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Letter to the Editor

Structural transmission at line junctions: a benchmarking exercise

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1. Introduction

Structural vibration transmission calculations are routinely carried out as part of the calculation of sound and vibration transmission through structures. If two or more plates meet together to form a line junction then the properties of the junction can be given in terms of the transmission coefficient, τ, defined as the ratio of the power transmitted across the junction to the power incident upon it. This parameter can be expressed in dB as a transmission loss, $R = 10\log 1/\tau$ [1]. The power may be transmitted by a wave type that is different from the incident wave in which case there will be energy conversion from one wave type to another. In this paper, the plates are assumed to be isotropic, thin and flat and to meet at a continuous line junction. Such junctions are routinely found in buildings, ships, aircraft and other structures.

There are a number of theories for examining sound transmission at such junctions but the most common theory, which is the only one considered in this paper, is that where the transmission coefficient is determined by considering a wave to be incident on the junction formed by the intersection of a number of semi-infinite plates and the amplitude of the waves leaving the junction are computed.

When statistical energy analysis is being used to predict sound and vibration transmission it is usual to assume that the vibration field on each plate is diffuse and that all angles of incidence are equally likely. It is, therefore, the angular averaged transmission coefficient that is of interest.

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The theory of structural vibration transmission has been developed over many years. The earliest calculations were carried out by Cremer [2] in 1948 and subsequently incorporated into the well-known text of Cremer et al. [1]. Due to computational limitations, their examples were restricted to two plates and to either normal incidence with the generation of all waves or random incidence at junctions with no lateral displacement (pinned). Subsequent developments showed how the basic model could be adapted for any incident wave type at junctions where the plates were attached at arbitrary angles, possibly including elastic layers and beam elements. Such a junction can be modelled by considering the intersection of a number of semi-infinite plates which meet at a single line junction where there must be continuity of displacement and slope and equilibrium of moments and forces. Examples are given in joints 1, 2 and 3 in Fig. 1.

In some junctions, the semi-infinite plate elements meet at more than one line junction (or node) at each of which there is continuity and equilibrium. There are then short elements or "strip plates" connecting these nodes with continuity and equilibrium at all nodes. This approach was first considered in a special case by Bhattacharya et al. [3] in 1971 with subsequent developments by others for the more general cases. Examples of these types of junction can be seen in joints 4 and 5 in Fig. 1.

Although there is general agreement about the theory that should be used to model such systems, the conversion of the set of equations into a working computer model is far from routine and simple sign changes (in the several pages of mathematics) can still lead to consistent results that are wrong. New users can compare their numerical models with published data but it is very difficult to be sure that a large numerical model is correct by comparing results with a small published graph. It is, therefore, not uncommon for users to exchange numerical data and to cross-verify their models.

In 1999, a statistical energy analysis network (SEANET) was set by the EU to facilitate the transfer of information between users and one of the outputs from this network has been a formal benchmarking of numerical models for computing transmission coefficients both to provide reassurance to all users that their models were correct and to produce a reference benchmark. A comparison of the results also allows the estimation of the uncertainty that can be expected from the different numerical methods adopted.

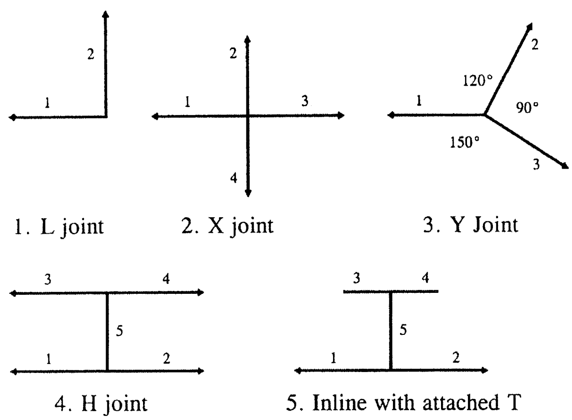


Fig. 1. Five different joints considered in this paper. Junctions 1, 2 and 3 have a single connection point and junctions 4 and 5 also have finite-width strip plates. Arrow heads at the end of lines indicate semi-infinite plates.

