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Parametric study of modal properties of damped two-degree-of-freedom crowd–structure dynamic systems

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Abstract

This paper investigates numerically the modal properties of damped 2-degree-of-freedom (d.o.f.) representations of crowd-occupied civil engineering structures, such as grandstands. In particular, it attempts to explain observations of some curious changes in natural frequencies and damping in measurements made by other researchers in the past when such structures were occupied compared with when they were empty. Natural frequencies, mode shapes, modal masses and damping ratios are examined parametrically for a range of ratios of frequency, mass and damping coefficients of two single-d.o.f. systems connected in series representing the 'human' and 'structural' vibration behaviour. It is found that a damped 2-d.o.f. model of a crowd–structure system can explain (1) damping increases, (2) additional modes of vibration and (3) increases as well as decreases of natural frequencies observed on real-life grandstand structures due to crowd occupation. Therefore, a mathematical framework for simplified dynamic response analysis of assembly structures based on equivalent 2-d.o.f. dynamic modelling of crowd–structure interaction may be a prudent way forward.

1. Introduction

Crowd–structure dynamic interaction [1–3] is currently a major issue in the safety and serviceability of civil engineering assembly structures [4,5]. It is now well established that crowds not only induce significant dynamic forces, but also alter the dynamic properties of the occupied structure. Consequently, there are two questions related to the crowd–structure dynamic

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interaction:

1. what are the dynamic forces applied by a crowd of people, and
2. what are the effects of a crowd on the dynamic properties (natural frequencies, modal mass, modal stiffness and damping) of the structure it occupies.

This paper deals with the latter issue only. This issue is particularly important in the case of structures, such as grandstands, where crowds can cause significant changes of the properties of vertical and horizontal modes of vibration [1]. Occupants often, but not always, reduce natural frequencies of civil engineering structures [1–3]. Most remarkably, in one case it was reported that they reduced the natural frequency of the fundamental vertical mode of a relatively light temporary grandstand from about 16 Hz to about 5 Hz [2]. However, crowds also have the potential to increase existing natural frequencies and even create new modes of vibration [1,2]. Furthermore, it is widely acknowledged that human occupants increase damping of civil engineering structures. A mathematical framework which would explain analytically all these experimental observations is, unfortunately, quite weak and inconsistent in the published literature. Therefore, the aim of this paper is to formulate a consistent but simple methodology for interpreting and modelling the effects of human–structure dynamic interaction on assembly structures.

Currently, it is common in civil engineering design practice to model human occupants just as additional mass. This can be represented as a rigid mass m_H added to the mass m_S of a s.d.o.f. system (Fig. 1a), which represents the relevant mode of vibration of an empty civil engineering assembly structure. This simple way of modelling humans can explain observed decreases of natural frequencies, but not increases, or the appearance of additional natural frequencies. This insufficiency has led to the proposal of a single-degrees-of-freedom (s.d.o.f.) occupant model defined not only by a lumped mass m_H but also by a stiffness k_H [2]. Combining this undamped s.d.o.f. occupant model with an undamped s.d.o.f. model of an empty structure leads to the human–structure model shown in Fig. 1b [2]. This model principally explains increases and decreases of natural frequencies as well as additional modes [2]. However, it has no damping capabilities associated with the structural and human d.o.f.s ($c_H = c_S = 0$). Therefore, it is

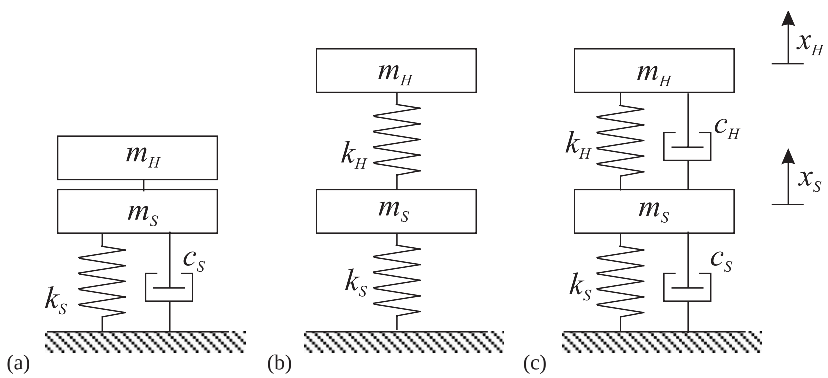

Fig. 1. Models of human-occupied structures. (a) Mass-only model; (b) undamped 2-d.o.f. model; (c) damped 2-d.o.f. model.

difficult to use this model to explain the significant increase in damping observed in real life. For example, in one case of a building floor with a fundamental natural frequency of 6.25 Hz, the presence of one and two occupants increased the measured damping ratio from 0.55% to 1.4% and 2.25%, respectively [6].

The human body is heavily damped [7] and, depending on the circumstances, it has the potential to affect the dynamic properties and responses of a human–structure system significantly. A key problem here is to establish the conditions under which humans act most effectively as an additional source of damping. This piece of information is currently missing in the published literature on human–structure interaction. To address this issue viscous dashpots c_S and c_H representing the damping capabilities of the empty structure and human occupants, respectively, may be added to the existing undamped human–structure model (Fig. 1b). This modification leads to the damped 2-d.o.f. human–structure model (Fig. 1c) that is investigated in this paper.

Attaching a damped (or undamped) s.d.o.f. crowd model to a structure model is somewhat similar to attaching a tuned mass damper (TMD) [8]. TMDs have been researched extensively [9] and are widely employed to reduce vibrations of mechanical and civil engineering structures [10]. They usually have a mass significantly smaller than the modal mass m_S of the structural mode they are designed to dampen. Furthermore, the natural frequency of a TMD is designed to be close to the natural frequency of the mode the TMD is dampening.

In contrast to manufactured TMDs, the properties of humans as 'TMDs' are impossible to control and, therefore, some important differences exist. Firstly, the mass of a crowd on a civil engineering structure (m_H) can be similar to the mass of the structure itself (m_S) [8]. Also, the (undamped) natural frequencies f_H and f_S of crowd and structure s.d.o.f. models, respectively, given by

$$f_H = \frac{1}{2\pi}\sqrt{\frac{k_H}{m_H}}, \tag{1}$$

$$f_S = \frac{1}{2\pi}\sqrt{\frac{k_S}{m_S}} \tag{2}$$

can be very different. Therefore, the extensive literature on TMDs is of limited use in understanding the possible behaviour of a 2-d.o.f. crowd–structure dynamic system. As there is a lack of published literature on the general behaviour of 2-d.o.f. systems consisting of one lightly and one heavily damped s.d.o.f. system connected in series, this paper contains a theoretical study of such systems.

First, the theory of damped 2-d.o.f. crowd–structure systems is outlined and possible properties systems are specified. Next, a parametric study of natural frequencies, mode shapes, modal masses and damping ratios of damped 2-d.o.f. crowd–structure systems is presented. Finally, representative frequency response functions (FRFs) are calculated and used to explain some of the effects of human occupants on civil engineering structures observed and reported in the literature.

2. Damped 2-d.o.f. crowd–structure systems

Modelling a crowd-occupied structure using a damped 2-d.o.f. model (Fig. 1c) is only valid if the structure and occupying crowds can both be modelled separately as damped s.d.o.f. systems. This simplification implies that occupants and the structure are linear and time-invariant dynamic systems. This assumption is generally valid for civil engineering structures under low levels of vibration and is thought to be true if occupants are stationary and in continuous contact with the structure. In this case, the modal properties of crowd–structure systems can be estimated using the well-known theory of analytical modal analysis [11].

