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Research on stability of periodic elastic motion of a flexible four bar crank rocker mechanism

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Abstract

In this paper, the stability of periodical elastic motion of a flexible four bar crank rocker mechanism is analyzed using the first approximation of Liapunov's stability theorem and Floquet theory. A procedure for predicting the stability is presented. Experimental investigation on the stability is carried out. Strain responses of the coupler up-surface midpoint of the mechanism are measured for several angular velocities of the crank. The stability of periodical elastic motion of the coupler for these angular velocities is determined based on the strain responses. The experimentally obtained conclusions agree well with the analytical conclusions presented in Section 4. This shows that the stability analysis of periodical elastic motion of the mechanism is correct.

1. Introduction

During high-speed operation, flexible mechanism systems exhibit obvious elastic vibratory deflections. This can reduce the motion accuracy and shorten the longevity of the systems. So research on elastic dynamics and elastic vibration stability of flexible mechanisms is very useful. Over the past two decades, a considerable amount of research has addressed problems associated with dynamics modelling, calculation and control of flexible mechanisms. However, only a few studies [1–7] have been conducted on elastic vibration stability of flexible mechanism systems. Zadoks and Midha [3,4] derived the non-linear equations of motion for an elastic two-degree-of-freedom machine system in torsional vibration, and linearized the equations about the system's steady state rigid-body response. Then the stability of the linearized equations (with time-periodic

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coefficients) was examined using Floquet theory. Mahyuddin et al. [5,6] developed a method to study parametric stability of flexible cam-follower systems based on Floquet theory. This method was applied to an automotive valve train which was modelled as a single-degree-of-freedom vibration system. The parametric vibration stability of the system was studied, and the results are presented in the form of parametric stability charts. Dynamic stability of parametrically excited flexible cam-follower systems was also investigated by Feng and Lan [7]. They derived the equations of approximate transition curves that separate the stable from the unstable solutions in the plane of the dimensionless frequency and excitation parameter by using the method of multiple scales and the technique of Fourier series expansion.

One of important research fields is stability study of periodical elastic motion of flexible linkage systems. This study can be referred to stability analysis of periodical solution of non-linear elastic motion differential equations for the systems. It has not been extensively dealt with so far. Yang and Sadler [8] and Farhang and Midha [9] indicated that steady state elastic motion response of flexible linkage systems operating at constant input rotational speed exhibits period characteristics. According to Liapunov theory [10], when periodical elastic motion of the systems is unstable, real elastic motion of the systems will leave far the neighborhood of periodical elastic motion and show as anomalistic severe elastic vibration. So the knowledge of stability of periodical elastic motion of flexible linkage systems is very useful to the engineer in avoiding the occurrence of the unstable periodical elastic motion.

In this paper, a flexible four bar crank rocker mechanism is taken as an object of study. Theoretical research on the stability of periodical elastic motion of the mechanism is performed, and a procedure for determining the stability is presented. On the basis of the theoretical research, experimental investigation on the stability of periodical elastic motion of the mechanism is carried out to verify theoretical conclusions.

2. Dynamic model

The model for study is a four bar crank rocker mechanism (as shown in Fig. 1). This mechanism can realize transformation from rotation of the crank to swing of the rocker. In order to realize the transformation, the length of the crank is designed much less than that of the coupler and the

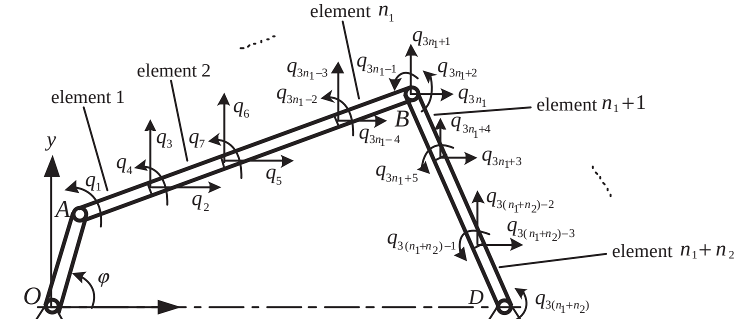

Fig. 1. Schematic diagram of a four bar crank rocker mechanism.

