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Determination of the steady state response of viscoelastically corner point-supported rectangular specially orthotropic plates under the effect of sinusoidally varying moment

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Abstract

Vibration of orthotropic rectangular plates having viscoelastic point supports at the corners under the effect of sinusoidally varying concentrated moment is analyzed. The Lagrange equation is used to examine the free vibration characteristics and the steady state response to a sinusoidally varying concentrated moment acting at the centre of a viscoelastically point-supported orthotropic elastic plate of rectangular shape. In the study, for applying the Lagrange equation, the trial function denoting the deflection of the plate is expressed in the polynomial form. By using the Lagrange equation, the problem is reduced to the solution of a system of algebraic equations. The influence of the mechanical properties, and of the damping of the supports on the mode shapes and the steady state response of the viscoelastically point-supported rectangular plates is investigated numerically, for a concentrated moment at the centre for various values of the mechanical properties which characterize the anisotropy of the plate material and for various damping ratios. The results of the natural frequencies are given for the first three antisymmetrical–symmetrical modes, and the steady state responses to a sinusoidally varying concentrated moment are determined for the frequency ranges of the first two antisymmetrical–symmetrical mode types. Convergence studies are made. The validity of the obtained results is demonstrated by comparing them with other solutions for free vibration analysis of point-supported or completely free rectangular plates for the first three antisymmetrical–symmetrical vibration modes based on the Kirchhoff–Love plate theory.

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1. Introduction

Free and forced vibrations of the point-supported plates are of considerable interest to the engineers designing panels at isolated points. Many researchers have investigated the free vibration analysis of rectangular isotropic and orthotropic plates supported at various points and based on the Kirchhoff–Love plate theory. However, it appears that there is only a limited number of studies on the steady state response of viscoelastically point-supported plates.

There are many studies concerning the free vibration analysis of rectangular isotropic and orthotropic plates that are completely free or supported at various points and based on Kirchhoff–Love plate theory (for example [1–7]). Although there are lots of studies on the free vibration analysis of rectangular plates supported at various points, there are only a limited number of studies on the steady state response of point-supported rectangular plates. The steady state response to a sinusoidally varying force was determined for a viscoelastically point-supported rectangular plate by Yamada et al. [8] by using the generalized Galerkin method. A generalization of this study to orthotropic rectangular plates was investigated by Kocatürk [9] by using the generalized Galerkin method, and by Kocatürk and Altintaş [10] by using an energy-based finite difference method. In the present study, the Lagrange equation is used to examine the free vibration characteristics and steady state response to a sinusoidally varying concentrated moment acting at the centre of a viscoelastically point-supported orthotropic elastic plate of rectangular shape. By analyzing the steady state response of the considered problem, the peak values of the moment transmissibilities are obtained. In the study, for applying the Lagrange equation, trial functions denoting the deflection of the plate are expressed in the polynomial form.

In many branches of modern industry, the structural elements, such as plates, are fabricated from composite materials. For this reason, the present investigation may be considered to be a problem of the mechanics of elements fabricated from composite materials. The purpose of the present work is to analyze the steady state response of a viscoelastically point-supported orthotropic plate to a sinusoidally varying moment for various values of the mechanical properties characterizing the anisotropy of the plate material by using the Lagrange equation. The problems considered are solved within the framework of the Kirchhoff–Love hypothesis. The convergence study is based on the numerical values obtained for various numbers of polynomial terms. In the numerical examples, the natural frequency parameters are determined for the first three antisymmetrical–symmetrical modes, and steady state responses to a sinusoidally varying moment are determined for the frequency ranges of the first two antisymmetrical–symmetrical mode types. Because there is no existing study on the steady state response of viscoelastically corner point-supported rectangular specially orthotropic plates under the effect of sinusoidally varying concentrated moment, the accuracy of the results is partially established by comparison with previously published accurate free vibration results of the first three antisymmetrical–symmetrical modes for the corner point-supported plates based on the thin plate theory.

2. Analysis

Consider a viscoelastically corner point-supported rectangular elastic orthotropic plate of side lengths a, b and thickness h under the effect of the sinusoidally varying concentrated moment $M(t)$

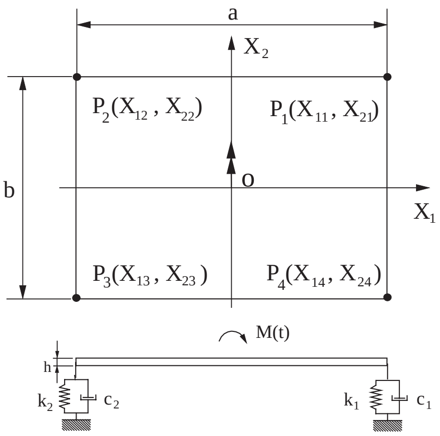


Fig. 1. Viscoelastically corner point-supported rectangular orthotropic plate subjected to an external force.

