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A hybrid mode/Fourier-transform approach for estimating the vibrations of beam-stiffened plate systems

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Abstract

In this paper, a hybrid Mode/Fourier-transform approach is described for estimating the vibration response of a structure such as a beam-stiffened plate with excitation applied to the beam. The beam is defined deterministically in terms of its modes, whereas the plate is treated approximately by assuming it extends to infinity. Equilibrium and continuity conditions are approximated along the interface between the beam and the plate in the wavenumber domain by a Fourier transform method. Consequently, both the dynamic response of the beam and the power transmitted to the plate can be simply estimated. Meanwhile, the dynamic interactions of the coupled system can be determined. These depend on the correlations between the modal properties of the beam and the wave motions of the plate. Expressions are given for the effective mass (density) and effective loss factor the plate applies to each mode of the beam. When a locally reacting plate approximation is incorporated into the Mode/Fourier-transform procedure, a simpler 'locally reacting impedance method' can be developed. The results are discussed and compared to those of fuzzy structure theory. Numerical examples are presented.

1. Introduction

Many practical engineering structures are built up from beams and plates. One such example is the machinery foundation of a ship, which is constructed from a collection of stiff beams and large flexible plates. When excited by external vibration sources, wave motions are generated in both beams and plates. Usually, the wavelengths in the stiff beams are relatively long compared to those in the plates. The differences in wavelengths may then present a number of challenges to

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predicting the vibrations of beam/plate built-up structures [1]. Since these problems are mainly caused by the limitations of low-frequency deterministic methods (e.g., finite element analysis) and high-frequency statistical methods (e.g., statistical energy analysis), they are typical of certain mid-frequency vibration problems. Although a number of model reduction and specialist methods [2–10] might be applied to this mid-frequency region, there are problems in that they are often limited to certain special cases or are very time consuming, especially for structures with continuous, line couplings. It is true to say that more research effort is required before generally accepted, systematic methodologies are fully developed.

This paper concerns a simple method for approximately modelling a certain class of beam/plate built-up structures. Being broadly representative of machinery foundations, a beam-stiffened plate system, such as that shown in Fig. 1, is considered with external excitation sources applied to the beam. The beam is assumed to be well-defined with a long-wavelength and/or a low modal density, while the plate has a relatively short-wavelength and/or a high modal density, perhaps with complex boundary conditions. Under such circumstances, therefore, the frequency–response–functions (FRFs) of the stiff beam and the power transmitted to the flexible plate are of interest.

In the following section, a hybrid Mode/Fourier-transform (FT) approach is described by simply approximating the large flexible plate receiver as if it extended uniformly to infinity. It is a combination of conventional modal analysis and FT methods, and is used to predict both the FRFs of the beam and the power transmitted to the plate. The analysis concerns the dynamic correlations between the modal properties of the beam and wave motion in the plate. Then in Section 3 a locally reacting plate approximation [4,5] is incorporated into the Mode/FT procedure. As a result, a so-called 'locally reacting impedance method' is provided. The results are then discussed and compared to fuzzy structure theory [9,10]. Numerical examples are presented in Section 4.

It is expected that the Mode/FT approach can provide a useful methodology for predicting the vibrations of beam/plate coupled structures, in that it is able to deal with the large dynamic mismatch between the beam and plate components and at the same time can overcome the practical difficulty in determining the exact dynamic properties of the large, flexible plate. In addition, it gives insight into the vibration and coupling of general built-up structures comprising substructures which are dynamically mismatched.

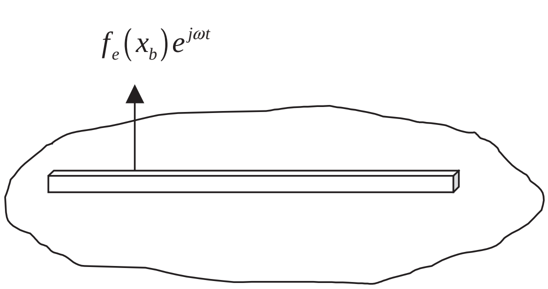

Fig. 1. Beam-stiffened plate.

2. Theoretical description of the mode/FT approach

It is quite common to estimate the vibration properties of a large flexible plate, especially in a frequency average sense, as if it extended uniformly to infinity. In the Mode/FT approach the vibration of a beam attached to an infinite plate is considered. Central to the Mode/FT approach is to approximately enforce the equilibrium and continuity boundary conditions along the interface between the beam and the plate in the wavenumber domain. The correlations between the modal properties of the beam and the wave motions of the plate can then be determined, and hence both the FRFs of the beam and power transmitted to the large flexible plate, as well as the dynamic interactions within the beam-stiffened plate system, can be found.

2.1. Modal analysis of the beam

Fig. 2 shows the beam and its force loadings. The beam is assumed to be straight and uniform. x_b is the local co-ordinate of the beam, and $f_e(x_b)$ and $f_I^b(x_b)$ are the amplitudes of the external and interface forces acting on the beam, respectively. (Appendix A contains a list of symbols.) Time-dependent behaviour of the form $\exp(j\omega t)$ is assumed, and the explicit time dependence will henceforth be suppressed. By conventional modal analysis [11], the beam displacement $w_b(x_b)$ can be defined in terms of its natural modes, as

$$w_b(x_b) = \sum_n w_{b,n}\phi_{b,n}(x_b), \tag{1}$$

where $\phi_{b,n}$ is the nth natural mode of the beam when it is separated from the plate, and $w_{b,n}$ is the corresponding modal amplitude. For convenience, normalized mode shape functions are used in the above equation, so that

$$\int_0^{L_b} \phi_{b,n}(x_b)\phi_{b,m}(x_b)\,\mathrm{d}x_b = \begin{cases} 1, n = m, \\ 0, n \neq m, \end{cases} \tag{2}$$

where L_b is the length of the beam. From Ref. [11], $w_{b,n}$ is given by

$$w_{b,n} = Y_{b,n}(f_{e,n} + f_{I,n}^b), \tag{3}$$

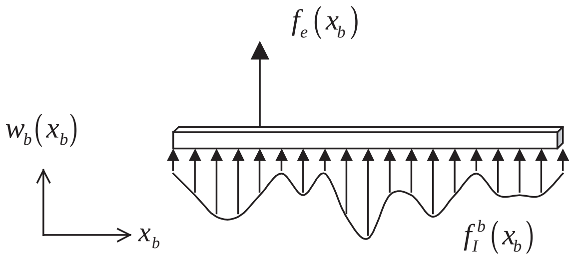

Fig. 2. The beam and its loadings.

