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Optimal analysis and experimental study on structures with viscoelastic dampers

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Abstract

In this paper, the simplex method, a synthetic optimization analysis method of structures with viscoelastic (VE) dampers, which is used to determine the optimal parameters and location of VE dampers, is presented. When applied to a shaking table test of the reinforced concrete structure with VE dampers, it is seen that the simplex method can act as the synthetic optimization method of structures with VE dampers. It is also found that the shock absorption effect of the VE dampers is best when the location of VE dampers is optimal.

1. Introduction

In recent years, both researchers and engineers have recognized that energy dissipation devices can provide an efficient means for reducing seismic response of structure induced by strong motion earthquakes. Among the available devices, viscoelastic (VE) dampers are one of the most common energy dissipation dampers, and have many applications due to their fine damping property, cheap cost and simple construction. Analytical and experimental studies on the behaviors of VE dampers are carried out [1–4]. At the same time, analytical investigations of the use of VE dampers in civil engineering structures have been processed [5,6], and experimental studies using shaking table have also been conducted [7–12]. Analytical and experimental results show that the damping of structures is notably increased and responses of structures due to strong earthquakes can be reduced significantly.

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Optimization analysis about structures with VE dampers, including optimization of VE dampers' parameters and optimization of VE dampers' location in the structure, is an important task, because rational parameters and location of VE dampers will lead to most effective shock absorption. Many investigators have investigated optimal designs for structure vibration control [13–18], and significant progress in this aspect has been made. However, there are few documents on the synthetic optimization method of structures with VE dampers. Synthetic optimization considering parameters and location of VE dampers can make dampers work more effectively.

In this paper, the simplex method, a synthetic optimization method for structures with VE dampers, by which the optimal parameters and location of VE dampers can be determined under the fixed goal, is introduced. Then, through a numerical example and a shaking table test on the reinforced concrete structure with VE dampers, the simplex method can act as the comprehensive optimization method for designing structures with VE dampers and the shock absorption effect of VE dampers is best when the location of VE dampers is optimal.

2. Properties of VE dampers

VE dampers consist of VE material and restrained steel plates. A typical VE damper is shown in Fig. 1, which is made up of three steel plates clamping two VE layers. VE material between the steel plates is a kind of high polymer, which has characteristics of spring and fluid [19]. Under the harmonic displacement excitation, one part of the damper's energy is stored as potential energy, the other part is dissipated as thermal energy. Generally, the storage modulus G_1 and the loss modulus G_2 are used to describe the storage character and the energy dissipation character of VE dampers, respectively [20].

Under a given harmonic strain or stress excitation, energy dissipation per cycle of VE dampers can be expressed as

$$E_d = \pi\gamma_0^2 G_1 \eta V, \tag{1}$$

where γ_0 is the shear strain amplitude, η is the loss factor ($\eta = G_2/G_1$), V is the volume of viscoelastic material ($V = n_v A_v h_v$), n_v is the number of viscoelastic layers (in this paper, $n_v = 2$), and A_v and h_v are the area and thickness of the viscoelastic layer, respectively. If the storage modulus G_1 and the loss factor η are determined, the stiffness k_d and the damping c_d of VE

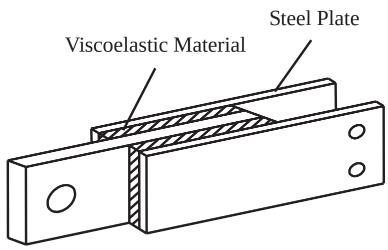

Fig. 1. The ordinary viscoelastic damper.

dampers can be written as

$$k_d = \frac{n_v G_1 A_v}{h_v}, \tag{2}$$

$$c_d = \frac{n_v G_1 \eta A_v}{\omega h_v}, \tag{3}$$

where ω is the excitation frequency.

3. Optimization analysis

3.1. The simplex method

The basic idea of the simplex method is that the function values of simplex apexes under the n-dimensional space are calculated and compared, the changing direction of the objective function is determined, then the worst apex is abandoned and at the same time a new apex is chosen, and iterative loop is computed until the minimum objective function value is reached [21]. The convergence ability of the simplex method is related to the constitution of the initial simplex and formation of the new simplex. The optimization process of the simplex method can be divided into three steps.

