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Letter to the Editor

Effect of soil interaction on the performance of tuned mass dampers for seismic applications

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1. Introduction

Tuned mass dampers (TMDs) are effective passive control devices for the mitigation of undesirable vibration in structures. Studies on the applicability of TMDs to structures subjected to seismic excitation have been carried out by researchers such as Villaverde and Koyoama [1], Rana and Soong [2], Lin et al. [3] and Wang et al. [4] amongst many others.

Design of a TMD for its proper performance requires knowledge of the natural frequencies and damping of the structure to which it is attached. Fujino and Abe [5] have shown that TMD efficiency in reducing structural response is greatly affected by the accuracy of tuning of the natural frequency of the TMD to the natural frequency of the structure. Generally, the properties of the structure used in the design of the TMD are those evaluated considering the structure to be of a fixed-base type. These properties of the structure may be significantly altered when the structure has a flexible base, i.e. when the foundation of the structure is supported on compliant soil and undergoes motion relative to the surrounding soil. In such cases, it is necessary to study the effects of soil-structure interaction (SSI) while designing the TMD for the desired vibration control of the structure. Xu and Kwok [6] have examined the frequency domain response of soil-structure–TMD linear systems subjected to wind loading. Takewaki [7] has developed a method for the response reduction of structures while considering soil interaction effects by a combination of viscous damper and TMD. However, the past studies on TMD used for seismic applications in structures, have not considered the effects of the altered properties of the structure due to SSI, on the performance of the damper.

In the present study, the behaviour of flexible-base structures with attached TMD, subjected to earthquake excitations has been investigated. The aspect of considering modified structural properties due to SSI has been covered in this study. The superstructure has been modelled as a linear, single-degree-of-freedom (s.d.o.f.) system having a natural frequency equal to the most

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predominant modal frequency of the structure (fundamental or otherwise). The mass damper would be tuned to this modal frequency in case of design of this damper for structures that are assumed to be fixed at the base. It is fairly common in simplified TMD design to control a single mode that makes the maximum contribution to the largest response of the structure (e.g. see Refs. [2,3]. An expression for the transfer function of the response of the flexible-base s.d.o.f. system with attached TMD has been obtained. This expression has been derived from the general formulation of Dey and Gupta [8] for the seismic response of secondary systems in flexible-base multi-degree-of-freedom structures. Through a numerical study in the frequency domain, the different effects of SSI on the performance of the TMD have been illustrated.

2. Transfer functions

The TMD-structure–foundation model investigated is shown in Fig. 1. In this, the TMD and the main structure are both modelled as s.d.o.f. systems. The foundation of the structure is considered to be a rigid slab of mass m_0, anchored to the surface of a homogenous, visco-elastic half-space through linear springs and viscous dashpots. The mass of the structure and the TMD are represented by $m_1$and m_2 respectively, both assumed to be concentrated at a height h from the foundation. Let $x(t)$ denote the horizontal displacement of the main structure relative to the foundation and $u(t)$ denote the horizontal displacement of the TMD relative to the structure. By neglecting the rocking component of the free-field ground excitation and the effects of kinematic interaction, the foundation input motion is assumed to be the same as the free-field ground translation, which is represented by $z(t)$. Let the foundation undergo translation, $z_0(t)$, and rotation, $\theta_0(t)$, relative to the soil medium. Assuming small displacements, the total horizontal displacement of the structure is given by $\{x(t) + z(t) + z_0(t) + h\theta_0(t)\}$. The interaction forces between the foundation and the underlying soil acting at the foundation-soil interface are represented by $V_s(t)$, the base shear, and $M_s(t)$, the base moment. The damping and stiffness of the structure are denoted by c_1 and k_1, respectively. The force transmitted from the TMD to the

