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Statistical energy analysis for complicated coupled system and its application in engineering

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Abstract

A new SEA method is developed in this paper to analyze the vibration and noise radiation from the complicated coupled systems. Firstly, some elements influence the energy transmission between the two coupled mechanical structures are separated and corresponding parameters are introduced to investigate those influences separately. Linking style coefficient is introduced to denote the rule of vibration energy transmission when structures are linked by one point, some points or a line. Non-conservative coupling coefficient is introduced to describe the influence of isolation or damping when structures are isolated or damped. Indirect coupling coefficient is introduced to research the property of vibration energy transmission when two structures are indirectly linked by other structure. Secondly, on the basis above, the gradation analysis is put forward to simplify the vibration analysis of complicated coupled system. Thirdly, vibration and noise radiation from an underwater vehicle is analyzed by the developed SEA. Levels of vibration and sound power induced by the underwater vehicle are predicted. The analysis results agree well with the experiment results. Finally, based on the analysis, the way to control the noise from the vehicle is pointed out.

1. Definitions of SEA parameters

Since non-conservatively coupling is very common in engineering, more and more studies of non-conservatively coupled system were carried out.

Some researchers [1] believe that non-conservatively coupling increases the internal losses of both the two coupled structures (Fig. 1). By apportioning the non-conservatively coupled loss to internal losses of the two coupled structures, one can express the energy balance equation by

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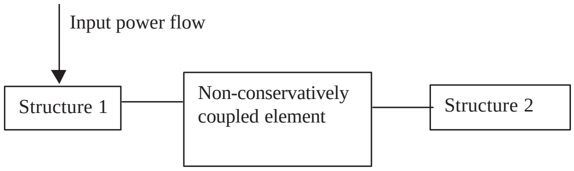

Fig. 1. Two non-conservatively coupled structures system.

means of SEA

$$P_1 = \omega(\eta_1' + \eta_{12})E_1 - \omega\eta_{21}E_2, \tag{1}$$

where $\eta_1' = \eta_1 + \Delta\eta_1$ is the equivalent internal loss factor, the increment $\Delta\eta_1$ suggests the effect of the non-conservative coupling. According to this change, the non-conservative coupling increases the equivalent internal loss factor. But in the later experiments, negative loss factor is obtained [2], which means equivalent internal loss factor may be less than zero in some times. The negative loss factor seems unreasonable, and was usually regarded as measurement errors. In fact, structures' coupling makes the transmission of vibration more complicated, and $\Delta\eta_1$ may not be an increment.

Some other researchers [3] suggest to introduce the coupling damping loss factor to express the characteristic of the non-conservative coupling. Thus the energy balance equation is rewritten as

$$P_1 = \omega(\eta_1 + \eta_{12} + \varsigma_{12})E_1 - \omega\eta_{21}E_2, \tag{2}$$

where internal loss factor still means the internal loss factor of structure, coupling loss factor still means the coupling loss factor of conservatively coupled system, and the coupling damping loss factor ς_{12} means the additional loss from the coupling.

Consequently, the authors proposed a new method by introducing the equivalent internal loss factor which is not gained by simply adding a positive increment to structure's internal loss factor. Here, the energy balance equation still works:

$$P_1 = \omega(\eta_1' + \eta_{12}')E_1 - \omega\eta_{21}'E_2, \tag{3}$$

where η_1' is the equivalent internal loss factor; η_{12}' and η_{21}' are the non-conservative coupling loss factors. The coupling loss factor of the non-conservatively coupled system indicates the transmission characteristic of power flow between the non-conservatively coupled structures. The equivalent internal loss factor adds a modified sect to the structures' internal loss factor and it reveals the difference of the energy transmission between two ends of the coupling. The modified sect can be positive or negative even make the equivalent internal loss factor negative.

2. Equivalent internal loss factor of non-conservatively coupled oscillators

Fig. 2 shows a two non-conservatively coupled oscillators system. Analysis model is set up as in Fig. 3. The energy balance equations of oscillators 1 and 2 can be expressed as

$$\begin{aligned} P_1 &= \omega_1(\eta_1 + \eta_{12}^{\mathrm{I}})E_1 - \omega_2\eta_{21}^{\mathrm{I}}E_2, \\ P_2 &= \omega_2(\eta_2 + \eta_{21}^{\mathrm{II}})E_2 - \omega_1\eta_{12}^{\mathrm{II}}E_1, \end{aligned} \tag{4}$$

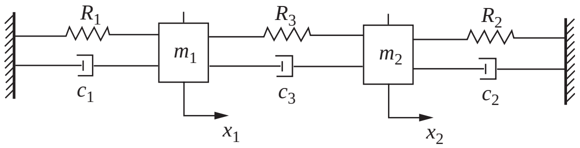

Fig. 2. Two non-conservatively coupled oscillators system.