where $Y_{b,n}$ is the nth modal receptance of the uncoupled beam, and $f_{e,n}$ and $f_{I,n}^b$ are, respectively, the nth modal forces corresponding to $f_e(x_b)$ and $f_I^b(x_b)$. These terms are given by

$$Y_{b,n} = \frac{1}{m_b} \frac{1}{(\omega_{b,n}^2(1 + \mathrm{j}\eta_{b,n}) - \omega^2)}, \tag{4}$$

$$f_{e,n} = \int_0^{L_b} f_e(x_b)\phi_{b,n}(x_b)\,\mathrm{d}x_b, \tag{5}$$

$$f_{I,n}^b = \int_0^{L_b} f_I^b(x_b)\phi_{b,n}(x_b)\,\mathrm{d}x_b. \tag{6}$$

Here m_b is the mass per unit length of the beam, and $\omega_{b,n}$ and $\eta_{b,n}$ are the nth natural frequency and modal loss factor of the uncoupled beam, respectively.

2.2. Line-impedance of an infinite plate

Fig. 3 shows the plate and its force loadings along the interface, where (x_p, y_p) are the local co-ordinates of the plate, and (x_p^I, y_p^I) and $f_I^p(x_p^I, y_p^I)$ are, respectively, the interface locations on the plate, and interface force distribution.

It is known [11,12] that a high mode-count structure is often very difficult or even impossible to define deterministically, especially when it has complex boundary conditions. It is more appropriate to describe such a structure approximately. One such approximation is as if it were extended uniformly to infinity, due to the fact that the dynamic response of a finite structure tends to that of the equivalent infinite structure as the modal overlap increases, at least in a frequency average sense. In this Mode/FT approach the plate receiver is simply treated as being infinite. Conventional FT methods can then be used to yield the line-impedance of an infinite plate in the wavenumber domain. The procedure is given below.

The one- and two-dimensional FTs are, respectively, defined as [13]

$$G(k) = \int_{-\infty}^{+\infty} g(x)\mathrm{e}^{-\mathrm{j}kx}\,\mathrm{d}x, \tag{7}$$

$$g(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} G(k)\mathrm{e}^{\mathrm{j}kx}\,\mathrm{d}k, \tag{8}$$

$$G(k_x, k_y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x, y)\mathrm{e}^{-\mathrm{j}k_x x}\mathrm{e}^{-\mathrm{j}k_y y}\,\mathrm{d}x\,\mathrm{d}y, \tag{9}$$

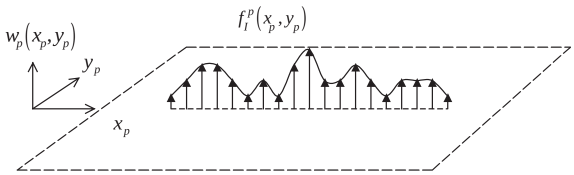

Fig. 3. The plate and its loadings.

$$g(x,y) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} G(k_x,k_y) e^{jk_x x} e^{jk_y y} \, dk_x \, dk_y. \tag{10}$$

It is assumed that the interface starts from the point $(x_p^I, y_p^I) = (x_p, y_p) = (0,0)$ and ends at $(x_p^I, y_p^I) = (x_p, y_p) = (L_b, 0)$ along the line $y_p^I = y_p = 0$. The interface force acting on the plate (per unit) coupling area can be expressed as

$$f_I^p(x_p,y_p) = \begin{cases} \bar{f}_I^p(x_p)\delta(y_p), & 0 \leqslant x_p \leqslant L_b, \\ 0, & x_p < 0 \cup x_p > L_b, \end{cases} \tag{11}$$

where $\bar{f}_I^p(x_p)$ has a unit of N/m. The equation of motion of the plate is given by [11]

$$D_p \nabla^4 w_p(x_p,y_p) - m_p \omega^2 w_p(x_p,y_p) = f_I^p(x_p,y_p), \tag{12}$$

where D_p and m_p are, respectively, the bending stiffness and the mass per unit area of the plate, and ∇ is a differential operator defined as

$$\nabla^4 = \frac{\partial^4}{\partial x_p^4} + 2\frac{\partial^4}{\partial x_p^2 \partial y_p^2} + \frac{\partial^4}{\partial y_p^4}. \tag{13}$$

Applying the two-dimensional FT of Eq. (9) to Eq. (12) gives

$$D_p(k_x^2 + k_y^2)^2 W_p(k_x,k_y) - m_p\omega^2 W_p(k_x,k_y) = F_I^p(k_x), \tag{14}$$

where $W_p(k_x,k_y)$ and $F_I^p(k_x)$ are, respectively, the FTs of $w_p(x_p,y_p)$ and $f_I^p(x_p,y_p)$. Eq. (14) then yields

$$W_p(k_x,k_y) = \frac{F_I^p(k_x)}{D_p(k_x^2+k_y^2)^2 - m_p\omega^2}. \tag{15}$$

The inverse FT of the plate displacement, by Eq. (10), is given by

$$w_p(x_p,y_p) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} W_p(k_x,k_y) e^{jk_x x_p} e^{jk_y y_p} \, dk_x \, dk_y. \tag{16}$$

The plate displacement along the interface can then be expressed as

$$w_p(x_p,0) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} W_p(k_x,k_y) e^{jk_x x_p} \, dk_x \, dk_y. \tag{17}$$

