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Letter to the Editor

Form factors for vibration control of beams using resistively shunted piezoceramics

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1. Introduction

The present work is motivated by a desire to investigate the effect of thickness and length ratios of the resistively shunted piezoceramic-host beam system on its ability to induce damping. Are there any optimal length and thickness ratios for which the additive damping due to piezoceramic resistive shunting is a maximum? Are these thickness and length ratios influenced by the boundary condition?

Electrical passive shunting of piezoceramics bonded to beams has been investigated in the past [1–3]. The analytic vibration models represent the damping and stiffness due to electrical shunting of the piezoceramic as a complex frequency-dependent modulus similar to that used in viscoelastic solids [2]. Steffen and Inman [4] have applied techniques developed for optimization of dynamic vibration absorbers to optimize the parameters of a resonantly shunted piezoceramic bonded to a beam. The analysis of damping in composite plates with multiple resistively shunted piezoelectric layers has been developed by Saravanos [5]. These studies have focused on analytical and experimental investigation of the additive damping, and change in resonance frequencies, due to resistive and resonant shunting. Chaudhry and Rogers [6] have studied the problem of optimal thickness ratio of the PZT-host beam in terms of actuator induced surface strain. For the static case, they arrive at a host beam/PZT thickness ratio of 2.75. However, they could not arrive at any conclusive results for the dynamic case, as well as investigate the effect of boundary conditions on the optimal thickness ratio.

The present work reviews the effect of thickness and length ratios in the specific context of maximizing additive damping due to resistive shunting of the PZT. The effect of boundary conditions on these optimal values of thickness and length ratios are also examined. Analytical results are presented supported by experimental evidence.

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2. Modelling of shunted piezoceramic materials

The modelling of shunted piezoceramics bonded to structures has been dealt with elsewhere [2], and here we present the essential steps. The mechanical impedance of the shunted piezoceramic can be obtained, in non-dimensional form, for uniaxial loading in the jth direction, as

$$Z_{jj}^{ME}(s) = \frac{A_j}{C_{jj}^{SU} L_j s}. \tag{1}$$

In the above, A_j is the area of the piezoceramic element whose normal is in the jth direction, L_j is the length of the piezoceramic, $C_{jj}^{SU} = (1 - k_{ij}^2 \bar{Z}_i^{EL}) C_{jj}^E$ is the mechanical compliance of the shunted piezoceramic wherein C_{jj}^E is the mechanical compliance with short-circuit electrical boundary conditions, and s is the Laplace variable. One can non-dimensionalize this as the ratio of the mechanical impedance of the shunted piezoceramic to the mechanical impedance of the piezoceramic with open-circuit electrical boundary condition:

$$\bar{Z}_{jj}^{ME} = \frac{Z_{jj}^{SU}(s)}{Z_{jj}^{OC}(s)} = \frac{(1 - k_{ij}^2)}{[1 - k_{ij}^2 \bar{Z}_i^{EL}(s)]}, \tag{2}$$

where k_{ij} is the electromechanical coupling coefficient, $\bar{Z}_{jj}^{EL}$ is the total electrical impedance of the PZT with the shunt impedance non-dimensionalized to its open-circuit values. The mechanical impedance, $\bar{Z}_{jj}^{ME}$, is in general complex and frequency dependent and can be represented as

$$\bar{Z}_{jj}^{ME} = \bar{E}_{jj}(\omega)[1 + i\eta_{jj}(\omega)]. \tag{3}$$

Shunting devices such as resistors act as an energy dissipater on the electrical side. Electrical resistive shunting of a piezoceramic bonded to a host structure is equivalent to a viscoelastic damping treatment of the same host structure [2]. The non-dimensional mechanical impedance of a resistively shunted piezoelectric is given by

$$Z_i^{SU}(s) = R_i, \quad \bar{Z}_i^{EL}(s) = \frac{Z_i^{EL}(s)}{Z_i^{OC}(s)} = \frac{sC_{pi}R_i}{1 + sC_{pi}R_i}, \quad \bar{Z}_{jj}^{ME}(\omega) = 1 - \frac{k_{ij}^2}{1 + i\rho_i}, \tag{4}$$

where C_{pi} is the capacitance of the piezoceramic, R_i is the resistance of the shunt resistor, and $\rho_i = R_i C_{pi} \omega$ is the non-dimensional frequency. The loss factor and the frequency-dependent storage modulus are

$$\eta_{jj}^{ME}(\omega) = \frac{\rho_i k_{ij}^2}{(1 - k_{ij}^2) + \rho_i^2}, \qquad \bar{E}_{jj}^{ME}(\omega) = 1 - \frac{k_{ij}^2}{(1 + \rho_i^2)}. \tag{5}$$