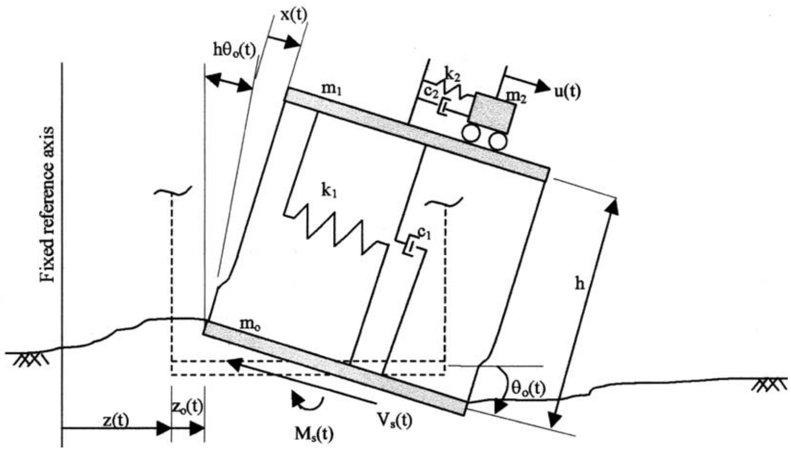

Fig. 1. Soil-structure–damper model.

structure is given by $(c_2\dot{u}(t) + k_2u(t))$, with c_2 and k_2 representing the damping and stiffness, respectively, of the mass–spring–dashpot model of the TMD.

The expression for the transfer function relating the displacement, relative to foundation, of a s.d.o.f. structural system with TMD and founded on compliant soil, to the free-field ground acceleration is given by [8]

$$H_x(\omega) = \frac{H_1(\omega)\left\{H^f_{zz}(\omega) + 1 + hH^f_{\theta z}(\omega)\right\}\left[H_2(\omega)\left\{H^f_{zu}(\omega) + hH^f_{\theta u}(\omega) - \chi(\omega)\right\} - 1\right]}{1 + H_2(\omega)\left\{H^f_{zu}(\omega) + hH^f_{\theta u}(\omega)\right\}\{1 + \omega^2 H_1(\omega)\} - \omega^2 H_1(\omega)H_2(\omega)\chi(\omega)} \tag{1}$$

where $H^f_{zz}(\omega)$ and $H^f_{\theta z}(\omega)$ denote the transfer functions relating the translational and rocking accelerations of the foundation to the free-field ground acceleration. $H^f_{zu}(\omega)$ and $H^f_{\theta u}(\omega)$ represent the transfer functions relating the foundation accelerations to the displacement of the TMD. The expressions for these transfer functions and for $\chi(\omega)$ are given in the Appendix. Further, in Eq. (1), $H_1(\omega)$ represents the transfer function relating the displacement of the fixed-base s.d.o.f. structural system alone to the input ground acceleration and is given by

$$H_1(\omega) = \frac{1}{(\omega_1^2 - \omega^2 + 2i\zeta_1\omega_1\omega)}. \tag{2}$$

Also, in Eq. (1), $H_2(\omega)$ is the transfer function relating the displacement of the TMD modelled as a fixed-base s.d.o.f. oscillator to the free-field ground acceleration and is given by

$$H_2(\omega) = \frac{1}{(\omega_2^2 - \omega^2 + 2i\zeta_2\omega_2\omega)}. \tag{3}$$

In Eqs. (2) and (3) ζ_1, ω_1 and ζ_2, ω_2 represent the damping ratio and natural frequency of the structure and TMD respectively.

For the purpose of comparison, the expression for the transfer function relating the displacement, relative to ground, of the fixed-base structure with attached TMD, to the ground acceleration is given by [8]

$$\bar{H}_x(\omega) = \frac{-H_1(\omega)\{1 + H_2(\omega)\chi(\omega)\}}{[1 - \omega^2 H_1(\omega)H_2(\omega)\chi(\omega)]}. \tag{4}$$

The above equation follows directly from Eq. (1) when the transfer functions relating the foundation accelerations to the ground acceleration, i.e. $H^f_{zz}(\omega)$, $H^f_{zu}(\omega)$, $H^f_{\theta z}(\omega)$, $H^f_{\theta u}(\omega)$, are negligibly small. This will occur when the soil stiffness is very high and the foundation can be assumed to be ‘fixed’. If the ground excitation is characterized by a band-limited white noise power spectral density function (PSDF) of intensity S_0, between frequencies Ω_1 and Ω_2, then the PSDF of the displacement response of the structure, denoted by $S_x(\omega)$, is expressed by [9]

$$S_x(\omega) = |H_x(\omega)|^2 S_0, \quad \Omega_1 \leqslant \omega \leqslant \Omega_2 \tag{5}$$

The root-mean-square (r.m.s.) value of the displacement response of the structure, relative to foundation, can be numerically evaluated by computing the square root of the area under the PSDF curve as given in Eq. (5).