By integration over k_y, the above equation can be re-written as

$$w_p(x_p,0) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} W_p(k_x) e^{jk_x x_p} \, dk_x, \tag{18}$$

where $W_p(k_x)$ is the FT of the plate displacement along the coupling line in the wavenumber domain k_x, and is given by

$$W_p(k_x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} W_p(k_x,k_y) \, dk_y. \tag{19}$$

Substituting Eq. (15) into Eq. (19), yields

$$W_p(k_x) = \frac{F_I^p(k_x)}{2D_p\sqrt{k_x^4 - k_p^4}\left(\sqrt{k_x^2 + k_p^2} + \sqrt{k_x^2 - k_p^2}\right)}. \tag{20}$$

The above equation gives the relation between the interface displacement and the interface force in the wavenumber domain. As a result, the line-impedance of the plate along the interface can be expressed as

$$Z_p(k_x) = \frac{1}{\mathrm{j}\omega}\left[2D_p\sqrt{k_x^4 - k_p^4}\left(\sqrt{k_x^2 + k_p^2} + \sqrt{k_x^2 - k_p^2}\right)\right]. \tag{21}$$

If the damping of the plate is negligible, the above equation is such that

$$\mathrm{Re}[Z_p(k_x)] = 0, \quad |k_x| \geqslant k_p. \tag{22}$$

Eq. (22) implies that only the waves propagating in the beam faster than the wave motion in the plate can transmit energy into the plate. Hence it is reasonable to assume that energy transmitting, non-reactive interaction between a beam and an infinite plate mainly involves wavenumbers within the range $|k_x| < |k_p|$.

2.3. Vibration response of the coupled system in the wavenumber domain

From the above description, it is seen that the local co-ordinates of the beam and plate are related such that $x_p^I = x_p = x_b = x$, where $0 \leqslant x \leqslant L_b$. The equilibrium and continuity conditions along the interface between the beam and the plate are

$$\bar{f}_I^p(x) = -f_I^b(x) \quad 0 \leqslant x \leqslant L_b, \tag{23}$$

$$w_p(x, 0) = w_b(x) \quad 0 \leqslant x \leqslant L_b. \tag{24}$$

A new set of orthogonal functions is now defined as

$$\hat{\phi}_{b,n}(x) = \begin{cases} \phi_{b,n}(x) & 0 \leqslant x \leqslant L_b, \\ 0 & x < 0 \cup x > L_b. \end{cases} \tag{25}$$

The interface force $\bar{f}_I^p(x)\delta(y)$ may then be decomposed into the form

$$\bar{f}_I^p(x)\delta(y) = \left[\sum_n f_{I,n}\hat{\phi}_{b,n}(x)\right]\delta(y). \tag{26}$$

Taking the FT of the above equation, gives

$$F_I^p(k) = \sum_n f_{I,n}\Phi_{b,n}(k), \tag{27}$$

where $F_I^p(k)$ is the FT of $\bar{f}_I^p(x)$ and $\Phi_{b,n}(k)$ is the FT of $\hat{\phi}_{b,n}(x)$.

From Eqs. (20) and (21), it is seen that the plate interface force and displacement response are related, in the wavenumber domain, by

$$W_p(k) = \frac{F_I^p(k)}{\mathrm{j}\omega Z_p(k)}, \tag{28}$$

where $W_p(k)$ is the FT of $w_p(x, 0)$, given by

$$W_p(k) = \int_{-\infty}^{+\infty} w_p(x, 0)\mathrm{e}^{-\mathrm{j}kx}\,\mathrm{d}x. \tag{29}$$

Substituting Eq. (27) into Eq. (28), it follows that

$$\sum_n f_{I,n}\Phi_{b,n}(k) = \mathrm{j}\omega Z_p(k)W_p(k). \tag{30}$$

From Eqs. (1) and (25), the displacement of the beam may be re-written as

$$w_b(x) = \sum_n w_{b,n}\hat{\phi}_{b,n}(x). \tag{31}$$

The above equation then yields

$$W_b(k) = \sum_n w_{b,n}\Phi_{b,n}(k), \tag{32}$$

where $W_b(k)$ is the FT of $w_b(x)$, given by

$$W_b(k) = \int_0^{L_b} w_b(x)\mathrm{e}^{-\mathrm{j}kx}\,\mathrm{d}x. \tag{33}$$

It is now assumed that the displacement of the plate outside of the interface region gives a negligible contribution to the integral in Eq. (29), so that in that integral

$$w_p(x, 0) \approx 0, \quad x < 0 \cup x > L_b. \tag{34}$$

Substituting Eq. (34) into Eq. (29), and then combining with Eqs. (24) and (33), gives the following approximation

$$W_p(k) \approx W_b(k). \tag{35}$$

The assumption of Eq. (34) is thus equivalent to assume that $W_p(k)$ is dominated by the contribution from $w_p(x, 0)$ in the range of $0 \leqslant x \leqslant L_b$. Substituting Eq. (32) into Eq. (35) and then into Eq. (30), yields

$$\sum_n f_{I,n}\Phi_{b,n}(k) \approx \mathrm{j}\omega Z_p(k)\sum_n w_{b,n}\Phi_{b,n}(k). \tag{36}$$

The orthogonality of $\hat{\phi}_{b,n}(x)$ in Eq. (25) gives, by [13]

$$\int_{-\infty}^{+\infty} \hat{\phi}_{b,n}(x)\hat{\phi}_{b,m}(x)\,\mathrm{d}x = \int_0^{L_b} \phi_{b,n}(x)\phi_{b,m}(x)\,\mathrm{d}x$$
$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \Phi^*_{b,n}(k)\Phi_{b,m}(k)\,\mathrm{d}k = \begin{cases} 1, n = m, \\ 0, n \neq m, \end{cases} \tag{37}$$

where $*$ denotes the complex conjugate. Let both sides of Eq. (36) be multiplied by $\Phi^*_{b,n}(k)$ and integrated over k from $-\infty$ to $+\infty$. If it is assumed that $Z_p(k)$ changes slowly compared to $\Phi_{b,n}(k)$ so that the cross couplings between the modes of the beam can be ignored, it follows that

$$f_{I,n} \approx \mathrm{j}\omega w_{b,n}Z_n, \tag{38}$$

where

$$Z_n = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |\Phi_{b,n}(k)|^2 Z_p(k)\,\mathrm{d}k. \tag{39}$$

Physically, Eq. (39) shows the coupling relations between the line-impedance of the plate $Z_p(k)$ and the mode shapes of the beam. Hence the interactions between the wave motion in the plate and the modal properties of the beam can be determined.