rocker. Therefore, the crank is usually made into a short thick rigid rod or a rigid disk with lesser radius. Thus, the crank can be considered as a rigid body in analytical modelling of the mechanism. However, the coupler and the rocker are slender rods usually compared to the crank, and their stiffness is much smaller. Hence, the effect of flexibility of the coupler and the rocker on motion of the mechanism must be taken into consideration. A finite element modelling of the mechanism results in the following non-linear differential equation of elastic motion [11]:

$$\mathbf{M}(\varphi)\ddot{\mathbf{q}} + \mathbf{C}(\varphi, \omega, \mathbf{q})\dot{\mathbf{q}} + \mathbf{K}(\varphi, \omega, \mathbf{q})\mathbf{q} = \mathbf{Q}^*(\varphi, \omega), \tag{1}$$

where φ and ω are the crank angle and the crank angular velocity, respectively, and $\mathbf{q}$ is the global deflection vector. The expansion of $\mathbf{q}$ is

$$\mathbf{q} = [q_1 \quad q_2 \cdots q_{3(n_1+n_2)}]^{\mathrm{T}}, \tag{2}$$

where q_1 is elastic rotation of left hand end of element 1, q_2 is elastic displacement of left hand end of element 2 in the x direction, q_3 is elastic displacement of left hand end of element 2 in the y direction, …, $q_{3(n_1+n_2)}$ is elastic rotation of right hand end of element $n_1 + n_2$. Eq. (1) is formulated on the assumption that the mechanism operates with constant input rotational speed, the crank is taken as a rigid body, the coupler and the rocker are considered as flexible beams. Large-deformation non-linearity, motion constraint and coupling terms between rigid body and elastic motion of flexible links are considered in the derivation of Eq. (1). Matrices $\mathbf{M}(\varphi)$, $\mathbf{C}(\varphi, \omega, \mathbf{q})$, $\mathbf{K}(\varphi, \omega, \mathbf{q})$ and vector $\mathbf{Q}^*(\varphi, \omega)$ were proved to be periodic functions of the crank angle φ, and the period equals 2π [11], i.e.,

$$\begin{aligned}
\mathbf{M}(\varphi) &= \mathbf{M}(\varphi + 2\pi), \\
\mathbf{C}(\varphi, \omega, \mathbf{q}) &= \mathbf{C}(\varphi + 2\pi, \omega, \mathbf{q}), \\
\mathbf{K}(\varphi, \omega, \mathbf{q}) &= \mathbf{K}(\varphi + 2\pi, \omega, \mathbf{q}), \\
\mathbf{Q}^*(\varphi, \omega) &= \mathbf{Q}^*(\varphi + 2\pi, \omega).
\end{aligned} \tag{3}$$

3. Stability analysis of periodical elastic motion

The boundary conditions for periodical elastic motion of the flexible four bar crank rocker mechanism are

$$\mathbf{q}(0) = \mathbf{q}(T), \quad \dot{\mathbf{q}}(0) = \dot{\mathbf{q}}(T), \tag{4}$$

where $T = 2\pi/\omega$ is the time taken by the crank to complete one revolution, the super dot (•) represents the time derivative. The study of the stability of periodical elastic motion of the mechanism can be referred to stability analysis of the solution satisfying Eq. (1) and periodicity boundary conditions (4). Additionally, it is most convenient for the stability analysis to consider the equations in the first order form. Let

$$\bar{\mathbf{q}}_1 = \mathbf{q}, \quad \bar{\mathbf{q}}_2 = \frac{\mathrm{d}\mathbf{q}}{\mathrm{d}\varphi} \tag{5}$$

and introduce notation

$$\bar{\mathbf{q}} = \begin{bmatrix} \bar{\mathbf{q}}_1 \\ \bar{\mathbf{q}}_2 \end{bmatrix}. \tag{6}$$

Then Eq. (1) can be rewritten as

$$\frac{\mathrm{d}\bar{\mathbf{q}}}{\mathrm{d}\varphi} = \bar{\mathbf{f}}(\varphi, \bar{\mathbf{q}}), \tag{7}$$

where

$$\bar{\mathbf{f}}(\varphi, \bar{\mathbf{q}}) = \begin{bmatrix} \bar{\mathbf{q}}_2 \\ \frac{1}{\omega^2}[M(\varphi)]^{-1}\left[\mathbf{Q}^*(\varphi,\omega) - \omega\mathbf{C}(\varphi,\omega,\bar{\mathbf{q}}_1)\bar{\mathbf{q}}_2 - \mathbf{K}(\varphi,\omega,\bar{\mathbf{q}}_1)\bar{\mathbf{q}}_1\right] \end{bmatrix}. \tag{8}$$

Periodical boundary conditions (4) can be rewritten as

$$\bar{\mathbf{q}}|_{\varphi=0} = \bar{\mathbf{q}}|_{\varphi=2\pi}. \tag{9}$$

So the study of the stability of periodical elastic motion of the mechanism can be referred to stability analysis of the solution satisfying Eq. (7) and periodicity boundary condition (9).

