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Letter to the Editor

Steiglitz–Mcbride type adaptive IIR algorithm for active noise control

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1. Introduction

Great progresses have been made in the last two decades in the field of active noise control (ANC) [1–3]. Most of the adaptive algorithms for ANC are based on FIR filters. Compared to FIR filters, IIR filters can model the physical system with much fewer coefficients due to its inherent zero-pole structure [4], especially in cases where the system is resonant or light damped. Several adaptive IIR algorithms have been proposed for active noise control e.g. the filtered-u LMS (FULMS) algorithm [5] and the filtered-v LMS (FVLMS) algorithm [6]. However, these algorithms cannot ensure global convergence due to the local minima on the error surface. Recently, Sun and Chen [7] proposed an algorithm that can ensure global convergence. However, considerable deviation may occur for Sun and Chen's algorithm when measurement noise exists or the filter order is not sufficient. In this letter, we proposed an algorithm which can ensure global convergence and is robust to measurement noise and order insufficient. It is shown that the proposed algorithm is an extension of the online Steiglitz–Mcbride algorithm [8].

This letter is organized as follows. In Section 2, some existing algorithms are briefly reviewed. In Section 3, the proposed algorithm is developed and discussed. Simulation results are given in Section 4. Section 5 is the conclusion.

Note that a mixed notation is used in the following part, i.e. if

$$H(z) = \sum_{k=-\infty}^{\infty} h_k z^{-k} \quad \text{then} \quad H(z)u(n) = \sum_{k=-\infty}^{\infty} h_k u(n-k).$$

2. Review of some existing algorithms

Although acoustic feedback [2] from the secondary source to the reference sensor may occurs in many ANC applications, it is assumed that an uncorrelated reference signal is available

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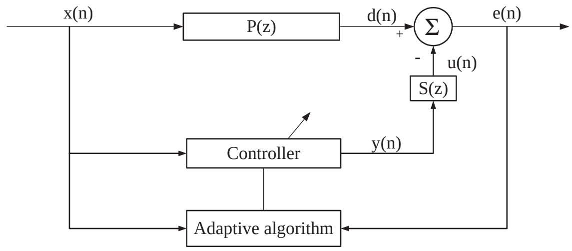

Fig. 1. Block diagram for active noise control.

throughout this letter, i.e. the reference signal is not influenced by the output of the controller. Methods to cancel the acoustic feedback are discussed in Ref. [2].

Fig. 1 depicts the block diagram of an adaptive ANC system, where $P(z)$, $S(z)$, $x(n)$, $e(n)$, $d(n)$, and $y(n)$ represent the primary path, the secondary path, the reference signal, the error signal, the primary signal, and the output of the controller, respectively. Suppose the controller is based on an adaptive IIR filter of order (M,N) with transfer function

$$\frac{B(n,z)}{1-A(n,z)}=\frac{b_0(n)+b_1(n)z^{-1}+\cdots+b_Nz^{-N}}{1-[a_1(n)z^{-1}+\cdots+a_Mz^{-M}]}, \tag{1}$$

then the error signal $e(n)$ can be expressed as

$$\begin{aligned} e(n) &= d(n)-S(z)y(n) \\ &= d(n)-S(z)\sum_{j=0}^{N} b_j(n)x(n-j)-S(z)\sum_{i=1}^{M} a_i(n)y(n-i). \end{aligned} \tag{2}$$

Suppose the filter coefficients vary slowly in the adaptive process, the above equation can be rewritten as

$$e(n)=d(n)-\sum_{j=0}^{N} b_j(n)[S(z)x(n-j)]-\sum_{i=1}^{M} a_i(n)[S(z)y(n-i)]. \tag{3}$$

