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Non-fragile H_∞ vibration control for uncertain structural systems

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Abstract

The paper deals with the robust non-fragile H_∞ control problem for uncertain structural systems with additive controller gain variations. The parameter uncertainties for the mass, damping and stiffness of the structural systems are unknown but norm bounded. Based on the H_∞ control theory and a linear matrix inequality formulation, a new method for designing a robust state-feedback control law is presented. The objective is to reduce the disturbance on the controlled output to a prescribed level for all admissible parametric uncertainties and controller gain variations. A four-degree-of-freedom building model subject to seismic excitation is used to illustrate the effectiveness of the approach through simulation.

1. Introduction

Active vibration control of structural systems such as large flexible space structures, tall and slender buildings, long bridges, etc. has become an increasingly important area in engineering practice. To date, a variety of control strategies based on H_2 (LQR) and H_∞ theories, neural networks, fuzzy logic, adaptive control, sliding mode, independent modal space, for instance, have been developed to attenuate the effects of structural vibration. New types of devices have been invented in order to implement these active control schemes in practical applications. Due to modelling errors, variation of materials properties, component non-linearities, and changing load environments, the system description for these structural systems inevitably contains uncertainties

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of different nature and levels [1]. These uncertainties can affect both the stability and performance of a control system. To accommodate such possible degradation of stability and performance, methods such as robust H_∞ control are often used. In many literature [1–4], the uncertainties in the mass matrix are modelled in an additive form in the inverse mass matrix which is an indirect and unnatural way to describe and reflect the structural uncertainty. Moreover, such an approach may lead to uncertainties appearing in the input and disturbance matrices which complicate the controller design procedures.

The application of the standard H_∞ control theory has an implicit assumption that the controller can be realized exactly. However, in practice, many physical limitations lead to a loss of precision in controller implementation, for example, the effects of finite word length in any digital systems, round-off errors in numerical arithmetic, inherent imprecision in analog devices, etc. Consequently, even though controllers are robust with respect to system uncertainties, they may be very sensitive to their own uncertainties (implementation errors). Recently, much attention have been paid to the so-called fragility problems of controllers since its initial presentation in Ref. [5], and then followed by many discussions in Refs. [6–10], and references therein. This controller fragility problem is basically the problem of performance deterioration of a feedback control system due to the inaccuracies in controller implementation [11]. In particular, several examples are presented in Ref. [5] to show that the existing H_2, H_∞, l_1, and μ designs could lead to very fragile controllers. Blanchini et al. [12] show that the searching for the optimal control policies for an ATM network is fragile if the delay time allowed to vary (possibly an arbitrarily small amount) with respect to the nominal value on which the design is based. Yee et al. [13] demonstrate that using the non-fragile H_∞ flight controllers are robustly stable and have H_∞ disturbance attenuation bounds with respect to some admissible controller gain variations while, on the contrary, the standard H_∞ flight controllers are unstable under the same controller gain variations. As we know, one of the main shortcomings of active control for structural systems is the possible failure of the controller, since it will cause serious damage even than that of no control to be used. Hence, this brings a new control issue to controller synthesis such that, for a given structural system, the resulting controller must be resilient or non-fragile with respect to its gain variations. During the last few years, many efforts have been made to tackle the non-fragile controller design problem for linear systems, see, e.g., Refs. [11,13–15], and references therein. Specifically, Dorato [11] gave an overview of non-fragile controller design for linear systems. A state feedback non-fragile H_∞ controller design method with respect to additive norm-bounded controller gain variations is given in Ref. [16] by using the Riccati inequality approach and the corresponding problem for designing output feedback controller is given in Ref. [17]. Non-fragile H_∞ controller problem for the case of multiplicative gain variations is addressed in Ref. [14], and non-fragile guaranteed cost control for linear systems is studied in Refs. [18,19], respectively. These efforts benefit us to further consider the controller fragility problems for active vibration control of structural systems.

This paper is mainly concerned with two aspects of vibration control for structural systems. One is that robust H_∞ disturbance attenuation for structural systems with parametric uncertainties, especially in the mass, damping, and stiffness matrices which are unknown but bounded, is considered. The uncertainties in the mass matrix do not require a commonly used inverse mass matrix perturbation description. Thus, the uncertainty can be described more naturally and does not introduce uncertainties into the control and disturbance matrices unnecessarily. Another aspect is that a non-fragile H_∞ state feedback controller is considered to deal with additive controller gain variations. The results developed in this paper are given in terms of the feasibility of some linear matrix inequalities (LMIs) which can be easily solved using standard numerical software.

The rest of this paper is organized as follows. In Section 2, the structural system under consideration is introduced. A non-fragile H_∞ state feedback controller design method (dealing with parametric uncertainties in the structural system and additive gain variations in the controller) based on LMIs is given in Section 3. A numerical example using seismic excitation data is provided in Section 4 to illustrate the effectiveness of the proposed technique. Finally, concluding remarks are given in Section 5.

Notation. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ the set of all $n\times m$ real matrices, $\|\cdot\|$ refers to either the Euclidean vector norm or the induced matrix 2-norm. For a real symmetric matrix W, the notation of $W>0$ ($W<0$) is used to denote its positive (negative) definiteness. Also, I is used to denote the identity matrix of appropriate dimensions. To simplify notation, $*$ is used to represent a block matrix which is readily inferred by symmetry.

2. Description of structural system

Now consider the following uncertain structural system:

$$(M+\Delta_M)\ddot{x}(t)+(C+\Delta_C)\dot{x}(t)+(K+\Delta_K)x(t)=Bu(t)+B_w w(t), \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the displacement, $u(t)$ is the control input, and $w(t)$ is the external disturbance or excitation. $M\in\mathbb{R}^{n\times n}$, $C\in\mathbb{R}^{n\times n}$ and $K\in\mathbb{R}^{n\times n}$ are the mass, damping, and stiffness matrices; Δ_M, Δ_C and Δ_K are corresponding perturbations, $B\in\mathbb{R}^{n\times m}$ is the input matrix and $B_w\in\mathbb{R}^{n\times p}$ is the disturbance matrix. By using $q(t)=[x^{\mathrm{T}}(t)\ \ \dot{x}^{\mathrm{T}}(t)]^{\mathrm{T}}$, Eq. (1) can be written as

$$\begin{bmatrix} I & 0 \\ 0 & M+\Delta_M \end{bmatrix}\dot{q}(t)=\begin{bmatrix} 0 & I \\ -K-\Delta_K & -C-\Delta_C \end{bmatrix}q(t)+\begin{bmatrix} 0 \\ B \end{bmatrix}u(t)+\begin{bmatrix} 0 \\ B_w \end{bmatrix}w(t). \tag{2}$$