Assuming that $\bar{\mathbf{q}}^* = \bar{\mathbf{q}}^*(\varphi)$ is the solution satisfying Eq. (7) and periodicity boundary condition (9), and $\bar{\mathbf{y}} = \bar{\mathbf{y}}(\varphi)$ is the perturbation from the solution, we have

$$\bar{\mathbf{y}} = \bar{\mathbf{q}} - \bar{\mathbf{q}}^*. \tag{10}$$

Introducing Eq. (10) into Eq. (7), we obtain

$$\frac{\mathrm{d}\bar{\mathbf{y}}}{\mathrm{d}\varphi} = \mathbf{f}(\varphi, \bar{\mathbf{q}}) - \mathbf{f}(\varphi, \bar{\mathbf{q}}^*), \tag{11}$$

where

$$\mathbf{f}(\varphi, \bar{\mathbf{q}}) = \begin{bmatrix} \bar{\mathbf{q}}_2 \\ -\frac{1}{\omega^2}[\mathbf{M}(\varphi)]^{-1}[\omega\mathbf{C}(\varphi,\omega,\bar{\mathbf{q}}_1)\bar{\mathbf{q}}_2 + \mathbf{K}(\varphi,\omega,\bar{\mathbf{q}}_1)\bar{\mathbf{q}}_1] \end{bmatrix}. \tag{12}$$

Substituting equation $\bar{\mathbf{q}} = \bar{\mathbf{q}}^* + \bar{\mathbf{y}}$ into Eq. (11) yields

$$\frac{\mathrm{d}\bar{\mathbf{y}}}{\mathrm{d}\varphi} = \mathbf{f}(\varphi, \bar{\mathbf{q}}^* + \bar{\mathbf{y}}) - \mathbf{f}(\varphi, \bar{\mathbf{q}}^*) = \mathbf{J}(\varphi)\bar{\mathbf{y}} + \mathbf{O}(|\bar{\mathbf{y}}|^2), \tag{13}$$

where

$$\mathbf{J}(\varphi) = \left.\frac{\partial \mathbf{f}(\varphi, \bar{\mathbf{q}})}{\partial \bar{\mathbf{q}}}\right|_{\bar{\mathbf{q}}=\bar{\mathbf{q}}^*=\bar{\mathbf{q}}^*(\varphi)}. \tag{14}$$

$\mathbf{O}(|\bar{\mathbf{y}}|^2)$ is higher order term in the perturbation. Thus the stability characteristic of null solution $\bar{\mathbf{y}} = 0$ of Eq. (13) is the same as the stability characteristic of periodical solution $\bar{\mathbf{q}}^* = \bar{\mathbf{q}}^*(\varphi)$ of Eq. (7). So the study of the stability of periodical elastic motion of the mechanism can be referred to the stability analysis of null solution of Eq. (13). Assuming that the perturbation $\bar{\mathbf{y}}$ is sufficiently small for the higher order term $\mathbf{O}(|\bar{\mathbf{y}}|^2)$ to be neglected, we obtain the first approximation of Eq. (13)

$$\frac{d\bar{\mathbf{y}}}{d\varphi} = \mathbf{J}(\varphi)\bar{\mathbf{y}}, \tag{15}$$

where coefficient matrix $\mathbf{J}(\varphi)$ is periodic function of the crank angle φ, and the period equals 2π, i.e., $\mathbf{J}(\varphi) = \mathbf{J}(\varphi + 2\pi)$. It can be proved as follows:

Substituting the first three equations of Eqs. (3) into Eq. (12), we obtain

$$\mathbf{f}(\varphi, \bar{\mathbf{q}}) = \mathbf{f}(\varphi + 2\pi, \bar{\mathbf{q}}). \tag{16}$$

