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Letter to the Editor

Comments on "Interpreting proper orthogonal modes of randomly excited linear vibration systems"

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In a recent article, Feeny and Liang [1] have interpreted the proper orthogonal modes (POMs) for linear vibration systems under random excitation. More precisely, they have shown that the POMs converge to the linear normal modes (LNMs) of distributed systems under the two following assumptions:

- The mass distribution must be known.
- The random excitation must not contain any sustained sinusoidal components and each mode is to be excited uniformly.

If the problem is discretized in time and space, then the number of snapshots must also be "large enough" and the spatial discretization must be evenly spaced. This study is the continuation of a series of papers dedicated to the physical interpretation of the POMs in structural dynamics [2–5].

First of all, the authors are to be commended for their paper which paves the way for the practical application of POD as a modal analysis tool.

Secondly, the present letter intends to pursue the discussion by highlighting an interesting contribution due to North [6]. In this study, it is also attempted to provide some insight into the physical interpretation of the POMs, referred to as empirical orthogonal functions (EOFs), of linear stochastically forced systems. An important feature of the excitation is that it should provide equal forcing variance to all the normal modes.

The author starts by considering the following class of systems:

$$H\left(\frac{\partial}{\partial t}, \nabla, \mathbf{x}\right)\boldsymbol{\psi}(\mathbf{x}, t) = f(\mathbf{x}, t) \qquad (1)$$

and continues by Fourier analyzing this equation:

$$H_\omega \boldsymbol{\psi}_\omega(\mathbf{x}) = f_\omega(\mathbf{x}) \quad \text{with } H_\omega = H(\mathrm{i}\omega, \nabla, \mathbf{x}), \qquad (2)$$

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where for instance, H_ω is equal to $-\omega^2 - c^2\nabla^2$ or $-i\omega - \kappa\nabla^2$ for the wave and diffusion equations, respectively. The discussion is then restricted to the case of Hermitian operators H_ω for which the eigenvectors are orthogonal and the eigenvalues are real. After a few developments, it is shown that Hermitian mechanical systems excited by a white noise have their EOFs coinciding with their normal modes defined as the eigenfunctions of H_ω. This is consistent with the results obtained in Ref. [1].

For the detailed demonstration, the reader is referred to Ref. [6]. It should however be emphasized that the demonstration is realised by introducing the original concept of four-dimensional EOFs which amounts to finding the eigenvectors of the cross-spectrum matrix. This results in eigenfunctions and eigenvalues depending upon frequency. North notes that for Hamiltonian-type systems $H_\omega = -\omega^2 + h(\mathbf{x}, \nabla)$ there will be no frequency dependence of the EOFs because ω appears as an additive scalar in H_ω. Nevertheless, this formulation allows to consider a broader class of systems, e.g., the vorticity equation, for which the EOFs depend upon frequency.

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