3. Numerical study

To demonstrate the effects of SSI on the performance of the TMD when the structure is subjected to earthquake excitations, an example system has been considered in which the structure has a mass of 7.0×10^5 kg, lumped at a height of 14 m above the base. The fixed-base natural time period of the structure is $0.7\,\mathrm{s}(= 8.98\,\mathrm{rad/s})$. The ratio of the mass of the TMD to that of the structure is 0.05. The damping ratio of the structure and that of the TMD have been assumed to be 0.02 and 0.05 respectively. The example structure has been considered to rest on a rigid square foundation with a half side-width of 5 m. The mass of the foundation and the centroidal mass moments of inertia of both the foundation and structure have been assumed to be negligible. The foundation has been assumed to be bonded to a uniform, viscoelastic half-space. The impedance functions characterizing the dynamic response of the foundation have been taken from the results presented by Wong and Luco [10]. The soil is characterized by the shear modulus G, mass density ρ, the Poisson ratio v and hysteretic damping ratio ζ. The values of the soil parameters that have been considered for this study are given below:

$$\text{Mass density} \quad (\rho) = 1500\,\mathrm{kg/m^3},$$

$$\text{The Poisson ratio} \quad (v) = 0.3$$

$$\text{Hysteretic damping ratio} \quad (\zeta) = 0.02$$

To assess the extent of SSI effects on the performance of the TMD, the soil stiffness has been varied by choosing three different values of the shear wave velocity, $v_s\left(= \sqrt{G/\rho}\right)$, as 100 m/s, 200 m/s and 300 m/s

Using the expressions in Section 2, the transfer function for the displacement response of the example structure, relative to the base, has been evaluated and presented in Fig. 2. Here, the transfer function has been evaluated for the different shear wave velocities and compared with that of the fixed-base structure. In each case, the mass damper has been exactly tuned (i.e. tuning ratio equals unity) to the natural frequency of the fixed-base structure. As expected, the curve for

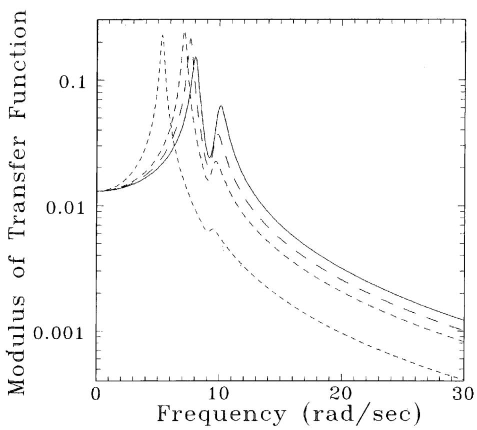

Fig. 2. Displacement transfer function of 0.7 s period structure with TMD, for different soil conditions: $v_s = 100$ m/s (- - - - -) , $v_s = 200$ m/s (– – –), $v_s = 300$ m/s (——–) and fixed-base (——).

the fixed-base structure shows a deep trough at the natural frequency of the structure. This indicates the reduction in response of the structure, achieved due to the presence of the TMD. As the soil becomes less stiff, as represented by decreasing values of the shear wave velocity, the peak of the curve shifts to the left and the trough at the natural frequency of the structure alone gradually smoothens out. This demonstrates that the natural frequency of the structure–foundation system has been lowered due to SSI effects. Also, as the mass damper gradually loses its tuning to the natural frequency of the structural system, the effectiveness in suppressing the vibrational response of the structure reduces. Further, it is observed that the shifted peaks have all registered an increase in their amplitude. This indicates that as compared to the fixed-base condition, the effects of SSI may cause an increase in the response of the structure with attached TMD. This aspect is illustrated in Fig. 3. Here, the r.m.s value of the displacement response of the structure with TMD, to white noise input having a spectral intensity of $1\,m^2/s^3$, banded between 0–50 rad/s, has been plotted for a range of s.d.o.f. systems with natural periods ranging from 0.2 s to 1.5 s. An increase in the response of the structure as compared to that for the fixed-base condition is noted for most of the cases. This can be attributed to two factors, namely, (1) increased flexibility of the structure–foundation system due to SSI effects, and (2) reduced effectiveness of the TMD. The decrease in the response for some of the systems, as compared to the fixed-base condition, when $v_s = 100\,m/s$, may be explained by the above factors being offset by the considerable dissipation of vibrational energy due to significant SSI effects.

