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Letter to the Editor

Experimental study of resonant frequency of thick rubber tubes

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1. Introduction

Vibro-acoustography is a new non-contact imaging method based on the radiation force of ultrasound [1]. Our objective is to apply this new technique for imaging and property characterization of arterial vessels. For this the arterial wall is first excited remotely by ultrasound at one of its natural resonant frequencies, usually the fundamental frequency. The vibration of the wall can be measured with non-contact laser or ultrasound Doppler, while the acoustic emission from the wall can be measured with hydrophone. From these measurements images or material properties of the vessel wall can be determined with appropriate theories.

A vast amount of research has been done since the last century on blood flow, mechanics of arterial vessels, and wave propagation in arteries [2,3]. However, theoretical models for the vibration analysis of arterial walls are rarely seen. In this "letter to the editor", we show some preliminary experimental results on a thick rubber tube, and compare it with our newly developed theory. In this theory we use a thick shell theory, the first order shear deformation theory (FSDT) [4], for the arterial wall because the ratio of thickness to radius h/R is relatively large for most arterial walls. A mean value of 0.15 for h/R was obtained by Peterson et al. [2]. Compared to thin shells widely used in engineering, for which the ratio is generally less than 0.05, the arterial wall is relatively thick, therefore, the thick shell theory should be applied to arterial walls. A new method, named wave propagation approach [5], is applied for easily solving the resonant frequency of the tube with various boundary conditions.

2. Experimental results and discussion

Experimental studies were carried out on thick cylindrical tubes. Natural rubber latex tubing (Kent Elastomer Products, Inc.) was chosen for the experiment because the material of latex

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tubing is close to arterial walls. A tube was fixed on the ends with interior solid plastic rods and clamped with clips on the exterior surface. The tube and the exciter were fixed on a specially designed table that could be rotated so that the vibration in the circumferential direction could be measured. The experimental setup is shown in Fig. 1(a). The length, outer diameter and