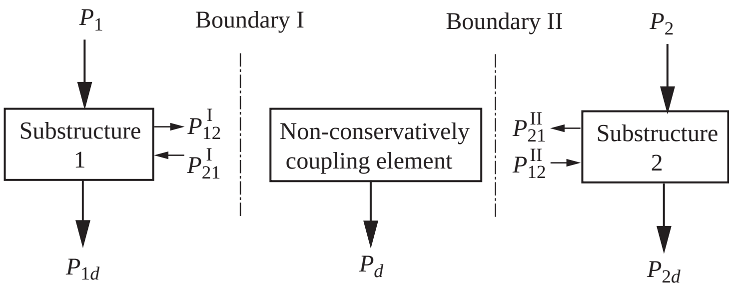

Fig. 3. SEA model of non-conservatively coupled system.

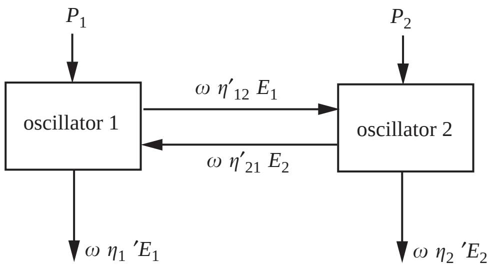

Fig. 4. The effective SEA model of non-conservatively coupled system.

where η_{ij}^{I} means the coupling loss factor occurred in boundary I from substructure i to substructure j; η_{ij}^{II} has a meaning similar to that which occurs on boundary II, and $i,j = 1, 2$.

The effective SEA model is put forward in this paper, shown in Fig. 4. Equations can be derived as

$$\begin{aligned} P_1 &= \omega_1(\eta_1' + \eta_{12}')E_1 - \omega_2\eta_{21}'E_2, \\ P_2 &= \omega_2(\eta_2' + \eta_{21}')E_2 - \omega_1\eta_{12}'E_1. \end{aligned} \tag{5}$$

From Eqs. (4) and (5), one can obtain the equivalent internal loss factor and the non-conservative coupling loss factor

$$\begin{aligned} \eta_{12}' &= \eta_{12}^{\mathrm{II}}, \qquad \eta_1' = \eta_1 + (\eta_{12}^{\mathrm{I}} - \eta_{12}^{\mathrm{II}}), \\ \eta_{21}' &= \eta_{21}^{\mathrm{I}}, \qquad \eta_2' = \eta_2 + (\eta_{21}^{\mathrm{II}} - \eta_{21}^{\mathrm{I}}). \end{aligned} \tag{6}$$

The equivalent internal loss factor of the non-conservative coupled system is not equal to the structures' internal loss factor. Coupling characteristic has important influence to equivalent internal loss factor. Only when the coupling damping is small enough, the equivalent internal loss factor is close to the internal loss factor of the substructure.

In order to make numerical analysis easier, a model of two non-conservatively coupled oscillators system is set-up, shown in Fig. 4. Assuming that external exciting frequency is f and oscillator's parameters are decided, i.e., for oscillator 1, mass $m_1 = 1.5$ kg, stiffness $K_1 = 400$ N/m, damping coefficient $c_1 = 0.5$ Ns/m, for oscillator 2, mass $m_2 = 2.0$ kg, stiffness $K_2 = 200$ N/m, damping coefficient $c_2 = 1.0$ Ns/m.

Fig. 5 displays the curve of the equivalent internal loss factor η'_1 influenced by c_3. There is an abrupt change of equivalent internal loss factor at resonance frequency of oscillator 2. A valley of η'_1 occurs when exciting frequency f is a little lower than the nature frequency of oscillator 2, f_2, and a peak occurs when f is a little higher than f_2. Abrupt change is more obvious when the coupling damping is larger. But beyond the resonance frequency range, equivalent internal loss factor is higher when coupled damping is larger, as one can foresee.