The results of the exercise showed that the various models in use did indeed give the same answers though not all users were able to model all junctions as different applications generally have different types of structural junction. This paper presents the results of this exercise.

2. Outline of theory

The plates that make up the junctions are all assumed to be isotropic, thin (in the sense that the thickness is small compared to the wavelength) and flat. The plates are either semi-infinite or have a finite width but are infinitely long. Each plate supports bending waves, longitudinal and transverse shear in-plane waves and is assumed to be loss-less.

The theory used to determine the transmission loss is well described in the literature throughout the last 40 years and so is not described here. Some of the literature describes the calculation method in some detail, often with reference to particular junctions or applications. Other papers are more general with less detail, aimed at experienced users. The most appropriate reference for any reader will depend on the junction of interest and previous knowledge.

Langley and Heron [4] give the general solution for junctions with only semi-infinite plates (a single connection point) and Heron [5] describes the theory (again in a general manner) for modelling "strip plates" as used in junctions 4 and 5.

In addition to comparing numerical results with those of Tables 2–6 there are three self-consistency checks that can be carried out. The simplest check is that the power leaving the junction must equal the power incident as the junction itself does not dissipate power. This requires that the sum of the transmission coefficients be 1.0. A second check is that transmission along any single path is the same in each direction so that $\tau_{ij}(\theta_i) = \tau_{ji}(\theta_j)$, where θ is the angle of incidence [6]. Finally, the angular-averaged transmission coefficients are related through the relationship $k_i\tau_{ij} = k_j\tau_{ji}$, where k is the wavenumber [6].

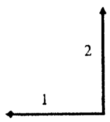
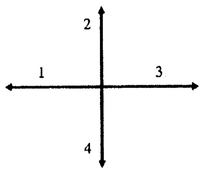
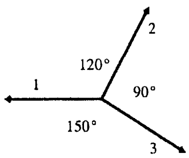
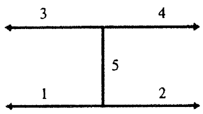
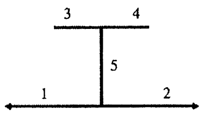
3. Test junctions

Five different test junctions were examined as shown in Fig. 1. The first joint is a corner joint which consists of two plates at right angles each of which has different properties. The second joint is a cross-junction with all plates at right angles and in which opposite plates have the same thickness and material properties and is common in buildings. This symmetry leads to some wave types not being generated. For example, an incident bending wave does not generate a reflected longitudinal or transverse shear wave.

The third junction has three plates that meet in a Y formation with an angle of 120° between plates 1 and 2, 90° between 2 and 3 and 150° between plates 1 and 3. All plates have different properties and thicknesses. Although this junction is less common in real structures, it does allow general models to be tested.

The fourth junction consists of two in-line plates connected by a short coupling element (sometimes called a strip plate) to give an H-type junction and is commonly found in double leaf partitions. Although the connecting element can sometimes be modelled as a beam, it is modelled here as a finite width plate. There are, therefore, waves travelling in both directions in plate

Table 1
Plate properties: density (ρ); elastic modules (E); thickness (h); the Poisson ratio (μ)

Junction number	Junction type	Plate number	ρ (kg/m^3)	$E \times 10^9$ (N/m^3)	h (m)	μ	Width of "strip" element (m)
1		1	8000	200	0.002	0.30	
		2	5000	100	0.001	0.25	
2		1	2000	20	0.150	0.20	
		2	3000	40	0.200	0.25	
		3	2000	20	0.150	0.20	
		4	3000	40	0.200	0.25	
3		1	8000	200	0.002	0.30	
		2	5000	100	0.001	0.25	
		3	6000	150	0.003	0.30	
4		1	8000	200	0.002	0.30	
		2	8000	200	0.002	0.30	
		3	5000	100	0.001	0.25	
		4	5000	100	0.001	0.25	
		5	6000	50	0.001	0.30	0.05
5		1	8000	200	0.002	0.30	
		2	5000	100	0.001	0.25	
		3	6000	150	0.001	0.30	0.05
		4	6000	150	0.001	0.30	0.10
		5	6000	180	0.003	0.20	0.15

element 5 with continuity and equilibrium at the junction between plates 1, 2 and 5 and between plates 3, 4 and 5. Plates 1 and 2 and plates 3 and 4 have the same thickness and material properties.