2.1. Estimation of modal properties

The equation of free vibration of a general viscously damped 2-d.o.f. model, as shown in Fig. 1c, is

$$\begin{bmatrix} m_S & 0 \\ 0 & m_H \end{bmatrix} \begin{Bmatrix} \ddot{x}_S(t) \\ \ddot{x}_H(t) \end{Bmatrix} + \begin{bmatrix} c_S + c_H & -c_H \\ -c_H & c_H \end{bmatrix} \begin{Bmatrix} \dot{x}_S(t) \\ \dot{x}_H(t) \end{Bmatrix} + \begin{bmatrix} k_S + k_H & -k_H \\ -k_H & k_H \end{bmatrix} \begin{Bmatrix} x_S(t) \\ x_H(t) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}. \quad (3)$$

Solving the corresponding non-proportionally damped eigenproblem:

$$\left(\lambda_r^2 \begin{bmatrix} m_S & 0 \\ 0 & m_H \end{bmatrix} + \lambda_r \begin{bmatrix} c_S + c_H & -c_H \\ -c_H & c_H \end{bmatrix} + \begin{bmatrix} k_S + k_H & -k_H \\ -k_H & k_H \end{bmatrix} \right) \begin{Bmatrix} \psi_{Sr} \\ \psi_{Hr} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (4)$$

leads to two modes of vibration ($r = 1, 2$). Each mode is defined by its complex eigenvalue λ_r and mode shape $\{\Psi\}_r$, which is also complex. These two key modal properties have been calculated numerically and are used in the parametric studies presented later in this paper.

Eigenvalues λ_1 and λ_2 define the (damped) natural frequencies f_1 and f_2:

$$f_r = \frac{1}{2\pi} |\lambda_r| \quad (r = 1, 2) \quad (5)$$

and the damping ratios ζ_1 and ζ_2:

$$\zeta_r = \frac{-\mathrm{Re}(\lambda_r)}{|\lambda_r|} \quad (r = 1, 2) \quad (6)$$

of the damped 2-d.o.f. dynamic model.

Both properties (natural frequencies and damping ratios) are considered in the following parametric study. Additionally, the generally complex mode shapes $\{\boldsymbol{\psi}\}_1$ and $\{\boldsymbol{\psi}\}_2$ of the damped 2-d.o.f. crowd–structure model are presented in unity-normalized form, which means that the ‘maximum value’ is $1 + 0\mathrm{i}$. Such unity-normalized mode shapes are used to calculate the modal masses m_1 and m_2 using the following equation [11]:

$$\begin{Bmatrix} \psi_S \\ \psi_H \end{Bmatrix}_r^{\mathrm{T}} \begin{bmatrix} m_S & 0 \\ 0 & m_H \end{bmatrix} \cdot \begin{Bmatrix} \psi_S \\ \psi_H \end{Bmatrix}_r^{*} = m_r \quad (r = 1, 2). \quad (7)$$

2.2. Definition and range of parameters

In this paper, four parameters are used to define a range of damped 2-d.o.f. crowd–structure models, which are reasonably expected to occur in practice. The parameters are:

(1) the mass ratio α

$$\alpha = \frac{m_H}{m_S}, \tag{8}$$

(2) the frequency ratio f_H/f_S

$$\frac{f_H}{f_S} = \sqrt{\frac{k_H}{m_H}\frac{m_S}{k_S}}, \tag{9}$$

(3) the damping ratio ζ_S of the s.d.o.f. structure model

$$\zeta_S = \frac{c_S}{4\pi m_S f_S}, \tag{10}$$

and

(4) the damping ratio ζ_H of the s.d.o.f. human model

$$\zeta_H = \frac{c_H}{4\pi m_H f_H}. \tag{11}$$

Grandstands and floors with natural frequencies below 6 Hz are of particular concern in the design of civil engineering structures for vibration serviceability [5]. Their mass per square meter can easily be as little as 500 kg/m^2, or even less for composite steel–concrete constructions. Crowd densities of 6 or more people/m^2 are possible and have been observed during sports and concert events. Therefore, mass ratios α of 10%, 50% and 100% are quite realistic and have been used in this paper.

The human body is a complex non-linear dynamic system. Its dynamic properties depend on posture, the level of vibration and many other parameters. Reported natural frequencies f_H of the whole human body range from 1 Hz [12] to 16 Hz [13]. In biomechanics, natural frequencies of about 1–3 Hz and natural frequencies of 4–6 Hz are associated with horizontal and vertical vibrations of the sitting human body [7,12], respectively. These values correspond to levels of vibration of 0.5–2.5 m/s^2. Levels of vibration encountered in civil engineering are typically lower than 0.5 m/s^2. Therefore, higher natural frequencies f_H are likely in civil engineering applications because the human body tends to stiffen with decreasing level of vibration [7]. For illustration purposes, natural frequencies f_H of 5 and 6 Hz are repeatedly used in the following parametric studies. Note also that higher natural frequencies f_H can be expected for standing than for sitting people.

Natural frequencies f_S of civil engineering structures experiencing serviceability problems caused by human-induced resonant vibrations range from 0.5 Hz [14] to about 10 Hz. Considering the whole range of reported natural frequencies f_H of the whole human body, frequency ratios f_H/f_S ranging from 0.1 (1/10) to 32 (16/0.5) are to be expected. However, the studies presented in this paper concentrate on frequency ratios f_H/f_S from close to 0 to 15 Hz, as this range was considered to be most realistic and relevant.

Empty civil engineering structures such as floors and grandstands typically have damping ratios of 1–2% [15]. At the design stage, damping cannot be predicted easily and, therefore, it is conservative to assume a low value. Therefore, a damping ratio ζ_S of 1% is used in this paper.

However, significantly higher damping ratios are quoted for the human body in biomechanical research [7]. They typically range from 30% to 50%. Both limiting values are used in this paper for the damping ratio ζ_H of a s.d.o.f. crowd model.

3. Parametric study of modal properties

Using the four parameters α, f_H/f_S, ζ_S and ζ_H specified above, the modal properties of damped 2-d.o.f. crowd–structure systems (Fig. 1c) are investigated parametrically. This leads to various sets of natural frequencies f_1 and f_2, mode shapes $\{\psi\}_1$ and $\{\psi\}_2$, modal masses m_1 and m_2, and modal damping ratios ζ_1 and ζ_2 that are presented in this section.

3.1. Natural frequencies

For three different mass ratios α, Figs. 2 and 3, respectively, present the first and second natural frequencies f_1 and f_2 of damped 2-d.o.f. crowd–structure systems where $\zeta_H = 30\%$. These two frequencies are presented normalized to the natural frequency f_S of the s.d.o.f. empty structure model. Almost identical graphs can be obtained for $\zeta_H = 50\%$ indicating that increasing human

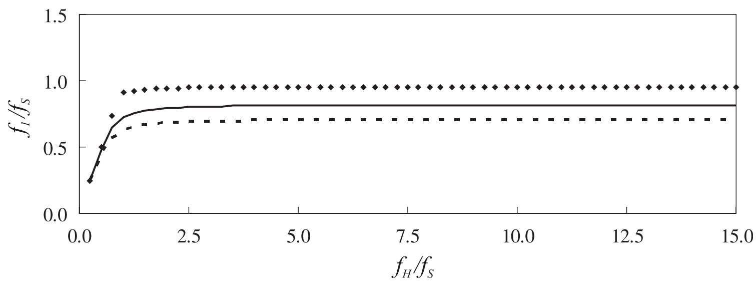

Fig. 2. Normalized natural frequencies f_1/f_S of a damped 2-d.o.f. crowd–structure model ($\zeta_S = 1\%$). ◆◆, $\alpha = 10\%$; —, $\alpha = 50\%$; – –, $\alpha = 100\%$ for $\zeta_H = 30\%$.