Using LMS method to minimize the mean square error $E\{e^2(n)\}$, the filter coefficients is updated as [2]

$$a_k(n+1)=a_k(n)-\mu e(n)\frac{\partial e(n)}{\partial a_k(n)} \quad (k=1,2,\ldots,M), \tag{4}$$

$$b_k(n+1)=b_k(n)-\mu e(n)\frac{\partial e(n)}{\partial b_k(n)} \quad (k=0,1,\ldots,N). \tag{5}$$

The above two equations can be rewritten as

$$\begin{aligned} \boldsymbol{\theta}(n+1) &= \boldsymbol{\theta}(n)-\frac{\mu}{2}\frac{\partial e^2(n)}{\partial \boldsymbol{\theta}(n)} \\ &= \boldsymbol{\theta}(n)-\mu e(n)\frac{\partial e(n)}{\partial \boldsymbol{\theta}(n)}, \end{aligned} \tag{6}$$

where $\boldsymbol{\theta}(n) = \begin{bmatrix} b_0(n) & b_1(n) & \cdots & b_N(n) & a_1(n) & \cdots & a_M(n) \end{bmatrix}^{\mathrm{T}}$ represents the coefficients vector. It follows from Eq. (3)

$$\frac{\partial e(n)}{\partial b_k(n)} = -S(z)\left[x(n-k) + \sum_{j=1}^{M} a_j(n)\frac{\partial y(n-j)}{\partial b_k(n)}\right] \quad (k = 0, 1, \ldots, N), \tag{7}$$

$$\frac{\partial e(n)}{\partial a_k(n)} = -S(z)\left[y(n-k) + \sum_{j=1}^{M} a_j(n)\frac{\partial y(n-j)}{\partial a_k(n)}\right] \quad (k = 1, 2, \ldots, M). \tag{8}$$

Assuming [2]

$$\frac{\partial y(n-j)}{\partial b_k(n)} \approx 0 \quad (k = 0, 1, \ldots, N, \quad j = 1, 2, \ldots, M) \tag{9}$$

and

$$\frac{\partial y(n-j)}{\partial a_k(n)} \approx 0 \quad (k = 1, 2, \ldots, M, \quad j = 1, 2, \ldots, M), \tag{10}$$

one can get

$$\frac{\partial e(n)}{\partial b_k(n)} = -S(z)x(n-k) \quad (k = 0, 1, \ldots, N), \tag{11}$$

$$\frac{\partial e(n)}{\partial a_k(n)} = -S(z)y(n-k) \quad (k = 1, 2, \ldots, M). \tag{12}$$

Substituting Eqs. (11) and (12) into Eqs. (4) and (5), respectively, yields

$$b_k(n+1) = b_k(n) + \mu e(n)[S(z)x(n-k)] \quad (k = 0, 1, \ldots, N), \tag{13}$$

$$a_k(n+1) = a_k(n) + \mu e(n)[S(z)y(n-k)] \quad (k = 1, 2, \ldots, M), \tag{14}$$

or equivalently,

$$\boldsymbol{\theta}(n+1) = \boldsymbol{\theta}(n) + \mu e(n)[S(z)\,\boldsymbol{\varphi}(n)], \tag{15}$$

where $\boldsymbol{\varphi}(n) = \begin{bmatrix} x(n) & x(n-1) & \cdots & x(n-N) & y(n-1) & \cdots & y(n-M) \end{bmatrix}^{\mathrm{T}}$. Eqs. (2) and (15) express the FULMS algorithm, which was firstly proposed by Eriksson [5]. Implementations of the FULMS algorithm are shown in Fig. 2, where $\hat{S}(z)$ represents the secondary path model.