(1) *The initial simplex constitution*: If the objective function $J(x)$ is an n-dimensional function, the simplex must choose $n+1$ apexes (i.e., $x_0, x_1, \ldots, x_n$), and the vectors $x_1 - x_0, x_2 - x_0, \ldots, x_n - x_0$ are linearly independent. Edge lengths of the simplex can take different values. For simplicity, the regular simplex is chosen. Thus the ith apex co-ordinates can be determined as

$$x_i = x_0 + h_s e_i, \tag{4}$$

where h_s is the step size of the initial simplex, e_i is the ith column vector in $n \times n$ unit matrix, $e_i = [0\ 0\ \ldots\ 1\ \ldots\ 0]^{\mathrm{T}}$.

(2) *The new apex of the simplex*: For n variables, provided the objective function value of the apex H is maximum (i.e., the worst apex) and the objective function value of the apex L is minimum (i.e., the best apex), the apex H should be discarded, as shown in Fig. 2. The co-ordinate

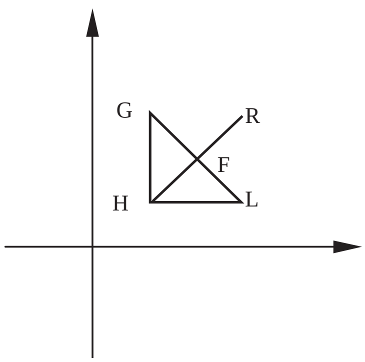

Fig. 2. Apexes hunt of the simplex method.

of barycentre is $x_F = (1/n)(\sum_{i=1}^{n} x_i - x_H)$ after the apex H is discarded, and the co-ordinate of the reflection apex R (the new apex) is $x_R = 2x_F - x_H$.

(3) *Compression and expansion of the simplex*: If $J(x_R) \geqslant J(x_G)$, compression is carried out according to the equation $x_S = (1-\lambda)x_H + \lambda x_R$, where $J(x)$ is the n-dimensional objective function, x_G is the co-ordinate of the apex G, x_S is the co-ordinate of the new compression apex, λ is the compression factor, $0 < \lambda < 1$, and $\lambda \neq 0.5$.

If $J(x_S) \geqslant J(x_G)$, compression is failing. So the simplex must be narrowed, and co-ordinates of the initial simplex apexes should be reduced according to the equation $x_i = (x_i + x_L)/2$ $(i = 1, 2, \ldots, n)$. Then the calculation goes back to the second step.

If $J(x_S) < J(x_G)$, compression is successful, and if $x_H = x_S$, calculation goes back to the second step.

If $J(x_R) < J(x_G)$, expansion is carried out according to the equation $x_E = (1-\mu)x_H + \mu x_R$, where x_E is the co-ordinate of the new expansion apex, μ is the expansion factor, $\mu > 1$, generally and $\mu = 1.2 \sim 2$.

If $J(x_E) \leqslant J(x_L)$, expansion is successful, and if $x_H = x_E$, calculation goes back to the second step.

If $J(x_E) > J(x_L)$, expansion is failing, and if $x_H = x_R$, calculation goes back to the second step.

Iterative process is carried out until precision discriminant $|(J(x_H) - J(x_L))/J(x_L)| \leqslant \varepsilon$ is satisfied, where ε is the given convergence error.

3.2. Optimal analysis for structures with VE dampers

For frame structures, VE dampers are usually attached to braces. In consideration of the stiffness of braces, the damper–brace system can be treated as a damper and a spring being connected in series. In order to assure that VE dampers function effectively, the stiffness of braces is usually strong. Accordingly, the stiffness of braces can be neglected for simplifying calculation, and the equations of motion of the structure with VE dampers can be written as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\Gamma\ddot{x}_g - \mathbf{B}\mathbf{f}_d, \tag{5}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices of the structure, respectively, $\mathbf{x}$ is the vector of the relative displacements of each floor, Γ is the column vector of ones, $\ddot{x}_g$ is the earthquake acceleration excitation, $\mathbf{B}$ is the matrix determined by the placement of VE dampers in the structure, $\mathbf{f}_d = [f_{d1}, f_{d2}, \ldots, f_{dn}]^{\mathrm{T}}$ is the vector of control forces produced by VE dampers and f_{dn} is the control force of the nth floor.