Here, $w(t)$ is assumed to be an energy-bounded signal (i.e., $w(t)\in L_2[0,\infty)$). The output or measurement signal $z(t)$ to be controlled is given by

$$z(t)=C_d x(t)+C_v\dot{x}(t), \tag{3}$$

where $C_d\in\mathbb{R}^{q\times n}$ and $C_v\in\mathbb{R}^{q\times n}$. The system can be rewritten as

$$(\mathscr{E}+\Delta_{\mathscr{E}})\dot{q}(t)=(\mathscr{A}+\Delta_{\mathscr{A}})q(t)+\mathscr{B}u(t)+\mathscr{B}_w w(t), \tag{4}$$

$$z(t)=\mathscr{C}q(t), \tag{5}$$

where

$$\mathscr{E}=\begin{bmatrix} I & 0 \\ 0 & M \end{bmatrix},\quad \Delta_{\mathscr{E}}=\begin{bmatrix} 0 & 0 \\ 0 & \Delta_M \end{bmatrix},\quad \mathscr{A}=\begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix},\quad \Delta_{\mathscr{A}}=\begin{bmatrix} 0 & 0 \\ -\Delta_K & -\Delta_C \end{bmatrix},$$

$$\mathscr{B} = \begin{bmatrix} 0 \\ B \end{bmatrix}, \quad \mathscr{B}_w = \begin{bmatrix} 0 \\ B_w \end{bmatrix}, \quad \mathscr{C} = [C_d \;\; C_v].$$

If all of the output signals can be chosen and measured independently, the matrix $\mathscr{C}$ could be identity matrix. The uncertainty Δ_M is assumed to satisfy the following bound:

$$\|\Delta_M M^{-1}\| \leqslant \delta < 1, \tag{6}$$

which implies that $\|\Delta_{\mathscr{E}}\mathscr{E}^{-1}\| \leqslant \delta < 1$. Notice that the condition in Eq. (6) ensures that $\mathscr{E} + \Delta_{\mathscr{E}}$ is non-singular. Also, we have

$$\Delta_K = L_k F_k E_k, \tag{7}$$

$$\Delta_C = L_c F_c E_c, \tag{8}$$

where $\|F_k\| \leqslant 1$, $\|F_c\| \leqslant 1$, L_k, L_c, E_k, E_c are known constant matrices which characterize how the uncertain parameters in F_k, F_c enter the nominal damping and stiffness matrices C and K, respectively. The uncertainties in structural system (1) satisfying (6)–(8) are said to be *admissible*. Therefore,

$$\Delta_{\mathscr{A}} = -\begin{bmatrix} 0 \\ L_k \end{bmatrix} F_k [E_k \;\; 0] - \begin{bmatrix} 0 \\ L_c \end{bmatrix} F_c [0 \;\; E_c] = \mathscr{L}_k F_k \mathscr{E}_k + \mathscr{L}_c F_c \mathscr{E}_c \tag{9}$$

with

$$\mathscr{L}_k = -\begin{bmatrix} 0 \\ L_k \end{bmatrix}, \quad \mathscr{E}_k = [E_k \;\; 0], \quad \mathscr{L}_c = -\begin{bmatrix} 0 \\ L_c \end{bmatrix}, \quad \mathscr{E}_c = [0 \;\; E_c].$$

The control input, utilizing both position and velocity feedback signals, is given by

$$u(t) = (F_d + \Delta_{fd})x(t) + (F_v + \Delta_{fv})\dot{x}(t), \tag{10}$$

where $F_d \in \mathbb{R}^{n \times m}$, $F_v \in \mathbb{R}^{n \times m}$ are the feedback gain matrices for the displacement and the velocity, respectively, and Δ_{fd}, Δ_{fv} their corresponding uncertainties. This can be rewritten as

$$u(t) = (\mathscr{F} + \Delta_{\mathscr{F}})q(t), \tag{11}$$

where $\mathscr{F} \in \mathbb{R}^{m \times 2n}$ is the state feedback gain to be designed, $\mathscr{F} = [F_d \;\; F_v]$, and $\Delta_{\mathscr{F}} = [\Delta_{fd} \;\; \Delta_{fv}]$ is a norm-bounded gain variation in the form of [15,16],

$$\Delta_{\mathscr{F}} = \mathscr{L}_f F_f \mathscr{E}_f, \tag{12}$$

where $\mathscr{L}_f$, $\mathscr{E}_f$, are known constant matrices of appropriate dimensions describing the uncertainty structure, and $\|F_f\| \leqslant 1$.

Suppose $T_{zw}(s)$ denotes the closed-loop transfer function from disturbance $w(t)$ to measurement $z(t)$. The objective of this paper is to determine a state feedback controller gain matrix $\mathscr{F}$ such that the H_∞ norm of $\|T_{zw}(s)\|_\infty$ is less than a prescribed level $\gamma > 0$ for all admissible uncertainties in the structural system and gain variations in $\mathscr{F}$. Such a controller is referred to as a *non-fragile* H_∞ *state feedback controller*.

3. Robust non-fragile H_∞ state feedback control

Here, we develop a solution for the problem of robust non-fragile H_∞ state feedback control for the structural system (4)–(5) in which both robust closed-loop stability and robust H_∞ performance are achieved in spite of parametric uncertainties and controller gain variations.

System (4)–(5) with the state feedback control law (11) becomes

$$(\mathscr{E} + \Delta_{\mathscr{E}})\dot{q}(t) = (\mathscr{A} + \Delta_{\mathscr{A}})q(t) + \mathscr{B}(\mathscr{F} + \Delta_{\mathscr{F}})q(t) + \mathscr{B}_w w(t), \tag{13}$$

$$z(t) = \mathscr{C}q(t). \tag{14}$$

According to the assumption in Eq. (6), we can write Eq. (13) as

$$\begin{aligned}\dot{q}(t) &= (\mathscr{E} + \Delta_{\mathscr{E}})^{-1}(\mathscr{A} + \Delta_{\mathscr{A}})q(t) + (\mathscr{E} + \Delta_{\mathscr{E}})^{-1}\mathscr{B}(\mathscr{F} + \Delta_{\mathscr{F}})q(t) + (\mathscr{E} + \Delta_{\mathscr{E}})^{-1}\mathscr{B}_w w(t) \\ &= [\bar{\mathscr{A}} + \bar{\mathscr{B}}(\mathscr{F} + \Delta_{\mathscr{F}})]q(t) + \bar{\mathscr{B}}_w w(t),\end{aligned} \tag{15}$$

where

$$\bar{\mathscr{A}} = (\mathscr{E} + \Delta_{\mathscr{E}})^{-1}(\mathscr{A} + \Delta_{\mathscr{A}}), \quad \bar{\mathscr{B}} = (\mathscr{E} + \Delta_{\mathscr{E}})^{-1}\mathscr{B}, \quad \bar{\mathscr{B}}_w = (\mathscr{E} + \Delta_{\mathscr{E}})^{-1}\mathscr{B}_w.$$

According to the Bounded Real Lemma [20], for systems (13) and (14), the following statements are equivalent:

(1) $\|T_{zw}(s)\|_\infty < \gamma$, $\gamma > 0$, where $T_{zw}(s) = \mathscr{C}[sI - \bar{\mathscr{A}} - \bar{\mathscr{B}}(\mathscr{F} + \Delta_{\mathscr{F}})]^{-1}\bar{\mathscr{B}}_w$ is the transfer function from w to z.
(2) There exists a matrix $P > 0$ such that

$$\begin{bmatrix} [\bar{\mathscr{A}} + \bar{\mathscr{B}}(\mathscr{F} + \Delta_{\mathscr{F}})]^{\mathrm{T}}P + P[\bar{\mathscr{A}} + \bar{\mathscr{B}}(\mathscr{F} + \Delta_{\mathscr{F}})] & P\bar{\mathscr{B}}_w & \mathscr{C}^{\mathrm{T}} \\ \bar{\mathscr{B}}_w^{\mathrm{T}}P & -\gamma I & 0 \\ \mathscr{C} & 0 & -\gamma I \end{bmatrix} < 0. \tag{16}$$