To assess the effect of the reduced effectiveness of the TMD in case of Fig. 3, consider Figs. 4–6. The range of s.d.o.f. systems studied and the input at the base of the structure remain the same as in Fig. 3. In Fig. 4, the r.m.s values of the displacement response of the fixed-base structures have been presented with and without TMD attached to the structures. The curves show that the TMD achieves a response reduction of 36%. In Figs. 5 and 6, the r.m.s values of the displacement response of the structures with and without TMD have been plotted for $v_s = 100\,m/s$ and $v_s = 300\,m/s$, respectively. The curves indicate that while for medium soft soil with $v_s = 300\,m/s$, the TMD is still able to control the vibration of the structure by about 18%–25% (depending on the

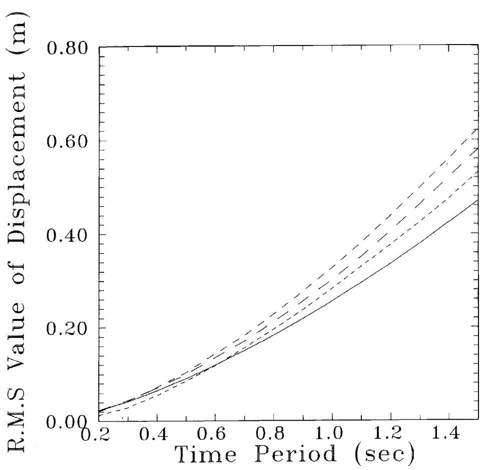

Fig. 3. R.M.S values of the displacement response of structures with TMD, to white noise input, for different soil conditions : fixed-base (——), $v_s = 100\,m/s$ (- - -), $v_s = 200\,m/s$ (– – – –), and $v_s = 300\,m/s$ (— — —).

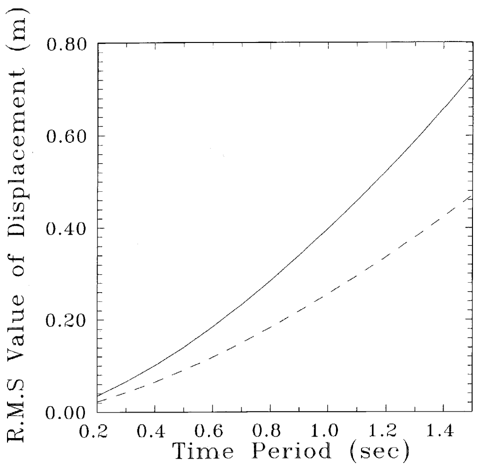

Fig. 4. R.M.S values of the displacement response of structures with TMD (- - -) and without TMD (——), to white noise input, for fixed-base condition

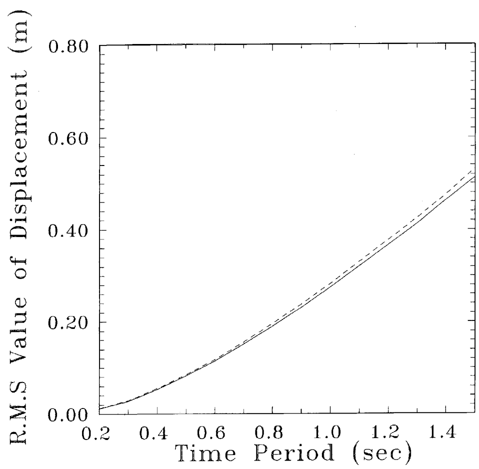

Fig. 5. R.M.S values of the displacement response of structures with TMD (- - -) and without TMD (——), to white noise input, for $v_s = 100\,\text{m/s}$.

period of the structure), for $v_s = 100\,\text{m/s}$, the TMD is totally ineffective in controlling the vibrational response of the structure.