The influence curve of the coupling stiffness k_3 to the equivalent internal loss factor η'_1 can be seen in Fig. 6. It reveals that when the coupling stiffness becomes larger, fall of the peak to the valley becomes greater, which means coupling stiffness has catalysis equivalent to internal loss factor. Furthermore, analyses [4] tell that the peak of η'_1 corresponds to the valley of η'_2, vice versa.

Thus some important conclusion can be obtained, i.e., the equivalent internal loss factor of non-conservatively coupled system reveals not only the internal loss factor of the structure, but also the difference of the power flow between two ends of non-conservative coupling. For conservatively coupled system, the difference of power flow between two ends is zero, and the equivalent internal loss factor equal to the internal loss factor of the structure.

This conclusion is useful to SEA of non-conservatively coupled system. However, practical system consists of continuous multi-modal structures with complex vibration response characteristic different from oscillator systems. Thus the conclusion derived from the study of oscillator's power flow cannot be used directly in SEA of practical system. The characteristic of power flow of structural coupled system is studied as follows.

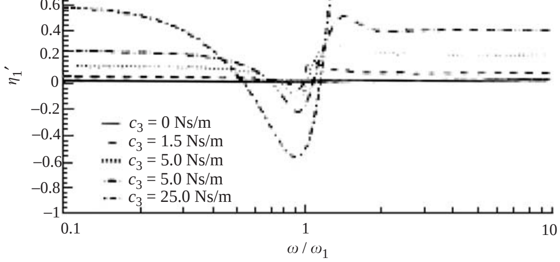

Fig. 5. The influence of coupling damping on effective loss factor.

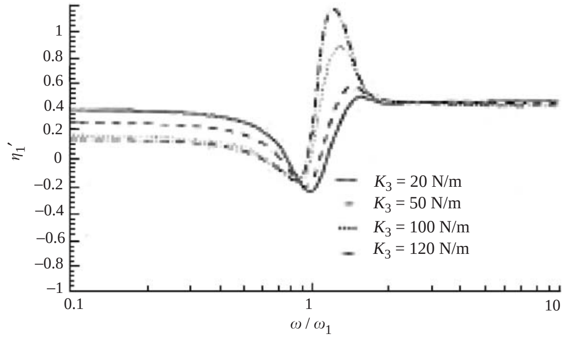

Fig. 6. The influence of coupling stiffness on effective loss factor.

3. Point-like and line-like joint systems

For a structure with mass m and loss factor η, suppose that the real part of the average driving point mobility is G and the average transfer point mobility is y. One gets [5]

$$G = \frac{n}{4M}, \quad |y|^2 = \frac{G}{\omega \eta M}, \tag{7}$$

where n is the modal density.

In a conservatively coupled system, two structures are connected by a single point. In the case when there is an external force acting at structure 1, average transfer mobility from structure 1 to structure 2 by using four ports network approach is given by

$$y_{12} = \frac{y_1 y_2}{Y_1 + Y_2}, \tag{8}$$

where y_{12} is the transfer mobility between two structures; Y_1 and Y_2 are driving mobilities of structures 1 and 2, respectively; y_1 and y_2 are average transfer mobilities of structures 1 and 2, respectively. If only structure 1 is excited, the ratio of average vibration energy of structure 2 to structure 1 can be rewritten as

$$E_{21}^{(1)} = \frac{G_2}{\omega \eta_2 M_1 |Y_1 + Y_2|^2}. \tag{9}$$

Similarly, the ratio of the average vibration energy of structure 1 to structure 2, $E_{12}^{(2)}$ can be acquired if only structure 2 is excited. From the energy balance equations of SEA, coupling loss factor may be expressed as

$$\begin{aligned} \eta_{12} &= \frac{\eta_2}{E_{21}^{(1)} - \eta_1 E_{12}^{(2)}}, \\ \eta_{21} &= \frac{\eta_1}{E_{12}^{(2)} - \eta_2 E_{21}^{(1)}}. \end{aligned} \tag{10}$$

If there is a conservatively coupled system jointed by two continuous structures via N separated points, making use of four ports network approach, the transfer mobility between two structures is given as

$$y'_{12} = \frac{y_{12}}{p}. \tag{11}$$

Defining p as the linking style coefficient,

$$p = \frac{1}{N} + \frac{N-1}{N} \cdot \frac{y_1 + y_2}{Y_1 + Y_2}. \tag{12}$$

Supposing that the transfer mobility is far less than the driving mobility [5], one gets

$$E_{21}^{(1)'} = \left|\frac{1}{p}\right|^2 \cdot E_{21}^{(1)}. \tag{13}$$

For a line joint system, the number of connecting points N can be expressed as [5]

$$N = \frac{\pi}{4} \frac{L}{\lambda}, \tag{14}$$

where L means the length of connecting line and λ means the wavelength.