By using the uncertainty structure in Eq. (12) for $\Delta_{\mathscr{F}}$ and considering Lemma 3 in Appendix A, it follows that inequality (16) is implied by the existence of a constant $\varepsilon > 0$ such that

$$\begin{bmatrix} (\bar{\mathscr{A}} + \bar{\mathscr{B}}\mathscr{F})^{\mathrm{T}}P + P(\bar{\mathscr{A}} + \bar{\mathscr{B}}\mathscr{F}) + \varepsilon P\bar{\mathscr{B}}\mathscr{L}_f\mathscr{L}_f^{\mathrm{T}}\bar{\mathscr{B}}^{\mathrm{T}}P + \varepsilon^{-1}\mathscr{E}_f^{\mathrm{T}}\mathscr{E}_f & P\bar{\mathscr{B}}_w & \mathscr{C}^{\mathrm{T}} \\ \bar{\mathscr{B}}_w^{\mathrm{T}}P & -\gamma I & 0 \\ \mathscr{C} & 0 & -\gamma I \end{bmatrix} < 0. \tag{17}$$

Define the new variable $X \triangleq P^{-1}$ in Eq. (17). Pre- and post-multiplying (17) by $\mathrm{diag}(X, I, I)$ and its transpose, respectively, then substituting $Y = \mathscr{F}X$, and then, pre- and post-multiplying it by

$$\mathrm{diag}(\mathscr{E} + \Delta_{\mathscr{E}}, I, I)$$

and its transpose, respectively, and followed by applying the Schur complement, we obtain

$$\begin{bmatrix} \Theta & \mathscr{B}_w & (\mathscr{E}+\Delta_{\mathscr{E}})X\mathscr{C}^{\mathrm{T}} \\ \mathscr{B}_w^{\mathrm{T}} & -\gamma I & 0 \\ \mathscr{C}X(\mathscr{E}+\Delta_{\mathscr{E}})^{\mathrm{T}} & 0 & -\gamma I \end{bmatrix} < 0, \tag{18}$$

where

$$\Theta = (\mathscr{E}+\Delta_{\mathscr{E}})X(\mathscr{A}+\Delta_{\mathscr{A}})^{\mathrm{T}} + (\mathscr{A}+\Delta_{\mathscr{A}})X(\mathscr{E}+\Delta_{\mathscr{E}})^{\mathrm{T}} + (\mathscr{E}+\Delta_{\mathscr{E}})Y^{\mathrm{T}}\mathscr{B}^{\mathrm{T}} + \mathscr{B}Y(\mathscr{E}+\Delta_{\mathscr{E}})^{\mathrm{T}} + \varepsilon\mathscr{B}\mathscr{L}_f\mathscr{L}_f^{\mathrm{T}}\mathscr{B}^{\mathrm{T}} + \varepsilon^{-1}(\mathscr{E}+\Delta_{\mathscr{E}})X\mathscr{E}_f^{\mathrm{T}}\mathscr{E}_fX(\mathscr{E}+\Delta_{\mathscr{E}})^{\mathrm{T}}. \tag{19}$$

Here, no matrix inverse of $\mathscr{E}+\Delta_{\mathscr{E}}$ is involved in the inequality.

For a given scalar $\eta > 0$, according to Lemma 1 in Appendix A, there always exists

$$(\mathscr{A}+\Delta_{\mathscr{A}})X(\mathscr{E}+\Delta_{\mathscr{E}})^{\mathrm{T}} + (\mathscr{E}+\Delta_{\mathscr{E}})X(\mathscr{A}+\Delta_{\mathscr{A}})^{\mathrm{T}} \leqslant \eta(\mathscr{A}+\Delta_{\mathscr{A}})X(\mathscr{A}+\Delta_{\mathscr{A}})^{\mathrm{T}} + \eta^{-1}(\mathscr{E}+\Delta_{\mathscr{E}})X(\mathscr{E}+\Delta_{\mathscr{E}})^{\mathrm{T}}. \tag{20}$$

Furthermore, notice that, for any scalars $\mu_1 > 0$, $\mu_2 > 0$, $\mu_3 > 0$, $Q > 0$, $\mu_1 I - \mathscr{E}_cX\mathscr{E}_c^{\mathrm{T}} > 0$, $\mu_2 I - \mathscr{E}_kX\mathscr{E}_k^{\mathrm{T}} > 0$, $\mu_3 I - \mathscr{E}_kQ\mathscr{E}_k^{\mathrm{T}} > 0$, Lemma 1 in Appendix A also has

$$\begin{aligned}
(\mathscr{A}+\Delta_{\mathscr{A}})X(\mathscr{A}+\Delta_{\mathscr{A}})^{\mathrm{T}} &= [(\mathscr{A}+\mathscr{L}_kF_k\mathscr{E}_k)+\mathscr{L}_cF_c\mathscr{E}_c]X[(\mathscr{A}+\mathscr{L}_kF_k\mathscr{E}_k)+\mathscr{L}_cF_c\mathscr{E}_c]^{\mathrm{T}} \\
&= (\mathscr{A}+\mathscr{L}_kF_k\mathscr{E}_k)X(\mathscr{A}+\mathscr{L}_kF_k\mathscr{E}_k)^{\mathrm{T}} + (\mathscr{A}+\mathscr{L}_kF_k\mathscr{E}_k)X\mathscr{E}_c^{\mathrm{T}}(\mu_1 I - \mathscr{E}_cX\mathscr{E}_c^{\mathrm{T}})^{-1} \\
&\quad \times \mathscr{E}_cX(\mathscr{A}+\mathscr{L}_kF_k\mathscr{E}_k)^{\mathrm{T}} + \mu_1\mathscr{L}_c\mathscr{L}_c^{\mathrm{T}} \\
&\leqslant (\mathscr{A}+\mathscr{L}_kF_k\mathscr{E}_k)X(\mathscr{A}+\mathscr{L}_kF_k\mathscr{E}_k)^{\mathrm{T}} + (\mathscr{A}+\mathscr{L}_kF_k\mathscr{E}_k)Q(\mathscr{A}+\mathscr{L}_kF_k\mathscr{E}_k)^{\mathrm{T}} + \mu_1\mathscr{L}_c\mathscr{L}_c^{\mathrm{T}} \\
&\leqslant \mathscr{A}X\mathscr{A}^{\mathrm{T}} + \mathscr{A}X\mathscr{E}_k^{\mathrm{T}}(\mu_2 I - \mathscr{E}_kX\mathscr{E}_k^{\mathrm{T}})^{-1}\mathscr{E}_kX\mathscr{A}^{\mathrm{T}} + \mu_2\mathscr{L}_k\mathscr{L}_k^{\mathrm{T}} + \mathscr{A}Q\mathscr{A}^{\mathrm{T}} \\
&\quad + \mathscr{A}Q\mathscr{E}_k^{\mathrm{T}}[\mu_3 I - \mathscr{E}_kQ\mathscr{E}_k^{\mathrm{T}}]^{-1}\mathscr{E}_kQ\mathscr{A}^{\mathrm{T}} + \mu_3\mathscr{L}_k\mathscr{L}_k^{\mathrm{T}} + \mu_1\mathscr{L}_c\mathscr{L}_c^{\mathrm{T}},
\end{aligned} \tag{21}$$

where

$$X\mathscr{E}_c^{\mathrm{T}}(\mu_1 I - \mathscr{E}_cX\mathscr{E}_c^{\mathrm{T}})^{-1}\mathscr{E}_cX < Q. \tag{22}$$