The results are now presented for the mass damper tuned to the fundamental frequency of the structure–foundation system when the effects of SSI are significant ($v_s = 100\,\text{m/s}$). The transfer function curves for the displacement response (relative to the base) of the example structure considered in Fig. 2, (a) without damper, (b) with mass damper tuned to the natural frequency of the structure alone and (c) with mass damper tuned to the fundamental frequency of the structure-foundation system, have been compared in Fig. 7. These curves clearly demonstrate that to obtain vibrational control of the structure when the SSI effects are prominent, the mass damper must be tuned to the natural frequency of the structure–foundation system. The r.m.s values of the

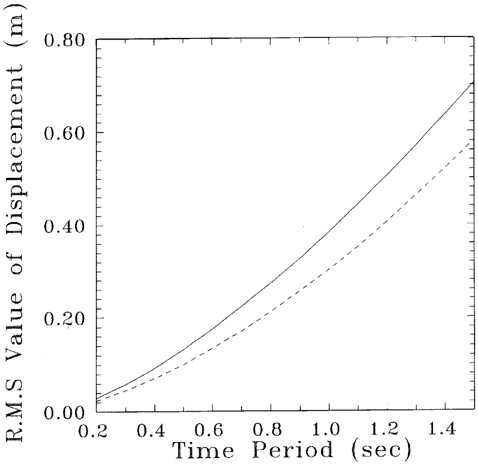

Fig. 6. R.M.S values of the displacement response of structures with TMD (- - -) and without TMD (——), to white noise input, for $v_s = 300\,\mathrm{m/s}$.

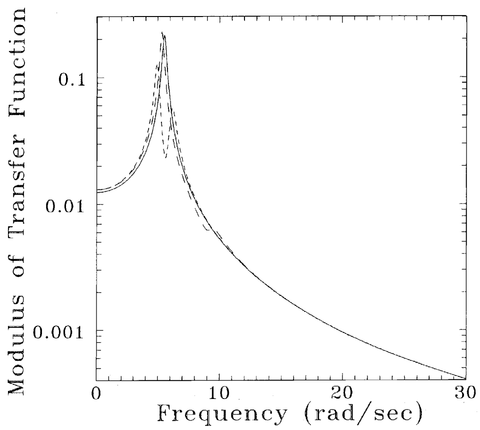

Fig. 7. Displacement transfer function of 0.7 s period structure, (a) without damper (——), (b) with mass damper tuned to natural frequency of structure alone (– – –), and (c) with mass damper tuned to fundamental frequency of structure–foundation system (- - - -), for $v_s = 100\,\mathrm{m/s}$.

displacement response for the range of oscillators and excitation previously considered in Fig. 3 have been evaluated with the damper tuned to the fundamental frequency of the structure–foundation system when the soil is very soft with $v_s = 100\,\mathrm{m/s}$, and when the soil is medium soft with $v_s = 300\,\mathrm{m/s}$ (see Figs. 8 and 9). Both figures indicate that this mass damper is an effective vibration control device for all the systems considered. However, the extent of the response reduction varies considerably for the cases studied. A quantitative evaluation of this variation has been done in Fig. 10, by plotting the percent reduction in r.m.s values of the displacement response that has been achieved by the use of this mass damper. As is evident from the curves, the percent reduction when the soil is medium soft shows lesser variation (28.4%–37.6%) than that

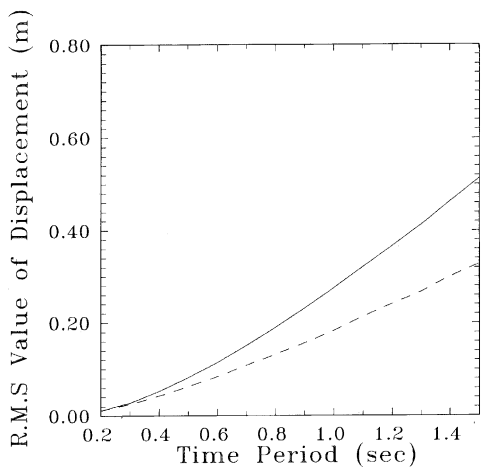

Fig. 8. R.M.S values of the displacement response of structures without TMD (——) and with mass damper tuned to fundamental frequency of structure–foundation system (– – –), to white noise input, for $v_s = 100\,\mathrm{m/s}$.