Instead of Eqs. (9) and (10), Crawford and Stewart adopted a more accuracy gradient estimation as follows:

$$\frac{\partial y(n-j)}{\partial b_k(n)} \approx \frac{\partial y(n-j)}{\partial b_k(n-j)} \quad (j = 1, 2, \ldots, M), \tag{16}$$

$$\frac{\partial y(n-j)}{\partial a_k(n)} \approx \frac{\partial y(n-j)}{\partial a_k(n-j)} \quad (j = 1, 2, \ldots, M) \tag{17}$$

which results the FVLMS algorithm [6]:

$$\boldsymbol{\theta}(n+1) = \boldsymbol{\theta}(n) + \mu e(n)[S(z)\,\mathbf{V}(n)], \tag{18}$$

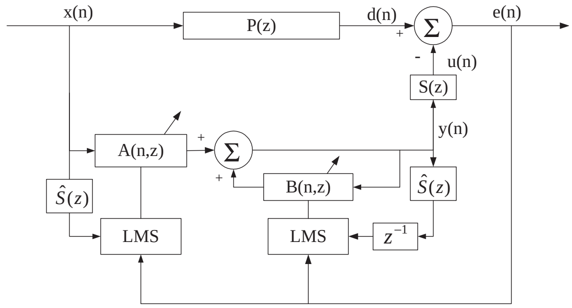

Fig. 2. Block diagram for filtered-u LMS algorithm.

where

$$\mathbf{V}(k) = \boldsymbol{\varphi}(n) + \sum_{i=1}^{M} a_i \mathbf{V}(n-i). \tag{19}$$

The FULMS and FVLMS algorithms may converge to a local minimum since the error surface for $E\{e^2(n)\}$ is multimodal, which can be seen from Eq. (3).

Sun and Chen [7] proposed to minimize the mean square equation error $E\{\xi^2(n)\}$ instead of the mean square output error $E\{e^2(n)\}$, where

$$\xi(n) = [1 - A(n,z)]e(n) = e(n) - \sum_{i=1}^{M} a_i(n)e(n-i). \tag{20}$$

It follows from Eqs. (3) and (20)

$$\xi(n) = d(n) - \sum_{j=0}^{N} b_j[S(z)x(n-j)] - \sum_{i=1}^{M} a_i[S(z)y(n-i) + e(n-i)]. \tag{21}$$

Note that $\hat{d}(n-k) = S(z)y(n-k) + e(n-k)$ is independent of the filter coefficients, the error surface for $E\{\xi^2(n)\}$ is parabolic. Using LMS method to optimize $E\{\xi^2(n)\}$, one can get

$$b_k(n+1) = b_k(n) + \mu\xi(n)[S(z)x(n-k)] \quad (k = 0, 1, \ldots, N), \tag{22}$$

$$a_k(n+1) = a_k(n) + \mu\xi(n)[S(z)y(n-k) + e(n-k)] \quad (k = 1, 2, \ldots, M). \tag{23}$$

Eqs. (20), (22) and (23) express Sun and Chen's algorithm. Sun and Chen's algorithm can ensure global convergence in model-matching and noise free cases. However, when measurement noise exists or the filter order is not sufficient, it may converge to a biased solution. Implementations of Sun and Chen's algorithm are shown in Fig. 4. The practical version of Sun and Chen's algorithm can thus be expressed as

$$b_k(n+1) = b_k(n) + \mu\xi(n)\left\lfloor \hat{S}(z)x(n-k) \right\rfloor \quad (k = 0, 1, \ldots, N), \tag{24}$$

$$a_k(n+1) = a_k(n) + \mu\xi(n)\hat{d}(n-k) \quad (k = 1, 2, \ldots, M), \tag{25}$$

where $\hat{d}(n) = \hat{S}(z)y(n) + e(n)$ is the estimated primary noise.

3. Development of the proposed algorithm

Sun and Chen's algorithm tries to minimize the mean square output error $E\{e^2(n)\}$ by minimizing the mean square equation error $E\{\xi^2(n)\}$, as a consequence, a biased convergence occurs when the measurement noise exists or the filter order is not sufficient. The idea of the proposed algorithm is to directly minimize the mean square output error $E\{e^2(n)\}$, but the mechanism of Sun and Chen's algorithm is inherently included in the proposed algorithm.