For any scalar $\mu_4 > 0$, $\mu_4 I - \mathscr{E}X\mathscr{E}^{\mathrm{T}} > 0$, there exists

$$\begin{aligned}
(\mathscr{E}+\Delta_{\mathscr{E}})X(\mathscr{E}+\Delta_{\mathscr{E}})^{\mathrm{T}} &\leqslant \mathscr{E}X\mathscr{E}^{\mathrm{T}} + \mathscr{E}X\mathscr{E}^{\mathrm{T}}(\mu_4 I - \mathscr{E}X\mathscr{E}^{\mathrm{T}})^{-1}\mathscr{E}X\mathscr{E}^{\mathrm{T}} + \mu_4\Delta_{\mathscr{E}}\mathscr{E}^{-1}(\Delta_{\mathscr{E}}\mathscr{E}^{-1})^{\mathrm{T}} \\
&\leqslant \mathscr{E}X\mathscr{E}^{\mathrm{T}} + \mathscr{E}X\mathscr{E}^{\mathrm{T}}(\mu_4 I - \mathscr{E}X\mathscr{E}^{\mathrm{T}})^{-1}\mathscr{E}X\mathscr{E}^{\mathrm{T}} + \mu_4\delta^2 I
\end{aligned} \tag{23}$$

and similarly, for any $\varepsilon_1 > 0$,

$$\begin{aligned}
(\mathscr{E}+\Delta_{\mathscr{E}})Y^{\mathrm{T}}\mathscr{B}^{\mathrm{T}} + \mathscr{B}Y(\mathscr{E}+\Delta_{\mathscr{E}})^{\mathrm{T}} &= \mathscr{E}Y^{\mathrm{T}}\mathscr{B}^{\mathrm{T}} + \mathscr{B}Y\mathscr{E}^{\mathrm{T}} + \Delta_{\mathscr{E}}Y^{\mathrm{T}}\mathscr{B}^{\mathrm{T}} + \mathscr{B}Y\Delta_{\mathscr{E}}^{\mathrm{T}} \\
&\leqslant \mathscr{E}Y^{\mathrm{T}}\mathscr{B}^{\mathrm{T}} + \mathscr{B}Y\mathscr{E}^{\mathrm{T}} + \varepsilon_1^{-1}\mathscr{B}Y\mathscr{E}^{\mathrm{T}}\mathscr{E}Y^{\mathrm{T}}\mathscr{B}^{\mathrm{T}} + \varepsilon_1\Delta_{\mathscr{E}}\mathscr{E}^{-1}\mathscr{E}^{-\mathrm{T}}\Delta_{\mathscr{E}}^{\mathrm{T}} \\
&\leqslant \mathscr{E}Y^{\mathrm{T}}\mathscr{B}^{\mathrm{T}} + \mathscr{B}Y\mathscr{E}^{\mathrm{T}} + \varepsilon_1^{-1}\mathscr{B}Y\mathscr{E}^{\mathrm{T}}\mathscr{E}Y^{\mathrm{T}}\mathscr{B}^{\mathrm{T}} + \varepsilon_1\delta^2 I.
\end{aligned} \tag{24}$$

By Lemma 2 in Appendix A, with any $\varepsilon_2 > 0$, the following inequality also holds:

$$\begin{bmatrix} \# & (\mathscr{E}+\Delta_{\mathscr{E}})X\mathscr{C}^{\mathrm{T}} \\ \mathscr{C}X(\mathscr{E}+\Delta_{\mathscr{E}})^{\mathrm{T}} & * \end{bmatrix} \leqslant \begin{bmatrix} \# + \varepsilon_2 \Delta_{\mathscr{E}}\mathscr{E}^{-1}\mathscr{E}^{-\mathrm{T}}\Delta_{\mathscr{E}}^{\mathrm{T}} & \mathscr{E}X\mathscr{C}^{\mathrm{T}} \\ \mathscr{C}X\mathscr{E}^{\mathrm{T}} & * + \varepsilon_2^{-1}\mathscr{C}X\mathscr{E}^{\mathrm{T}}\mathscr{E}X\mathscr{C}^{\mathrm{T}} \end{bmatrix}$$
$$\leqslant \begin{bmatrix} \# + \varepsilon_2\delta^2 I & \mathscr{E}X\mathscr{C}^{\mathrm{T}} \\ \mathscr{C}X\mathscr{E}^{\mathrm{T}} & * + \varepsilon_2^{-1}\mathscr{C}X\mathscr{E}^{\mathrm{T}}\mathscr{E}X\mathscr{C}^{\mathrm{T}} \end{bmatrix}, \tag{25}$$

where # and $*$ denote some submatrices in Eq. (18). Therefore, by using the bounding results in Eqs. (20)–(25), to ensure Eq. (18), it suffices to have

$$\begin{bmatrix} \Theta_0 & \mathscr{B}_w & \mathscr{E}X\mathscr{C}^{\mathrm{T}} & (\mathscr{E}+\Delta_{\mathscr{E}})X\mathscr{E}_f^{\mathrm{T}} \\ \mathscr{B}_w^{\mathrm{T}} & -\gamma I & 0 & 0 \\ \mathscr{C}X\mathscr{E}^{\mathrm{T}} & 0 & -\gamma I + \varepsilon_2^{-1}\mathscr{C}X\mathscr{E}^{\mathrm{T}}\mathscr{E}X\mathscr{C}^{\mathrm{T}} & 0 \\ \mathscr{E}_f X(\mathscr{E}+\Delta_{\mathscr{E}})^{\mathrm{T}} & 0 & 0 & -\varepsilon I \end{bmatrix} < 0, \tag{26}$$

where

$$\begin{aligned}\Theta_0 = {} & \eta[\mathscr{A}X\mathscr{A}^{\mathrm{T}} + \mathscr{A}Q\mathscr{A}^{\mathrm{T}} + \mu_1\mathscr{L}_c\mathscr{L}_c^{\mathrm{T}} + \mu_2\mathscr{L}_k\mathscr{L}_k^{\mathrm{T}} + \mu_3\mathscr{L}_k\mathscr{L}_k^{\mathrm{T}} + \mathscr{A}X\mathscr{E}_k^{\mathrm{T}}(\mu_2 I - \mathscr{E}_k X\mathscr{E}_k^{\mathrm{T}})^{-1}\mathscr{E}_k X\mathscr{A}^{\mathrm{T}} \\ & + \mathscr{A}Q\mathscr{E}_k^{\mathrm{T}}(\mu_3 I - \mathscr{E}_k Q\mathscr{E}_k^{\mathrm{T}})^{-1}\mathscr{E}_k Q\mathscr{A}^{\mathrm{T}}] + \eta^{-1}[\mathscr{E}X\mathscr{E}^{\mathrm{T}} + \mathscr{E}X\mathscr{E}^{\mathrm{T}}(\mu_4 I - \mathscr{E}X\mathscr{E}^{\mathrm{T}})^{-1}\mathscr{E}X\mathscr{E}^{\mathrm{T}} + \mu_4\delta^2] \\ & + \mathscr{E}Y^{\mathrm{T}}\mathscr{B}^{\mathrm{T}} + \mathscr{B}Y\mathscr{E}^{\mathrm{T}} + \varepsilon_1^{-1}\mathscr{B}Y\mathscr{E}^{\mathrm{T}}\mathscr{E}Y^{\mathrm{T}}\mathscr{B}^{\mathrm{T}} + \varepsilon_1\delta^2 I + \varepsilon_2\delta^2 I + \varepsilon\mathscr{B}\mathscr{L}_f\mathscr{L}_f^{\mathrm{T}}\mathscr{B}^{\mathrm{T}}.\end{aligned} \tag{27}$$