The control force f_{dn} can be calculated according to the Kelvin model [22], in which VE dampers are assumed to have a spring component augmented by a Newtonian viscosity component, and its relationship between force and displacement can be expressed as

$$f_{dn} = k_{dn}\Delta_{dn} + c_{dn}\dot{\Delta}_{dn}, \tag{6}$$

$$\begin{aligned} \Delta_{dn} &= \Delta_n \cos\theta, \\ \dot{\Delta}_{dn} &= \dot{\Delta}_n \cos\theta, \end{aligned} \tag{7}$$

where k_{dn} and c_{dn} are the stiffness and the damping of VE dampers in the nth floor, which can be determined by Eqs. (2) and (3), respectively. Δ_{dn} and $\dot{\Delta}_{dn}$ are the displacement and velocity, respectively, produced by VE dampers in the nth floor. Δ_n and $\dot{\Delta}_n$ are the inter-story drift and inter-story velocity of the nth floor, respectively. θ is the angle between the braces and the horizontal axis.

For the structure with VE dampers, if the location and dimensions of VE dampers are regarded as optimization parameters, the pole of the structure with VE dampers can be found by the simplex method. Under some constraints, a series of optimization parameters are sought so that the objective function value is minimum and the structure system is optimal. The aims of optimal design for the structure with VE dampers are mainly the following two aspects: (1) When the dynamic responses of the structure with VE dampers satisfies the given demand, the number of VE dampers should be as few as possible and (2) in order to assure that VE dampers function effectively, the stiffness of VE dampers cannot be too strong compared with that of braces. According to these aims the objective function should be written as

$$J = \alpha_1 \frac{\theta_m}{[\theta]} + \alpha_2 \frac{\Delta_m}{[\Delta]} + \alpha_3 \frac{\sum_{i=1}^{n} n_{d_i}}{n_{d_0}} + \alpha_4 \frac{k_d}{k_{d_0}}, \tag{8}$$

where θ_m and Δ_m are the maximum inter-story displacement angle and the maximum inter-story displacement, respectively, $[\theta]$ and $[\Delta]$ are the limits of elastic inter-story displacement angle and elastic inter-story displacement, respectively, n_{d_i} is the number of VE dampers in the ith floor, n_{d_0} is the sum of the initial setting VE dampers, k_{d_0} is the initial setting stiffness of VE dampers, which is determined by experience according to the stiffness of structures without dampers, α_1, α_2, α_3 and α_4 are the weight coefficients, for which we assign 0.28, 0.22, 0.27 and 0.23, respectively, in this paper. It must be noted that the initial location of VE dampers is based on good distribution of the stiffness of each floor of structures with dampers, and it can be determined by calculating roughly the stiffness of structures without dampers and the initial setting stiffness of VE dampers.

The constraint condition, which the frame structures must satisfy, is $\theta_i \leqslant 1/450$, and the constraint condition, which the VE damper must satisfy, is $k_d \leqslant \alpha \cdot \min(k_i)$, where α is the coefficient and in this paper, $\alpha = 0.1$.

Firstly, the seismic responses of the structure with VE dampers expressed as Eq. (5) can be calculated by the time–history analysis method [23] under the initial setting location and parameters of VE dampers. Then, in accordance with the objective function and constraint conditions, the new parameters can be determined by the simplex method. Under the new parameters, the dynamic responses of the structure can be calculated and the objective function value can be obtained. Iterative process is carried out until the system pole and the minimum objective function value are found. Some programs for this method are developed in Matlab language by the authors.

3.3. Numerical example

Consider an eight-story shear building with a mass of 1.08×10^6 kg, a stiffness of 6.72×10^8 N/m, story height of 5 m for the first floor and a mass of 9.5×10^5 kg, a stiffness of 8.4×10^8 N/m and story height of 3.3 m for the others. The ordinary VE damper as shown in Fig. 1 is adopted, whose storage modulus G_1 is 1.0×10^7 N/m^2 and loss modulus G_2 is 1.4×10^7 N/m^2. The working temperature is 25°C, the initial location of VE dampers is [12,10,8,4,2,1,1,1], and the initial area and thickness of VE layers are $A_v = 8 \times 10^{-3}$ m^2 and $h_v = 12 \times 10^{-3}$ m, respectively.