For an experimental system shown in Fig. 7, the measured results show good coincidence with the theoretical prediction. The experimental system is made of two steel plates linked by bolts. Fig. 8 gives out the predictions and the experimental measured results of vibration energy ratio. The curves show that the structure's measured vibration energy ratio fluctuates around the theoretical predicted values. On this account, the analysis method and formula studied above are reliable and applicable in vibration analysis of multi-point coupled system.

4. Non-conservatively coupled system

Non-conservatively coupled system is very common in engineering systems. In order to make SEA applicable widely in practice, study of the vibration analysis and prediction of non-conservatively coupled systems is necessary [6].

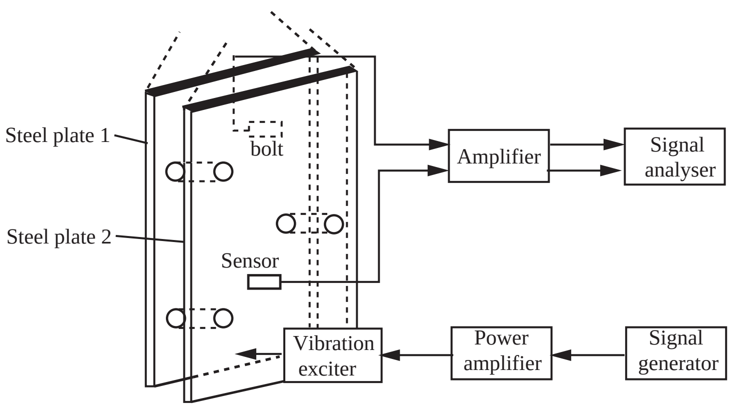

Fig. 7. Experimental system.

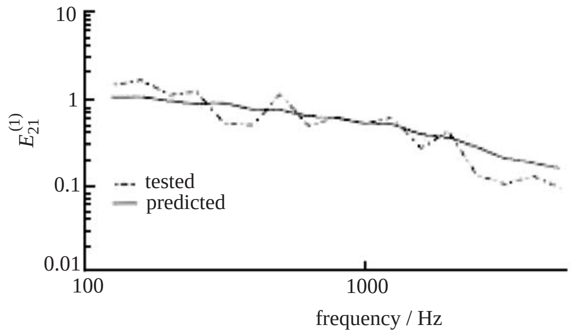

Fig. 8. Energy ratio of structures.

Fig. 9 gives a SEA analysis model of structural non-conservatively coupled systems. Energy balance equation can be expressed as Eqs. (5). Assuming that only structure 1 is excited, Eqs. (5) can be rewritten as

$$\eta_{1s} = \eta'_1 + \eta'_{12} - \eta'_{21}E_{21}^{(1)},$$
$$0 = \eta'_2 + \eta'_{21} - \eta'_{12}/E_{21}^{(1)}, \tag{15}$$

where η_{1s} means the total loss factor of substructure 1. Similar equations can be got if only structure 2 is excited. Combining these equations, one can acquire equivalent internal loss factor and coupling loss factor as follows:

$$\eta'_1 = \frac{\eta_{1s} + \eta_{2s}E_{21}^{(1)}}{1 - E_{21}^{(1)}E_{12}^{(2)}}, \quad \eta'_2 = \frac{\eta_{2s} + \eta_{1s}E_{12}^{(2)}}{1 - E_{21}^{(1)}E_{12}^{(2)}},$$
$$\eta'_{12} = \frac{E_{21}^{(1)}}{1 - E_{21}^{(1)}E_{12}^{(2)}}\eta_{2s}, \quad \eta'_{21} = \frac{E_{12}^{(2)}}{1 - E_{21}^{(1)}E_{12}^{(2)}}\eta_{1s}. \tag{16}$$

From Eqs. (15) and (16), one finds that to calculate the equivalent internal loss factor and coupling loss factor of non-conservatively coupled system, it is necessary to figure out the structure's vibration energy ratio and total loss factors with single excitation which can be obtained by mobility analyzing.