Similarly, among Eq. (26), by Lemma 2 in Appendix A, with any $\varepsilon_3 > 0$, there exists

$$\begin{bmatrix} \# & (\mathscr{E}+\Delta_{\mathscr{E}})X\mathscr{E}_f^{\mathrm{T}} \\ \mathscr{E}_f X(\mathscr{E}+\Delta_{\mathscr{E}})^{\mathrm{T}} & * \end{bmatrix} \leqslant \begin{bmatrix} \# + \varepsilon_3(\Delta_{\mathscr{E}}\mathscr{E}^{-1})(\Delta_{\mathscr{E}}\mathscr{E}^{-1})^{\mathrm{T}} & \mathscr{E}X\mathscr{E}_f^{\mathrm{T}} \\ \mathscr{E}_f X\mathscr{E}^{\mathrm{T}} & * + \varepsilon_3^{-1}\mathscr{E}_f X\mathscr{E}^{\mathrm{T}}\mathscr{E}X\mathscr{E}_f^{\mathrm{T}} \end{bmatrix}$$
$$\leqslant \begin{bmatrix} \# + \varepsilon_3\delta^2 I & \mathscr{E}X\mathscr{E}_f^{\mathrm{T}} \\ \mathscr{E}_f X\mathscr{E}^{\mathrm{T}} & * + \varepsilon_3^{-1}\mathscr{E}_f X\mathscr{E}^{\mathrm{T}}\mathscr{E}X\mathscr{E}_f^{\mathrm{T}} \end{bmatrix}, \tag{28}$$

where # and $*$ the corresponding submatrices in Eq. (26). Applying the results of Eq. (26) together with Eq. (28), and the Schur complement followed by some rearrangement of matrix sub-blocks, inequality Eq. (26) can be expressed as an LMI. Also, Eq. (22) can be expressed by an LMI by using Schur complement. Therefore, we conclude that, for the uncertain system (4)–(5) with given $\gamma > 0$ and $\eta > 0$, a state feedback control of form (11) can be constructed which could tolerate the system uncertainties Δ_M, Δ_C, Δ_K, and controller gain variations $\Delta_{\mathscr{F}}$ such that the resulting closed-loop system is robustly stable with disturbance attenuation γ provided that there exists matrices $X > 0$, $Q > 0$, Y and scalars $\varepsilon > 0$, $\varepsilon_i > 0$, $i = 1, \ldots, 3$, $\mu_i > 0$, $i = 1, \ldots, 4$, satisfying the following LMIs:

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} \\ * & \Omega_{22} & 0 & 0 \\ * & * & \Omega_{33} & 0 \\ * & * & * & \Omega_{44} \end{bmatrix} < 0, \tag{29}$$

$$
\begin{aligned}
\Omega_{11} = {} & \eta(\mathscr{A}X\mathscr{A}^{\mathrm{T}} + \mu_1\mathscr{L}_c\mathscr{L}_c^{\mathrm{T}} + \mu_2\mathscr{L}_k\mathscr{L}_k^{\mathrm{T}} + \mu_3\mathscr{L}_k\mathscr{L}_k^{\mathrm{T}} + \mathscr{A}Q\mathscr{A}^{\mathrm{T}}) \\
& + \eta^{-1}(\mathscr{E}X\mathscr{E}^{\mathrm{T}} + \mu_4\delta^2 I) + \mathscr{E}Y^{\mathrm{T}}\mathscr{B}^{\mathrm{T}} + \mathscr{B}Y\mathscr{E}^{\mathrm{T}} + \varepsilon_1\delta^2 I + \varepsilon_2\delta^2 I + \varepsilon_3\delta^2 I + \varepsilon\mathscr{B}\mathscr{L}_f\mathscr{L}_f^{\mathrm{T}}\mathscr{B}^{\mathrm{T}},
\end{aligned}
$$

$$\Omega_{12} = [\mathscr{B}_w \;\; \eta\mathscr{A}Q\mathscr{E}_k^{\mathrm{T}} \;\; \eta\mathscr{A}X\mathscr{E}_k^{\mathrm{T}} \;\; \mathscr{E}X\mathscr{E}^{\mathrm{T}} \;\; \mathscr{B}Y\mathscr{E}^{\mathrm{T}}],$$

$$\Omega_{22} = \mathrm{diag}[-\gamma I, -\eta(\mu_3 I - \mathscr{E}_k Q\mathscr{E}_k^{\mathrm{T}}), -\eta(\mu_2 I - \mathscr{E}_k X\mathscr{E}_k^{\mathrm{T}}), -\eta(\mu_4 I - \mathscr{E}X\mathscr{E}^{\mathrm{T}}), -\varepsilon_1 I],$$

$$\Omega_{13} = [I \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]^{\mathrm{T}}\mathscr{E}X\mathscr{C}^{\mathrm{T}}[I \;\; 0],$$

$$\Omega_{33} = \begin{bmatrix} -\gamma I & \mathscr{C}X\mathscr{E}^{\mathrm{T}} \\ * & -\varepsilon_2 I \end{bmatrix},$$

$$\Omega_{14} = [I \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]^{\mathrm{T}}\mathscr{E}X\mathscr{E}_f^{\mathrm{T}}[I \;\; 0],$$

$$\Omega_{44} = \begin{bmatrix} -\varepsilon I & \mathscr{E}_f X\mathscr{E}^{\mathrm{T}} \\ * & -\varepsilon_3 I, \end{bmatrix}$$

and

$$\begin{bmatrix} -Q & X\mathscr{E}_c^{\mathrm{T}} \\ \mathscr{E}_c X & -(\mu_1 I - \mathscr{E}_c X\mathscr{E}_c^{\mathrm{T}}) \end{bmatrix} < 0. \tag{30}$$

Moreover, a desired robust non-fragile H_∞ state feedback control gain matrix is given by $\mathscr{F} = YX^{-1}$.