at the centre of the plate as shown in Fig. 1, where k_i is the spring constant, c_i is the damping coefficient, $P_i(X_{1i}, X_{2i})$ is the support force of a point support at the ith support. The axes of the elastic symmetry of the plate material coincide with the OX_1- and OX_2-axis. Also, the co-ordinate axes OX_1 and OX_2 are oriented along the edges of the plate with the origin at O. Therefore the plate is specially orthotropic. Because the plate is orthotropic and the supports are viscoelastic, there are lots of parameters to be considered: For this reason, although it is possible to take lots of point supports at arbitrary points, in the numerical investigations here, for brevity of the study, it will be considered that the plate is supported symmetrically at the four corner points and k_i and c_i are taken to have the same respective values at all the supports denoted by $k_i = k_s$ and $c_i = c_s$. Thus, in the considered loading and support conditions, only antisymmetrical–symmetrical vibrations arise in the plate. Under the above-mentioned conditions, the steady state responses of the viscoelastically corner point-supported plate to a sinusoidally varying concentrated moment for various orthotropy ratios, damping and spring constant values, will be determined by using the Lagrange equation.

For a plate undergoing sinusoidally varying concentrated moment $M(t) = Q.\mathrm{e}^{\mathrm{i}\omega t}$, where ω is the radian frequency, the strain energy of bending in Cartesian co-ordinates is given by

$$U = \frac{1}{2}\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\left[D_{11}\left(\frac{\partial^2 W}{\partial X_1^2}\right)^2 + 2D_{11}v_{21}\frac{\partial^2 W}{\partial X_1^2}\frac{\partial^2 W}{\partial X_2^2} + D_{22}\left(\frac{\partial^2 W}{\partial X_2^2}\right)^2 + 4D_{66}\left(\frac{\partial^2 W}{\partial X_1\,\partial X_2}\right)^2\right]\mathrm{d}X_1\,\mathrm{d}X_2. \tag{1}$$

In Eq. (1), D_{11}, D_{22}, D_{66} are expressed as follows:

$$D_{11} = \frac{E_1 h^3}{12\left(1 - v_{21}^2/e\right)}, \quad D_{22} = \frac{E_2 h^3}{12\left(1 - v_{21}^2/e\right)}, \quad D_{66} = \frac{G_{12}h^3}{12}, \tag{2}$$

where G_{12} is the shear modulus. In Eq. (2), the following definitions are used:

$$\frac{v_{12}}{E_1} = \frac{v_{21}}{E_2}, \quad e = \frac{E_2}{E_1}. \tag{3}$$

Here E_1, E_2 are Young's moduli in the OX_1- and OX_2-directions, respectively, and ν_{21} is the Poisson ratio for the strain response in the X_1 direction due to an applied stress in the X_2 direction. The potential energy of the external concentrated moment is

$$F_e = -M(t)\frac{\partial W(0,0,t)}{\partial X_1}. \tag{4}$$

With rotary inertia neglected, the kinetic energy of the vibrating plate is

$$T = \frac{1}{2}\int_{-a/2}^{a/2}\int_{-b/2}^{b/2} \rho h \left(\frac{\partial W}{\partial t}\right)^2 \mathrm{d}X_1\, \mathrm{d}X_2, \tag{5}$$

where ρ is the mass density per unit volume. The additive strain energy and dissipation function of viscoelastic supports are

$$F_s = \frac{1}{2}\sum_{i=1}^{4} k_i W_{Si}^2,$$

$$D = \frac{1}{2}\sum_{i=1}^{4} c_i(\dot{W}_{Si})^2. \tag{6}$$

Introducing the following non-dimensional parameters

$$x_1 = \frac{X_1}{a}, \quad x_2 = \frac{X_2}{b}, \quad \alpha = \frac{a}{b}, \quad \bar{w}(x_1, x_2, t) = W/a \tag{7}$$

the above energy expressions can be written at time t as

$$U = \frac{D_{11}}{2}\int_{-1/2}^{1/2}\int_{-1/2}^{1/2}\left[\frac{1}{\alpha}\left(\frac{\partial^2 \bar{w}}{\partial x_1^2}\right)^2 + 2\nu_{21}\alpha\frac{\partial^2 \bar{w}}{\partial x_1^2}\frac{\partial^2 \bar{w}}{\partial x_2^2} + e\alpha^3\left(\frac{\partial^2 \bar{w}}{\partial x_2^2}\right)^2 + 4\frac{D_{66}\alpha}{D_{11}}\left(\frac{\partial^2 \bar{w}}{\partial x_1^2\, \partial x_2^2}\right)^2\right] \mathrm{d}x_1\, \mathrm{d}x_2, \tag{8a}$$

$$T = \frac{a^3 b}{2}\int_{-1/2}^{1/2}\int_{-1/2}^{1/2} \rho h\left(\frac{\partial \bar{w}}{\partial t}\right)^2 \mathrm{d}x_1\, \mathrm{d}x_2, \tag{8b}$$

$$F_s = \frac{a^2}{2}\sum_{i=1}^{4} k_i \bar{w}_i^2, \quad D = \frac{a^2}{2}\sum_{i=1}^{4} c_i(\dot{\bar{w}}_i)^2, \quad F_e = -M(t)\frac{\partial \bar{w}(0,0,t)}{\partial x_1}. \tag{8c–e}$$