Substituting Eq. (38) into Eq. (26) then into Eq. (23) and finally into Eqs. (6) and (3), the nth modal amplitude of the beam, $w_{b,n}$, after coupling to the plate, can be expressed as

$$w_{b,n} \approx \frac{Y_{b,n}}{(1+\mathrm{j}\omega Y_{b,n} Z_n)} f_{e,n} \approx \frac{1}{(1/Y_{b,n}+\mathrm{j}\omega Z_n)} f_{e,n}. \tag{40}$$

Eq. (40) shows that the nth modal impedance of the beam, after coupling to the plate, is increased by Z_n. Hence Z_n may be called the 'plate-loaded modal impedance', which depends on both the plate properties k_p and D_p, and the beam property $\phi_{b,n}(x)$, by the relations given in Eqs. (21) and (39). Note that in Eq. (40) it is implicitly assumed that the plate loads each beam mode independently and hence that the cross-mode loading is negligible.

Combining Eqs. (1) and (40), the beam displacements can then be expressed as

$$w_b(x) \approx \sum_n \frac{f_{e,n}}{(1/Y_{b,n}+\mathrm{j}\omega Z_n)}\phi_{b,n}(x). \tag{41}$$

The power transmitted from the beam to the plate is given by

$$P = \frac{1}{2}\mathrm{Re}\left\{\int_0^{L_b}\left[\sum_n f_{I,n}\phi_{b,n}(x)\right]^* \mathrm{j}\omega\left[\sum_n w_{b,n}\phi_{b,n}(x)\right]\mathrm{d}x\right\}. \tag{42}$$

Combining Eqs. (37)–(39), the transmitted power can be expressed as

$$P \approx \frac{1}{2}\mathrm{Re}\left\{\mathrm{j}\omega\sum_n f_{I,n}^* w_{b,n}\right\} \approx \frac{1}{2}\omega^2\sum_n |w_{b,n}|^2\,\mathrm{Re}\{Z_n\}. \tag{43}$$

From Eqs. (21) and (39), it is seen that

$$\mathrm{Re}\{Z_n\} \approx 0, \quad |k| \geqslant k_p. \tag{44}$$

Therefore, Eq. (44) indicates that only the components of the mode shapes of the beam with wavelengths larger than the plate wavelength can transmit significant power to the plate. Otherwise, the components generally only cause near-field wave motions in the plate.

In summary, the Mode/FT approach can be briefly divided into the following steps:

(1) The beam model is defined in terms of its uncoupled natural modes (Section 2.1), while the plate model is described in the wavenumber domain by a line-impedance (Section 2.2).

(2) By introducing a new set of orthogonal functions $\hat{\phi}_{b,n}$ in the range $-\infty < x < +\infty$ based on the beam modes $\phi_{b,n}$ in the range $0 < x < L_b$ (Eq. (25)), the plate interface force can then be decomposed in terms of $\hat{\phi}_{b,n}$ (Eq. (26)). As a result, a new relation between the plate interface force and displacement can be given (Eq. (30)).

(3) When the plate displacement outside of the interface region is ignored (Eq. (34)), an estimate can then be made of the relation between displacements of the plate (along the interface) and beam in the wavenumber domain (Eq. (35)). Consequently, an approximate relation between the plate interface force and the beam displacement can be established (Eq. (36)).

(4) The line-impedance of the plate is assumed to change slowly with k compared to $\Phi_{b,n}(k)$. By the orthogonality properties of $\phi_{b,n}$ and hence $\hat{\phi}_{b,n}$, the interface force and displacement relation can then be simply estimated (Eq. (38)). Finally, the beam displacement and the power transmitted to the plate can be predicted in a simple manner.

In step (4) the cross-mode coupling interaction has been ignored. Such a simplification is quite reasonable if the uncoupled mode shape $\phi_{b,n}$ has a strong sinusoidal component at a given wavenumber (as it generally will for a uniform, straight beam), so that $|\Phi_{b,n}(k)|^2$ is sharply peaked in the wavenumber domain. These cross-mode coupling terms are expected to be less important for the higher modes of the beam.

2.4. Dynamic interactions between the beam and the plate

The dynamic interactions between the beam and the plate are given by Eq. (41). The modal receptance $Y_{b,n}$ of an uncoupled beam is given in Eq. (4), while the modal receptance $Y'_{b,n}$ of the beam after coupling to the plate becomes

$$Y'_{b,n} \approx \frac{Y_{b,n}}{(1 + \mathrm{j}\omega Y_{b,n} Z_n)}. \tag{45}$$

Substituting Eq. (4) into Eq. (45), it follows that

$$Y'_{b,n} \approx \frac{1}{[m_b \omega_{b,n}^2 (1 + \mathrm{j}\eta_{b,n}) - m_b \omega^2] + \mathrm{j}\omega Z_n}. \tag{46}$$