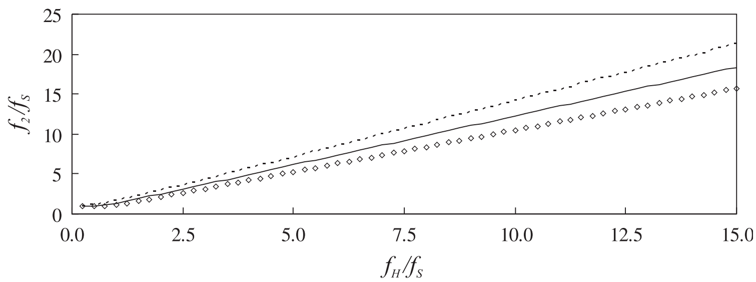

Fig. 3. Normalized natural frequencies f_2/f_S of a damped 2-d.o.f. crowd–structure model ($\zeta_S = 1\%$). ◇◇, $\alpha = 10\%$; —, $\alpha = 50\%$; – –, $\alpha = 100\%$ for $\zeta_H = 30\%$.

damping ratio from $\zeta_H = 30\%$ to 50% has little effect on changing the natural frequencies f_1 and f_2 corresponding to these two high damping values.

Fig. 2 also shows that the higher the natural frequency f_H of the human d.o.f. with respect to the natural frequency f_S of the structure, the more the human d.o.f. acts as an additional rigid mass attached to the structure, which corresponds to the mass-only model (Fig. 1a), as would be expected. This is to be expected as it can be shown that when $f_H \to \infty$, the fundamental frequency f_1 of the 2-d.o.f. system becomes:

$$f_1 = \sqrt{\frac{1}{1+\alpha}} f_S. \tag{12}$$

The natural frequency f_1 of the joint human–structure dynamic system is close to this upper limit if $f_H/f_S > 5$ (Fig. 2), which is the case for a structure with a natural frequency of, say, $f_S < 1$ Hz assuming $f_H = 5$ Hz. It is interesting to note in Fig. 3 that the natural frequency f_2 is in this case an approximately linear function of the ratio f_H/f_S.

Often, the natural frequency of vertical modes of civil engineering assembly structures is about 5 Hz. Thus, it is similar to the natural frequency of vertical vibrations of a sitting person. Therefore, damped 2-d.o.f. crowd–structure models are likely to have frequency ratios f_H/f_S of about 1. The natural frequencies f_1 and f_2 of such systems are not very clear in Figs. 2 and 3. Therefore, normalized natural frequencies f_1/f_S and f_2/f_S of crowd–structure systems characterized by frequency ratios f_H/f_S within the range from 0.25 to 1.25 are presented in Figs. 4a and b for $\zeta_H = 30\%$ and 50%, respectively.

A damped 2-d.o.f. crowd–structure model defined by $f_S = 16$ Hz and $f_H = 5$ Hz has a frequency ratio f_H/f_S of about 0.3. The natural frequency f_2 of this system is similar to the natural frequency f_S of the empty structure for both values of ζ_H (Fig. 4). More importantly, the

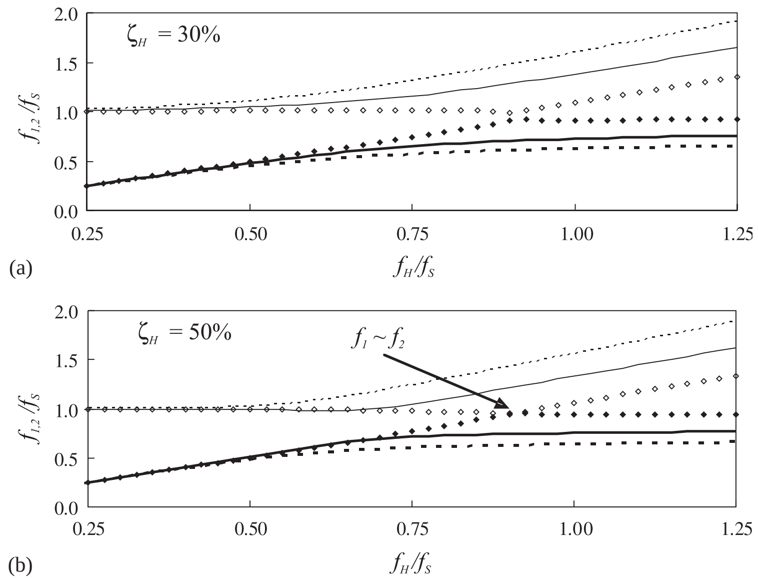

Fig. 4. Normalized natural frequencies f_1/f_S and f_2/f_S of a damped 2-d.o.f. crowd–structure model ($\zeta_S = 1\%$). f_1/f_S : ♦♦, $\alpha = 10\%$; —, $\alpha = 50\%$; – –, $\alpha = 100\%$. f_2/f_S : ◇◇, $\alpha = 10\%$; —, $\alpha = 50\%$; …, $\alpha = 100\%$.

fundamental frequency f_1 of the crowd–structure model is slightly less than 5 Hz. Thus, such a damped 2-d.o.f. model of a crowd-occupied structure explains the significant reduction of the fundamental frequency of an assembly structure observed by Ellis and Ji [2] mentioned at the beginning of the paper.

Fig. 2 demonstrates that the fundamental frequency f_1 of all investigated crowd–structure models is smaller than the natural frequency f_S of the corresponding structure. However, it is important to realise that the vibration responses of an occupied structure can be affected significantly and even be dominated by the second mode of the crowd–structure system.

Both modes of crowd–structure systems were identified by Ellis and Ji [2] on a real-life grandstand. Based on their experimental data (Table 1) and using an undamped 2-d.o.f. crowd–structure model (Fig. 1b), they estimated the natural frequency f_H to be between 5.5 and 5.8 Hz by back analysis. In another real-life civil engineering structure, standing occupants were found to increase the natural frequency of a horizontal mode from 3.05 to 3.30 Hz [1]. Interestingly, the natural frequency decreased to 1.71 Hz if the occupants were sitting. The changes of this and other modes (Table 2) demonstrate the influence of posture and direction of vibration on f_H. These observations emphasise that it is important for accurate occupant models to be used.

The question arises as to when and why only the first, the second, or both modes of the 2-d.o.f. crowd–structure system affect the vibrations of the occupied structure. This will become clear in Section 3.2 of this paper that presents mode shapes.

Before this, it should be noted that the natural frequencies f_1 and f_2 of a damped 2-d.o.f. crowd–structure system can be practically identical (Fig. 4b). This happens when $f_H/f_S = 0.9$, i.e., when, say, $\zeta_H = 50\%$, $\alpha = 10\%$, $f_H = 6$ Hz and $f_S = 6.7$ Hz. In such a case, it is likely that an analysis of experimental data would identify only one and not two modes. This might have been the case when 400 people occupied a floor and reduced its fundamental frequency from 7.28 to 6.60 Hz [3].