It is known that some expressions satisfying geometrical boundary conditions are chosen for $\bar{w}(x_1, x_2, t)$ and by using the Lagrange equations with the trial function; the natural boundary conditions are also satisfied. By using the Lagrange equations, by assuming the displacement $\bar{w}(x_1, x_2, t)$ to be representable by a linear series of admissible functions and adjusting the coefficients in the series to satisfy the Lagrange equation, an approximate solution is found for the displacement function. For applying the Lagrange equation, the trial function $\bar{w}(x_1, x_2, t)$ is approximated by space-dependent polynomial terms $x_1^0, x_1^1, x_1^2, \ldots, x_1^M$ and $x_2^0, x_2^1, x_2^2, \ldots, x_2^N$, and time-dependent generalized displacement co-ordinates $\bar{A}_{mn}(t)$. Thus

$$\bar{w}(x_1, x_2, t) = \sum_{m=0}^{M}\sum_{n=0}^{N} \bar{A}_{mn}(t) x_1^m x_2^n, \tag{9}$$

where $\bar{w}(x_1, x_2, t)$ is the steady state response (the transverse deflection) of the plate to a sinusoidally varying concentrated moment $M(t) = Qe^{i\omega t}$. Each term, x_1^m and x_2^n must satisfy the geometrical boundary conditions. However, in the considered problem, there is no geometrical boundary condition to be satisfied. As it is known, there is no need for these functions to satisfy the natural boundary conditions. However, if the natural boundary conditions are also satisfied when selecting the functions, then the rate of convergence will be high.

The function $\bar{w}(x_1, x_2, t)$ that is given by Eq. (9), is substituted in Eqs. (8a–e). Then, application of Lagrange equation yields a set of linear algebraic equations. The Lagrange equation for the considered problem is given as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{\bar{A}}_{mn}}\right) - \frac{\partial (T - U)}{\partial \bar{A}_{mn}} + \frac{\partial D}{\partial \bar{A}_{mn}} + \frac{\partial F_s}{\partial \bar{A}_{mn}} + \frac{\partial F_e}{\partial \bar{A}_{mn}} = 0, \tag{10}$$

where the overdot stands for the partial derivative with respect to time. Introducing the following non-dimensional parameters,

$$\kappa_j = \frac{k_j a^3}{b D_{11}}, \quad \gamma_j = \frac{c_j a}{b\sqrt{\rho h D_{11}}}, \quad \lambda^2 = \frac{\rho h \omega^2 a^4}{D_{11}}, \quad q = \frac{Q}{D_{11}} \tag{11}$$

and considering that when the moment is expressed as $M(t) = Qe^{i\omega t}$, then the time-dependent generalized functions can be expressed as follows:

$$\bar{A}_{mn}(t) = A_{mn} e^{i\omega t}. \tag{12}$$

In Eq. (12), A_{mn} is a complex variable containing a phase angle. Then, dimensionless complex amplitude of the displacement of a point of the plate can be expressed as

$$w(x_1, x_2) = \sum_{m=0}^{M}\sum_{n=0}^{N} A_{mn} x_1^m x_2^n. \tag{13}$$

By using Eq. (10), the following set of linear algebraic equations is obtained which can be expressed in the following matrix form

$$[\mathbf{A}]\{A_{mn}\} + i\lambda\gamma[\mathbf{B}]\{A_{mn}\} - \lambda^2[\mathbf{C}]\{A_{mn}\} = \{\mathbf{q}\}, \tag{14}$$

where [**A**], [**B**] and [**C**] are coefficient matrices obtained by using Eq. (10).

For free vibration analysis, when the external force and damping of the supports are zero in Eq. (14), this situation results in a set of linear homogeneous equations that can be expressed in the following matrix form:

$$[\mathbf{A}]\{A_{mn}\} - \lambda^2[\mathbf{C}]\{A_{mn}\} = \{\mathbf{0}\}. \tag{15}$$

By increasing the polynomial terms, the accuracy can be increased.

The maximum moment caused by the couple of the reaction forces of the supports is given by

$$M_{r\,max} = a^2 \sum_{j=1}^{2}(k_j + ic_j\omega)\sum_{m=0}^{M}\sum_{n=0}^{N} A_{mn} x_{1j}^m x_{2j}^n \tag{16}$$

and therefore the moment transmissibility is determined by

$$T_M = \sum_{j=1}^{4} M_{r\,max}/Q = \sum_{j=1}^{2} (\kappa_j + \mathrm{i}\gamma_j\lambda) \sum_{m=0}^{M} \sum_{n=0}^{N} A_{mn} x_{1j}^m x_{2j}^n/(\alpha q). \tag{17}$$

The number of unknown coefficients is $M \times N$. Again, the number of equations which can be written for each A_{mn} coefficient by using Eq. (10) is $M \times N$, which is given in matrix form by Eq. (14). Therefore, the total number of these equations is equivalent to the total number of unknown displacements and these unknowns can be determined by using the above-mentioned equations.