According to Eq. (14), $\mathbf{J}(\varphi + 2\pi)$ can be expressed as

$$\mathbf{J}(\varphi + 2\pi) = \left.\frac{\partial \mathbf{f}(\varphi + 2\pi, \bar{\mathbf{q}})}{\partial \bar{\mathbf{q}}}\right|_{\bar{\mathbf{q}}=\bar{\mathbf{q}}^*(\varphi+2\pi)}. \tag{17}$$

Considering that the period of function $\bar{\mathbf{q}}^*(\varphi)$ is 2π, Eq. (17) can be rewritten as

$$\mathbf{J}(\varphi + 2\pi) = \left.\frac{\partial \mathbf{f}(\varphi + 2\pi, \bar{\mathbf{q}})}{\partial \bar{\mathbf{q}}}\right|_{\bar{\mathbf{q}}=\bar{\mathbf{q}}^*(\varphi)}. \tag{18}$$

Substituting Eq. (16) into Eq. (18) yields

$$\mathbf{J}(\varphi + 2\pi) = \left.\frac{\partial \mathbf{f}(\varphi, \bar{\mathbf{q}})}{\partial \bar{\mathbf{q}}}\right|_{\bar{\mathbf{q}}=\bar{\mathbf{q}}^*(\varphi)}. \tag{19}$$

Comparing Eq. (14) with Eq. (19), we obtain

$$\mathbf{J}(\varphi) = \mathbf{J}(\varphi + 2\pi). \tag{20}$$

So Eq. (15) is linear differential equation with periodic coefficient.

According to Liapunov's theorem on the stability in the first approximation [10], when system (15) possess significant behavior, the stability characteristic of null solution of system (13) is the same as that of null solution of system (15). Because system (15) is a linear system with periodic coefficient, we can determine the stability characteristic of null solution of system (15) using Floquet's theory [12]. Assuming that the standard fundamental matrix of Eq. (15) is $\mathbf{X} = \mathbf{X}(\varphi)$, i.e., $\mathbf{X} = \mathbf{X}(\varphi)$ is the solution of matrix differential equation

$$\frac{d\mathbf{X}}{d\varphi} = \mathbf{J}(\varphi)\mathbf{X},$$
$$\mathbf{X}(0) = \mathbf{I}, \tag{21}$$

where $\mathbf{I}$ is identity matrix. According to Floquet's theory, the stability characteristic of null solution of system (15) can be determined by the eigenvalues λ_i of matrix $\mathbf{X}(2\pi)$. These eigenvalues are obtained from the following characteristic equation:

$$\det(\lambda\mathbf{I} - \mathbf{X}(2\pi)) = 0. \tag{22}$$

According to Floquet's theory the statements concerning the stability of system (15) can be summarized as follows: If $\max|\lambda_i| < 1$, null solution of system (15) is asymptotically stable; if $\max|\lambda_i| > 1$, null solution of system (15) is unstable.

According to Liapunov's theorem on the stability in the first approximation, we can conclude that: if $\max|\lambda_i| < 1$, null solution of system (13) is asymptotically stable. If $\max|\lambda_i| > 1$, null solution of system (13) is unstable. If $\max|\lambda_i| = 1$ (i.e. system (15) exhibit critical behavior), null solution of system (13) can be either stable or unstable, depending on the higher order term $\mathbf{O}(|\bar{\mathbf{y}}|^2)$.

Considering that the study of the stability of periodical elastic motion of the mechanism can be referred to analysis of the stability characteristics of null solution of Eq. (13), so the conclusions concerning the stability of periodical elastic motion of the mechanism can be summarized as follows:

1. If $\max|\lambda_i| < 1$, periodical elastic motion of the mechanism is asymptotically stable.
2. If $\max|\lambda_i| > 1$, periodical elastic motion of the mechanism is unstable.

From the conclusions, the eigenvalues of matrix $\mathbf{X}(2\pi)$ must be determined in order to analyze the stability of periodical elastic motion of the mechanism. So the procedure for predicting the stability can be summarized as follows:

1. Determine the numerical solution $\bar{\mathbf{q}}^*(\varphi)$ satisfying Eq. (7) and periodicity boundary condition (9) using the shoot method, where $0 \leqslant \varphi \leqslant 2\pi$.
2. Determine the numerical solution $\mathbf{J}(\varphi)$ from Eq. (14), where $0 \leqslant \varphi \leqslant 2\pi$.
3. Integrate matrix differential equation (21) using the Runge–Kutta method until matrix $\mathbf{X}(2\pi)$ is obtained.
4. Determine all eigenvalues λ_i of matrix $\mathbf{X}(2\pi)$.
5. Compute $\max|\lambda_i|$.
6. Compare $\max|\lambda_i|$ with 1, if $\max|\lambda_i| < 1$, periodical elastic motion of the mechanism is asymptotically stable. If $\max|\lambda_i| > 1$, periodical elastic motion of the mechanism is unstable.