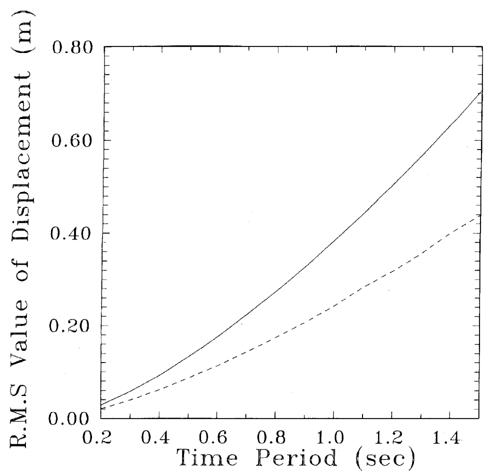

Fig. 9. R.M.S values of the displacement response of structures without TMD (——) and with mass damper tuned to fundamental frequency of structure–foundation system (– – –), to white noise input, for $v_s = 300\,\mathrm{m/s}$.

when the soil is very soft (4.4%–36.1%). Also, the stiffer structures undergo less response reduction by use of the mass damper. A possible explanation for these observations is that for the stiffer structures the effect of SSI is more pronounced. Hence, even without damper, the response of the structure reduces by a considerable extent due to increased damping in the system resulting from SSI effects. Consequently, the further response reduction achieved by the presence of the mass damper may not be as significant as compared to that for the relatively flexible structures. This is all the more applicable for the case where the soil is very soft ($v_s = 100\,\mathrm{m/s}$), enhancing the effects of SSI. However, the above reasoning may be more likely to be pertinent when the seismic excitation is broad-banded in its frequency content.

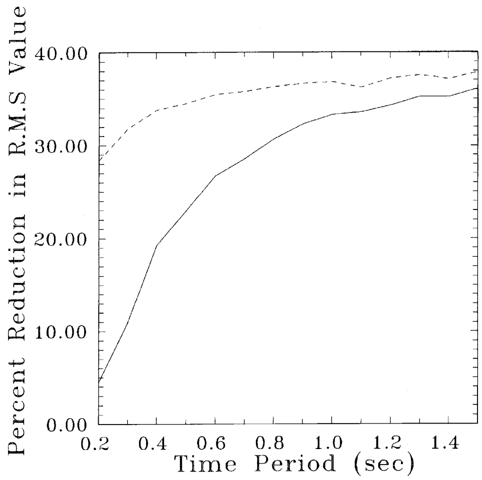

Fig. 10. Percent reduction in r.m.s values of the displacement response of structures by use of mass damper tuned to fundamental frequency of structure–foundation system, to white noise input, for different soil conditions, $v_s = 100\,\mathrm{m/s}$ system (——), $v_s = 300\,\mathrm{m/s}$ (- - -).

When the ground excitation is narrow banded in its energy content, then the effects of SSI on the performance of the TMD will depend on the actual frequency content of the excitation. The case where the excitation band is situated away from the natural frequency of the fixed-base structure, as well as from the frequency of the structure–foundation system, is not of interest as the response is low. When the energy is concentrated at the natural frequency of the fixed-base structure, softening of the soil may cause the predominant frequency of the structure–foundation system to shift outside the excitation frequency band. This will automatically cause substantial reduction in the response and again the presence of the TMD will have no significant impact on the response. However, if the frequency content of the excitation lies in a band such that due to SSI effects the frequency of the structure-foundation system shifts towards this band, then control of structural response is important. In this case, the conventional TMD (tuned to the natural frequency of the fixed-base structure) will be rendered ineffective and it will be necessary to tune the damper to the natural frequency of the structure–foundation system. There is a possibility of the peak of the narrow banded excitation matching with either of the two frequencies at which peaks are formed in the transfer function due to the tuning of the damper to the fundamental frequency of the system (whether fixed or flexible-base). As an illustration, consider the example structure in Fig. 7, subjected to a narrow banded excitation characterized by a Kanai–Tajimi type PSDF defined by