As shown in Fig. 4, if the reference signal is prefiltered by an all pole transfer function $1/(1 - A(n,z))$, then under the condition of slow adaption, the following two observations can be made:

(i) $x'(n)$ is independent of the coefficients $a_i(n)$ $(i = 1, 2, \ldots, M)$. In this case the blocks at the right side of the dashed line can be regarded as Sun and Chen's algorithm with reference signal $x'(n)$, and thus the global convergence can still be guaranteed.
(ii) The effects of $1/(1 - A(n,z))$ and $1 - A(n,z)$ counteract each other. As a result, the output $\xi'(n)$ in Fig. 4 is equal to $e(n)$ in Fig. 2. So it is the mean square output error that is minimized in Fig. 4.

According to the above two observations, the algorithm depicted in Fig. 4 can ensure global convergence, but compared to Sun and Chen's algorithm, the algorithm shown in Fig. 4 will be much more robust to measurement noises and order insufficient for its directly minimizing the mean square output error $E\{e^2(n)\}$. Fig. 4 depicts the mechanism of the proposed algorithm. Comparing Fig. 4 with Fig. 3, one can see that

$$\hat{d}'(n) = \frac{\hat{d}(n)}{1 - A(n,z)}, \tag{26}$$

$$x'(n) = \frac{x(n)}{1 - A(n,z)}. \tag{27}$$

Replacing $x(n)$, $\hat{d}(n)$ and $\xi(n)$ in Eqs. (20) and (21) with $x'(n)$, $\hat{d}'(n)$ and $e(n)$, respectively, yields

$$b_k(n+1) = b_k(n) + \mu e(n)\left[\frac{\hat{S}(z)}{1 - A(n,z)} x(n-k)\right] \quad (k = 0, 1, \ldots, N), \tag{28}$$

$$a_k(n+1) = a_k(n) + \mu e(n)\left[\frac{\hat{S}(z) y(n-k) + e(n-k)}{1 - A(n,z)}\right] \quad (k = 1, 2, \ldots, M) \tag{29}$$

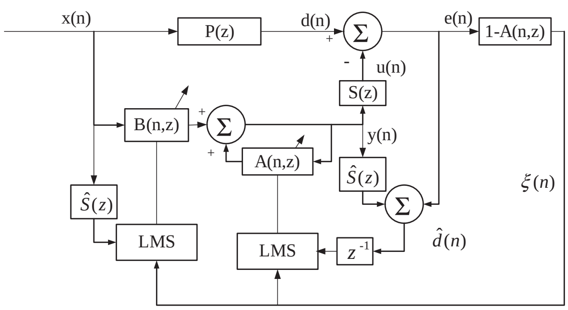

Fig. 3. Block diagram for Sun and Chen's algorithm.

which give the expressions for the proposed algorithm. Note that the block diagram in Fig. 4 cannot be implemented in ANC practice, it is rearranged as shown in Fig. 5.

The FULMS algorithm, Sun and Chen's algorithm, and the proposed algorithm can be regarded as extensions of the output error algorithm [4], equation error algorithm [4], and Steiglitz–Mcbride algorithm [8], respectively. As the secondary path is equal to 1, it is interesting to see that the FULMS algorithm as well as the FVLMS algorithm degenerates into the output error algorithm, Sun and Chen's algorithm degenerates into the equation error algorithm, and the proposed algorithm degenerates into the online Steiglitz–Mcbride algorithm. It is well known that the output error algorithm often converge to a local minima [4], the equation error algorithm often has a biased convergence due to measurement noise and order insufficient. The online Steiglitz–Mcbride algorithm is attractive for its global convergence and robustness to measurement noise [8].

4. Simulation results

Two simulation examples are given in this section to verify the effects of the proposed algorithm.