The eigenvalues (characteristic values) λ are found from the condition that the determinant of the system of equations given by Eq. (15) must vanish.

3. Numerical results

The steady state response to a sinusoidally varying concentrated moment $M(t)$ acting at the centre of an orthotropic square plate, viscoelastically supported at four points which are symmetrically located at the corners is calculated numerically. The parameters κ_i and γ_i are taken as having the same respective values at all the supports denoted by $\kappa_i = \kappa_s$ and $\gamma_i = \gamma_s$. Because of the symmetry of the structure and of the viscoelastic point supports, under the considered sinusoidally varying moment, only antisymmetrical–symmetrical vibrations arise in the plate. The symbol AS represents symmetrical vibration with respect to x_1 axis and antisymmetrical vibration with respect to x_2 axis.

A short investigation of the free vibration of an elastically point-supported plate is made for comparing the obtained results with the existing results. The natural frequencies of the elastically point-supported plate are determined by calculating the eigenvalues λ of the frequency Eq. (15). The parameter κ_i is taken as having the same value at all the supports denoted by $\kappa_i = \kappa_s$. In the frequency equation, m and n are odd or even integers depending on the vibration mode. For example, the AS mode is symmetric about the x_1 axis and antisymmetric about the x_2 axis. Therefore, for the AS mode, $m = 1, 3, 5, \ldots$ and $n = 0, 2, 4, \ldots$. In the numerical calculations, G_{12} is assumed as follows [11]:

$$G_{12} \approx \frac{E_1\sqrt{e}}{2(1 + \nu_{21}\sqrt{1/e})}. \tag{18}$$

It is possible to simulate infinite lateral support stiffness by setting the translational stiffness coefficient equal to 1×10^8 at all the supports for comparing the obtained results with the existing results of the point supported plates. Also, by setting the translational stiffness coefficient equal to zero at all the supports, a completely free plate situation is obtained. In Table 1, the calculated frequency parameters λ are compared with those of the other researchers for the AS-1, AS-2, AS-3 natural frequencies of an isotropic square plate supported at the corners for $\nu_{21} = 0.3$. Also, the convergence is tested in the table by taking the number of terms $M \times N = 3 \times 3,\ 4 \times 4,\ 5 \times 5,\ 6 \times 6$. The corresponding determinant size becomes $9 \times 9,\ 16 \times 16,$

25×25, 36×36, respectively. It is seen that the present converged values show excellent agreement with those of Refs. [6,7].

In Table 1b, the Poisson ratio is taken as $\nu_{21} = 0.333$ and the obtained results are compared with those of Gorman [4,5] and Narita [6]. It is shown that the convergence with respect to the number of the polynomial terms is excellent in the considered cases. As it is observed from Table 1, the frequency parameter decreases as the number of the polynomial terms increases: It means that the convergence is from above. By increasing the number of the polynomial terms, the exact value can be approached from above. It should be remembered that energy methods always overestimate the fundamental frequency, so with more refined analyses the exact value can be

Table 1
(a) Comparison of the obtained results with the existing results and convergence study of frequency parameters λ for corner point supported square plates, $\nu_{21} = 0.3$, $\alpha = 1$, $e = 1$

	Determinant size	AS-1	AS-2	AS-3
Present study $\kappa_s = 1 \times 10^8 \approx \infty$	9×9	15.7716	51.6850	85.5248
	16×16	15.7703	50.3820	80.4778
	25×25	15.7703	50.3768	80.3625
	36×36	15.7702	50.3767	80.3609
Narita [6]		15.7702	—	—
Venkateswara Rao et al. [7]		15.7702	—	—
Kerstens [1]		15.64	—	—

(b) Comparison of the obtained results with the existing results and convergence study of frequency parameters λ for corner point supported square plates, $\nu_{21} = 0.333$, $\alpha = 1$, $e = 1$

	Determinant size	AS-1	AS-2	AS-3
Present study $\kappa_s = 1 \times 10^8$	9×9	15.5438	51.1148	85.1211
	16×16	15.5426	48.8345	80.1487
	25×25	15.5426	49.8295	80.0349
	36×36	15.5426	49.8294	80.0333
Narita [6]		15.5426	—	—
Gorman [4]		15.550	49.92	80.08
Gorman [5]		15.564	50.00	80.20

Table 2
Comparison of the obtained results with the existing results and convergence study of frequency parameters λ for a completely free square plate, $\nu_{21} = 0.25$, $\kappa_s = 0$, $\alpha = 1$, $e = 1$

	Determinant size	AS-1	AS-2	AS-3
Present study $\kappa_s = 0$	9×9	0.00	36.1059	63.1936
	16×16	0.00	35.6024	61.3069
	25×25	0.00	35.6019	61.2898
	36×36	0.00	35.6019	61.2897
Gorman [3]		0.00	35.6	61.28