Here, Y_c means the mobility of non-conservative coupling, the transfer mobility between two structures can be shown to be given by

$$y_{12} = \frac{y_1 y_2}{Y_1 + Y_2 + Y_c}. \tag{17}$$

Define the non-conservative coupling coefficient by

$$q = \frac{y_{12}(con)}{y_{12}(incon)} = 1 + \frac{Y_c}{Y_1 + Y_2}. \tag{18}$$

The transfer mobility between two structures is far less than driving mobility of structure for most mechanic systems, so the computing of the total loss factor and vibration energy ratio can be

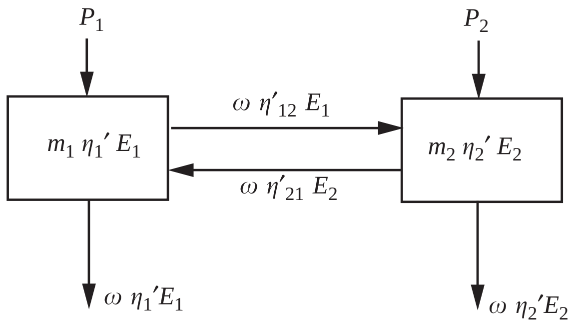

Fig. 9. SEA model of non-conservatively coupled machine system.

simplified as

$$\eta_{1s} = \frac{\eta_1}{\left|1 - \frac{1}{q} \cdot \frac{y_1}{Y_1 + Y_2}\right|^2},$$
$$\eta_{2s} = \frac{\eta_2}{\left|1 - \frac{1}{q} \cdot \frac{y_2}{Y_1 + Y_2}\right|^2}, \tag{19}$$

and

$$E_{21}^{(1)}(incon) = \frac{1}{|q|^2} E_{21}^{(1)}(con),$$
$$E_{12}^{(2)}(incon) = \frac{1}{|q|^2} E_{12}^{(2)}(con). \tag{20}$$

The non-conservative coupling coefficient is a bridge between the conservatively and non-conservatively coupled systems. In disposing a conservatively coupled system by vibration isolating or damping, non-conservatively coupled coefficient is introduced to describe the influence of isolation or damping easily.

Experiment is carried out to validate the theory's correction, the experimental system is made of two steel plates connected with damper as shown as Fig. 10. Fig. 11 presents the experimental results that show good consistency compared with theoretically predicted results.

5. Series coupled system

The development of SEA includes power flow research in coupled oscillators and SEA analysis of continuous structures. Many research works had been performed about oscillators, from two conservatively coupled oscillators to two non-conservatively coupled oscillators, from three series conservatively coupled oscillators, to three series non-conservatively coupled oscillators, and many important conclusions have been gained. But studies on power flow of structural coupled systems are relatively few.

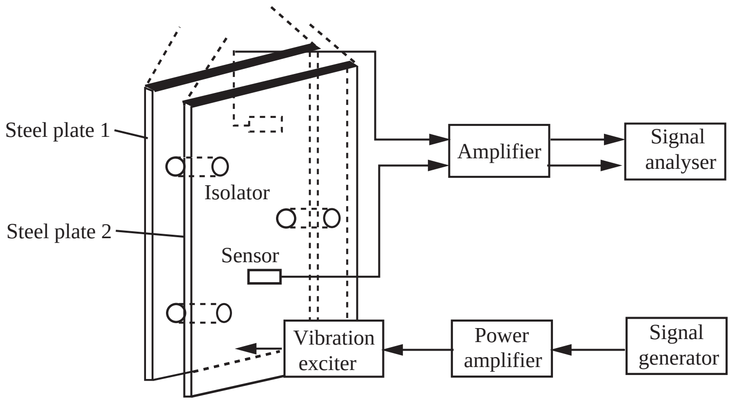

Fig. 10. Experimental system.