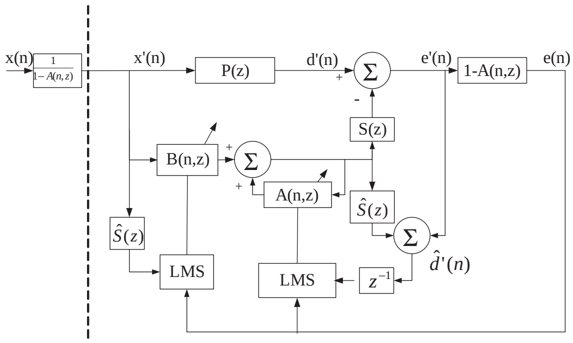

Fig. 4. Development of the proposed algorithm.

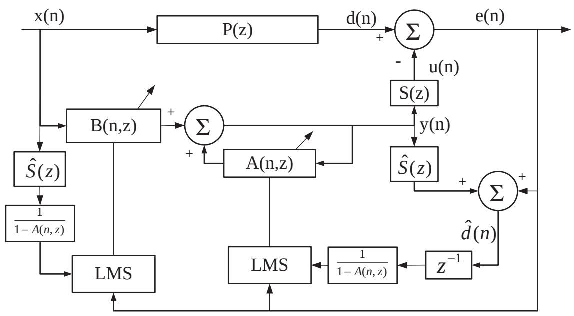

Fig. 5. Block diagram for the proposed algorithm.

Example 1. In this example, transfer functions are measured from experimental setup, which are available in the disk attached to Ref. [2]. The frequency responses for the primary path and the secondary path are shown in Figs. 6 and 7, respectively. In the simulation, the order of the adaptive IIR filter are chosen as $M = 15$ and $N = 15$, the noise source are chosen as a white noise with variance of 1. The measurement noise is adopted as a white noise with variance of 0.1. Trends of mean square error of the residual noise is given in Fig. 8, from which it can be seen that The proposed algorithm presents slightly better convergence and steady state performance than FULMS algorithm and Sun and Chen's algorithm.

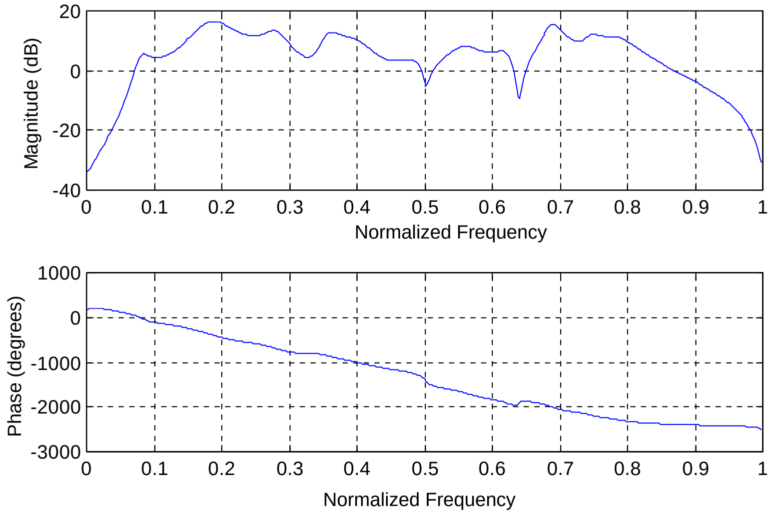

Fig. 6. Frequency response of the primary path.