4. Simulation

This section gives a numerical example to demonstrate the applicability of the proposed approach in Section 3. The robust stability of the steady state motion of an uncertain four-degree-of-freedom 4-d.o.f. building model with controller gain variations is considered. The building model is shown in Fig. 1, where x_i, m_i, c_i, k_i, $i = 1, \ldots, 4$ are the relative displacement, mass, damping and stiffness of each storey, respectively, and $m_1 = m_2 = 2 \times 1.05$ (10^6 kg), $m_3 = m_4 = 1.05$ (10^6 kg), $k_1 = k_2 = 2 \times 350$ (10^6 N/m), $k_3 = k_4 = 350$ (10^6 N/m), $c_1 = c_2 = c_3 = c_4 = 1.575$ (10^6 N s/m). The basic structural system has been used in Refs. [1,21] for vibrational control simulation. The dynamic equation of the system is given as in Eq. (1) with system matrices

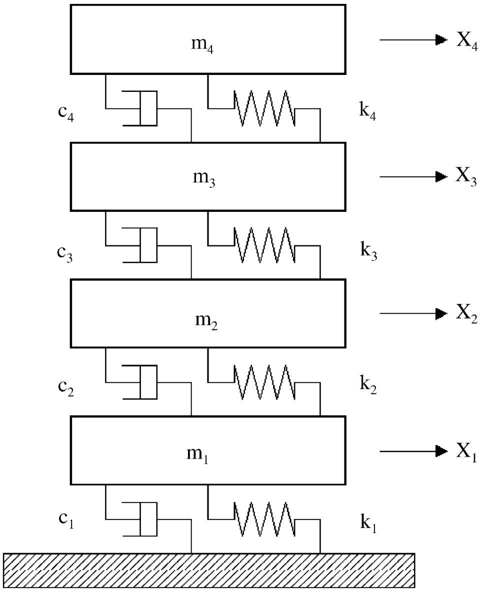

Fig. 1. 4-d.o.f. building model.

given by

$$M = \begin{bmatrix} 2.1 & 0 & 0 & 0 \\ 0 & 2.1 & 0 & 0 \\ 0 & 0 & 1.05 & 0 \\ 0 & 0 & 0 & 1.05 \end{bmatrix} \ (10^6\ \mathrm{kg}), \quad K = \begin{bmatrix} 1.4 & -0.7 & 0 & 0 \\ -0.7 & 1.05 & -0.35 & 0 \\ 0 & -0.35 & 0.7 & -0.35 \\ 0 & 0 & -0.35 & 0.35 \end{bmatrix} \ (10^9\ \mathrm{N/m}),$$

$$C = \begin{bmatrix} 3.15 & -1.575 & 0 & 0 \\ -1.575 & 3.15 & -1.575 & 0 \\ 0 & -1.575 & 3.15 & -1.575 \\ 0 & 0 & -1.575 & 3.15 \end{bmatrix} \ (10^6\ \mathrm{N\ s/m}).$$

It is assumed that each storey of the building model has a controller and $B = I$ in (1) so that $\mathscr{B} = [0 \ \ I]^{\mathrm{T}}$ in Eq. (4). The external disturbance corresponds to the earthquake excitation force given in Ref. [1] and

$$\mathscr{B}_w = [0 \ \ 0 \ \ 0 \ \ 0 \ \ 1.250 \ \ 1.250 \ \ 0.625 \ \ 0.625]^{\mathrm{T}}$$

in Eq. (4). The output variables are chosen to be the displacements and velocities of each storey, therefore, $\mathscr{C} = I$.

The uncertainties in the mass, damping, and stiffness matrices are, respectively, modelled as

$$\|\Delta_M M^{-1}\| \leqslant \delta = 0.1, \quad \Delta_K = L_k F_k E_k \equiv (0.1K) F_k(I), \quad \Delta_c = L_c F_c E_c = (0.1C) F_d(I).$$

Assume that the controller gain variation has structure

$$\mathscr{L}_f = a_f[1\ \ 1\ \ 1\ \ 1]^{\mathrm{T}}, \quad \mathscr{E}_f = [1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1],$$

where a_f is an adjustable parameter to describe the level of gain variation (in the range of $\pm 10^7$). In this simulation, a time history of acceleration from 1940 El Centro (California) earthquake is applied to the base of the building model [1,3].

As a means for comparison, we design an H_∞ state feedback controller for the nominal system (i.e., the system has no parametric uncertainties) and no consideration is given to controller gain variations. When $\gamma = 0.1$, a controller gain matrix $\mathscr{F}_0$ is rounded to four decimal places to reflect its finite word length implementation:

$$\mathscr{F}'_0 = 10^9 \begin{bmatrix} 1.3961 & -0.6994 & -0.0005 & 0.0003 & -0.0025 & -0.0006 & -0.0006 & 0.0004 \\ -0.7000 & 1.0465 & -0.5491 & -0.0000 & -0.0016 & -0.0020 & -0.2629 & 0.0000 \\ 0.0028 & 0.0024 & 0.6976 & -0.3501 & 0.0039 & 0.5829 & 0.0004 & -0.0018 \\ -0.0006 & 0.0002 & -0.3500 & 0.3477 & -0.0010 & 0.0004 & -0.0016 & -0.0009 \end{bmatrix}.$$

This results in an equivalent perturbation equals

$$\Delta'_{\mathscr{F}_0} = 10^4 \begin{bmatrix} 4.2346 & -1.7259 & 4.1840 & 2.6815 & -3.6102 & -0.4524 & 0.1069 & 3.0021 \\ 4.3950 & -3.8218 & -0.2554 & -3.9238 & -1.1490 & -2.7658 & -4.8989 & -0.6790 \\ -2.6115 & 2.0351 & -2.0762 & 0.3787 & 4.3559 & 3.2244 & -3.5548 & 0.7404 \\ 0.5419 & 2.9434 & -1.8495 & 0.5553 & 1.3732 & -1.7500 & 0.3116 & -4.9466 \end{bmatrix}$$

such that $\mathscr{F}_0 = \mathscr{F}'_0 + \Delta'_{\mathscr{F}_0}$. When $\mathscr{F}_0$ is used (that is, a full-digit realization), the output of uncontrolled and controlled displacements of the fourth storey and the first storey are shown in

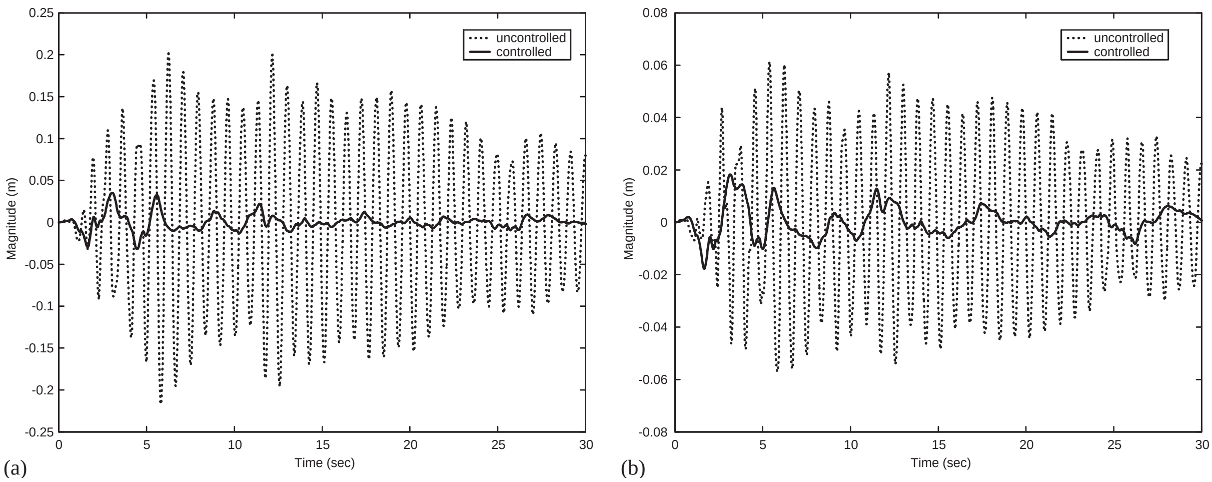

Fig. 2. Displacements with $\mathscr{F}_0$ for nominal system: (a) displacements of the fourth storey and (b) displacements of the first storey.