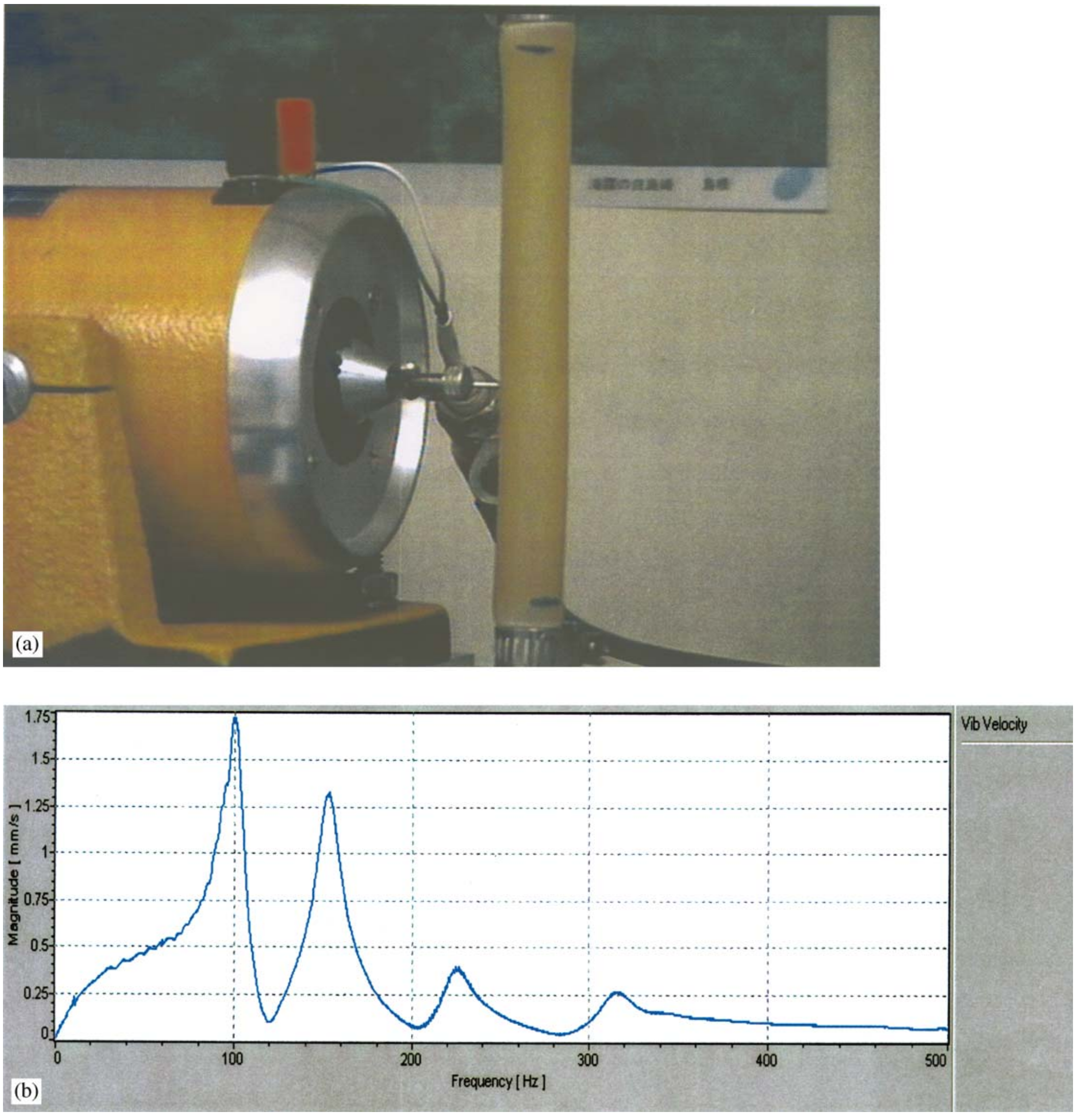

Fig. 1. (a) Experimental setup. A natural rubber latex tube was fixed with interior solid plastic rods and clamped with clips on the exterior surface at both ends. The tube was excited with a mechanical exciter. The vibration responses of the tube were measured with a laser vibrometer system; (b) Experimental results of the velocity frequency response at a point. The frequency range of interest was DC to 500 Hz with a frequency resolution of 0.3125 Hz. In the frequency range of interest, four resonant peaks can be identified which are the first four natural frequencies of the tube. They are measured, respectively, as 100.0, 153.1, 225.6 and 315.0 Hz.

thickness of the tube are, respectively, 100, 14 and 2.5 mm. The tube material has mass density $\rho = 960\,\text{kg/m}^3$, the Poisson ratio $\nu = 0.44$ and Young's modulus $E = 1.1 \times 10^6\,\text{N/m}^2$.

A periodic chirp waveform was generated by a HP 33120A function generator, which was then amplified and sent to the shaker. The tube was connected with the shaker by a thin steel rod that was cemented to the tube with cyanoacrylate adhesive. The vibration responses of the tube were measured with a laser vibrometer system (Polytec VibraScan Laser Vibrometer®, Polytec PI, Inc., Tustin, CA). The system consists of an OFV056 Vibrometer Scanning Head and an OFV 3001S Vibrometer Controller. With this system the objects can be scanned automatically at pre-described points. In our experiment, a scan along the axial direction of the tube was automatically done with the laser vibrometer system. Twenty measurement points were used for axial scanning along the tube length. Measurements in the circumferential direction of the tube were made by manually adjusting the experimental table. The frequency range of interest was DC to 500 Hz. The frequency resolution was 0.3125 Hz in the experiments. Fig. 1(b) shows one measurement of the velocity frequency response at the point $(x, \theta) = (40\,\text{mm}, \pi)$ which is diametrically opposite the excitation point $(x, \theta) = (40\,\text{mm}, 0)$. In the frequency range of interest, four resonant peaks can be identified that are the first four natural frequencies of the tube. They are measured, respectively, as 100.0, 153.1, 225.6 and 315.0 Hz.