In order to study the effectiveness of piezo-resistive shunting in controlling the dynamics of a vibrating system, the dynamics of the host structure is modelled by a single vibration mode. The piezoceramic is then coupled in parallel to this one degree-of-freedom (1-DOF) system. The modal velocity of the vibrating system with piezoceramic can be expressed in the Laplace domain as

$$v(s) = \frac{F(s)}{Ms + (K/s) + Z_{jj}^{ME}(s)}, \tag{6}$$

where Ms is the impedance associated with modal mass of the host structure, K/s, is the impedance associated with the modal stiffness of the host structure, and $Z_{jj}^{RES}(s)$ is the impedance associated with the resistively shunted piezoceramic. The above modelling of the resistively shunted piezoceramic bonded to the host structure assumes a linear electro-mechanical coupling leading to a linear visco-elastic model of the overall structural dynamics.

3. Results and discussions

Analytical as well as experimental studies were carried out to study the variation of additive damping due to resistive shunting as a function of thickness and length ratios. Analytically, the additive damping was evaluated from the poles of the response transfer function equation (6). In Fig. 1, the variation of additive damping, at optimal shunt resistance, as a function of the ratio of beam thickness (t_b) to piezo thickness (t_c) is shown for different ratios of beam length (l) to piezoceramic length (L). These values of additive damping are for the first mode of vibration of the cantilever beam. In all these studies, the location of the piezoceramic on the beam was not varied. In Fig. 1, it is clearly seen that greater the length of the beam, higher is the strain induced in the PZT, and consequently, the added damping is more. However, for a given ratio of piezoceramic length to beam length, the additive damping is a maximum at a thickness ratio (t_b/t_c) of 2.73. In order to validate the above analytical results experimentally, five different duralumin cantilevered beam models were tested with surface bonded piezoceramic

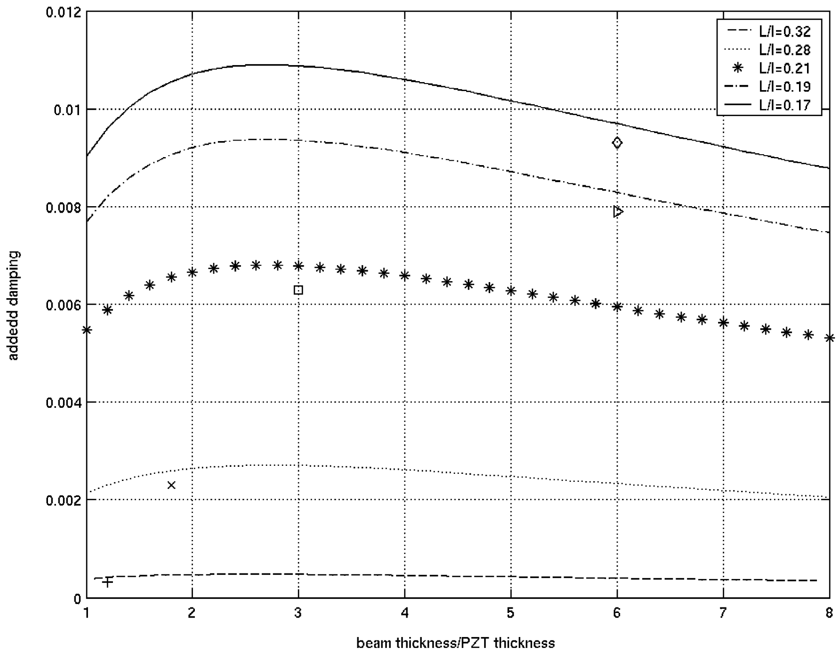


Fig. 1. Variation of added damping for different (beam/PZT) thickness ratios with variation in the PZT length (L)–beam length (l) ratio.