4. Example

To illustrate the stability analysis of periodical elastic motion of flexible four bar crank rocker mechanism, we consider the model illustrated in Fig. 2. The properties of this model, used for the theoretical and experimental analysis, are listed in Table 1. The crank is assumed to be rigid. The

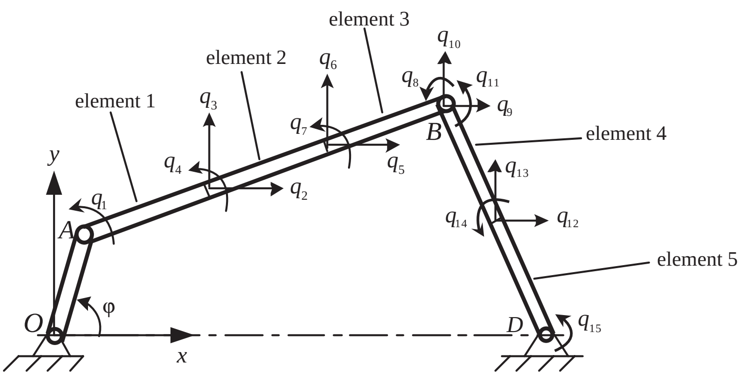

Fig. 2. Example stability problem—a flexible four bar crank rocker mechanism model.

Table 1
Flexible four bar crank rocker mechanism parameters

	Length (mm)	Width (mm)	Height (mm)	Mass density (kg/m^3)	Modulus of elasticity (N/m^2)
Crank	100	—	—	7.8×10^3	—
Coupler	330	24	2	7.8×10^3	2.1×10^{11}
Rocker	260	24	3	7.8×10^3	2.1×10^{11}

Distance between ground pivots = 400 mm.
Lumped mass of the bearing assembly = 0.046 kg.

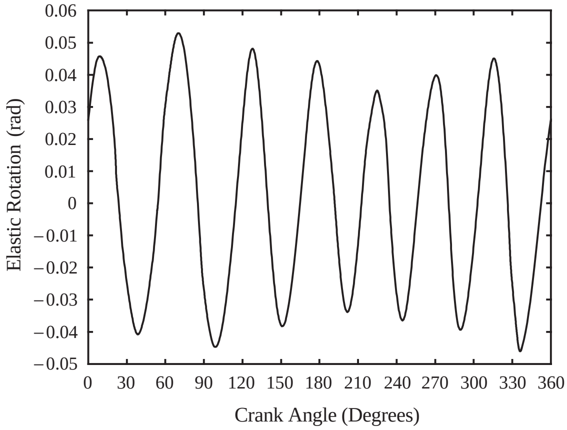

Fig. 3. Elastic rotation of left hand end of element 1 versus crank angle for a specified crank angular velocity of 65 rad/s.

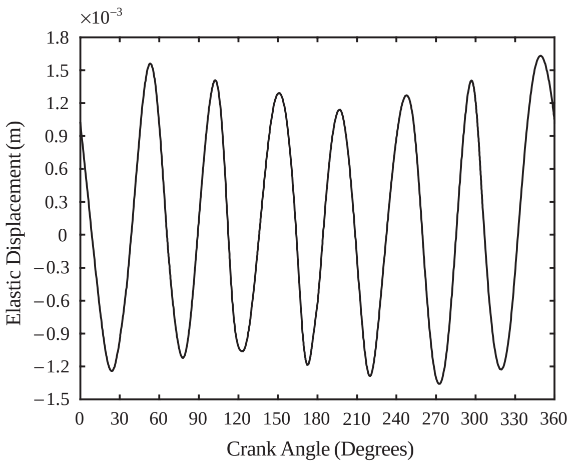

Fig. 4. Elastic displacement of left hand end of element 2 in the x direction versus crank angle for a specified crank angular velocity of 65 rad/s.

