12] Y.-H. Lin, C.-L. Chu, Active flutter control of a cantilever tube conveying fluid using piezoelectric actuators, Journal of Sound and Vibration 196 (1) (1996) 97–105 doi: 10.1006/jsvi.1996.0470.
[13] Y.-H. Lin, C.-L. Chu, A new design for independent modal space control of general dynamic systems, Journal of Sound and Vibration 180 (2) (1995) 351–361 doi: 10.1006/jsvi.1995.0083.
[14] L. Meirovitch, H. Baruh, H. Öz, A comparison of control techniques for large flexible systems, Journal of Guidance and Control 6 (1983) 302–310.
[15] Y.-K. Tsai, Y.-H. Lin, Adaptive modal vibration control of a fluid-conveying cantilever pipe, Journal of Fluids and Structures 11 (1997) 535–547 doi: 10.1006/jfls.1997.0092.
[16] Y.-H. Lin, Y.-K. Tsai, Non-linear active vibration control of a cantilever pipe conveying fluid, Journal of Sound and Vibration 202 (4) (1997) 477–490 doi: 10.1006/jsvi.1996.0858.
[17] Y.-H. Lin, M.W. Trethewey, Active vibration suppression of beam structures subjected to moving loads: a feasibility study using finite elements, Journal of Sound and Vibration 166 (3) (1993) 383–395 doi: 10.1006/jsvi.1993.1302.
[18] C.-L. Chu, Y.-H. Lin, Finite element analysis of fluid-conveying Timoshenko pipes, Shock and Vibration 2 (1995) 247–255.
[19] L. Meirovitch, H. Baruh, Optimal control of damped flexible gyroscopic systems, Journal of Guidance and Control 4 (2) (1981) 157–163.
[20] B.D.O. Anderson, J.B. Moore, Linear Optimal Control, Prentice-Hall, Inc., New Jersey, 1971.
[21] L. Meirovitch, D. Ghosh, Control of flutter in bridges, American Society of Civil Engineers Journal of Engineering Mechanics 113 (5) (1987) 720–736.
[22] R.W. Clough, J. Penzien, Dynamics of Structures, 2nd Edition, McGraw-Hill, Inc., New York, NY, 1993.
[23] L. Meirovitch, Dynamics and Control of Structures, John Wiley & Sons, Inc., New York, NY, 1992.