patches. Table 1 lists the dimensions and physical properties of the aluminum alloy beams and PZT patches. Sine-sweep and random excitation tests were conducted to determine resonance frequencies and change in the natural frequencies of the beam with open- and short-circuited piezoceramic layer. The results of these experiments are summarized in Table 2. The damping was estimated by computing the energy dissipated in one vibration cycle when the beam was excited by an electro-mechanical shaker at the first resonance frequency [7]. The damping estimated by this technique was also compared with that obtained from the circle-fit method [8, pp.158–162], and the values obtained by integrating the force–velocity over one vibration cycle compared very favorably with the circle-fit method (the error was less than 10%). The experimental values of added damping due to optimal resistive shunting are shown as data points in Fig. 1. These experimental values agree well with analytical results shown there. In order to investigate whether the optimal thickness ratio is dependent on the mode of vibration, we analytically simulated the variation of additive damping due to resistive shunting as a function of the thickness ratio for the second and third modes of a cantilever beam. The placement of the piezoceramic patches for the second and third mode are as shown in Fig. 2. The results of this simulation are shown in Fig. 3. In Fig. 3(a), we have shown the effect of thickness ratio on additive damping for the first mode of the cantilever beam with the PZT patches near the root, A1, and at another location A2. Note that though the maximum values of additive damping are different, the thickness ratio at which this maximum value occurs is still the same, namely $T = 2.73$. Figs. 3(b) and (c) show the variation of additive damping as a function of thickness ratio for mode number two and three.

Table 1
Cantilever beam and PZT dimensions and properties

Parameter	Beam-1	Beam-2	Beam-3	Beam-4	Beam-5	PZT
Material	Duralumin	Duralumin	Duralumin	Duralumin	Duralumin	SP-5H (Sparkler)
Length (m)	0.155	0.178	0.242	0.270	0.291	0.050
Width (m)	0.030	0.030	0.027	0.026	0.026	0.025
Thickness (m)	0.0009	0.0009	0.0015	0.003	0.003	0.0005
Young's modulus (GPa)	70	70	70	70	70	69
Density (kg/m^3)	2700	2700	2700	2700	2700	7500
Capacitance (μF)	—	—	—	—	—	0.058
Coupling coefficient (k_{31})	—	—	—	—	—	0.36

Table 2
Experimentally measured parameters for resistive shunting (I-mode)

Parameter	Beam-1	Beam-2	Beam-3	Beam-4	Beam-5
Natural frequency (shorted) (Hz)	28.13	21.13	21.37	38.10	30.27
Natural frequency (open) (Hz)	28.25	21.25	21.60	38.33	30.53
Loss factor (with PZT shorted)	0.0186	0.0194	0.0213	0.0156	0.0224
Coupling coefficient k_{ij} (theory)	0.079	0.102	0.165	0.182	0.197
Optimal resistance (KΩ)	90	120	120	70	80