Table 1
Natural frequencies at Twickenham stadium [2]

Frame structure	f_S (Hz)	Occupied structure f_1 (Hz)	Occupied structure f_2 (Hz)
Truss 5	8.55	5.44	8.72
Truss 9	7.32	5.41	7.91
Truss 11	7.24	5.13	7.89

Table 2
Natural frequencies of an assembly structure [1]

Mode description	f_S (Hz)	Occupied structure: Occupants sitting (Hz)	Occupied structure: Occupants standing (Hz)
Front-to-back mode	3.05	1.71	3.30
Side-to-side mode	3.66	1.83	3.54
Vertical mode	13.6	9.03	9.16

Note also that natural frequencies f_1 and f_2 can, but do not have to, be within the range bound by the natural frequencies f_H and f_S of the two subsystems. However, this is not the case for the natural frequencies $f_{1,2}^{(UM)}$ (UM stands for undamped model) of an undamped 2-d.o.f. crowd–structure model:

$$f_{1,2}^{(UM)} = \left(\sqrt{\left(\frac{f_H}{f_S}+1\right)^2+\alpha\left(\frac{f_H}{f_S}\right)^2} \mp \sqrt{\left(\frac{f_H}{f_S}-1\right)^2+\alpha\left(\frac{f_H}{f_S}\right)^2}\right)\frac{f_S}{2} \tag{13}$$

that always have to satisfy the following condition:

$$f_1^{(UM)} < (f_S f_H) < f_2^{(UM)} \tag{14}$$

as demonstrated by Ellis and Ji [2]. Therefore, it is important to realise that damped and undamped 2-d.o.f. human–structure dynamic models may have considerably different behaviour.

3.2. Mode shapes

Damping of a viscously damped 2-d.o.f. crowd–structure model is generally non-proportional and, therefore, its mode shapes $\{\psi\}_1$ and $\{\psi\}_2$ are complex. Therefore, in this paper they are represented by magnitudes $|\{\psi\}_r|$ and phases $\arg(\{\psi\}_r)$. The mode shapes of 2-d.o.f. crowd–structure models with damping ratios $\zeta_S = 1\%$ and $\zeta_H = 30\%$ are parametrically studied to improve the understanding of modal masses and damping ratios.

3.2.1. Mode shape of the first mode

Magnitudes of the unity-normalized first mode at the human d.o.f. $|\psi_{H1}|$ and the structural d.o.f. $|\psi_{S1}|$ are shown in Fig. 5a for a range of α and f_H/f_S ratios. Fig. 5b presents the absolute values of the corresponding phase differences $|\arg(\psi_{H1}) - \arg(\psi_{S1})|$ denoted as $\arg\{\psi\}$.

Fig. 5a demonstrates that the human d.o.f. experiences stronger movements in the first mode of vibration than the structure for all investigated crowd–structure models. In other words:

$$|\psi_{S1}| < |\psi_{H1}|. \tag{15}$$

For example, the structural movements of a 2-d.o.f. crowd–structure model based on a structure with a natural frequency $f_S = 16$ Hz and assuming $f_H = 5$ Hz are close to 0 ($f_H/f_S < 0.4$ in Fig. 5a). Such a mode dominated by the human d.o.f. has a natural frequency f_1 close to the natural frequency f_H of the s.d.o.f. crowd model (Fig. 4).

It can be seen in Fig. 5a that the lower the natural frequency f_S of the empty civil engineering structure and the more people on it (that is f_H/f_S and α both increasing), the stronger the participation of the structure in the first mode of the 2-d.o.f. model. For $f_H/f_S > 3$, which corresponds to, say, $f_H = 6$ Hz and $f_S < 2$ Hz, both d.o.f.s of the crowd–structure model move in phase (Fig. 5b) with nearly the same amplitude (Fig. 5a). Such a 2-d.o.f. system behaves like a s.d.o.f. system (Fig. 1a) and has a natural frequency f_1 slightly below f_S and the value which can be approximated by Eq. (12).

Note that both the human and the structural d.o.f. tend to move almost in phase in the first mode of the 2-d.o.f. crowd–structure system when $f_H/f_S > 2$. Phase differences $|\arg(\psi_{H1}) - \arg(\psi_{S1})|$ are less than 90° in all cases considered (Fig. 5b). This indicates that some modeshape complexity is to be expected, particularly when $\alpha = 10\%$ and f_H/f_S is slightly less than 1.0.

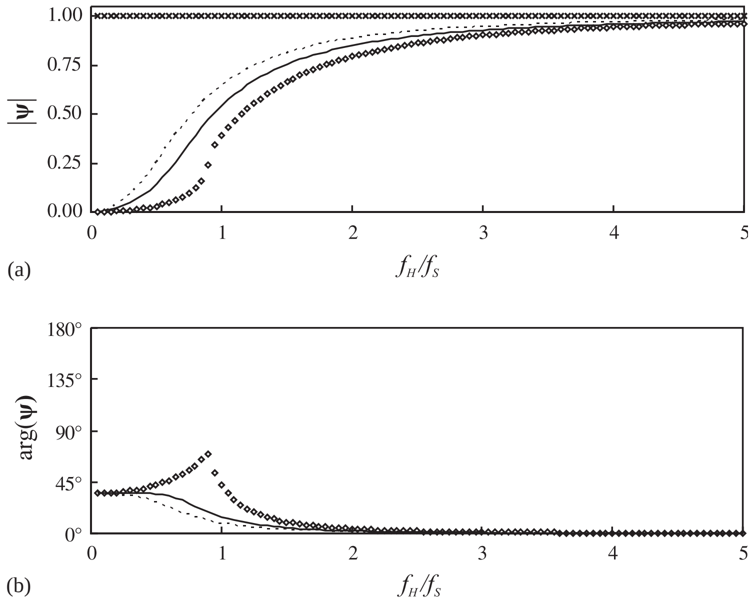

Fig. 5. Mode shape $\{\psi\}_1$ of 2-d.o.f. crowd–structure systems ($\zeta_S = 1\%$, $\zeta_H = 30\%$). (a) Modulus, structural d.o.f.: ◇ ◇, $\alpha = 10\%$; —, $\alpha = 50\%$; $\cdots$, $\alpha = 100\%$. Modulus, human d.o.f.: xxx for all $\alpha = 10\%, 50\%, 100\%$. (b) Phase difference. ◇ ◇, $\alpha = 10\%$; —, $\alpha = 50\%$; $\cdots$, $\alpha = 100\%$.

3.2.2. Mode shape of the second mode

The mode shape amplitudes $|\psi_{S2}|$ and $|\psi_{H2}|$ as well as the phase difference $|\arg(\psi_{H2}) - \arg(\psi_{S2})|$ of the second mode of damped 2-d.o.f. crowd–structure models are presented in Fig. 6.

Fig. 6c demonstrates that the second mode generally has phase difference $|\arg(\psi_{H2}) - \arg(\psi_{S2})|$ greater than 90°. The structural and human d.o.f. move practically 180° out of phase when $f_H/f_S > 2$ (Fig. 6c). When this happens, the structural d.o.f. has lower amplitude than its human counterpart when $\alpha < 100\%$, as shown in Fig. 6a.

If the natural frequency f_S of a vertical mode of an unoccupied civil engineering structure exceeds 10 Hz, it can be expected that $f_H/f_S < 0.6$. The second mode of such a crowd–structure system is dominated by structural movements (Fig. 6) and its natural frequency f_2 is, as mentioned before, close to f_S (Fig. 4). Thus, a surprising increase in the relatively high natural frequency of an empty structure (for which $f_S = 18.7$ Hz), noted by Ellis and Ji [2], can be expected if a person is present on the structure.