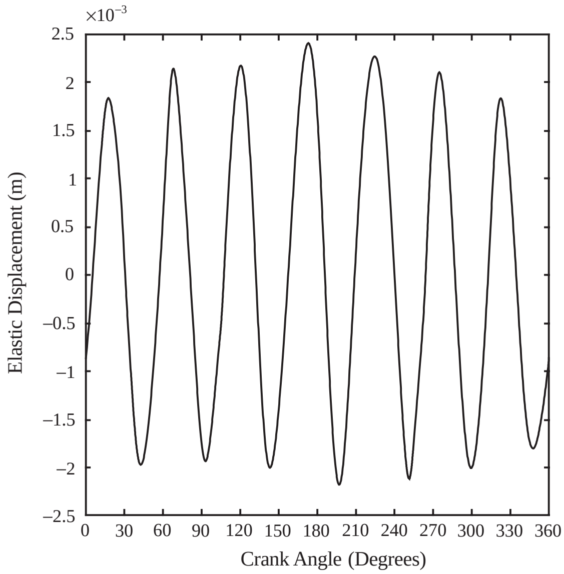

Fig. 5. Elastic displacement of left hand end of element 2 in the y direction versus crank angle for a specified crank angular velocity of 65 rad/s.

Table 2
The values of $\max|\lambda_i|$ for a number of crank angular velocities in the range of 50–70 rad/s

ω (rad/s)	50	51	52	53	54	55	56
$\max\|\lambda_i\|$	4.002×10^{-2}	7.641×10^{-2}	8.501×10^{-2}	1.189×10^{-1}	1.654×10^{-1}	2.241×10^{-1}	2.886×10^{-1}
ω (rad/s)	57	58	59	60	61	62	63
$\max\|\lambda_i\|$	3.616×10^{-1}	4.386×10^{-1}	5.204×10^{-1}	6.802×10^{-1}	7.001×10^{-1}	8.402×10^{-1}	1.139
ω (rad/s)	64	65	66	67	68	69	70
$\max\|\lambda_i\|$	1.306	1.415	1.505	1.574	1.628	1.664	1.703

coupler and the rocker are assumed to be flexible, and are modelled with beam elements. A total of five elements were employed, three for the couple and two for the rocker.

The periodical elastic motion response of the mechanism for a specified constant crank angular velocity $\omega = 65$ rad/s is obtained by using the shoot method. Figs. 3–5 show the periodical responses of the first three generalized coordinates q_1, q_2 and q_3, respectively. All eigenvalues λ_i of matrix $\mathbf{X}(2\pi)$ are determined by using QR method, where $\max|\lambda_i| \approx 1.415 > 1$. This indicates that the periodical elastic motion of the mechanism for the crank angular velocity $\omega = 65$ rad/s is unstable. The values of $\max|\lambda_i|$ for other crank angular velocities can be analogously determined. Table 2 lists the values of $\max|\lambda_i|$ for a number of crank angular velocities in the range of 50–70 rad/s.

From Table 2 it can be seen that $\max|\lambda_i| < 1$ when $\omega = 50-62$ rad/s and $\max|\lambda_i| > 1$ when $\omega = 63-70$ rad/s. So the following conclusions can be obtained:

1. When the mechanism is operated at the input crank angular velocity $\omega = 50-62$ rad/s, the periodical elastic motion of the mechanism is stable.
2. When the mechanism is operated at the input crank angular velocity $\omega = 63-70$ rad/s, the periodical elastic motion of the mechanism is unstable.

5. Experimental investigation on the stability of periodical elastic motion

The characteristics of elastic motion of flexible links in mechanisms can be reflected by the histories of flexible links strains [11]. A flexible four bar crank rocker mechanism is taken as an object of experimental investigation in this paper. Strain responses of the upper surface midpoint of the coupler for a number of crank angular velocities are recorded. From the characteristics of the responses, the stability of periodical elastic motion of the mechanism for these crank angular velocities is determined. The experimentally obtained conclusions are then compared with the analytical conclusions presented in Section 4 to verify the analytical methods.