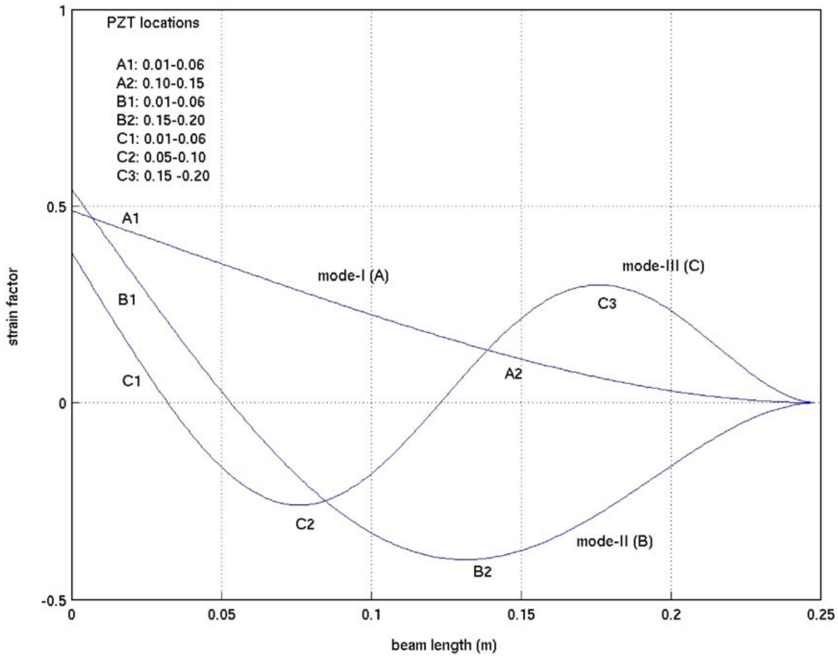


Fig. 2. Strain mode shapes of cantilever beam with locations of PZT patches on them.

The PZT patches were bonded at different locations as shown in Fig. 2. Once again the maximum additive damping values obtained are different, but the thickness ratio at which this maximum value occurs is still $T = 2.73$. From Fig. 3(c), note that the additive damping due to optimal shunt resistance is relatively higher than for mode numbers one and two. In general the values of additive damping due to optimal shunt resistance of the PZT are dependent on the mode in which the beam is vibrating, the location of the PZT, and the dimensions of the PZT. However, the optimal thickness ratio at which this maximum value of additive damping is realized is independent of all these.

The simulations were now repeated for beams on simple supports, and beams with both ends fixed. Fig. 4 shows the location of the PZT patches for the beam on simple supports. Figs. 5(a)–5(c), show the variation of additive damping as a function of thickness ratio for mode numbers one, two, and three, respectively. Once again, the results follow the same trend as that of the cantilever beam. The optimal thickness ratio is still $T = 2.73$. Fig. 6 likewise show the location of the piezoceramic patches for a fixed-fixed beam, and Fig. 7 details the variation of additive damping as a function of thickness ratio. In this case too, the optimal thickness ratio is $T = 2.73$.

The results arrived here agree in part with those of Chaudhry and Rogers [6]. They consider the problem of static strain actuation using a piezoceramic, and arrive at the optimum thickness ratio of 2.75 in terms of the bending surface strain induced on the substructure. This result is almost the same as what we have obtained. However, they seem to be inconclusive about the effect of inertia loads on the optimum thickness ratios in terms of frequency response of the beam with PZT bonded to it. The effect of boundary conditions is equated with the application of an applied load, and optimum thickness ratio for maximum surface strain is stated to decrease with the increase in