Table 3
The frequencies and dimensionless damping coefficients at which the peak values of the force transmissibilities occur: $\nu_{21} = 0.3$, $\kappa = 0$, 10, 100, 200

Modes	e	$\gamma = 0$	$\gamma = 1$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$	$\gamma = 2000$
$\kappa = 0$, $\nu_{21} = 0.3$, $x_{1s} = x_{2s} = 0.5$								
AS-1	1.0	0	15.14	15.74	15.76	15.77	15.77	15.77
	0.8	0	13.97	14.43	14.45	14.45	14.46	14.46
	0.6	0	12.48	12.80	12.82	12.82	12.83	12.83
AS-2	1.0	0	—	51.46	50.75	50.47	50.40	50.38
	0.8	0	—	50.66	49.49	49.06	48.96	48.93
	0.6	0	—	—	47.93	47.09	46.90	46.84
$\kappa = 10$, $\nu_{21} = 0.3$, $x_{1s} = x_{2s} = 0.5$								
AS-1	1.0	9.16	10.00	15.035	15.53	15.71	15.76	15.77
	0.8	8.90	9.75	13.80	14.24	14.40	14.44	14.46
	0.6	8.51	9.29	12.27	12.63	12.78	12.81	12.83
AS-2	1.0	38.61	—	51.37	50.71	50.46	50.40	50.38
	0.8	35.96	—	50.60	49.44	49.05	48.96	48.93
	0.6	32.87	—	—	47.89	47.08	46.90	46.84
$\kappa = 100$, $\nu_{21} = 0.3$, $x_{1s} = x_{2s} = 0.5$								
AS-1	1.0	14.52	14.54	14.68	14.90	15.35	15.64	15.77
	0.8	13.48	13.49	13.59	13.76	14.10	14.34	14.46
	0.6	12.14	12.15	12.21	12.31	12.54	12.73	12.83
AS-2	1.0	47.60	48.67	50.09	50.28	50.36	50.38	50.38
	0.8	45.80	47.41	48.86	48.91	48.92	48.93	48.93
	0.6	43.45	—	—	47.24	46.92	46.86	46.84
$\kappa = 200$, $\nu_{21} = 0.3$, $x_{1s} = x_{2s} = 0.5$								
AS-1	1.0	15.11	15.12	15.14	15.19	15.34	15.57	15.77
	0.8	13.95	13.95	13.97	14.00	14.11	14.28	14.46
	0.6	12.47	12.47	12.48	12.50	12.57	12.69	12.83
AS-2	1.0	48.95	49.07	49.60	49.97	50.25	50.35	50.38
	0.8	47.31	47.45	48.07	48.48	48.80	48.89	48.93
	0.6	45.07	45.28	46.11	46.51	46.75	46.82	46.84

approached from above. Since the energy methods always yield upper bounds and the convergence study indicates that the calculated values are converged to within five significant figures, it seems that the present results, as well as those in Ref. [7] are closer to the exact ones than the results of Gorman [4,5].

In Table 2, the obtained numerical results are compared with those of obtained by Gorman [3] for a completely free orthotropic plate and it is observed from that table that the results are in good agreement.

For obtaining the steady state response results of the viscoelastically corner point-supported rectangular specially orthotropic plates under the effect of sinusoidally varying concentrated moment given in Figs. 3–8 and Table 3, Eqs. (14) and (17) are used. From here on, in the calculation of the results of the present study, 4×4 terms of the polynomial series are used, namely the size of the determinant is 16×16.

In tables, the values $\kappa_s = 0$ and $\kappa_s = 10^8 \approx \infty$, respectively, represent the frequency parameters of an unconstrained free plate and a simply point-supported plate. Although the AS-1 mode existing in the case of simple point supports does not occur in the completely free plate, it occurs for every value of stiffness parameter, which is different from zero. Therefore, the frequency value of the first AS mode of the completely free plate is taken as zero and the frequency value after the zero value is assumed as the frequency value of the second mode for convenience while tabulating the results for stiffness parameters varying from zero to 10^8. However, the first frequency value, which is different from zero, is taken as the frequency value of the first AS mode in the studies on the completely free plates. For $\kappa_s = 0$ and 10, by increasing the damping parameter γ_s, the