The final junction consists of two semi-infinite plates together with three strip elements that form a T junction. This junction is typical of ribs on a ship. The junction has some symmetry. Continuity and equilibrium has to be considered at the intersection of plates 1, 2 and 5 and plates 3, 4 and 5 as with junction 4 but also at the free end of the plates 3 and 4.

The material properties of each plate at each junction is given in Table 1.

Each participant carried out the calculations independently without knowledge of the other participants or their results. Apart from formatting problems (such as columns transposed) all the results agreed without any revisions to the computer models.

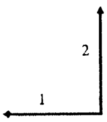
4. Results

The results that were obtained are given in Tables 2–6 for each junction. The results are all given as transmission loss in dB and in a common format. The first column gives the type of incident wave (bending, longitudinal or transverse) which excites each plate in turn. The second column gives the plate to which power is being transmitted and the last three columns the transmission loss to that plate for bending, longitudinal and transverse waves, respectively. Thus, on the bottom line under the longitudinal column of Table 2 is the value of 16.8295. This is the transmission loss for an incident transverse wave on plate 2 being reflected back as a longitudinal wave on plate 2 and the number above (12.2006) is the transmitted longitudinal wave to plate 1. All the results are for a single frequency of 500 Hz.

5. Expected accuracy

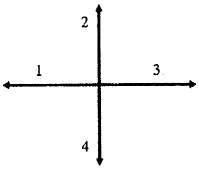
Once the transmission coefficient has been computed at a single angle, it is necessary to numerically integrate the answers to give an angular averaged value. The calculation process takes some time and so different strategies are adopted to optimize the integration. One of the simplest

Table 2
Transmission loss in dB computed for a corner junction (Bosmans and Vermeir, Cabos, Craik, Heron, Sarradj, Steel)



Corner joint	500 Hz Semi-infinite plates	Angles computed 1,000,000		
	$\rho 1 = 8000$ $\rho 2 = 5000$	$E1 = 200 \times 10^9$ $E2 = 100 \times 10^9$	$h1 = 0.002$ $h2 = 0.001$	$\mu 1 = 0.3$ $\mu 2 = 0.25$
Incident wave	Plate no.	Bending	Longitudinal	Transverse
Source on plate 1				
Bending	1	0.5414	50.5885	43.1418
	2	9.5139	26.8806	24.8735
Longitudinal	1	33.2842	3.1403	4.9462
	2	19.7567	11.4341	9.5215
Transverse	1	28.1171	7.2258	3.4402
	2	20.6543	11.5197	5.5738
Source on Plate 2				
Bending	1	11.2937	38.8408	37.4587
	2	0.3379	39.7031	36.9663
Longitudinal	1	10.1255	11.9833	9.7892
	2	21.1683	1.5926	14.6996
Transverse	1	10.2482	12.2006	5.9732
	2	20.5613	16.8295	2.4947

Table 3
Transmission loss in dB computed for a cross junction (Bosmans and Vermeir, Cabos, Craik, Heron, Sarradj, Steel)