Finally, note in Fig. 6a that mode shapes $\{\psi\}_2$ of crowd–structure systems with frequency ratios $0.6 < f_H/f_S < 1.5$, which are likely to occur in real situation depend strongly on the mass ratio α and the frequency ratio f_H/f_S. This affects the modal mass m_2 as shown later.

3.3. Modal masses

Modal masses m_1 and m_2 of 2-d.o.f. crowd–structure systems have been calculated using unity-normalized complex mode shapes (Eq. (7)). They are presented for crowd–structure systems with frequency ratios $f_H/f_S \leqslant 5$ in Fig. 7.

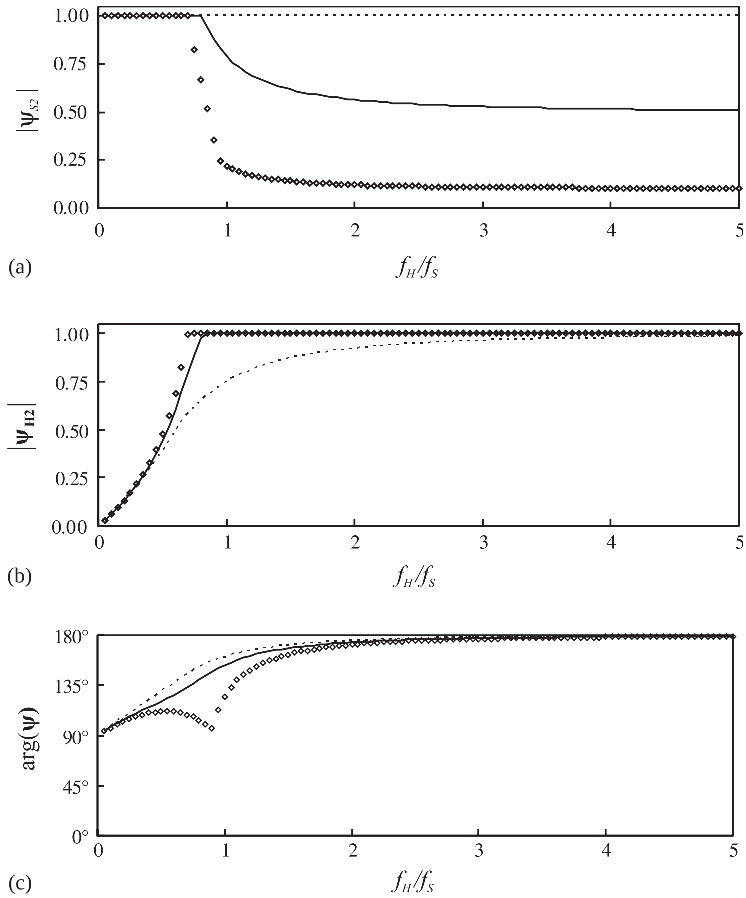

Fig. 6. Mode shape $\{\psi\}_2$ of 2-d.o.f. crowd–structure systems ($\zeta_S = 1\%$, $\zeta_H = 30\%$). (a) Modulus $|\psi_{S2}|$. (b) Modulus $|\psi_{H2}|$. (c) Phase difference $|\arg(\psi_{H2}) - \arg(\psi_{S2})|$. ◇◇, $\alpha = 10\%$; —, $\alpha = 50\%$; ···, $\alpha = 100\%$.

3.3.1. Modal mass of the first mode

The human d.o.f. dominates the first mode (Fig. 5a). Therefore, the modal mass m_1 comprises m_H plus a mode shape dependent contribution of m_S (Fig. 7a). Consequently, the modal mass m_1 has m_H as the lowest and $m_H + m_S$ has the highest possible value. This will be explained here.

The lower limit of m_1 corresponds to human occupation of high-frequency structures (Fig. 7a):

$$\lim_{f_S \to \infty} m_1 = m_H = \alpha m_S, \tag{16}$$

whose first mode is practically a movement of the human d.o.f. only (Fig. 5a for low f_H/f_S ratios).

The human and the structural d.o.f. move together in case of civil engineering structures with very low natural frequencies ($f_S \ll f_H$). In this case, the modal mass m_1 approaches its upper limit (Fig. 7a):

$$\lim_{f_H \to \infty} m_1 = m_H + m_S = (1 + \alpha)m_S. \tag{17}$$

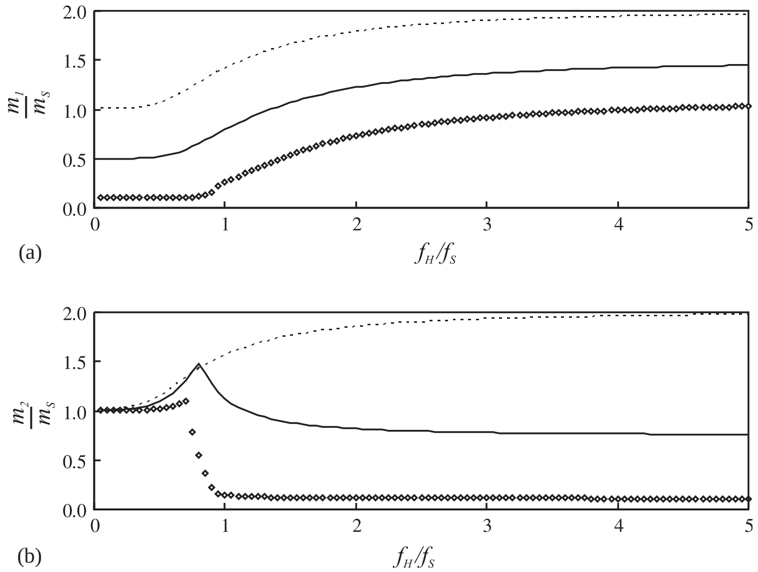

Fig. 7. Modal masses of 2-d.o.f. crowd–structure systems ($\zeta_S = 1\%$, $\zeta_H = 30\%$). ◇ ◇, $\alpha = 10\%$; —, $\alpha = 50\%$; – – –, $\alpha = 100\%$.

3.3.2. Modal mass of the second mode

The modal mass m_2 of the second mode (Fig. 7b) shows a more complicated dependence on mass and frequency ratios α and f_H/f_S than m_1 (Fig. 7a).

In the design of civil engineering structures against human-induced vibrations, damped 2-d.o.f. models with frequency ratios $f_H \approx f_S$ are of particular interest. Such systems, especially with f_H slightly lower than f_S ($f_H/f_S < 1$) can be expected to be characterized by a complex second mode (Fig. 6). This leads to modal masses m_2 as high as the physical limit of $(m_S + m_H)$ particularly for smaller α ($\alpha = 10\%$ or 50%) (Fig. 7b).

3.4. Damping ratios

The modal damping ratios of damped 2-d.o.f. crowd–structure systems defined by $\zeta_S = 1\%$ and $\zeta_H = 30\%$ or 50% are presented in Fig. 8. Firstly, the modal damping ratios ζ_1 and ζ_2 are discussed. Then, cases when similar damping ratios ζ_1 and ζ_2 occur are considered.