The term $\mathrm{j}\omega Z_n$ can be separated into real and imaginary parts so that

$$\mathrm{j}\omega Z_n = -K_{n1} + \mathrm{j}K_{n2}, \tag{47}$$

where K_{n1} and K_{n2} are given, respectively, as

$$\begin{aligned} K_{n1} = {} & \frac{D_p}{\pi} \int_{-\infty}^{+\infty} |\Phi_{b,n}(k)|^2 (k_p^2 - k^2) \sqrt{k_p^2 + k^2}\, \mathrm{d}k \\ & + \frac{D_p}{\pi} \int_{-\infty}^{-k_p} |\Phi_{b,n}(k)|^2 (k_p^2 + k^2) \sqrt{k^2 - k_p^2}\, \mathrm{d}k \\ & + \frac{D_p}{\pi} \int_{+k_p}^{+\infty} |\Phi_{b,n}(k)|^2 (k_p^2 + k^2) \sqrt{k^2 - k_p^2}\, \mathrm{d}k, \end{aligned} \tag{48}$$

$$K_{n2} = \frac{D_p}{\pi} \int_{-k_p}^{+k_p} |\Phi_{b,n}(k)|^2 (k_p^2 + k^2) \sqrt{k_p^2 - k^2}\, \mathrm{d}k. \tag{49}$$

Eq. (46) can then be re-written as

$$Y'_{b,n} \approx \frac{1}{(m_b\omega_{b,n}^2[1 + \mathrm{j}(\eta_{b,n} + \eta_n)] - (m_b + m_n)\omega^2)}, \tag{50}$$

where

$$m_n \approx \frac{K_{n1}}{\omega^2}, \tag{51}$$

$$\eta_n \approx \frac{K_{n2}}{m_b\omega_{b,n}^2}. \tag{52}$$

Eq. (50) indicates that the plate in effect adds mass m_n and damping η_n to each mode of the beam. The energy dissipated by the induced effective damping corresponds to the energy transmitted from the beam to the plate. It is seen from Eq. (52) that $\eta_n \to \infty$ when $\omega_{b,n} = 0$. This means that the rigid-body modes of the beam can be greatly damped by the plate.

2.4.1. Approximations for K_{n1} and K_{n2}

Eqs. (51) and (52) give the approximations for the effective mass and damping added to the beam by the plate. These two expressions can be further simplified under certain circumstances, which are described below. This gives insight into the coupling behaviour and allows comparisons with the locally reacting models of Refs. [4,5].

Suppose that the uncoupled mode shape $\phi_{b,n}$ of a uniform, straight beam has a strong sinusoidal component at a given wavenumber, e.g.,

$$\phi_{b,n}(x) = \sqrt{\frac{2}{L_b}}\sin\left(\frac{n\pi x}{L_b} + \theta\right), \tag{53}$$

where θ represents a phase constant which is determined by the exact boundary conditions of the beam. Under such circumstances $|\Phi_{b,n}(k)|^2$ tends to be sharply peaked in the wavenumber domain around the value of $k_{b,n} \approx \sqrt[4]{m_b\omega_{b,n}^2/D_b}$ but converge quickly to zero as $|k| \to \infty$. If it is assumed that the other terms in the integrands in Eqs. (48) and (49) vary slowly with k compared to $|\Phi_{b,n}(k)|^2$, K_{n1} and K_{n2} can, respectively, be simply approximated as

$$K_{n1} \approx 2D_p(k_p^2 - k_{b,n}^2)\sqrt{k_p^2 + k_{b,n}^2}, \tag{54}$$

$$K_{n2} \approx 2D_p(k_p^2 + k_{b,n}^2)\sqrt{k_p^2 - k_{b,n}^2}. \tag{55}$$

Here the orthogonality condition of Eq. (37) has been applied.

As a special case, when the beam is relatively very stiff compared to the plate such that $k_{b,n} \ll k_p$, the above equations become

$$K_{n1} \approx K_{n2} \approx 2D_pk_p^3. \tag{56}$$

The effective mass and damping in Eqs. (51) and (52) can then be estimated in a much simpler manner. Physically, Eq. (56) represents a 'locally reacting' case [4,5]. The relevant details will be discussed in the section below.

3. Discussion

In the above section, a hybrid Mode/FT approach is used to provide simple estimates of the vibrations of beam-stiffened plate systems. In this section, the limiting case where the beam-stiffened plate system has a very big dynamic mismatch will be considered, i.e., the plate is relatively very much more flexible than the beam. It then behaves as a 'locally reacting' model [4,5] or a set of 'fuzzy attachments' [9,10] to the beam. The discussions illustrate various features of the vibrations and coupling of a beam-stiffened plate.

3.1. Locally reacting impedance method

In previous studies [4,5], it was seen that when the plate wavenumber is much bigger than that of the beam (typically $k_p/k_b > 2$), the plate can be considered as being locally reacting. The locally reacting impedance is given by

$$Z'_p \approx \frac{2m_p\omega}{k_p}(1 + \mathrm{j}). \tag{57}$$

Such a plate model was then incorporated into a standard substructuring procedure to predict the vibration response of a beam-stiffened plate system by splitting the coupled structure into a beam attached to a set of independent narrow strips of the plate [5]. The corresponding solution, however, may still be very time-consuming since the calculation still requires many connecting points between the beam and the plates. However, this locally reacting plate model can be incorporated easily into the Mode/FT approach to predict the response of the beam-stiffened plate system.

When Eq. (57) is substituted into Eq. (39), it follows that

$$Z_n \approx Z'_p \approx \frac{2m_p\omega}{k_p}(1 + \mathrm{j}). \tag{58}$$

The above equation just corresponds to the results given by Eqs. (47) and (56). It implies that the plate-loaded impedance (using a locally reacting plate model) for each mode of the beam is approximately constant. It depends only on the plate properties k_p and m_p, regardless of the beam properties, since the beam is of relatively very high impedance. Substituting Eq. (58) into Eq. (40), the modal amplitudes of the beam are given approximately by

$$w'_{b,n} \approx \frac{Y_{b,n}}{1 + \mathrm{j}\omega Y_{b,n} Z'_p} f_{e,n} \approx \frac{1}{1/Y_{b,n} + \mathrm{j}\omega Z'_p} f_{e,n}. \tag{59}$$

Consequently, the power transmitted to the plate can be simply estimated by

$$P' \approx \frac{\omega^3 m_p}{k_p} \sum_n |w'_{b,n}|^2. \tag{60}$$

From the above it is seen that the locally reacting impedance method can, under these circumstances, provide estimates of the vibration response of a beam/plate system in a much simpler manner.