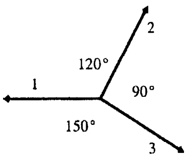
Cross joint	500 Hz Semi-infinite plates	Angles computed 1,000,000		
	$\rho 1 = 2000$ $\rho 2 = 3000$ $\rho 3 = 2000$ $\rho 4 = 3000$	$E1 = 20 \times 10^9$ $E2 = 40 \times 10^9$ $E3 = 20 \times 10^9$ $E4 = 40 \times 10^9$	$h1 = 0.15$ $h2 = 0.20$ $h3 = 0.15$ $h4 = 0.20$	$\mu 1 = 0.2$ $\mu 2 = 0.25$ $\mu 3 = 0.2$ $\mu 4 = 0.25$
Incident wave	Plate no.	Bending	Longitudinal	Transverse
Source on plate 1				
Bending	1	0.7730	*	*
	2	13.3072	19.9243	18.0249
	3	17.5007	*	*
	4	13.3072	19.9243	18.0249
Longitudinal	1	*	8.8978	8.0543
	2	8.0905	18.5572	15.9121
	3	*	5.1538	17.0385
	4	8.0905	18.5572	15.9121
Transverse	1	*	10.0440	4.8104
	2	10.5409	17.1290	10.5292
	3	*	19.0282	7.8046
	4	10.5409	17.1290	10.5292
Source on plate 2				
Bending	1	12.3444	14.0037	14.4645
	2	2.8784	*	*
	3	12.3444	14.0037	14.4645
	4	6.6372	*	*
Longitudinal	1	12.3721	17.8810	14.4631
	2	*	17.2659	16.0463
	3	12.3721	17.8810	14.4631
	4	*	1.3838	20.3371
Transverse	1	12.6025	17.3658	9.9931
	2	*	18.1762	11.6784
	3	12.6025	17.3658	9.9931
	4	*	22.4669	2.4847

*No wave transmitted.

methods is to use either the Trapezium or Simpsons Rule. With these methods, the length of time required for the calculation is directly proportional to the number of angles computed and as more angles are computed the answers become more accurate. Other methods examine features of the functions being integrated in order to reduce the number of calculations required without decreasing the overall accuracy.

Table 4
Transmission loss dB computed for a Y junction (Bosmans and Vermeir, Cabos, Craik and Steel, Heron, Sarradj)

Y joint

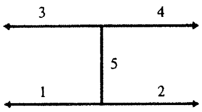


Y joint	500 Hz Semi-infinite plates	Angles computed 1,000,000		
	$\rho 1 = 8000$	$E1 = 200 \times 10^9$	$h1 = 0.002$	$\mu 1 = 0.3$
	$\rho 2 = 5000$	$E2 = 100 \times 10^9$	$h2 = 0.001$	$\mu 2 = 0.25$
	$\rho 3 = 6000$	$E3 = 150 \times 10^9$	$h3 = 0.003$	$\mu 3 = 0.3$

Incident wave	Plate no.	Bending	Longitudinal	Transverse
Source on plate 1				
Bending	1	1.2296	35.3168	34.3006
	2	18.9199	27.2439	24.9445
	3	6.4556	31.4149	29.6059
Longitudinal	1	18.0125	13.7086	38.5004
	2	27.2547	3.8487	27.2824
	3	16.1879	3.0435	22.7827
Transverse	1	19.2759	40.7801	15.2964
	2	25.0341	19.9415	4.8906
	3	16.5868	24.2153	2.2529
Source on plate 2				
Bending	1	20.6997	46.3387	41.8378
	2	0.1085	51.9916	45.7981
	3	18.0086	41.9269	38.2782
Longitudinal	1	10.4888	4.3979	18.2110
	2	33.4568	7.9488	13.7075
	3	11.1597	6.0708	22.8414
Transverse	1	10.3192	29.9615	5.2900
	2	29.3903	15.8374	5.3481
	3	10.9610	32.2214	6.7688
Source on plate 3				
Bending	1	5.5752	32.6117	30.7310
	2	15.3484	27.0343	24.7058
	3	1.6359	33.8149	33.0198
Longitudinal	1	14.1106	3.0435	21.9357
	2	22.8429	5.5216	29.5424
	3	17.3910	8.6707	17.4946
Transverse	1	14.5812	25.0624	2.2529
	2	21.4744	24.5719	6.3694
	3	18.8756	19.7742	9.9146

Table 5
Transmission loss in dB computed for a inline H junction (Cabos, Craik, Heron, Steel)