It must be noted that the initial location and parameters of VE dampers are determined by the characteristics of their structures and the design experience. Generally speaking, the stiffness of VE dampers added in a frame should be less than the stiffness of the frame. When the parameters of VE dampers are determined, the initial location of VE dampers can be calculated according to the homogeneous distribution of stiffness. The structure is subjected to the north–south component of the 1940 El Centro earthquake with 200 gal acceleration amplitude.

The optimal location [16, 13, 9, 3, 0, 0, 0, 0], the area of VE layer $A_v = 4.3 \times 10^{-3}$ m^2 and the thickness of VE layer $h_v = 6.2 \times 10^{-3}$ m can be obtained by the simplex method. Given the values of the area and thickness of the VE layer, the optimal location can be determined by the time–history analysis method or the random vibration method [24]. Suppose the area A_v and the thickness h_v of VE layer are 5.0×10^{-3} m^2 and 8.0×10^{-3} m, respectively. When the structure is subjected to 200 gal El Centro earthquake and Taft earthquake, the results calculated by the time–history analysis method are [16, 16, 12, 5, 0, 0, 0, 0] and [16, 15, 12, 5, 0, 0, 0, 0], respectively, and the results calculated by the random vibration method are [15, 15, 10, 4, 0, 0, 0, 0]. It can be seen that the optimal location results calculated by the time–history analysis method and the random vibration method are consistent. When the structure is subjected to different earthquake excitation, the numerical results have slight difference.

The first location is the optimal location [16, 13, 9, 3, 0, 0, 0, 0] calculated by the simplex method; the second location is [6, 5, 5, 5, 5, 5, 5, 5], which represents dampers' average distribution and the third location is [7, 7, 9, 4, 4, 3, 3, 4], which represents dampers' random location. Sum of VE dampers in the three locations is all 41. Under the 0.2g El Centro earthquake excitation, responses of the structure with VE dampers of the three locations are calculated. Figs. 3(a) and (b) show comparison between the maximum displacement and the maximum acceleration of each floor under the three locations, respectively.

It can be shown from Fig. 3(a) that the displacement responses of the structure with VE dampers under the optimal location are smaller than those under the other locations. It can also

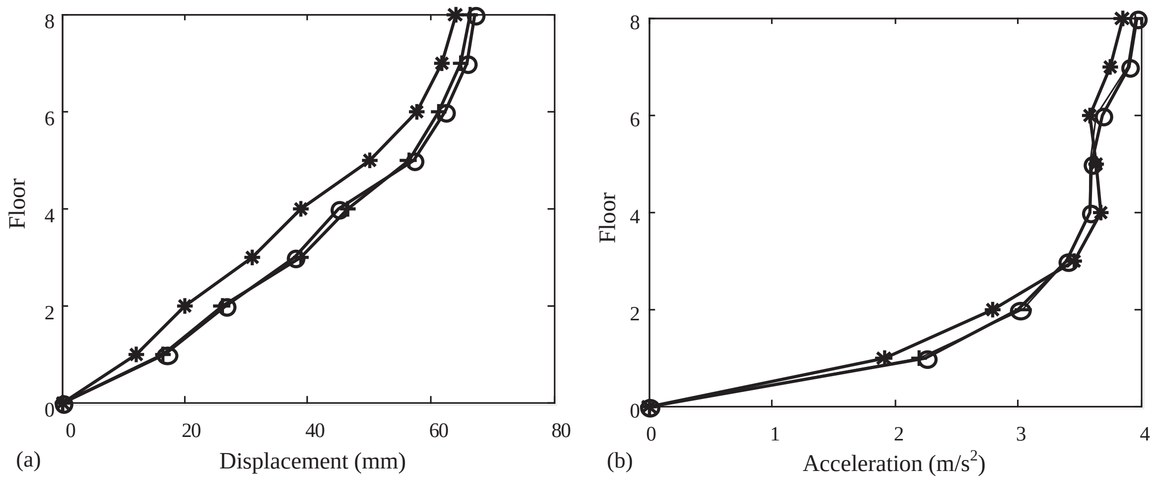

Fig. 3. Comparison of seismic response under the three locations: (a) comparison of the maximum displacements; (b) comparison of the maximum accelerations; —∗—, the optimum location; —◦—, the second location; —+— the third location.