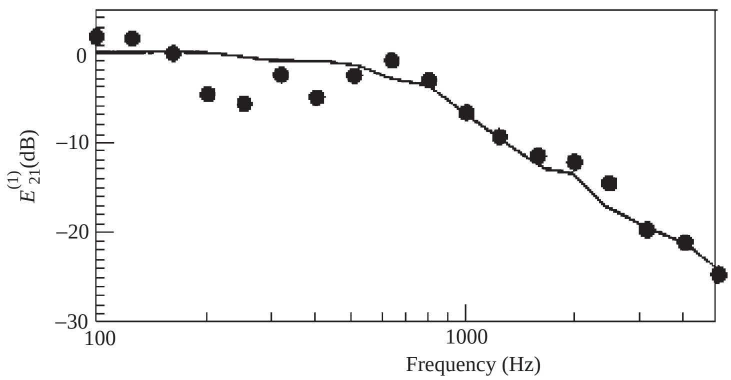

Fig. 11. Energy ratio of non-conservatively coupled structures.

For a three conservatively coupled structures system, making use of four ports network approach, transfer mobilities are given by

$$y_{12} = y_{21} = \frac{y_1 y_2 (Y_2 + Y_3 - y'_2)}{(Y_1 + Y_2)(Y_2 + Y_3) - y'^2_2},$$
$$y_{23} = y_{32} = \frac{y_2 y_3 (Y_1 + Y_2 - y'_2)}{(Y_1 + Y_2)(Y_2 + Y_3) - y'^2_2},$$
$$y_{13} = y_{31} = \frac{y_1 y_2 y_3}{(Y_1 + Y_2)(Y_2 + Y_3) - y'^2_2}. \tag{21}$$

The two boundaries of structure 2 are connected with structure 1 and structure 3, respectively. y'_2 in Eqs. (21) is on behalf of transfer mobility between the two boundaries and y_2 represents the average transfer mobility of structure 2.

The indirect coupling coefficient is introduced here to relate two substructures system and complicated coupled system. Define indirect coupling coefficient by

$$t_{1-2-3} = \frac{y_2'}{y_2}. \tag{22}$$

Since transfer mobility is far less than driving mobility, taking the advantage of indirectly coupling coefficient, the relationship of the vibration energy ratio between the series coupled system and the simplest two substructures system is expressed as

$$E_{31}^{(1)} = E_{21}^{(1)}(two) \cdot E_{32}^{(2)}(two) \cdot |t_{1-2-3}|^2. \tag{23}$$

Similarly, for two arbitrary structures in a N-substructure series coupled system, vibration energy ratio of them can be written as

$$E_{mi}^{(i)}(\cdots - i - j - k - l - \cdots) = E_{ji}^{(i)}(i-j) \cdot E_{kj}^{(j)}(j-k) \cdot E_{lk}^{(k)}(k-l) \cdot |t_{i-j-k}|^2 \cdot |t_{j-k-l}|^2. \tag{24}$$

One finds that it is easier to acquire the vibration energy ratio under single excitation in terms of indirectly coupling coefficient and vibration energy ratio is vital in computing the primary parameters of SEA.

Performing the experiment as in Fig. 12 to verify the theory, two steel plates were coupled through a steel beam. Result is shown in Fig. 13. Theoretical computing results of vibration energy ratio of two indirect coupled structures are compared with experiment results. The outcome shows that theoretical prediction and experimental measure are accordant except for a little error, and furthermore the analysis testifies that it is caused by beam's shape error.

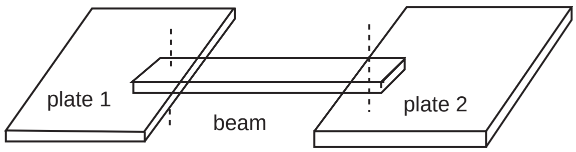

Fig. 12. Experimental system.

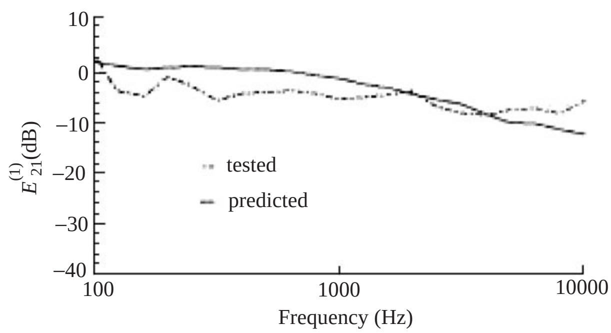

Fig. 13. Energy ratio of indirectly coupled structures.