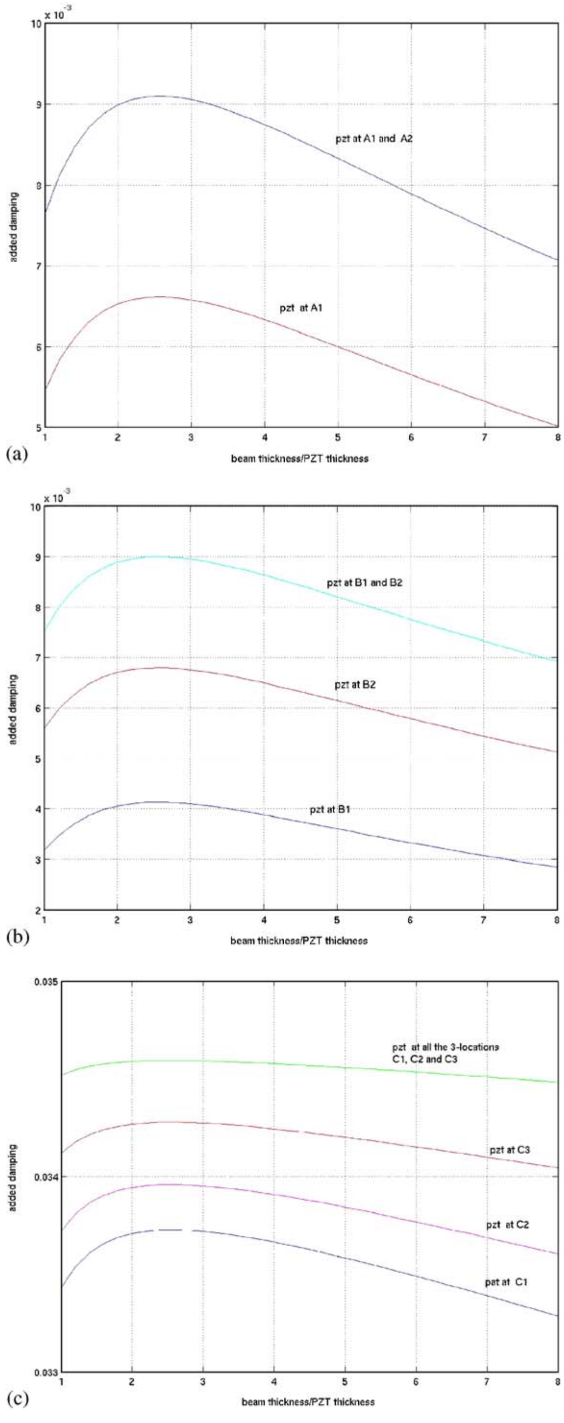


Fig. 3. (a) Added damping as a function of thickness ratio for cantilever beam 1st mode with PZT at, different locations, (b) added damping as a function of thickness ratio for cantilever beam 2nd mode with PZT at different locations, (c) added damping as a function of thickness ratio for cantilever beam 3rd mode with PZT at different locations.

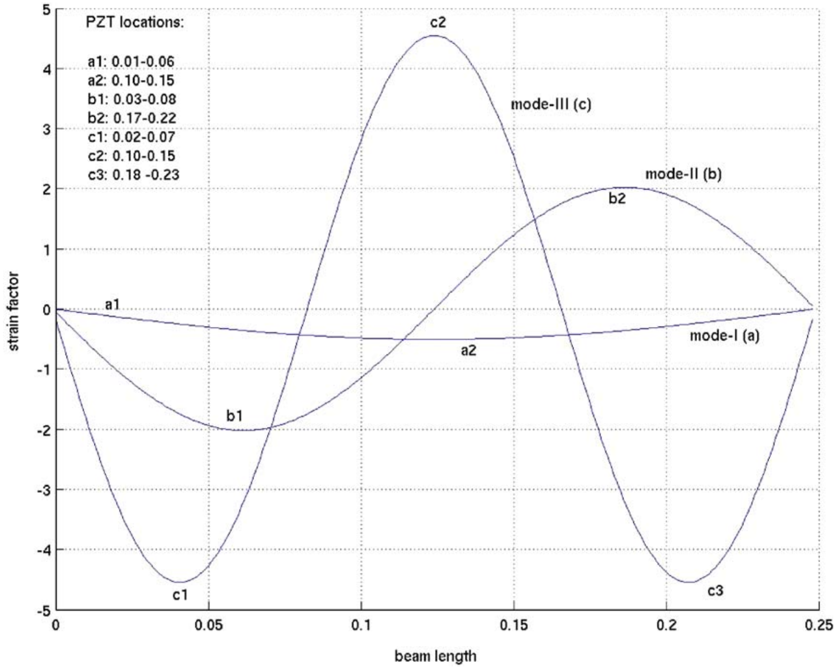


Fig. 4. Strain mode shapes of simply supported beam with locations of PZT patches on them.