We compared experimental data with our theoretical model for calculation of natural frequencies. In the experiment the tube is clamped at its both ends, but it is actually difficult to know how "clamped" it is. However, our theoretical model can easily predict the natural frequencies of various boundary conditions [6]. Therefore, we can simulate the natural frequencies with appropriate boundary conditions which best fit the experimental results. This feature is very important for later experimental analyses of blood vessels because the boundary conditions for a branched vessel are difficult to find. In the following simulations, both ends of the tube are constrained with torsion springs, as shown in Fig. 2. The boundary conditions are ranging from simply supported–simply supported (SS–SS) to clamped–clamped (C–C) depending on stiffness of the torsion springs. Five kinds of boundary conditions are considered. A non-dimensional stiffness parameter kL/EI is used, where k is the rotation stiffness of spring, L is the length, E is Young's modulus, and I is the area moment of inertia of the tube. $(k_1L/EI,\ k_2L/EI) = (0,0)$

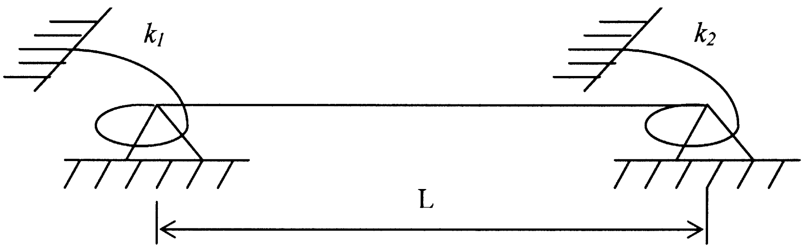

Fig. 2. For theoretical calculation the boundary conditions are assumed to be torsion springs at both ends of the tube. The rotation stiffness of the spring is expressed respectively k_1 and k_2 for the left and right ends. A non-dimensional stiffness parameter kL/EI is used, where k is the rotation stiffness of spring, L is the length, E is Young's modulus, and I is the area moment of inertia of the tube. $(k_1L/EI,\ k_2L/EI) = (0,0)$ corresponds to simply supported–simply supported (SS–SS) boundary conditions while $(k_1L/EI,\ k_2L/EI) = (\infty, \infty)$ corresponds to clamped–clamped (C–C) boundary conditions. The other three boundary conditions are between the SS–SS and C–C. With the wave propagation approach various complex boundary conditions can be easily handled in solving for the natural frequencies.

Table 1
Comparison of first four natural frequencies between the experiment and the theory

Boundary condition (k_1L/EI, k_2L/EI)	Fequency (Hz) Modal order			
	1	2	3	4
Experiment	100.0	153.1	225.6	315.0
(0,0)	93.489	122.02	180.93	255.64
(1,1)	94.565	124.15	183.12	257.54
(10,10)	98.93	134.42	195.3	269.22
(100,100)	102.76	145.87	212.46	289.34
(∞, ∞)	103.01	147.79	215.96	294.05
Comparison (%)	3.0	3.5	4.3	6.6

The comparison between the experiment and the theory is for the C–C boundary conditions and shown in percentage.

corresponds to simply supported–simply supported (SS–SS) boundary conditions while (k_1L/EI, k_2L/EI) = (∞, ∞) corresponds to clamped–clamped (C–C) boundary conditions. The other three boundary conditions are between the SS–SS and C–C.

Table 1 shows the comparison of the first four natural frequencies calculated by the present theory with five boundary conditions and with the experiment. If we only compare the first frequency, the boundary condition between (k_1L/EI, k_2L/EI) = (10,10) and (100,100) best fits the experiment result. However, if we take the first four natural frequencies into consideration, the C–C boundary conditions best fit the experimental results. Therefore, we use the C–C boundary conditions in our theoretical model for comparison with experiment and further analyses. The relative differences between the experiment and theory are expressed in percentage and presented on the last row of Table 1. They are within 7%. It can be seen that the experimental results agree fairly well with the theory. Also it is shown that the theoretical model can easily handle complex boundary conditions, which may be difficult for other closed form methods.

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