6. Complicated coupled system

Here one goes into line joint, non-conservatively coupled and multi-structures series systems gradually. Based on these changes, complex coupled system is dealt with. Namely, it combines characteristics of line joint, non-conservatively coupled, and in series or parallel connection. For all know that a system always consist of many structures and there are all kinds of connection styles in real world beside in simple series. Sometimes it is complicated like a net. The key point is to simplify the system convenient but keeping certain precision. By this token, the authors bring forward a gradation analysis model of complicated coupled system to meet this need. See Fig. 14.

In gradation analysis model, the structure directly excited by external force is called the source since it is the vibration source of all the other structures in this system. Those structures, whether connected with each other or not, its connected with source is named structure in the first layer. Structure in the second layer is connected with source indirectly by the medium of structures in the first layer. They can be joint to each other or joint to multi-structures in the first layer at the same time. Same as structure in the third layer, the fourth layer, and so on. Stated differently, complicated coupled system like a net, where structures are nodes and the connections of structures are lines of the net, is formed. Lifting one node, just like the source of gradation analysis model, the layers below clearly appear.

Energy balance equation of complicated coupled system gives

$$P_i = \omega \left(\eta'_i + \sum_{\substack{j=1 \\ j \neq i}}^{N} \eta'_{ij} \right) E_i - \omega \sum_{\substack{j=1 \\ j \neq i}}^{N} (\eta'_{ji} E_j). \tag{25}$$

Before analyzing the vibration of a system composed by N substructures, one must ascertain N equivalent internal loss factors and $N \times (N-1)$ coupled loss factors. Coupling loss factor of two arbitrary structures is acquired by means of the equation

$$\sum_{\substack{j=1 \\ j \neq k}}^{N} \{ [E_{jk}^{(i)} - E_{jk}^{(k)}] \cdot \eta'_{jk} \} = \eta_{ks} \tag{26}$$

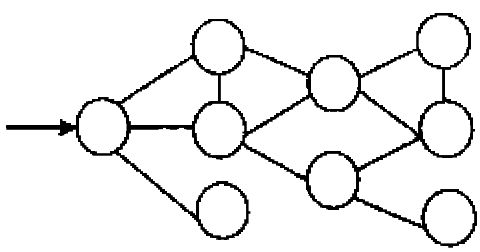

Fig. 14. Gradation analysis model of complicated coupled system.

while equivalent internal loss factor is computed by the equation

$$\eta_i' + \sum_{\substack{j=1 \\ j \neq i}}^{N} \eta_{ij}' = \eta_{ks} + \sum_{\substack{j=1 \\ j \neq k}}^{N} [E_{jk}^{(k)} \eta_{jk}']. \tag{27}$$

In this paper, the subject is universal. Thus the conclusion has wide applicability so as assuming that transfer mobility is far less than driving mobility.

7. Vibrations and noise prediction of vehicle

Most of the noise radiation of vehicle is from structural vibration. Here, the property of the vibration and noise radiation of an vehicle is analyzed by the developed SEA and compared with experimental results. The SEA model of the vehicle is shown as Fig. 15. Fig. 16 gives the vibration transfer loss of the vehicle under working condition by both theoretical prediction and experimental measure, and the result predicted by conservative SEA and the developed SEA are both shown in the figure. The result shows more preferable accordance between the developed SEAs compared to conservative SEA with measured data. In Fig. 17, the noise radiation predicted by the developed SEA is compared with experiment results. Additionally, in Figs. 16 and 17, the results show in the way of relative vibration transmission and sound pressure level.

More analysis indicates that two very effective ways can be used to reduce the noise radiation of the vehicle. The best way is to cut down the vibration energy transmission from the engines to the shells, and vibration isolation can be taken in practical work. The other way is to reduce the vibration levels of the engines, shells outside and near the engines by damping disposal. Through gradation analysis, it is also found that little effect may be induced by damping with shells far away from the engines.

8. Conclusion

The main contributions of studies stated in this paper are

1. A bridge between the complex coupled structures system with the two conservatively coupled structures system is built up by introducing three coefficients. Linking style coefficient is introduced to denote the rule of vibration energy transfer when structures are linked at one point, some points or a line. Non-conservative coupling coefficient is introduced to describe the influence of isolation or damping when structures are isolated or damped. Indirect coupling coefficient is introduced to research the property of vibration energy transfer when two structures are indirectly linked by other structure.
2. An approach of gradation analysis is put forward to simplify the vibration analysis of complicated coupled system. By this approach, a complex system may be simplified conveniently. Accordingly, the energy balance equation of the complicated coupled system is given.