3.4.1. Damping ratio of the first mode

As mentioned before, only the human d.o.f. is engaged in the first mode of 2-d.o.f. models representing human occupation of very stiff civil engineering structures ($f_S \to \infty$ in Fig. 5). Therefore, the damping ratio ζ_1 of such systems corresponds to the high damping ratio ζ_H of the human s.d.o.f. model (Figs. 8a and b):

$$\lim_{f_S \to \infty} \zeta_1 = \zeta_H. \tag{18}$$

However, if the structural frequency $f_S < f_H$ (that is $f_H/f_S > 1$), modal damping ratio of the first mode is significantly less than the 30% or 50%, which are percentages associated with the human

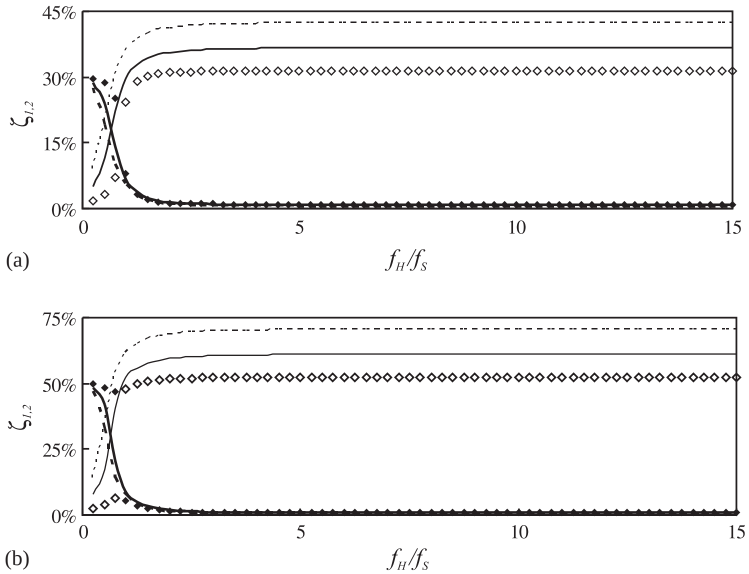

Fig. 8. Damping ratios ζ_1 and ζ_2 of 2-d.o.f. crowd–structure systems ($\zeta_S = 1\%$) for (a) $\zeta_H = 30\%$ and (b) $\zeta_H = 50\%$. ζ_1: ◆◆, $\alpha = 10\%$; —, $\alpha = 50\%$; …, $\alpha = 100\%$. ζ_2: ◇◇, $\alpha = 10\%$; —, $\alpha = 50\%$; …, $\alpha = 100\%$.

body (Fig. 8). This is because the human and the structural d.o.f. move in phase with approximately the same mode shape amplitude (Fig. 5). Thereby, occupants act primarily as an additional mass and the viscous dashpot c_H of the s.d.o.f. structure model (Fig. 1c) is not engaged significantly. Paradoxically, this configuration can actually lead to damping ratios ζ_1 smaller than damping of the empty structure $\zeta_S = 1\%$, regardless of whether $\zeta_H = 30\%$ or 50% (Figs. 9a and b). This situation may occur in the case of structures having natural frequencies f_S below 2 Hz assuming f_H of about 6 Hz ($f_H/f_S > 3$). The effect is more pronounced for higher mass ratios α. With regard to this, the authors are not aware of any publication reporting a reduction in damping of a civil engineering structure caused by human occupants. However, this might be due to the lack of good quality experimental data quantifying the effect of occupants on damping of large structures with very low natural frequencies.

Although the damping ratio ζ_1 can be smaller than ζ_S, ζ_1 is theoretically always higher than the damping ratio of a human-occupied structure where occupants are represented by the mass-only model (Fig. 1a). Nevertheless, ζ_1 of the 2-d.o.f. crowd–structure model approaches the latter value in case of very flexible structures (Fig. 9) when $f_H \to \infty$:

$$\lim_{f_H \to \infty} \zeta_1 = \frac{1}{\sqrt{1+\alpha}} \sqrt{\zeta_S}. \tag{19}$$

Fig. 8 indicates that there are large variations in damping ratios ζ_1 and ζ_2 when f_H/f_S is lower than approximately 1.5. Such systems correspond roughly to $f_S < 10$ Hz and represent realistic cases of crowd-occupied civil engineering structures. Modal damping ratios ζ_1 and ζ_2 of such systems, which cannot clearly be seen in Fig. 8, are presented in Fig. 10.

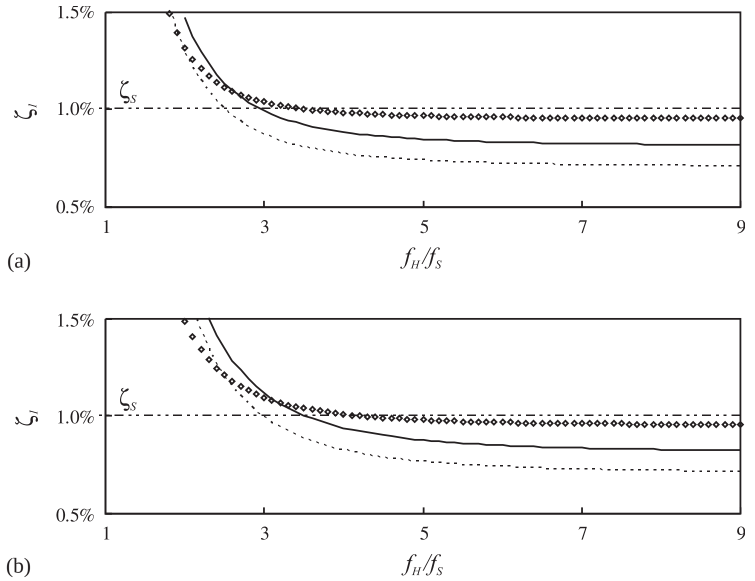

Fig. 9. Damping ratios ζ_1 of a damped 2-d.o.f. crowd–structure model ($\zeta_S = 1\%$). (a) $\zeta_H = 30\%$, (b) $\zeta_H = 50\%$. ◇◇, $\alpha = 10\%$; —, $\alpha = 50\%$; …, $\alpha = 100\%$.

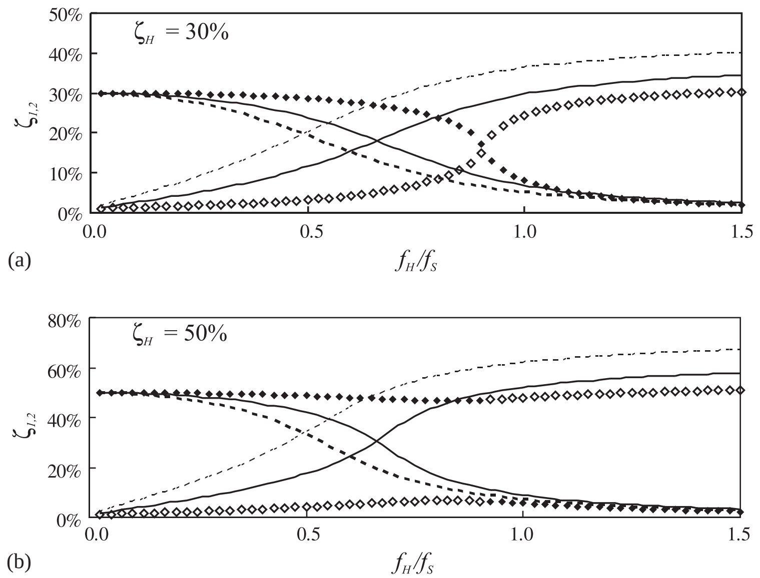

Fig. 10. Damping ratios ζ_1 and ζ_2 of 2-d.o.f. crowd–structure systems ($\zeta_S = 1\%$). ζ_1 : ◆◆, $\alpha = 10\%$; —, $\alpha = 50\%$; - -, $\alpha = 100\%$. ζ_2 : ◇◇, $\alpha = 10\%$; —, $\alpha = 50\%$; …, $\alpha = 100\%$.