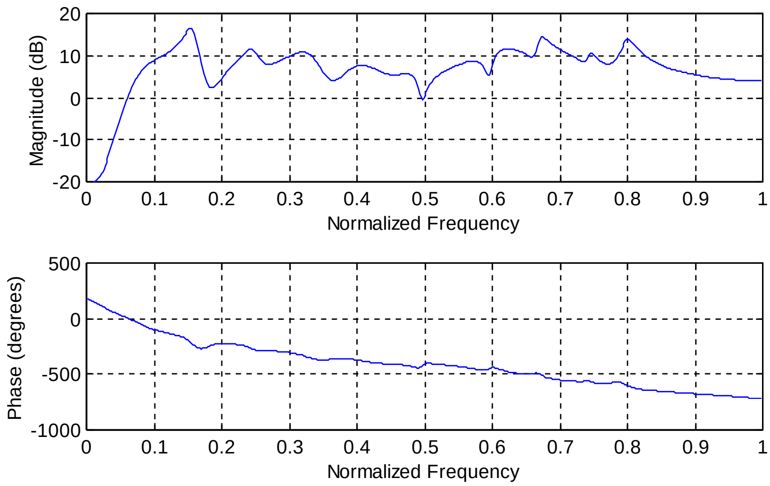

Fig. 7. Frequency response of the secondary path.

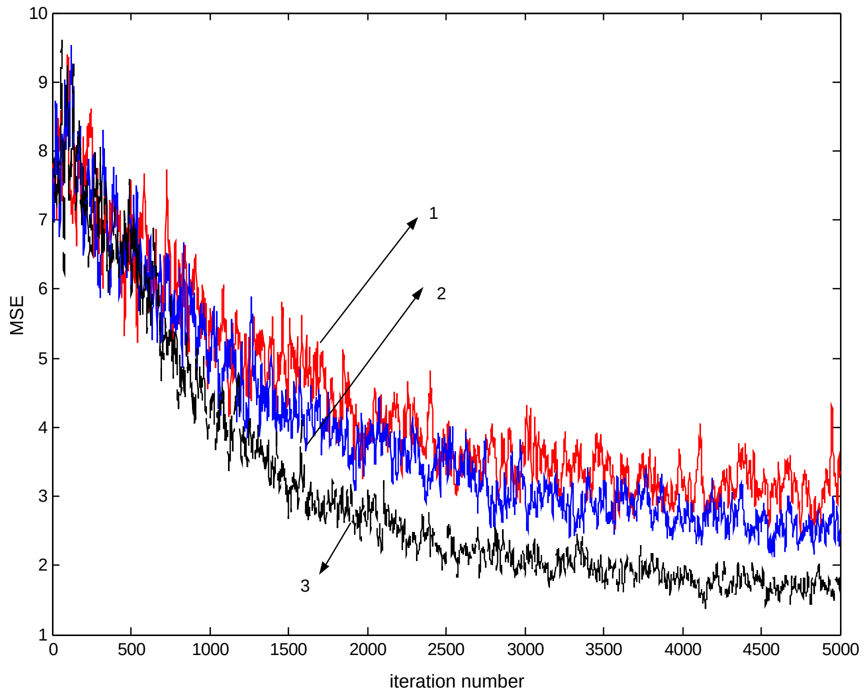

Fig. 8. Trends of MSEs in Example 1 for (1) FULMS algorithm, (2) Sun and Chen's algorithm and (3) the proposed algorithm.

Example 1 is somewhat not very convincing to verify the effects of the proposed algorithm since the other two algorithms also present good noise reduction effects. In the following, we give an extreme case where the proposed algorithm presents significantly improved noise reduction than the other two algorithms.

Example 2. Parameters in this example are given according to the example presented in Ref. [9] with some modifications.

The primary transfer function is adopted as $P(z) = (0.05z^{-3} - 0.4z^{-4})/(1 - 1.1314z^{-1} + 0.25z^{-2})$, and the secondary path is a pure time delay: $S(z) = z^{-3}$. Order of the adaptive IIR filter are chosen as $M = 0, N = 1$ such that its transfer function has the form of $W(z) = b_0/(1 - a_1 z^{-1})$. The reference signal and the measurement noise is the same as Example 1. Initial values of the IIR filters are all set to 0. When stepsize is adopted as 5×10^{-4}, trends of MSEs are shown in Fig. 9, from which one can see that the proposed algorithm has a much better convergence property than the other two algorithms.