Inline H Joint



500 Hz — Angles computed 1,000,000
Semi-infinite plates

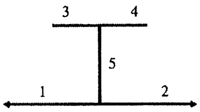
$\rho 1 = 8000$	$E1 = 200 \times 10^9$	$h1 = 0.002$	$\mu 1 = 0.3$
$\rho 2 = 8000$	$E2 = 200 \times 10^9$	$h2 = 0.002$	$\mu 2 = 0.3$
$\rho 3 = 5000$	$E3 = 100 \times 10^9$	$h3 = 0.001$	$\mu 3 = 0.25$
$\rho 4 = 5000$	$E4 = 100 \times 10^9$	$h4 = 0.001$	$\mu 4 = 0.25$

Element 5: $L = 0.05$, $\rho = 6000$, $E = 50 \times 10^9$, $h = 0.001$, $\mu = 0.3$
L is the spacing between the two parallel plates

Incident wave	Plate no.	Bending	Longitudinal	Transverse
Source on plate 1				
Bending	1	1.7879	61.6327	55.5433
	2	5.3300	60.8197	55.9770
	3	17.4609	54.6554	45.7072
	4	15.7886	52.9792	51.8384
Longitudinal	1	44.3284	20.1409	21.0420
	2	43.5154	0.5117	21.0452
	3	37.8182	18.4681	15.4647
	4	37.0659	18.4371	15.4803
Transverse	1	40.5186	23.3217	16.6601
	2	40.9523	23.3248	1.0452
	3	34.0494	18.6042	11.1473
	4	36.6999	18.6346	11.0809
Source on plate 3				
Bending	1	19.2406	56.9023	50.8436
	2	17.5683	56.1500	53.4858
	3	1.2182	64.7081	54.9724
	4	6.6724	61.9664	60.2539
Longitudinal	1	37.9003	19.0173	16.8737
	2	36.2241	18.9863	16.9041
	3	46.1732	11.7044	11.3808
	4	43.4316	1.4264	11.3628
Transverse	1	31.0816	18.1438	11.5467
	2	37.2117	18.1594	11.4803
	3	38.5565	13.5106	5.8948
	4	43.8071	13.4927	3.1837

Each person taking part in the benchmarking exercise was asked to provide two sets of data corresponding to 100 angles and 10,000 angles (where possible) when undertaking the angular average. Although different integration strategies were adopted, there was remarkable agreement between the individual results often to better than six decimal places. This was much better than

Table 6
Transmission loss in dB computed for an inline junction with attached T (Cabos, Craik, Heron, Steel)

Inline joint with attached T	500 Hz	Angles computed 1,000,000		
	Semi-infinite plates			
	$\rho 1 = 8000$	$E1 = 200 \times 10^9$	$h1 = 0.002$	$\mu 1 = 0.3$
	$\rho 2 = 5000$	$E2 = 100 \times 10^9$	$h2 = 0.001$	$\mu 2 = 0.25$

Finite width elements L is the plate width
Element 3: $L = 0.05$, $\rho = 6000$, $E = 150 \times 10^9$, $h = 0.001$, $\mu = 0.3$
Element 4: $L = 0.1$, $\rho = 6000$, $E = 150 \times 10^9$, $h = 0.001$, $\mu = 0.3$
Element 5: $L = 0.15$, $\rho = 6000$, $E = 180 \times 10^9$, $h = 0.003$, $\mu = 0.2$

Incident wave	Plate no.	Bending	Longitudinal	Transverse
Source on plate 1				
Bending	1	0.2127	30.3400	27.8634
	2	13.5645	34.9392	30.4075
Longitudinal	1	13.0357	8.9121	8.3260
	2	17.6823	1.8895	19.7780
Transverse	1	12.8387	10.6057	6.5418
	2	17.1870	19.5187	2.1539
Source on plate 2				
Bending	1	15.3442	36.7663	33.9914
	2	0.1329	41.4627	36.2458
Longitudinal	1	18.1841	2.4387	17.7882
	2	22.9278	6.2495	8.0808
Transverse	1	15.7823	22.4571	2.5533
	2	19.8408	10.2107	5.1316

expected as all models are likely to have some numerical rounding errors in addition to different integration strategies—even if all the equations were identical.