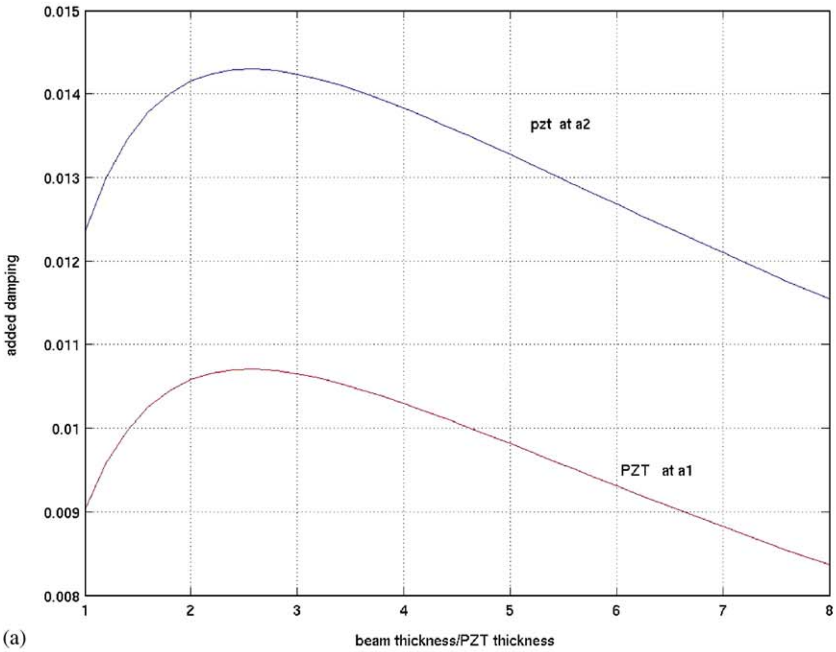
external load for a given actuation voltage. However, one should be careful in applying these conclusions to the case of electrical shunting of piezoceramics since here one is interested in the mechanical strain induced at the beam–piezo interface due to mechanical motion alone. That is, we are not considering the actuator induced surface strain in the substructure.

In order to explain the optimal thickness ratio for maximum additive damping due to resistive shunting, at a given PZT to host beam length ratio (L/l), we investigated the effect of surface strain on the beam due to bending as a function of thickness ratio. Note that the electric field induced in the piezoceramic is directly proportional to the strain induced in it, which in turn is proportional to the curvature. The bending strain in the beam per unit bending moment, evaluated on the surface of the beam, at a given station x on the beam, for arbitrary boundary conditions, is given by

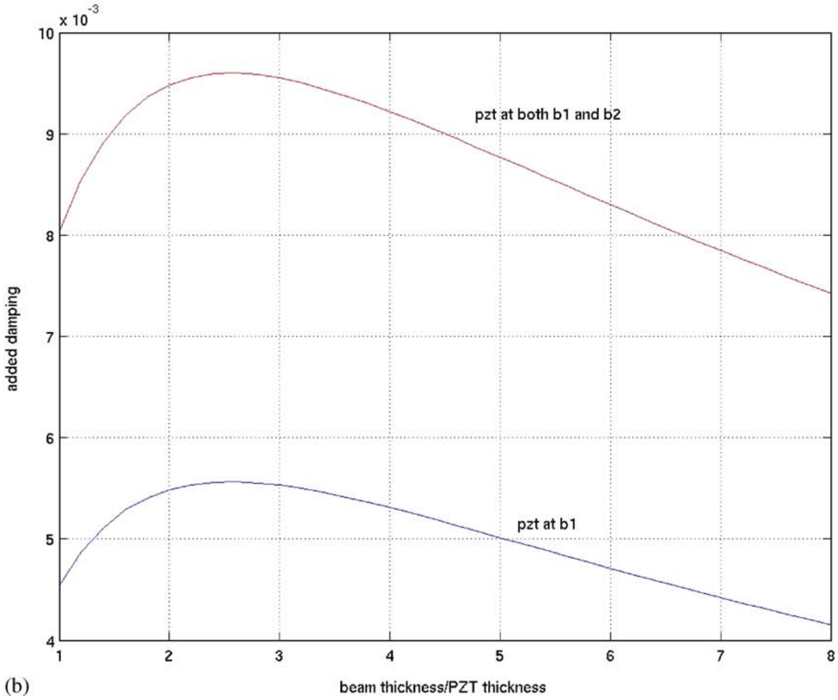
$$\frac{\kappa}{M(x,t)} = \frac{1 + 1/T}{6 + \bar{E}T + \dfrac{12}{T} + \dfrac{8}{T^2}}, \tag{7}$$

where κ is the curvature, $M(x,t)$ is the bending moment, $\bar{E} = E_b/E_c$, and $T = t_b/t_c$. This expression holds true whether the beam is statically excited or undergoing vibration. The maximum bending strain per unit moment as a function of thickness ratio is then

$$\frac{\partial(\kappa/M(x,t))}{\partial T} = 0 \quad \Rightarrow \quad \bar{E}T^4 + 2\bar{E}T^3 - 6T^2 - 16T - 8 = 0. \tag{8}$$



(a)



(b)

Fig. 5. (a) Added damping as a function of thickness ratio for simply-supported beam 1st mode with PZT at different locations, (b) added damping as a function of thickness ratio for simply-supported beam 2nd mode with PZT at different locations, (c) added damping as a function of thickness ratio for simply supported beam 3rd mode with PZT at different locations.