Similarly, the effective mass and loss factor loaded to each mode of the beam by the locally reacting plate model is given, from Eq. (58), by

$$m'_n \approx \frac{m_p \lambda_p}{\pi}, \tag{61}$$

$$\eta'_n \approx \frac{m_p \omega^2 \lambda_p}{m_b \omega_{b,n}^2 \pi}. \tag{62}$$

The above equations indicate that the effective loss factor increases with frequency but the effective mass decreases. Also the effective loss factor depends on both beam and plate properties whereas the effective mass depends on only the plate properties, regardless of the order of the beam mode.

3.2. Comparison to fuzzy structure theory

In Refs. [9,10], fuzzy structure theory was used to investigate the dynamic coupling relations between a large deterministic 'master' substructure and a continuous set of light oscillators, i.e., 'fuzzy attachments'. It was found that the attached items act mainly to provide damping to the master structure. Moreover, the level of this damping is independent of the dissipation factor of the attachments. Similar conclusions were also given in Ref. [7].

When the plate receiver is relatively much more flexible than the source beam, the former behaves like 'fuzzy attachments' to the latter. By Eq. (61), it is seen that in this case the ratio of m'_n/m_b can be very small since $m_p\lambda_p/m_b$ is usually very small for a fuzzy-like plate. As a result, the plate in effect only adds damping to each mode of the beam. Eq. (62) indicates that the effective damping is independent of the internal damping of the plate itself. These conclusions are consistent with those of fuzzy structure theory.

It is worth noting from Eq. (62) that the effective loss factor loaded to each mode of the beam by a fuzzy-like plate attachment can be simply estimated as $\eta'_n \approx m_p\lambda_p/m_b\pi$.

4. Numerical examples

The beam-stiffened plate system, comprising a free–free beam attached to a simply supported plate as shown in Fig. 4, is considered in this section. The relevant dimensions and the coupling

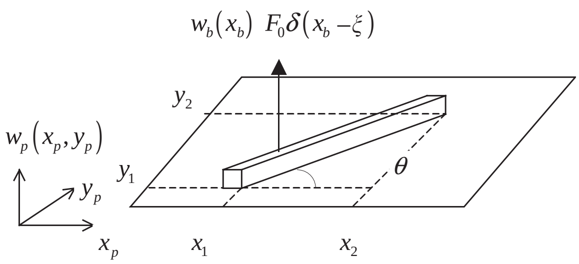

Fig. 4. Example plate-stiffened beam.

Table 1
System dimensions and coupling positions of Fig. 4

Structure	Beam	Plate
Dimension sizes (m)	Length $L_b = 2$, Width $t_b = 0.050$, Height $h_b = 0.020$	Length $L_x^{(p)} = 2$, Width $L_y^{(p)} = 0.9$, Thickness $h_p = 0.006, 0.002$
Coupling positions (m)	$x_1 = 0.03$, $y_1 = 0.3$, $\theta = 10^\circ$.	
Driving point (m)	$\xi = 0.73$	
Wavenumber ratio	$k_p/k_b = 1.8, 3.0$	

Table 2
Material properties of the beam-stiffened plate model

Young's modulus (GN/m^2)	Poisson ratio	Loss factor	Density (kg/m^3)
4.4	0.38	0.05	1152

locations are given in Table 1, and the material (perspex) properties are listed in Table 2. Two plate thicknesses 0.006 and 0.002 m are considered, corresponding to wavenumber ratios $k_p/k_b =$ 1.8 and 3.0, respectively. A time harmonic point force of unit amplitude acts at a distance ξ from one end of the beam. Results are shown for both the point mobility of the beam at the driving point and the power transmitted to the plate, as well as the effective mass and loss factor the 0.002 m thick plate adds to the first 3 modes of the beam.

4.1. Point mobility of the beam and the power transmitted to the plate

It is known in Ref. [1] that the FRF-based substructuring method can provide an 'exact' solution for the dynamic response of a beam-stiffened plate system when the line-coupling is simulated by many discrete point couplings. These points should be spaced at most a quarter of the plate wavelength apart. For example, at the frequency 1000 Hz the wavelengths in the 0.006 and 0.002 m thick plates are, respectively, about 0.152 and 0.088 m, and hence the connecting points required should be spaced at no more than 0.038 and 0.022 m apart at this frequency. In this section predictions made by the FRF-based substructuring method are compared to those of the Mode/FT approach and the locally reacting impedance method. The results are shown in Figs. 5–8, respectively, for $k_p/k_b = 1.8$ and 3.0. The first 30 modes of the beam are included for all calculations, whereas the exact solutions contain the first 1600 and 2400 modes of the 0.006 and 0.002 m thick plates, as well as 53 and 94 connecting points, respectively.

It is seen in Figs. 5 and 6 that when the wavelengths in the beam and the plate are comparable (e.g., $k_p/k_b = 1.8$), both the beam and the plate properties contribute significantly to the response of the coupled system. In this case, the Mode/FT approach, which approximates the plate receiver

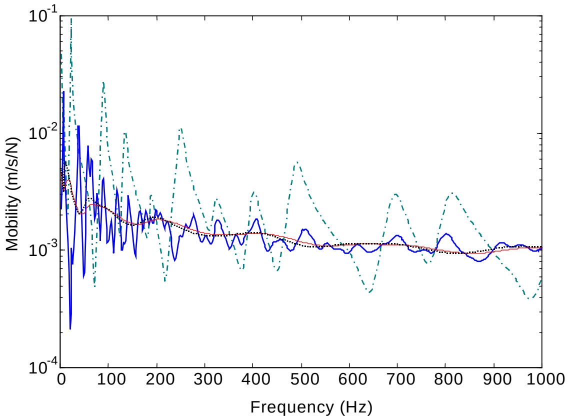

Fig. 5. Point mobilities of the beam at the driving point: before coupling to the plate (- · · · -, mode analysis method); after coupling to the 6 mm thick plate ($k_p/k_b = 1.8$); (——, FRF-based substructuring method; ——, Mode/FT method; ....., locally reacting impedance method).