In order to see the effect of increasing the number of angles on the accuracy of the results, one set of calculations was undertaken (using the computer model of Craik) with 10^6 angles as a reference and then a number of angles that varied from 100 to 10^6. The difference between each case and the reference is then plotted in Fig. 2.

The scale on the x-axis is the number of angles in the integration and the y-axis gives the absolute value of the difference in dB compared to the reference. The use of a log scale for the dB results is unusual but only a log scale spreads out the data so that the trends can be seen. This format allows the accuracy, as so many decimal places, to be determined for a given number of angles of incidence. It is interesting that most of the participants overestimated the accuracy of their results.

There is a marked trend for the "error" to reduce as the number of angles increases right up to 10^6 angles suggesting that there is, at least in this range, no practical limit caused by numerical

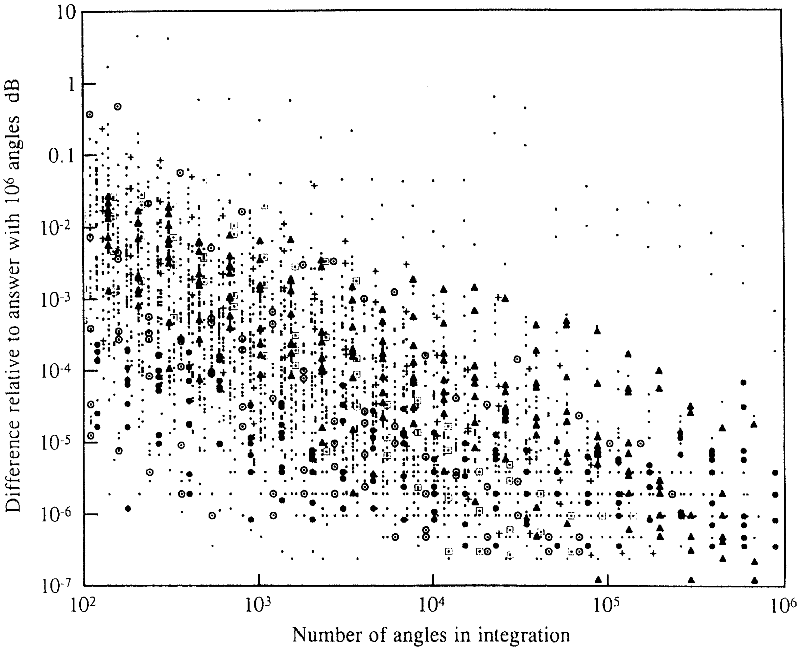


Fig. 2. Difference in the angular average transmission loss relative to the values computed from 10^6 angles. ·, individual result. For bending to bending only: ⊡, L joint; ⊙, X joint, ▲, Y joint; •, H joint; +, in-line joint with T.

rounding. There is, however, some rounding in the results when the difference is low (about 10^{-6}) resulting in the symbols tending to lie on horizontal lines.

Usually, the most important transmission, from a practical point of view, is from one bending wave to another bending wave and so bending to bending transmission loss values are shown as discrete symbols and all others are shown as small dots. There does not seem to be any clear trend with the large symbols covering the same area as the small dots and no symbol (junction type) that is consistently better or worse than any other. However, the largest difference is always associated with at least one in-plane wave (a small dot).

From the results, it can be seen that 100 angles will sometimes have a difference of 0.1–1.0 dB which is probably the minimum acceptable error. Increasing the angles to 10,000 will give answers that are generally better than 0.01 dB and will probably always be sufficient.

As the results for 10^6 angles have been computed, it is these specific values that are given in the tables. From Fig. 2, it can be seen that these results are only reliable to 0.0001 dB and so the data are given up to four decimal places.

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