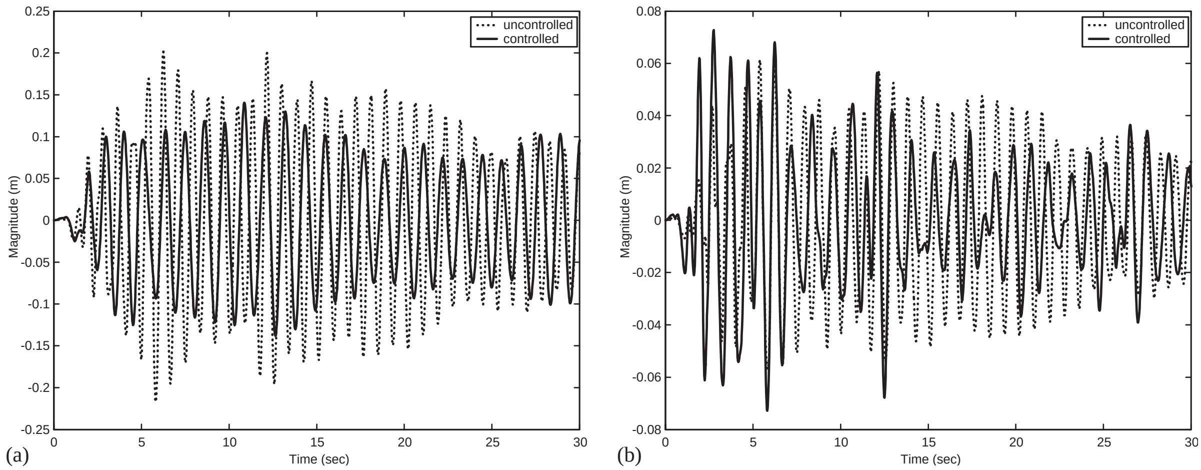

Fig. 3. Displacements with $\mathscr{F}_0$ for uncertain system: (a) displacements of the fourth storey and (b) displacements of the first storey.

Figs. 2(a)–(b), respectively. The displacements of the other two storeys have the similar varying trend, which are omitted here for brevity. In addition, the velocities output of the four storeys can give us the same information to explain our results, which are also omitted here for brevity. It can be seen that a very good reduction in the vibration can be obtained. When the full-digit controller $\mathscr{F}_0$ is used to control the structural system with above-mentioned parametric uncertainties and additive controller gain variations, in which $a_f = 2.43 \times 10^6$, the output of the simulation for the uncontrolled and controlled displacements of the fourth storey and the first storey are shown in Figs. 3(a)–(b), respectively. It can be observed that although the controller can control the nominal system very well, it cannot attenuate the disturbance when these uncertainties occur. In fact, the closed-loop control system becomes unstable when a_f is further increased. When we use the controller gain matrix $\mathscr{F}'_0$ instead of $\mathscr{F}_0$, which is also equivalent to adding a perturbation on the controller gain matrix, the output of the simulation for the uncontrolled and controlled displacements of the fourth storey and the first storey are shown in Figs. 4(a)–(b), respectively. It is obvious the ability to attenuate the vibration is significantly degraded and the level of vibration is unacceptable.

Finally, we design a controller using the proposed approach by solving the LMIs described in Eqs. (29) and (30) for the same uncertain system and consider the controller gain variations. When selecting $\gamma = 0.01$, $\eta = 17$, $\delta = 0.1$, we obtain a non-fragile controller gain matrix $\mathscr{F}$ and a four decimal place implementation is given by

$$\mathscr{F}' = 10^9 \begin{bmatrix} -2.1994 & -2.4947 & -2.1114 & -2.1074 & -2.2323 & -2.2315 & -2.2697 & -2.2697 \\ -0.3154 & -0.3539 & -0.9165 & -0.8812 & -0.6085 & -0.6098 & -0.5343 & -0.5345 \\ 0.1986 & 0.2384 & 0.6472 & 0.5451 & 0.4018 & 0.4027 & 0.3523 & 0.3527 \\ 0.0004 & 0.0004 & -0.0340 & 0.0329 & -0.0001 & -0.0001 & 0.0002 & 0.0000 \end{bmatrix}$$

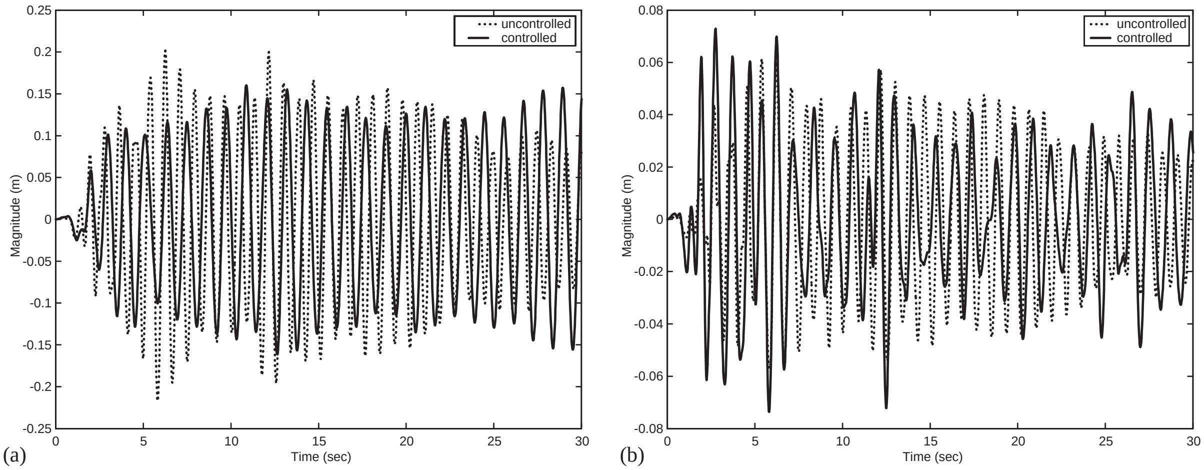

Fig. 4. Displacements with $\mathscr{F}'_0$ for uncertain system: (a) displacements of the fourth storey and (b) displacements of the first storey.