Fig. 10a shows that lower damping ratio ζ_1 corresponds to higher ratios f_H/f_S which happens in the case of relatively low natural frequency f_S. This is because with the increase of f_H/f_S the first mode changes from engaging mainly the human d.o.f. to engaging the structural d.o.f. (Fig. 5) that has less damping associated with it ($\zeta_S = 1\%$ as opposed to $\zeta_H = 30\%$). For example, in the case of a realistic crowd–structure system defined by: $f_H = 5.1$ Hz, $f_S = 6$ Hz, $\alpha = 10\%$ and $\zeta_H = 50\%$ ($f_H/f_S = 0.85$ in Fig. 10b), the damping ratio ζ_1 is about 46%.

However, if f_S is reduced to 5.3 Hz, for the same f_H the damping ratio ζ_1 would be only 6% ($f_H/f_S > 0.95$). This happens when the natural frequencies f_1 and f_2 are close (Fig. 4b) and there is strong interaction between the human and structural d.o.f.s.

3.4.2. Damping ratio of the second mode

Similarly to the damping ratio ζ_1, the damping ratio ζ_2 of 2-d.o.f. crowd–structure systems has upper and lower limits (Fig. 8). Of particular interest is the lower limit of ζ_2. It is reached if the second mode is dominated by strong movements of the structure, which happens in the case of a high-frequency structure when f_H/f_S is small.

Interestingly, the damping ratio ζ_2 exceeds ζ_H (30% or 50%) if $f_H > 2f_S$ (Figs. 8a and b), particularly if the structure is densely populated and, thus, the mass ratio α is high. However, such highly damped second modes of occupied low-frequency civil engineering structures (where, say, $f_S < 3$ Hz) are less relevant in the design of assembly structures against human-induced vibrations than the lightly damped ($\zeta_1 < \zeta_S$) fundamental modes.

3.4.3. High damping ratios of crowd–structure systems

A particularly interesting aspect of this parametric study is that both modes of a 2-d.o.f. crowd–structure system can be heavily damped at the same time. In fact, the damping ratios ζ_1 and ζ_2 can both exceed 10% (ten times the damping ratio ζ_S of the empty structure) simultaneously when $0.4 < f_H/f_S < 1$ (see Fig. 10a for $\alpha = 50\%$). Such cases correspond to close natural frequencies f_1 and f_2, particularly if the mass of the occupants is small compared to that of the structure (small mass ratio α). However, due to the close natural frequencies f_1 and f_2, it is possible that modal testing of an occupied full-scale structure identifies only a single mode, so experimental verification of this feature on, say, a real-life grandstand structure may be difficult.

4. Discussion

The parametric study of natural frequencies, mode shapes, modal masses and damping ratios has provided a valuable insight into the possible behaviour of damped 2-d.o.f. crowd–structure systems. To facilitate a discussion of the effect of a damped s.d.o.f. occupant model on the vibration behaviour of a damped s.d.o.f. model of a civil engineering structure, the analyzed behaviour will be presented in the form of point-accelerance FRFs corresponding to the excitation and response at the structural d.o.f. x_S (Fig. 1c).

The point-accelerance FRF $A_{SS}(f)$ of a damped 2-d.o.f. crowd–structure model (Fig. 1) can be calculated using the following closed form solution [16]:

$$A_{SS}(f) = \frac{-f^2(-m_H f^2 + \mathrm{i}c_H f + k_H)}{[(k_S + k_H - m_S f^2 + \mathrm{i}(c_S + c_H)f)(k_H - m_H f^2 + \mathrm{i}c_H f) - (\mathrm{i}c_H f + k_H)^2]}. \tag{20}$$

Table 3
Modal properties of two damped 2-d.o.f. crowd–structure models ($m_S = 10,000$ kg, $\zeta_S = 1\%$, $f_H = 6.0$ Hz, $\zeta_H = 30\%$, $\alpha = 50\%$)

Model No.	f_S (Hz)	f_H/f_S (dimensionless)	First mode f_1 (Hz)	First mode ζ_1 (%)	Second mode f_2 (Hz)	Second mode ζ_2 (%)
1	4.0	1.5	3.1	2.5	7.7	34.4
2	8.0	0.75	5.2	14.2	9.3	22.1

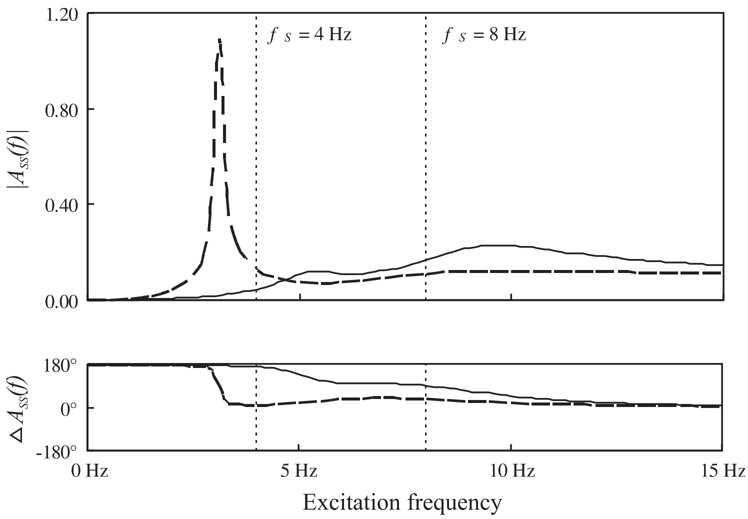

Fig. 11. Modulus and phase of point-accelerances $A_{SS}(f)$ in (mm s^{-2})/N ($m_S = 10,000$ kg, $\zeta_S = 1\%$, $f_H = 6$ Hz, $\zeta_H = 30\%$, $\alpha = 50\%$). - -, #1 ($f_S = 4$ Hz); —, #2 ($f_S = 8$ Hz).

On the other hand, the point-accelerance FRF $A_{s.d.o.f.}(f)$ of the corresponding empty structure s.d.o.f. model is defined by a well known formula [11]:

$$A_{s.d.o.f.}(f) = \frac{-f^2}{(k_S - m_S f^2 + \mathrm{i} c_S f)}. \tag{21}$$

Using $A_{SS}(f)$ of two realistic damped 2-d.o.f. crowd–structure models (Table 3), the influence of occupants on the vibration behaviour of civil engineering assembly structures is analysed. FRFs $A_{SS}(f)$ of both cases are presented in Fig. 11 by modulus and phase and as Nyquist plots in Fig. 12. The latter presentation is particularly valuable to identify closely spaced and heavily damped modes, which are likely in case of crowd–structure systems.