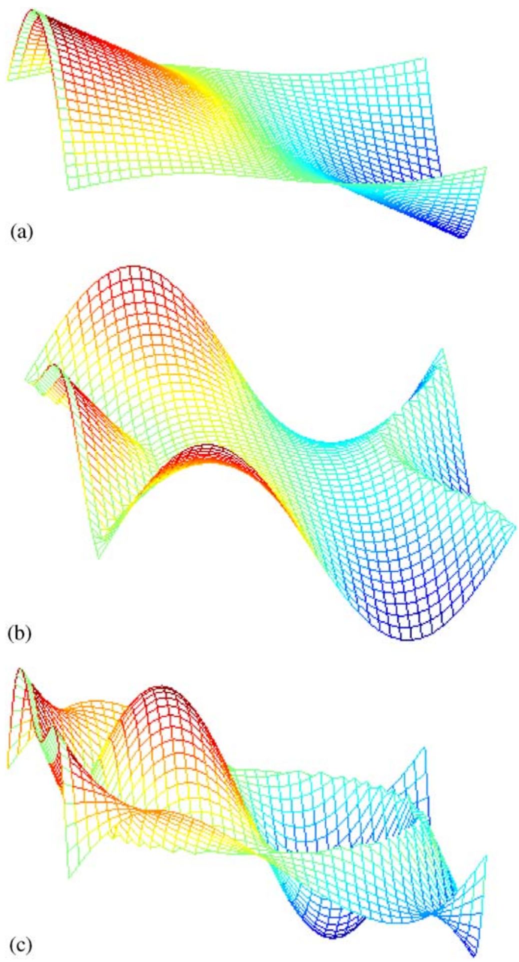


Fig. 2. Mode shapes of a corner point-supported plate for $E_2/E_1 = 1$ for (a) AS-1, (b) AS-2 and (c) AS-3.

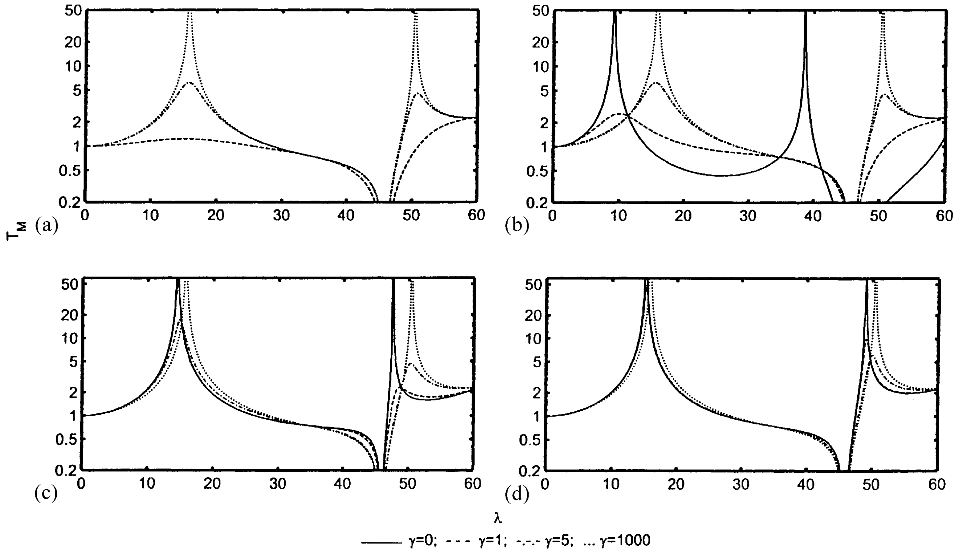


Fig. 3. The force transmissibilities for various values of γ_s for $E_2/E_1 = 1$ for (a) $\kappa_s = 0$, (b) $\kappa_s = 10$, (c) $\kappa_s = 100$ and (d) $\kappa_s = 200$.

frequency parameters of the resonant peaks for the AS-1 mode increase monotonically, the frequency parameters of the resonant peaks for the AS-2 decrease monotonically and ultimately become the values of a simply point-supported plate as seen in Table 3. For $\kappa_s = 100$ and 200, by increasing the damping parameter γ_s, the frequency parameters of the resonant peaks for the AS-1 and AS-2 modes increase monotonically and ultimately become the values of a simply point-supported plate as seen in Table 3. As far as the authors know, there are no values to compare the present obtained steady state results. Also, there are no values to compare the free vibration analyses results for values of the support stiffness κ_s different from zero and infinite.

The mode shapes of the vibration can be determined from Eq. (14) by taking a coefficient A_{mn} as known and calculating the eigenvectors corresponding to the eigenvalues. The mode shapes for the first three AS modes are shown in Fig. 2. The moment transmissibilities are determined for various damping parameters γ_s for various values of κ_s by using Eqs. (13) and (16). In all of the numerical calculations, ν_{21} is taken as 0.3 and the locations of the point supports are chosen at the corners of the plate.

Fig. 3a–d show the force transmissibilities for various values of γ_s for $E_2/E_1 = 1$, Fig. 4a–d show the force transmissibilities for $E_2/E_1 = 0.8$, Fig. 5a–d show the force transmissibilities for $E_2/E_1 = 0.6$, for various values of γ_s for $\kappa_s = 0, 10, 100, 200$, respectively. There are lots of parameters involved in the study. Therefore, for the brevity of the numerical results, the locations of the supports are taken only at the corners.