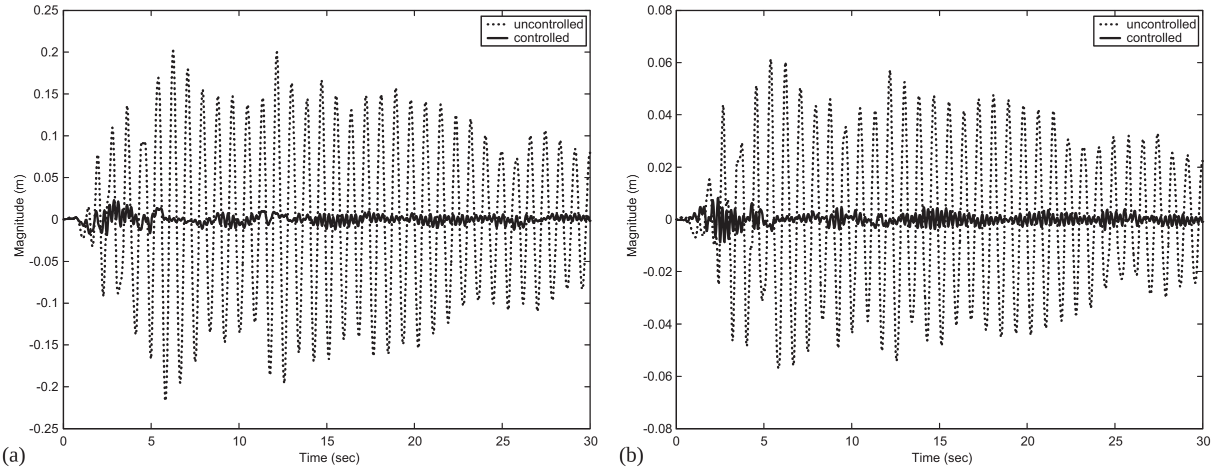

Fig. 5. Displacements with $\mathscr{F}$ for uncertain system: (a) displacements of the fourth storey and (b) displacements of the first storey.

and the round-off error matrix is

$$\Delta'_{\mathscr{F}} = 10^4 \begin{bmatrix} 1.0809 & -2.9042 & -0.2158 & -3.8307 & -0.2163 & -3.4901 & 2.3760 & -2.2790 \\ -0.8093 & 0.6264 & -3.4851 & 2.8223 & 3.8933 & 2.7184 & -2.8325 & 4.7743 \\ -4.0923 & 4.2116 & -3.4889 & 4.1823 & 1.8124 & -1.7037 & -1.9135 & -1.5830 \\ 0.9268 & 2.9315 & -2.3546 & 0.0024 & 3.0497 & 3.1137 & 2.9147 & -4.3585 \end{bmatrix},$$

where $\mathscr{F} = \mathscr{F}' + \Delta'_{\mathscr{F}}$. When $\mathscr{F}$ is used to control the vibration, the output of the uncontrolled and controlled displacements of the fourth storey and the first storey are shown in Figs. 5(a)–(b), respectively. It can be seen that although there are parametric uncertainties in the system and

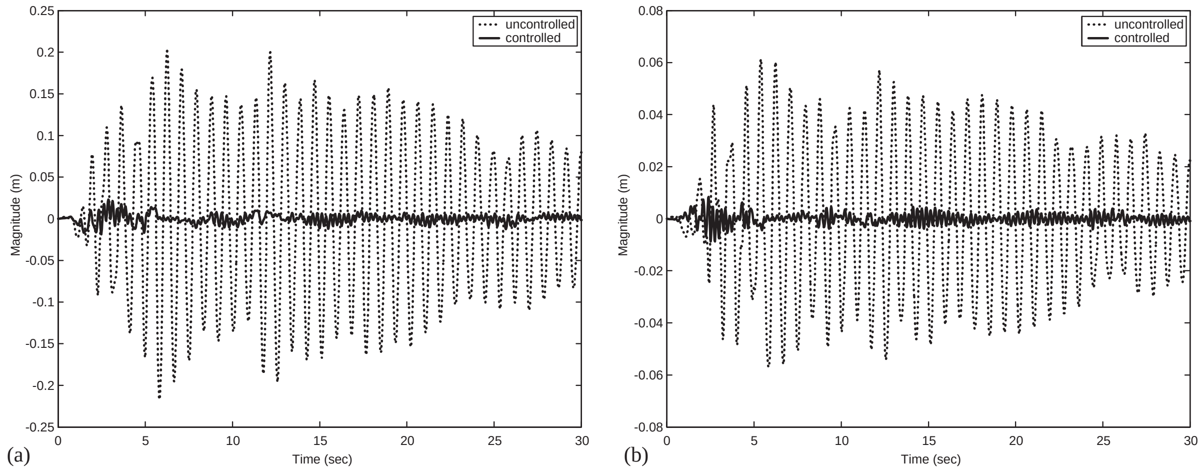

Fig. 6. Displacements with $\mathscr{F}'$ for uncertain system: (a) displacements of the fourth storey and (b) displacements of the first storey.

controller gain variations, the non-fragile controller can still attenuate the vibration due to the seismic disturbance very well in spite of uncertainties. As a matter of fact, a further increasing a_f does not lead to an unstable effect as seen in the previous case with $\mathscr{F}_0$. When the rounded-off non-fragile controller gain matrix $\mathscr{F}'$ is used, the output of the uncontrolled and controlled displacements of the fourth storey and first storey are depicted in Figs. 6(a)–(b), respectively. It can be observed that the effectiveness of vibration attenuation is still very well preserved. Thus, the non-fragile controller is very robust towards uncertainties since they are considered in advance in the design procedure, and this is important in practical engineering applications.

5. Conclusions

This paper presents a new approach to design a non-fragile H_∞ state feedback controller for structural systems with mass, damping, and stiffness uncertainties and controller gain variations based on a linear matrix inequality formulation. Due to no explicit inverse of mass matrix existing in this approach, the uncertainties in mass, damping and stiffness can be described more naturally and directly. Additive controller gain variations are also considered in this approach, which makes the approach more robust and more applicable in vibration engineering practice. A numerical example of four-degree-of-freedom building structure is used to illustrate the effectiveness of this approach. It can be concluded that the proposed method can successfully deal with the uncertainties in the structural system and its controller. Moreover, the control effectiveness is significantly better than the controller designed simply according to the nominal system.

Acknowledgements

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Appendix A

The following lemmas used in this paper can be found in Refs. [22,23].

Lemma 1. *Let* A, L, E *and* F *be real matrices of appropriate dimensions with* $\|F\| \leqslant 1$. *Then, for any real matrix* $P > 0$ *and scalar* $\mu > 0$ *such that* $\mu I - EPE^{\mathrm{T}} > 0$,

$$(A + LFE)P(A + LFE)^{\mathrm{T}} \leqslant APA^{\mathrm{T}} + APE^{\mathrm{T}}(\mu I - EPE^{\mathrm{T}})^{-1}EPA^{\mathrm{T}} + \mu LL^{\mathrm{T}}.$$

Lemma 2. *Let* M, N *be real matrices of appropriate dimensions, for any scalar* $\varepsilon > 0$,

$$\begin{bmatrix} 0 & NM^{\mathrm{T}} \\ MN^{\mathrm{T}} & 0 \end{bmatrix} \leqslant \begin{bmatrix} \varepsilon NN^{\mathrm{T}} & 0 \\ 0 & \varepsilon^{-1}MM^{\mathrm{T}} \end{bmatrix}.$$

Lemma 3. *Given matrices* Y, M *and* N *of appropriate dimensions, then*

$$Y + M\Delta N + N^{\mathrm{T}}\Delta^{\mathrm{T}}M^{\mathrm{T}} < 0$$

for all Δ *satisfying* $\|\Delta\| \leqslant 1$ *if and only if there exists a constant* $\varepsilon > 0$ *such that*

$$Y + \varepsilon MM^{\mathrm{T}} + \varepsilon^{-1}N^{\mathrm{T}}N < 0.$$

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