5.1. Experimental model

A photograph of the experimental apparatus is shown in Fig. 6. All links are constructed from steel. Table 1 lists the parameters of the experimental four bar crank rocker mechanism. The mechanism consists of a rigid crank, a flexible coupler and a flexible rocker. The input shaft is supported by a ball bearing. The crank-coupler, coupler-rocker, and rocker-ground connections are made via pins and small ball bearings. The mechanism is attached to a 3 cm thick oak foundation. The oak foundation is then bolted to a concrete support anchored to the floor in order to minimize external vibrations. The input crank is driven by a 1.1 kW dc motor. A 22 cm diameter sheave, serving as a flywheel, is mounted on the input shaft to minimize any angular velocity fluctuations of the crank. The crank angular velocity is controlled with a Bodine dc motor controller. The strain gage is located at the upper surface midpoint of the coupler. The strain is

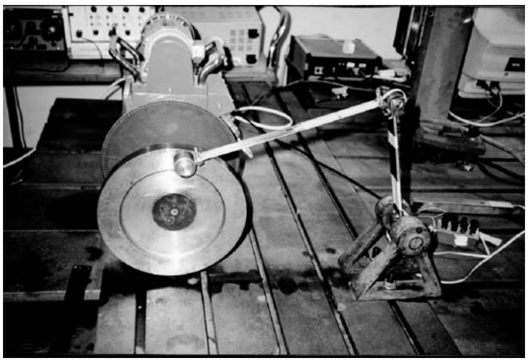

Fig. 6. The experimental apparatus.

measured with a four active-arm bridge circuit, using Micro Measurement EA-13-250BG-120 strain gages. The strain signals are recorded using an IBM Netvista personal computer. The mechanism is run at a number of crank angular velocities, between 55 and 65 rad/s. The crank zero-position is recorded using a GE H21A2 photon-coupled Interrupter Module with an 8 μs on-time. The photocell is triggered each time the crank passes through its zero-position by a rotating disk with a single small hole.

5.2. Experimental results

The experimental results are presented by plotting the strain measurements throughout a cycle of motion. Figs. 7–10 show the experimental strains at the upper surface midpoint of the coupler for four representative counter clockwise crank angular velocities of 55, 59, 62 and 65 rad/s.

From Figs. 7–9 it can be seen that the strains and the first time derivatives of strains at the beginning of each cycle are quite close to those at the end of the each cycle. This shows that responses of the upper surface midpoint strain of the coupler for crank angular velocities of 55, 59, and 62 rad/s exhibit periodic characteristics. The periodic characteristics of the strain responses indicate that the elastic motion responses of the coupler for the above crank angular velocities are also periodic. So we can conclude that periodic elastic motion responses of the coupler for above crank angular velocities are stable. This conclusion is in agreement with the analytical conclusion 1 presented in Section 4.

From Fig. 10 it can be seen that the amplitude of the strain signal for the crank angular velocity of 65 rad/s is much larger when compared to that of other strain signals for crank angular velocities of 55, 59, and 62 rad/s, and the strain response for the crank angular velocity of 65 rad/s exhibit severe non-periodic characteristics. This indicates that the periodic elastic motion of the coupler for the crank angular velocity 65 rad/s has not been realized. Therefore, periodic elastic motion of the coupler for this crank angular velocity is unstable. This conclusion agrees with the analytical conclusion 2 presented in Section 4.

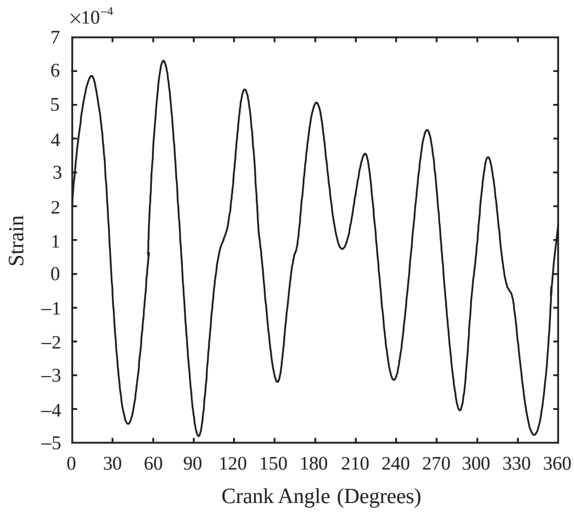

Fig. 7. Strain at the upper surface midpoint of the coupler for input crank angular velocity of 55 rad/s.