Figs. 3–5 show that within the frequency range of the figures, two resonant peaks appear for the AS-1 and AS-2 vibrations and also antiresonant peaks or lowest values appear between adjacent frequencies. In Figs. 3–5, the solid lines ($\gamma_s = 0$) represent the response curve of a plate with undamped elastic point supports and the dotted lines ($\gamma_s = 1000$) a plate with approximately simple point supports. As it is obvious and shown in Figs. 3–5, when κ_s and γ_s are both equal to

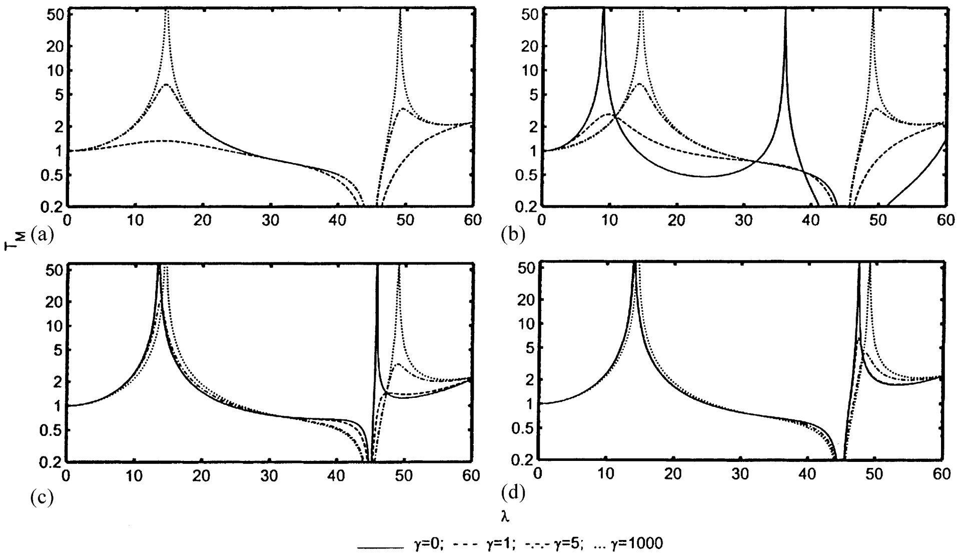


Fig. 4. The force transmissibilities for various values of γ_s for $E_2/E_1 = 0.8$ for (a) $\kappa_s = 0$, (b) $\kappa_s = 10$, (c) $\kappa_s = 100$ and (d) $\kappa_s = 200$.

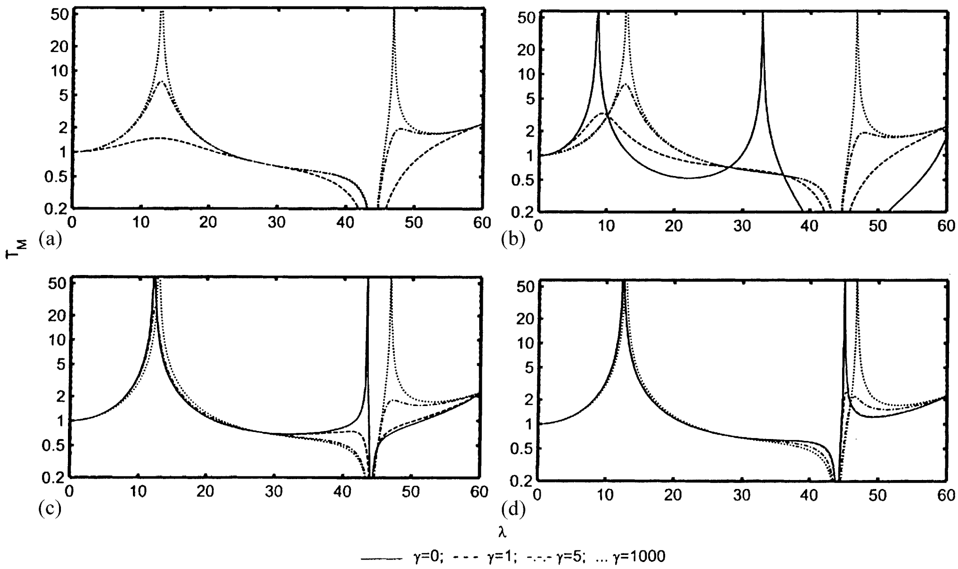


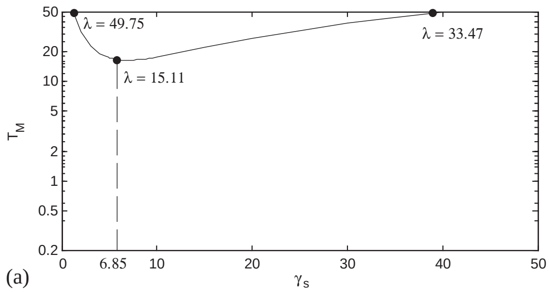
Fig. 5. The force transmissibilities for various values of γ_s for $E_2/E_1 = 0.6$ for (a) $\kappa_s = 0$, (b) $\kappa_s = 10$, (c) $\kappa_s = 100$ and (d) $\kappa_s = 200$.