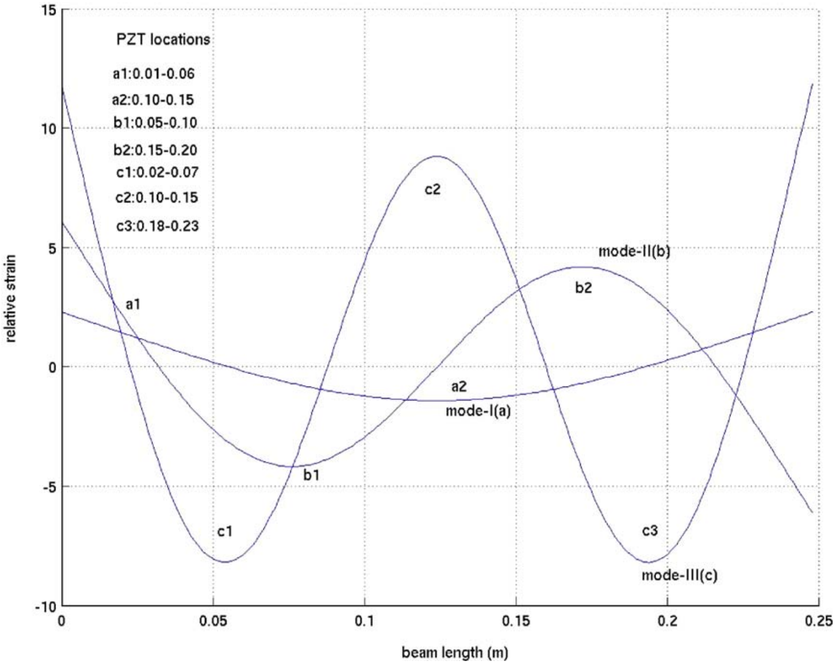


Fig. 6. Strain mode shapes of fixed-fixed beam with locations of PZT patches on them.

In the case of aluminum and PZT, $\bar{E}\approx 1$, and the optimal thickness ratio is then $T = 2.73$. Note that the voltage generated by the PZT is proportional to the strain induced on the surface of the beam that is then dissipated across the shunt resistor resulting in additive damping. So this analysis explains the results we obtained earlier by numerical and experimental simulations for various modes of vibration and with different boundary conditions for an Euler–Bernoulli beam. Therefore, for $\bar{E}\approx 1$, the maximum value of induced strain in the PZT will occur at $T = 2.73$. This optimal thickness ratio is independent of the mode of vibration as well as boundary condition. However, the induced strain value at this optimal thickness ratio will depend on the mode as well as the boundary condition. In the case of PZT bonded to the surface of a beam made of mild steel, $\bar{E}\approx 3$, and the optimal thickness ratio $T = 1.67$. However, the use of PZT with mild steel will result in lower induced strain per unit moment at the PZT–beam interface, relative to aluminum, since $\kappa/M(x,t)$ will decrease with increase in $\bar{E}$.

4. Conclusions

The effect of thickness and length ratios of piezoceramic-host beam on additive due to resistive shunting of the piezoceramic is investigated for different boundary conditions and mode shapes. It is shown that the additive damping, for a given beam length to PZT length ratio, is maximum for beam thickness to PZT thickness ratio equal to 2.73. This optimal thickness ratio is valid for all modes of vibration and all boundary conditions of the beam.