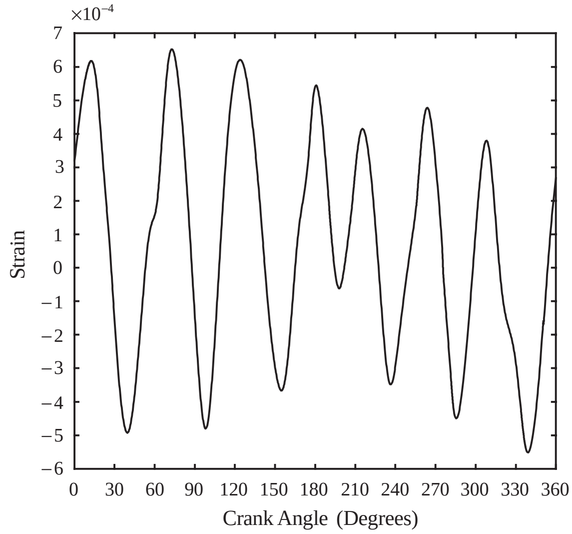

Fig. 8. Strain at the upper surface midpoint of the coupler for input crank angular velocity of 59 rad/s.

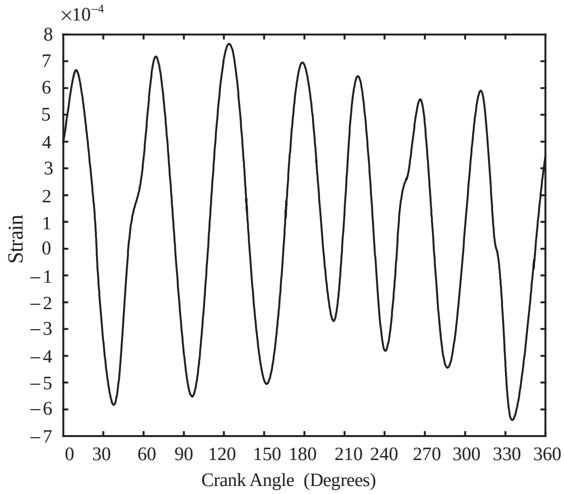

Fig. 9. Strain at the upper surface midpoint of the coupler for input crank angular velocity of 62 rad/s.

Agreement between above experimentally obtained conclusions and analytical conclusions presented in Section 4 indicates that the theoretical analysis of periodic elastic motion stability of the flexible four bar crank rocker mechanism is correct.

6. Conclusion remarks

In this work, a stability analysis of periodic elastic motion of flexible four bar crank rocker mechanism is performed. A procedure for predicting the stability is presented. To illustrate the

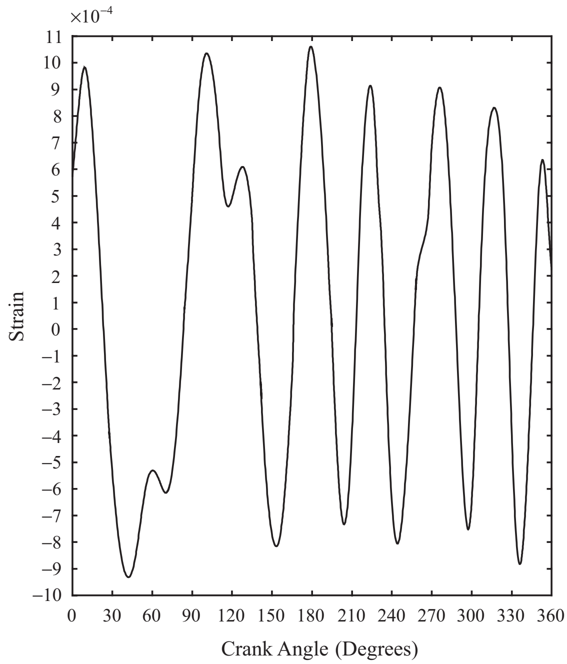

Fig. 10. Strain at the upper surface midpoint of the coupler for input crank angular velocity of 65 rad/s.

procedure, a particular flexible four bar crank rocker linkage system is taken as a case of study. Input crank angular velocity ranges corresponding to stable and unstable periodic elastic motion of the system are obtained, respectively.

Experimental investigation on the stability of periodic elastic motion of the system is carried out. The stability for four representative crank angular velocities is determined based on experimental results. The experimentally obtained conclusions agree well with the analytical conclusions presented in Section 4. This shows that the theoretical analysis of the periodical elastic motion stability of flexible four bar crank rocker mechanism is correct.

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