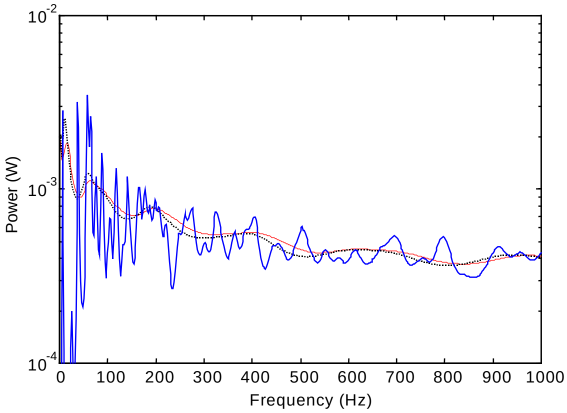

Fig. 6. Power transmitted from the beam to the plate ($k_p/k_b = 1.8$): ——, FRF-based substructuring method; ——, Mode/FT method; ....., locally reacting impedance method.

as if it were infinite, is too broad-brush to give accurate discrete frequency results. Nor is the locally reacting impedance method, which treats the plate as being locally reacting. However, in Figs. 7 and 8, where $k_p/k_b = 3.0$, it is seen that the main 'peaks' and 'troughs' of the response curves are largely controlled by the modal properties of the beam, while only the small 'wrinkles' appearing in the response curves are sensitive to the dynamics of the plate. These observations indicate, as expected, that the exact details regarding boundary conditions, size and shapes of a very flexible receiver, tend to be less important when estimating the broad features of the coupled response of the system as the dynamic mismatch of the system increases. As a result, the Mode/FT approach can be used to provide a fairly good estimate but with much lower computational cost (e.g., about 5% and 2% of the FRF-based substructuring method when $k_p/k_b = 1.8$ and $k_p/k_b = 3.0$, respectively) since relatively very few degrees of freedom are needed. This advantage is more noticeable as the flexibility of the plate increases, in that a very large number of connecting points are generally required by the FRF-based substructuring method.

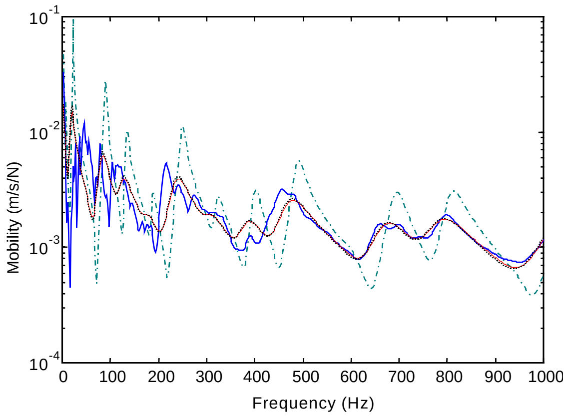

Fig. 7. Point mobilities of the beam at the driving point: before coupling to the plate (- · · · -, mode analysis method); after coupling to the 2 mm thick plate ($k_p/k_b = 3.0$); (——, FRF-based substructuring method; ——, Mode/FT method; ....., locally reacting impedance method).

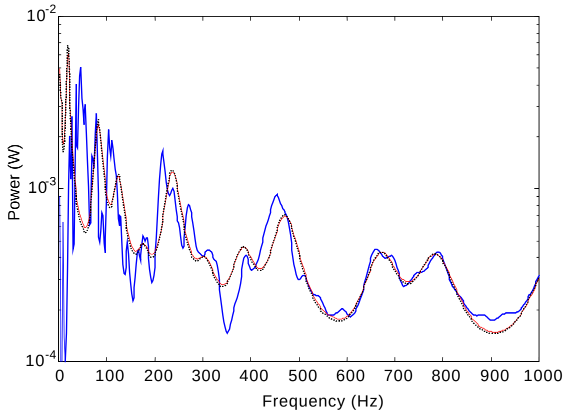

Fig. 8. Power transmitted from the beam to the plate ($k_p/k_b = 3.0$): ——, FRF-based substructuring method; ——, Mode/FT method; ....., locally reacting impedance method.

Figs. 5–8 also show that the locally reacting impedance method, being a special case of the Mode/FT approach, is most useful when the beam-stiffened plate system has a big dynamic mismatch, e.g., $k_p/k_b > 2$. The computational cost is only about 2% of the Mode/FT approach. This is because the plate-loaded modal impedance Z_n, in this case, can be taken as independent of the order of the beam mode.

It is worth noting that although the Mode/FT approach (locally reacting impedance method) was developed based on an infinite (locally reacting) plate approximation, it can be very useful to deal with beam/plate coupled structures where the exact dynamic properties of the plate receivers may not be available, due to property and boundary uncertainties, for example. Provided the plate receiver is flexible enough so that it exhibits non-resonant behaviour in the frequency range of interest, the Mode/FT approach can be used to give quite an accurate estimate of the vibration of the beam/plate coupled structure, at least in a frequency average sense.