be shown from Fig. 3(b) that the acceleration responses of most floors under the optimal location are smaller than those under other locations. For example, the displacements of the first floor and the top floor under the average location are 18.8 and 67.2 mm, respectivey, and those under the random location are 18.1 and 66.4 mm, respectively, while those under the optimal location are 13.6 and 62.8 mm, respectively. The accelerations of the first floor and the top floor under the average location are 2.34 and 3.92 m/s^2, respectively, and those under the random location are 2.28 and 3.91 m/s^2, respectively, while those under the optimal location are 1.90 and 3.76 m/s^2, respectively. Rational location of VE dampers can make the stiffness and the damping of the structure uniform, and avoid abrupt change in the stiffness and the damping of the structure. Thus the dynamic responses are reduced and the damage degree of the structure is alleviated more effectively.

4. Shaking table test of structures with VE dampers

4.1. Test general situation

The test frame models are two identical 1/5-scale three-story plain reinforced concrete frames, i.e., length similarity ratio $S_l = 0.2$. In order to strengthen the stability of the frame and make it convenient to add weight, two identical frames are adopted. Overall dimensions of each test frame are 1200 mm in plan, story height is 800 mm for the first story and 660 mm for the other two, as shown in Fig. 4. Dimensions of beams and columns are 50×100 and 80×80 mm^2, respectively. According to the model similarity relationship [25], i.e., mass similarity ratio $S_m = S_l^2 = 0.04$, the self-weights and added weights of the model frames are calculated. For one test frame model masses and stiffness of the structure obtained are follows: mass of each floor, $m = [576.9,\ 586.4,\ 548.1]$ kg and stiffness of each floor, $k = [2.868,\ 3.766,\ 3.766] \times 10^6$ N/m.

The ordinary VE damper as shown in Fig. 1 is adopted, and Fig. 5 shows the joint construction of the VE brace. 9050A material, which is made in Wuxi Shock Absorption Company in China, is used as VE material. For 9050A material, through the property test of VE dampers [24], the following obtained were: the storage modulus G_1, the loss modulus G_2 and the loss factor η are 0.510, 0.09 MPa and 0.18, respectively, when the temperature is 16.7°C, frequency is 0.2 Hz and the strain amplitude is 100%. The shear area $A_v = 2.42 \times 10^{-3}$ m^2, the thickness $h_v = 3.67 \times 10^{-3}$ m and the optimal location of VE dampers [2, 0, 0] can be acquired by the simplex method. According to the numerical results and actual manufacturing devices, the shear area A_v and the thickness h_v of VE layer are chosen as 60 mm × 50 mm and 5 mm, respectively.

Six seismograph apparatuses fixed in the structure are used to measure the displacements and accelerations of each floor and base slab, as shown in Fig. 4. The time-scaled El Centro wave record is used as the seismic inputs for the shaking table tests. The optimization test of the structure with VE dampers is carried out through installing or removing the bolts between dampers and braces. This test is performed in the Structure Laboratory of Xi'an Architecture and Technology University in China in 2000.

4.2. Test results and analysis

When the locations of dampers are [0, 0, 0], [0, 0, 2], [2, 0, 0], [2, 0, 2] and [2, 2, 2], the test results are analyzed under 0.2g El Centro wave. Figs. 6(a) and (b) show the comparison between the