$$S_0 = \hat{S}_0 \frac{\omega_g^2 + 4\zeta_g^2\omega_g^2\omega^2}{(\omega_g^2 - \omega^2)^2 + 4\zeta_g^2\omega_g^2\omega^2} \tag{6}$$

where ω_g and ζ_g are the natural frequency and damping ratio respectively of the soil idealized as a s.d.o.f. system. Here, it is assumed that the peak of the excitation is located at $\omega_g = 4.9\,\mathrm{rad/s}$, matching with the left peak in the transfer function curve marked (c) in Fig. 7. The value of ζ_g will determine the sharpness of the excitation peak. For soil sites, Lai [11] has given a value of ζ_g equal to 0.32. The term, $\hat{S}_o$, in Eq. (6) is a measure of the ground intensity and has been considered equal to $1\,\mathrm{m}^2/\mathrm{s}^3$. For the structure alone, the r.m.s value of the displacement response is evaluated as 0.242 m while for the structure with damper tuned to the structural frequency of the fixed-base structure, it is 0.261 m. For the case in which the damper is tuned to the fundamental frequency of the structure-foundation system, the response value is 0.182 m. This indicates that despite matching of the excitation peak with a peak in the transfer function peak of case (c) in Fig. 7, the reduction in the peak amplitude as compared to that in cases (a) and (b) has ensured a reduction in the vibrational response of the structure. It may be noted that the conventional tuning of the damper as in case (b) has led to an unconservative value of the system response. If the value of ζ_g is very small, then an eventuality may arise in which the presence of the damper causes an increase in the response of the structure. For example, when $\zeta_g = 0.05$, with other parameters unchanged the r.m.s. value of the structural response is obtained as 0.647 m, 0.846 m and 0.736 m for cases (a), (b) and (c), respectively. However, since the damper is being incorporated to cater not for a single excitation but for different types of excitation, it would be expedient to tune the damper to the frequency of the structure–foundation system.

Despite tuning the attached mass to the fundamental frequency of the structure–foundation system, subjected to a wide banded excitation, it is possible to have a condition in which the presence of the secondary mass causes an increased response of the structure. Consider the example structure in Fig. 2 with ζ_2 varying between 0% and 5% and founded on soft soil having $v_s = 100\,\mathrm{m/s}$. It is subjected to white noise input having a spectral intensity of $1\,\mathrm{m}^2/\mathrm{s}^3$, banded between 0 and 5 rad/s. The r.m.s. values of the displacement response, relative to the base, have been compared for the cases without damper and with damper tuned to the natural frequency of the structure-foundation system and plotted in Fig. 11. It is evident from the figure that there exists a critical value of the damping ratio in the TMD, below which the response of the system is more as compared to the case without damper.

It must be noted that the study in this paper is based on the linear behaviour of the structure and soil underlying the foundation. Trifunac et al. [12,13] have shown that non-linearity in the

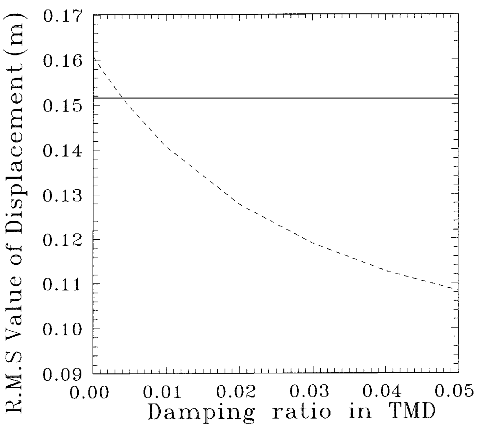

Fig. 11. R.M.S values of 0.7 s period structure with TMD (––––), for different damping ratio in TMD, and without TMD (——) in case of $v_s = 100\,\mathrm{m/s}$.

response of the foundation soil may cause the structure–foundation system frequency to change substantially from one earthquake to another as well as during a particular earthquake. Under such conditions, the concept of tuning the TMD to the structure–foundation system will not work and the TMD effectiveness will have to be studied by proper modelling of the soil, which caters for its non-linear behaviour.