4.2. Effective mass and loss factor

Figs. 9 and 10 show, respectively, the effective mass and loss factor added to the first 14 bending modes of the beam by the 0.002 m thick plate when it is either assumed to be infinite or modelled as locally reacting. In Fig. 9, a dimensionless mass is used which is defined as ratio of m_n/m_b or m'_n/m_b. It is seen that there are very small differences between the two sets of results in Fig. 9 but that the predictions are virtually identical in Fig. 10. Hence, Eqs. (61) and (62) can estimate the effective mass and damping the plate adds to the beam in the case of $k_p/k_b > 2$ in a simple manner. Meanwhile, it can also be observed from Figs. 9 and 10 that the effective mass decreases as frequency increases so that the plate can be taken as mainly to add effective damping to the beam in the high frequency range. Since the damping effects are really only important at the beam resonances, the effective damping η_n in Eq. (62) can be taken as $(m_p/m_b)(\lambda_{p,n}/\pi)$, where $\lambda_{p,n}$ corresponds to the plate wavelength at $\omega \approx \omega_{b,n}$. These values are shown in Fig. 10 by the points

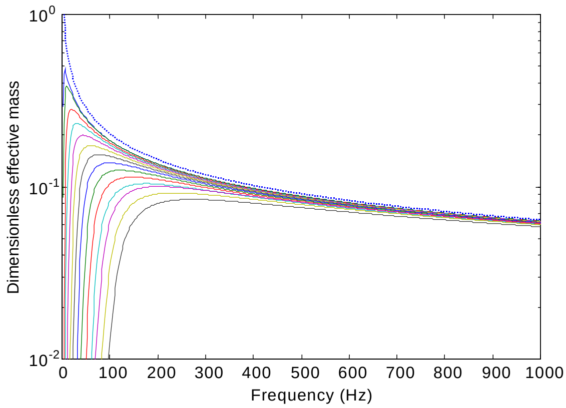

Fig. 9. Dimensionless effective mass induced to the first 14 bending modes of the beam: ——, Eq. (51); ....., Eq. (61).

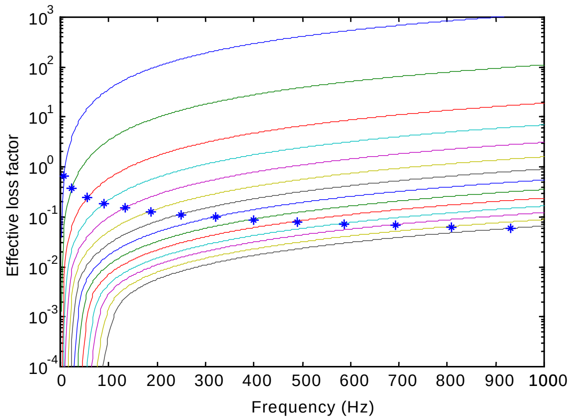

Fig. 10. Effective loss factor induced to the first 14 bending modes of the beam: ——, Eq. (52); ....., Eq. (62).

marked $*$ corresponding to the first 14 resonant frequencies. It indicates that the lower the order of the beam mode, the larger the effectively loaded loss factor.

5. Concluding remarks

In this paper, a hybrid Mode/FT approach was described for estimating the vibration response of a beam-stiffened plate system. Provided the plate receiver is flexible enough so that its vibration tends to exhibit non-resonant behaviour in the frequency range of interest, this approach can give a very simple and accurate estimate for both the FRFs of the beam and the power transmitted to the plate. When the plate receiver is much more flexible than the beam (e.g., $k_p/k_b > 2$), a locally reacting plate approximation was incorporated into the Mode/FT procedure to yield a locally reacting impedance method in an even simpler way. The performance of these two approximation approaches was demonstrated by numerical examples.

Meanwhile, it was seen that the dynamic interaction between the beam and the plate could be interpreted as the plate adding effective mass and damping to each mode of the beam. When the plate behaves like fuzzy attachments to the beam, the plate can be taken as mainly adding damping to each mode of the beam. Moreover, the effective damping is independent of the internal damping of the plate itself. These are in good agreement with the results of fuzzy structure theory. The numerical investigations also indicated that relatively more damping is added to the lower orders of modes of the beam.

Appendix A. List of symbols

D_b, D_p	bending stiffness of the beam and the plate
f_e	external force acting on the beam
f_I^b	interface force distribution on per unit length of the beam
$f_{e,n}$, $f_{I,n}^b$	nth modal forces corresponding to f_e and f_I^b
f_I^p	interface force distribution on per unit area of the plate
$\bar{f}_I^p$	force distribution on per unit length of the coupling line of the plate
F_I^p	Fourier transform of f_I^p (or $\bar{f}_I^p$)
k	wavenumber
k_b, k_p	wavenumbers of the beam and the plate
k_x, k_y	wavenumbers in x and y directions
L_b	length of the beam
m_b, m_p	mass distributions of the beam and the plate
m_n	effective mass added to the nth mode of the beam by the plate
m_n'	corresponding effective mass by loading a locally reacting plate
P	power transmitted from the beam to the plate
w_b, w_p	displacements of the beam and the plate
$w_{b,n}$	nth modal amplitude of the beam
W_b, W_p	Fourier transform of w_b and w_p
x_b	local co-ordinate of the beam

(x_p, y_p)	local co-ordinates of the plate
(x_p^I, y_p^I)	Interface locations on the plate
$Y_{b,n}$	*n*th modal receptance of the uncoupled beam
$Y'_{b,n}$	*n*th modal receptance of the coupled beam
Z_n	modal impedance added to the *n*th mode of the beam by the plate
Z'_n	modal impedance corresponding to a locally reacting plate
Z_p	line-impedance of an infinite plate
Z'_p	line-impedance of a locally reacting plate
$\eta_{b,n}$	*n*th modal loss factor of the beam
η_n	effective loss factor added to the *n*th mode of the beam by the plate
η'_n	effective loss factor corresponding to a locally reacting plate
θ	phase constant
λ_p	wavelength of the plate
$\lambda_{p,n}$	wavelength of the plate at the *nth* natural frequency of the beam
$\phi_{b,n}$	*n*th natural mode of the beam
$\hat{\phi}_{b,n}$	extended orthogonal function of $\phi_{b,n}$
$\Phi_{b,n}$	Fourier transform of $\hat{\phi}_{b,n}$
ω	radiation frequency
$\omega_{b,n}$	*n*th natural frequency of the beam

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