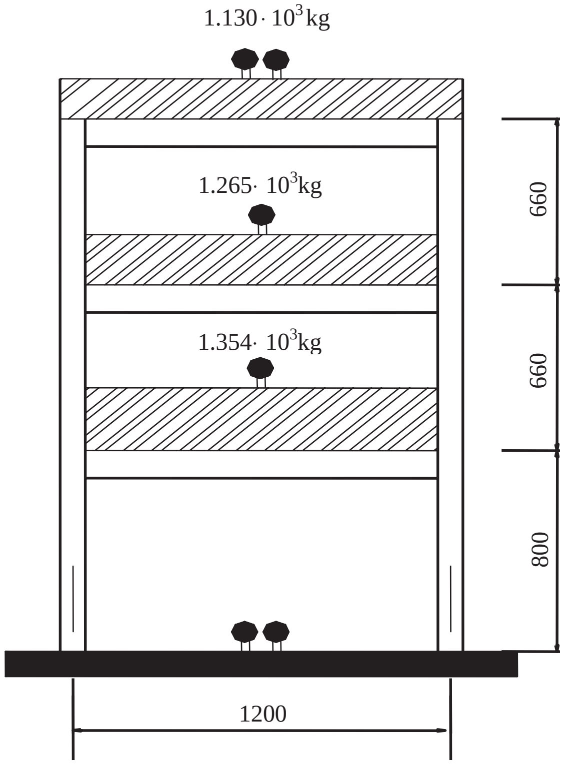

Fig. 4. The test frame model.

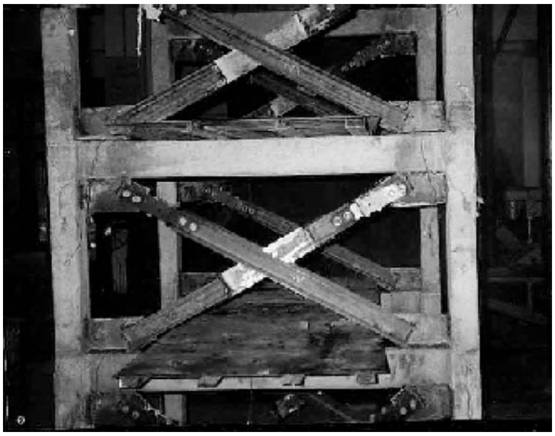

Fig. 5. The experimental viscoelastic braces.

displacement responses and the acceleration responses of the top floor between the locations of dampers [0, 0, 0] and [2, 2, 2], respectively. Fig. 7 shows comparison between the maximum displacement of each floor under different locations of VE dampers.

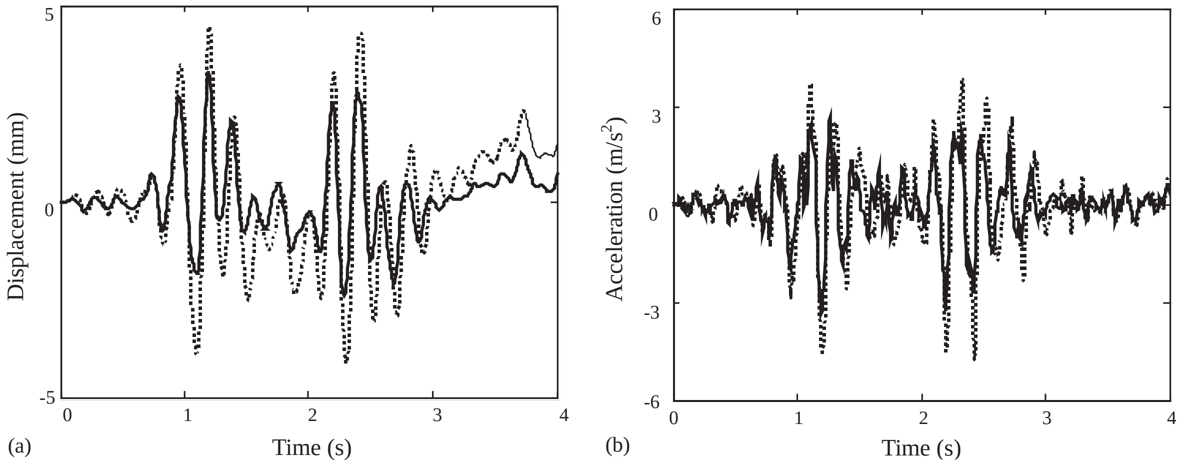

Fig. 6. Comparison between seismic responses of structures with dampers and without dampers: (a) comparison of the top floor displacement, (b) comparison of the top floor acceleration. ——, dampers' location [2, 0, 0], ····, dampers' location [0, 0, 0].