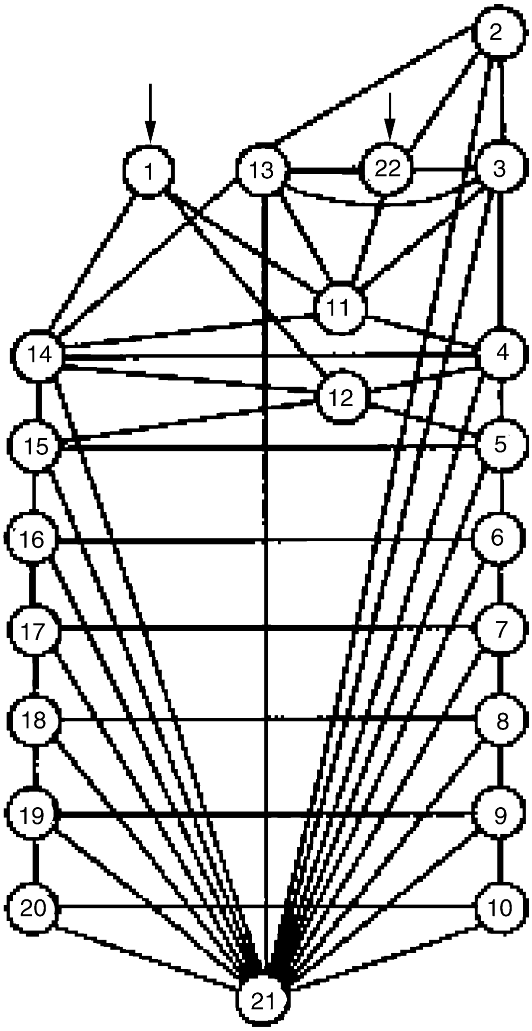

Fig. 15. SEA model of the vehicle structure 1: the shell of motor; structure 2–10: the shells from tail to head of the vechile; structure 11 and 12: plates; 13–20: the sound spaces in structures 2–10; 21: the sound space outside vechile; 22: axis.

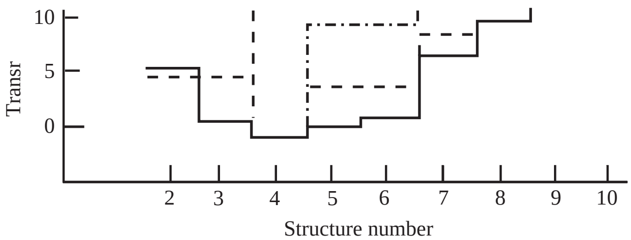

Fig. 16. Vibration transfer loss of vehicle (—) measured, (–·–) predicted by conservative SEA, (– –) predicted by advanced SEA.

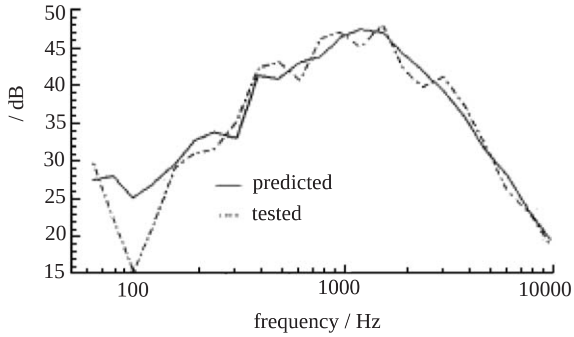

Fig. 17. Sound pressure lever of vehicle.

3. The vibration and noise radiation from an underwater vehicle is analyzed by the developed SEA, and the proper way to reduce noise radiation of the vehicle is pointed out. It is found that the best way to deduce the noise of the vehicle is isolating shells from engines, and damping shells outside and near the engines is also a good way.

Acknowledgements

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Appendix A. Nomenclature

c	damping coefficient
E_i	energy
$E_{ij}^{()}$	energy ratio
f	frequency
G	average driving point mobility
K	stiffness
L	length
M	mass
m	mass
N	modal number
n	modal density
P	power flow
p	linking style coefficient
q	non-conservatively coupling coefficient
t	indirect coupling coefficient
Y	driving mobility

y transfer mobility
η loss factor
λ wavelength
$\omega = 2\rho f$

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