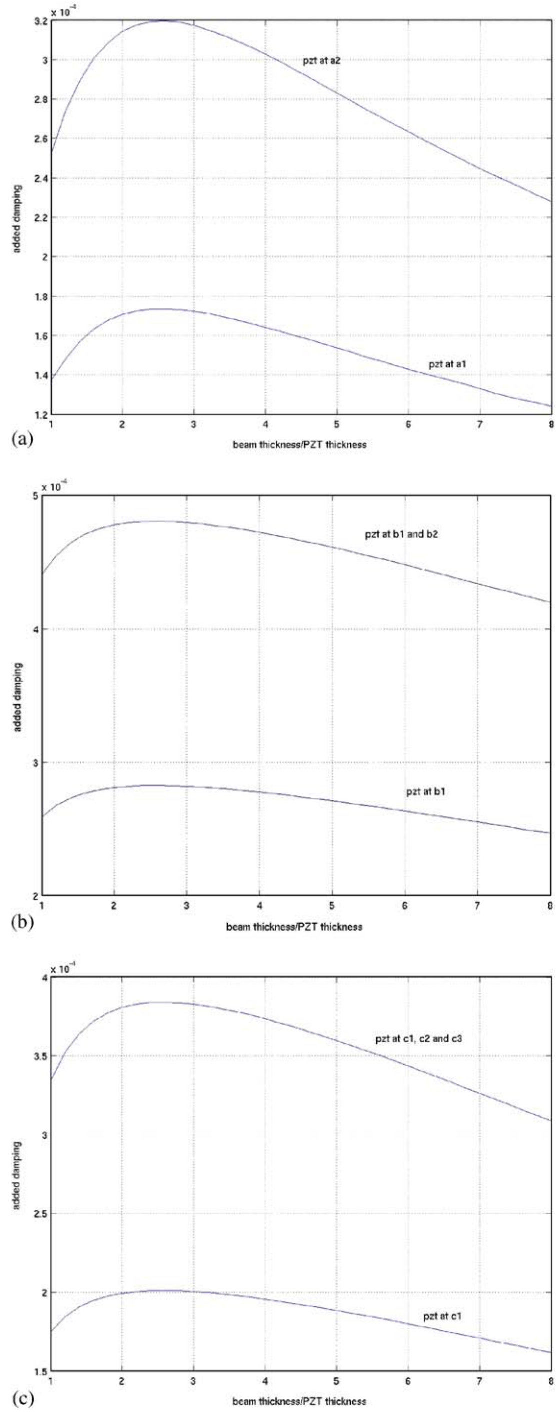


Fig. 7. (a) Added damping as a function of thickness ratio for fixed–fixed beam 1st mode with PZT at different locations, (b) added damping as a function of thickness ratio for fixed–fixed beam 2nd mode with PZT at different locations, (c) added damping as a function of thickness ratio for fixed–fixed beam 3rd mode with PZT at different locations.

References

[1] C.L. Davis, G.A. Lesieutre, A modal strain energy approach to the prediction of resistively shunted piezoceramic damping, *Journal of Sound and Vibration* 184 (1) (1995) 129–139.
[2] N.W. Hagood, A. von Flotow, Damping of structural vibrations with piezoelectric materials and passive electrical networks, *Journal of Sound and Vibration* 146 (2) (1991) 243–268.
[3] G.A. Lesieutre, Vibration damping and control using shunted piezoelectric materials, *Shock and Vibration Digest* 30 (3) (1998) 187–195.
[4] S. Valder Jr., D.J. Inman, Optimal design of piezoelectric materials for vibration damping in mechanical systems, *Journal of Intelligent Materials Systems and Structures* 10 (1999) 945–955.
[5] D.A. Saravanos, Damped vibration of composite plates with passive piezoelectric–resistor elements, *Journal of Sound and Vibration* 221 (3) (1999) 867–885.
[6] Z. Chaudhry, C.A. Rogers, Performance and optimization of induced strain actuated structures under external loading, *American Institute of Aeronautics and Astronautics Journal* 32 (6) (1994) 1289–1294.
[7] J.A. Powell, Phenomenological interpretation of energy loss and storage mechanisms in electrorheological fluids, *Journal of Intelligent Material Systems and Structures* 9 (1998) 679–689.
[8] D.J. Ewins, *Modal Testing: Theory and Practice*, Research Studies Press, Hertfordshire, UK, 1984.