For both the above examples, the proposed algorithm presents good noise reduction effects due to its global convergence. However, the FULMS presents very different noise reduction effects for the above two examples, which may be ascribed to the different geometric shapes of the error surfaces in the two examples. In Example 1, since the residual noise power at the local minimum is only slightly greater than that at the global minimum, good noise reduction effect can be achieved despite of the local convergence. In Example 2, the residual noise power at the local minimum

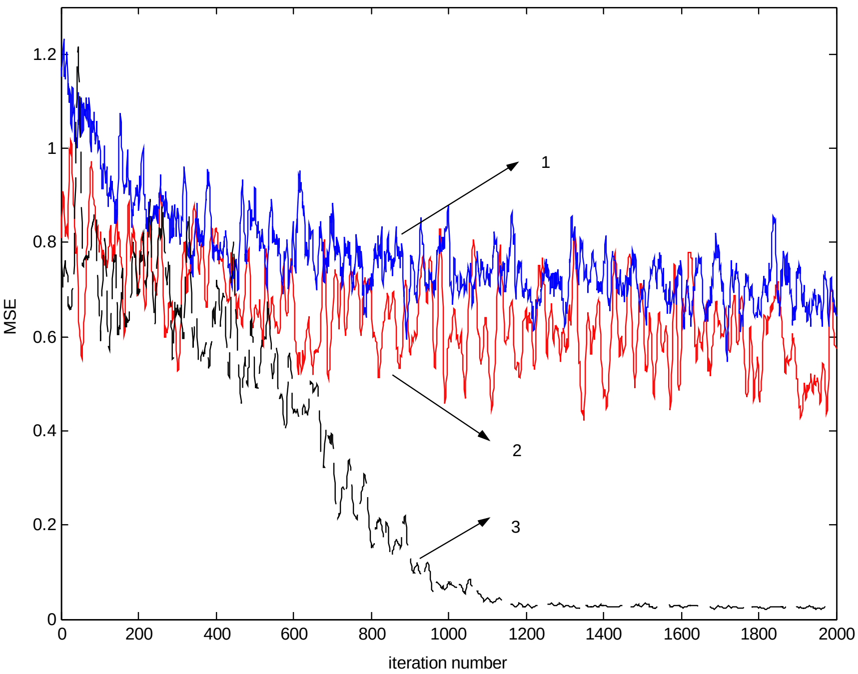

Fig. 9. Trends of MSEs in Example 2 for (1) Sun and Chen's algorithm, (2) FULMS algorithm and (3) the proposed algorithm.

approaches to the power of the primary noise, and is much greater than that at the global minimum (see Ref. [9] for details of the error surface in Example 2), so little noise reduction effect is achieved by the FULMS algorithm. In Example 2, Sun and Chen's algorithm presents a slightly worse noise reduction effect than the FULMS algorithm due to the measurement noise and the severe order insufficient.

5. Conclusion

In many practical applications, using IIR filters can greatly reduce the computational complexity of the algorithm. However, most of the existed algorithms based on IIR filters cannot guarantee global convergence. A Steiglitz–Mcbride type adaptive IIR algorithm is proposed for active noise control. If an uncorrelated reference signal is available, the algorithm can guarantee global convergence and is robust to measurement noise and order insufficient. Simulation examples are given to support the theoretical conclusion.

When the reference signal is correlated, just like the filtered-x LMS algorithm [2], FULMS algorithm, FVLMS algorithm, Sun and Chen's algorithm and many other adaptive algorithms for ANC, the proposed algorithm may fail to guarantee global convergence.

Note that stability of the adaptive IIR filter has been assumed for the proposed algorithm. In practice, the poles of the IIR filter may move out of the unit circle and instability may occur. Some proper measures should be taken to guarantee the stability of the adaptive IIR filter.

Acknowledgements

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