As noted before, it is widely reported that human-occupied structures have significantly higher damping than empty civil engineering structures. This conclusion can also be drawn if modal properties (and FRFs) of the crowd–structure models #1 and #2 are compared with those of the

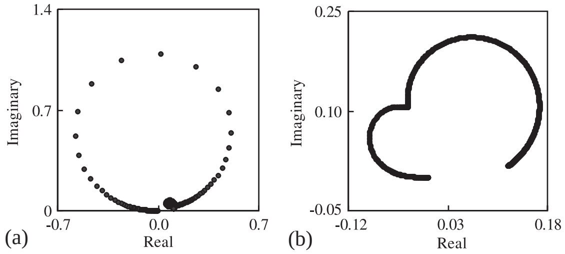

Fig. 12. Accelerances $A_{SS}(f)$ in (mm s^{-2})/N as Nyquist plot (frequency spacing 0.2 Hz), for: (a) model #1 and (b) model #2.

s.d.o.f. model of the empty structure (Table 3). In fact, the peak magnitudes of the FRF $|A_{SS}(f)|$ of both damped 2-d.o.f. crowd–structure models are only 1.1 and 0.2 (mm s^{-2})/N, respectively (Fig. 11), whereas their counterpart for the s.d.o.f. system is

$$|A_{s.d.o.f.}(f_S)| = \frac{1}{2\zeta_S m_S} = 5 \text{ (mm s}^{-2}\text{)/N}. \tag{22}$$

Hence, the same level of (near-) resonant excitation can lead to significantly higher responses of the empty than of the crowd-occupied structure. In other words, a crowd can have a very beneficial effect on reducing the excessive vibrations of the 'empty' structure, at least if occupants are stationary and in continuous contact with the structure. However, as mentioned before, crowds do not only increase damping but also have the potential to reduce natural frequencies significantly. Such a reduction in natural frequencies is adverse because civil engineering structures with lower natural frequencies are usually more likely to be excited by human-induced forces [15]. Therefore, considering how dramatic the changes of modal properties can be, it is becoming apparent how crucial it is to identify correctly the (relevant) modes of the crowd–structure system when designing assembly structures against human-induced vibrations.

Adding a damped s.d.o.f. crowd model to any s.d.o.f. empty structure model (produced, say, via a finite element modal analysis) will lead to an additional mode. However, an additional mode is clearly visible in the FRF $A_{SS}(f)$ of the 2-d.o.f. crowd–structure model only if:

1. both modes contribute sufficiently to the movement of the structure x_S, and
2. the natural frequencies of the two modes are well separated.

Conditions similar to those defining the crowd–structure model #1 (Table 3) would most likely lead to the conclusion that human occupants reduced the natural frequency. This is because the single sharp peak corresponding to the first mode occurs at a frequency about 1 Hz below the frequency of the empty structure $f_S = 4$ Hz (Fig. 11). In this case, the contribution of the second heavily damped mode at 7.7 Hz is very small (Fig. 12a) and, therefore, can easily be missed.

A frequency decrease, an additional mode and a frequency increase could all be deduced from the crowd–structure model #2. This 2-d.o.f. system model has two considerably damped modes (Table 3). They appear in the FRF $A_{SS}(f)$ as two blunt peaks at frequencies lower and higher, respectively, than the natural frequencies $f_H = 6$ Hz and $f_S = 8$ Hz of the two s.d.o.f. subsystems (Fig. 11). If both modes are identified, a reduction of the fundamental natural frequency (corresponding to the additional mode) and an increase of a natural frequency could be reported, depending on the exact situation and the corresponding strength of the two modes. However, in this particular case, the contribution of the first mode is smaller than that of the second mode (Figs. 11 and 12b). Therefore, it could be missed during the system identification, particularly if a s.d.o.f.—and not a m.d.o.f.—based identification algorithm was used. In this case, a frequency increase only would be identified. This, probably, led to quite a lot of confusion in the past.

In general, if $f_S > f_H$ (that is $f_H/f_S < 1$ as in model #2), the presence of occupants most likely leads to reports of increased natural frequencies and, possibly, additional (lower and heavily damped) modes. Assuming that f_H ranges from 4 to 6 Hz for vertical vibrations, this would be the case for empty structures with vertical natural frequencies higher than, say, 6 Hz.

In contrast, occupants on structures with $f_S < f_H$ ($f_H/f_S > 1$ as in model #1) reduce the fundamental natural frequency of the structure. In other words, low fundamental natural frequencies f_S (say, below 4 Hz) have the potential to be further reduced by human occupants. Such low-frequency civil engineering structures are susceptible to human-induced excitation, and therefore have often been investigated. This explains why reports of reduced natural frequencies are more widespread than reports of additional modes and/or frequency increases. It also explains the widespread and long-lasting acceptance of the mass-only model of occupants on civil engineering structures (Fig. 1a), which has a similar effect on the fundamental natural frequencies.

5. Conclusion

Parametric studies performed in this paper have demonstrated that a damped 2-d.o.f. crowd–structure dynamic model provides a good mathematical framework to explain the in situ vibration behaviour of assembly structures, such as grandstands, occupied by crowds. It is essential that the 'human' d.o.f. is damped and connected to the grounded 'structural' d.o.f. (representing a relevant mode of the empty structure) in series, as shown in Fig. 1c. By varying the parameters of such a 2-d.o.f. crowd–structure model, it has been possible to simulate the reduction and the increase of the fundamental natural frequency of the empty structure, as well as the appearance of an additional mode of vibration. All these phenomena have been observed on assembly structures when people were present.

The relationship between the natural frequencies and the mass and damping ratios of the s.d.o.f. models of the empty structure and the occupying crowd determine which of these three scenarios will occur. Overall, considering typical ranges of natural frequencies of empty structures and human bodies, decreases in natural frequency and increases in modal damping of the relevant mode(s) in the joint human–structure 2-d.o.f. system are the most likely observations. Nevertheless, people on high-frequency structures can, under certain circumstances, actually increase the relevant and observable natural frequency. In the case of very low-frequency structures (when $f_H/f_S > 3$), damping as well as the natural frequency of the relevant human–structure mode may be decreased below the values corresponding to the empty structure, even if damping ratio ζ_H of the human body s.d.o.f. is as large as 30% or 50%.

Unfortunately, mass, stiffness and damping properties of human bodies are difficult to ascertain for design purposes because they depend strongly on what the crowd is doing. Moreover, the natural frequency f_H of the crowd-related d.o.f. is difficult to specify because it depends on the posture of the members of the crowd, as well as on the direction and level of vibration. Nevertheless, some general indications on possible crowd properties exist in the published literature. A reasonable range of these can then be used in conjunction with the general mathematical framework described in this paper to parametrically evaluate the likely vibration behaviour of the joint crowd–structure system.

As described in the literature surveyed and further conclusively demonstrated in this paper, the effects of a crowd on the modal properties of an assembly structure can be so significant that they should not be neglected in the mathematical modelling and vibration serviceability design of slender assembly structures.

Acknowledgements

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Appendix A. Nomenclature

α	mass ratio
λ_r	eigenvalue of mode r of a damped 2-d.o.f. system
$\{\mathbf{\Psi}\}_r$	mode shape of r of a damped 2-d.o.f. system
ζ_r	damping ratio of mode r of a damped 2-d.o.f. system
ζ_H	damping ratio of a s.d.o.f. human model
ζ_S	damping ratio of a s.d.o.f. structure model
$A_{s.d.o.f.}(f)$	accelerance of a damped s.d.o.f. structure model
$A_{SS}(f)$	point-accelerance at the structural d.o.f. of a damped 2-d.o.f. crowd–structure model
c_H	viscous damping of a s.d.o.f. human model
c_S	viscous damping of a s.d.o.f. structure model
f_H	natural frequency of a s.d.o.f. human model
f_S	natural frequency of a s.d.o.f. structure model
f_r	natural frequency of mode r of a damped 2-d.o.f. system
$f_r^{(UM)}$	natural frequency of mode r of an undamped 2-d.o.f. system
k_H	stiffness of a s.d.o.f. human model
k_S	stiffness of a s.d.o.f. structure model
m_H	lumped mass of a s.d.o.f. human model
m_S	lumped mass of a s.d.o.f. structure model
m_r	modal mass of mode r of an undamped 2-d.o.f. system
x_H	displacement at the ‘human’ d.o.f. of a 2-d.o.f. crowd–structure model
x_S	displacement at the ‘structural’ d.o.f. of a 2-d.o.f. crowd–structure model

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