4. Conclusions

In this paper, the effects of SSI on the functioning of TMDs for seismic vibration control have been investigated in the frequency domain. The study has led to the following observations. As the soil becomes less stiff, allowing the foundation to undergo movement relative to the surrounding soil, the properties of the structure–foundation system change considerably from that of the fixed-base case. Then, the conventional tuning of the mass damper to the natural frequency of the fixed-base structure loses its effectiveness in controlling the vibrational response of the structure to base excitation. Ignoring the effects of SSI while designing the mass damper may even cause an increase in the response of the structure as compared to the case without damper. To avoid this and to ensure proper performance of the damper as a vibration control device, it is necessary to tune the damper to the fundamental frequency of the structure–foundation system. It is also essential to provide damping in the TMD greater than the critical damping to ensure response reduction of the structure. Tuning of the mass damper, having adequate damping ratio, to the structure–foundation frequency has proven effective for the cases studied (considering linear structure–soil model). However, the degree of control achieved depends on the time period of the structure and on the extent of the soil flexibility.

Appendix A

A.1. Transfer functions for foundation accelerations

$$H^f_{zz}(\omega) = \frac{-\omega^2}{\delta(\omega)}\left[\alpha(\omega)\left\{K_{MM}(\omega) - \omega^2\left(h^2\beta(\omega) + I_0 + I\right)\right\} + h\beta(\omega)\left\{K_{VM}(\omega) - \omega^2 h\beta(\omega)\right\}\right], \quad (A.1)$$

$$H^f_{\theta z}(\omega) = \frac{-\omega^2\beta(\omega)\chi(\omega)}{\delta(\omega)}\left[\left\{K_{MM}(\omega) - \omega^2\left(h^2\beta(\omega) + I_0 + I\right)\right\} - h\left\{K_{VM}(\omega) - \omega^2 h\beta(\omega)\right\}\right], \quad (A.2)$$

$$H^f_{zu}(\omega) = \frac{\omega^2}{\delta(\omega)}\left[\left\{K_{VM}(\omega) - \omega^2 h\beta(\omega)\right\}\alpha(\omega) + \left\{K_{VV}(\omega) + \omega^2\alpha(\omega)\right\}h\beta(\omega)\right], \quad (A.3)$$

$$H^f_{\theta u}(\omega) = \frac{\omega^2\beta(\omega)\chi(\omega)}{\delta(\omega)}\left[\left\{K_{VM}(\omega) - \omega^2 h\beta(\omega)\right\} - \left\{K_{VV}(\omega) + \omega^2\alpha(\omega)\right\}h\right], \quad (A.4)$$

with

$$\alpha(\omega) = -\{m_T + \omega^2 m_1 H_1(\omega)\}, \tag{A.5}$$

$$\beta(\omega) = m_1\{1 + \omega^2 H_1(\omega)\}, \tag{A.6}$$

$$\chi(\omega) = \mu(\mathrm{i}\omega 2\zeta_2\omega_2 + \omega_2^2), \tag{A.7}$$

$$\delta(\omega) = \{K_{VV}(\omega) + \omega^2\alpha(\omega)\}\{K_{MM}(\omega) - \omega^2\{h^2\beta(\omega) + I_0 + I\}\} - \{K_{VM}(\omega) - \omega^2 h\beta(\omega)\}^2. \tag{A.8}$$

In Eqs. (A.1)–(A.8), $K_{VV}(\omega)$, $K_{VM}(\omega)$ and $K_{MM}(\omega)$ denote the complex-valued impedance functions relating the interaction forces between with the foundation and underlying soil to the foundation displacements in the frequency domain. Also, μ denotes the ratio of the mass of the TMD to that of the main structure and $m_T(= m_1 + m_0)$ is the total mass of the structure–foundation system. Further, $I_T(= I_0 + I + m_1h^2)$ represents the total moment of inertia of the structure–foundation system about the central axis of the foundation, with I_0 and I denoting the centroidal moment of inertia of the foundation and structure, respectively.

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