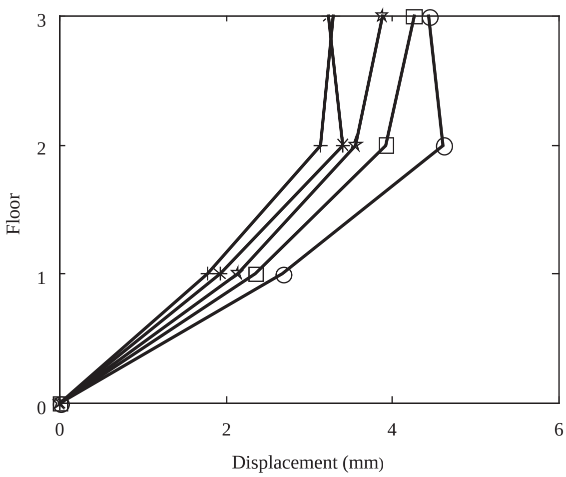

Fig. 7. Comparison between the experimental maximum displacement of each floor: —∗—, [2, 2, 2]; —◦—, [0, 0, 0]; —+—, [2, 0, 0]; —□—, [0, 0, 2]; —☆—, [2, 0, 2].

It can be seen clearly, from Fig. 6, that the displacement response and the acceleration response of the structure with VE dampers under the optimal location are smaller than those of the structure without VE dampers. The maximum displacement and acceleration response of the top floor of the structure without dampers is 4.56 mm and 4.89 m/s^2, respectively, while those of the top floor of the structure with VE dampers under the optimal location are 3.16 mm and 3.20 m/s^2, respectively. The displacement response is reduced by 30.7%, and the acceleration response is reduced by 34.6%, which shows that VE dampers can reduce seismic responses of the structure effectively. It can be seen from Fig. 7 that under 0.2 g El Centro wave the maximum displacement responses of the first floor and the second floor in the structure with optimal location dampers are smaller than those in the structure with dampers' location [2, 2, 2]. The maximum displacement responses of the first floor and the second floor in the structure under the optimal location are 89.8% and 87.2% of those under location [2, 2, 2], while the maximum displacement response of the top floor under the optimal location is larger than that under location [2, 2, 2] slightly, which is 100.9% of that under location [2, 2, 2]. It can also be seen from Fig. 7 that the maximum displacement responses under the optimal location are smaller than those under the other three locations. For the displacement response of the top floor, when the location of dampers is [2, 0, 2], the number of used dampers is two more than the number under the optimal location, while the maximum displacement response is 23.4% larger than that under the optimal location, and the maximum displacement responses under the location [0, 0, 2] and location [0, 0, 0] are 38.9% and 44.3% larger than that under the optimal location, respectively.

The maximum acceleration responses of the top floor of the structure under the [2, 2, 2], [2, 0, 2], [0, 0, 2] and [0, 0, 0] are 3.80, 3.81, 4.02 and 4.89 m/s^2, which are increased by 18.8%, 19.1%, 25.6% and 52.8%, respectively, comparing with the maximum acceleration response 3.20 m/s^2 of the top floor of the structure under the optimal location. It is shown that, rational location of VE dampers can make the stiffness and the damping of the structure distributed well and reduce seismic responses of the structure effectively. On the contrary, if the dampers are installed irrationally, the stiffness and the damping are not distributed well, seismic response cannot be reduced effectively, it may even be increased, and more VE dampers does not necessarily imply better shock absorption effect.

5. Conclusions

In this paper, the synthetic optimization method of the structure with VE dampers is introduced. An eight-story structure with VE dampers is analyzed. Through a shaking table test about a three-story structure with VE dampers, the analysis results of the simplex method are verified. Some conclusions can be obtained through analytical and experimental study on the structure with VE dampers:

(1) Optimization design of the structure with VE dampers can be achieved by the simplex method, i.e., the parameters and the location of VE dampers can be designed optimally in view of the fixed objective function and constrained conditions.

(2) VE dampers can increase the stiffness and the damping of structures, and reduce the seismic responses of structures effectively.

(3) When the structure is subjected to different earthquake excitations, the optimal location results are consistent.

(4) Rational location of VE dampers can reduce seismic responses of structures effectively due to the good distribution of stiffness and damping of structures. On the contrary, if the dampers are installed freely, the stiffness and the damping of structures may be increased irrationally, which will lead to the bad seismic mitigation effect.

(5) Increasing the number of VE dampers does not imply better shock absorption effect. The shock absorption effect of VE dampers is best when the location of VE dampers is optimal.

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