zero, then the moment transmissibility become zero. Figs. 3–5 show that, although there are no intersection points of the moment transmissibility curves for zero values of κ_s, with the increase of the κ_s value, all of the moment transmissibility curves for the AS-1 mode intersect at fixed point regardless of the damping parameters. Also, it is observed from these figures that, for low values of κ_s, in the resonant peak region, there is no fixed intersection point of the moment transmissibility curves for the AS-2 mode, Figs. 3a, b, 4a, b and 5a–c. For some κ_s and γ_s values, the AS-2 resonant peak does not occur: For example in Fig. 3b, for $\kappa_s = 10$ and $\gamma_s = 0$, 1, this situation occurs. By decreasing the E_2/E_1 value, for the AS-2 mode, the values of the resonant peaks become small and for some κ_s, γ_s values, the AS-2 resonant peak does not occur, Figs. 3c, 4c and 5c. When there is an intersection point in the resonant peak region, then by choosing a suitable value for the damping parameter γ_s, it is possible to reduce the peak values of the force transmissibilities to the values of the force transmissibilities which correspond to the intersection points shown in Figs. 3–5. Existence of such points is useful for an optimum design of a system by choosing appropriate damping parameter. By choosing appropriate damping parameters, resonant peaks of the moment transmissibilities disappear and the related peak quantities become small. Within a certain range of the frequencies, the moment transmissibilities are less than unity, which indicates the possibility of vibration isolation.

In Table 3, the frequencies at which the peak values of the moment transmissibilities for the AS-1, AS-2 modes occur are determined for various damping parameters γ_s for $\kappa_s = 0, 10, 100, 200$ by using Eqs. (16) and (17). The dash sign — in Table 3 shows that there is no resonant peak for the considered parameters.

When γ_s and κ_s are both zero, then, it is obvious that the moment transmissibility is zero. In the case of great γ_s values, the viscoelastically point-supported plate behaves like a simply point-supported plate. It is seen from Figs. 3–5 that when the value of κ_s is too big, then the effect of the damping coefficient γ_s is negligible. Similarly, it is seen from Figs. 3–5 that, when the value of the damping coefficient γ_s is too big, then the effect of the variation of κ_s is not effective on the behaviour of the system.

Figs. 6–8 show that with the variation of the damping parameter γ_s, a damping parameter can be obtained for which the peak values of the moment transmissibilities are minimum. The peak



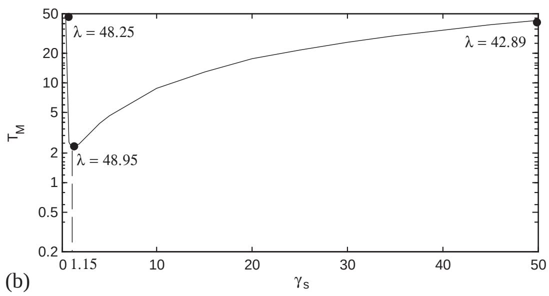


Fig. 6. The minimum resonant peak values with the variation of γ_s for (a) AS-1 mode, (b) AS-2 mode. $E_2/E_1 = 1.0$, $\kappa_s = 100$ and $\nu_{21} = 0.3$.

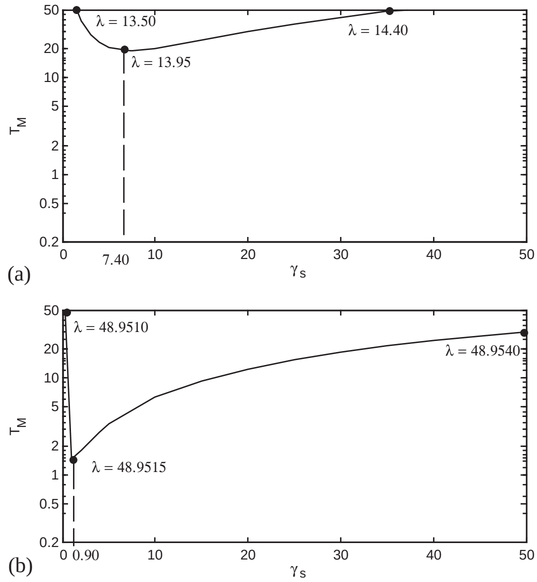


Fig. 7. The minimum resonant peak values with the variation of γ_s for (a) AS-1 mode, (b) AS-2 mode. $E_2/E_1 = 0.8$, $\kappa_s = 100$ and $\nu_{21} = 0.3$.

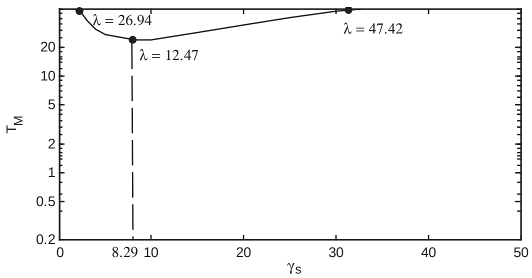


Fig. 8. The minimum resonant peak values with the variation of γ_s for AS-1 mode. $E_2/E_1 = 0.6$, $\kappa_s = 100$ and $\nu_{21} = 0.3$.

values of the moment transmissibilities occur at different values of λ while changing the damping parameter γ_s. However, the frequency parameter λ remains between the frequency parameters λ obtained for $\gamma_s = 0$ and ∞. Therefore, in Fig. 6, while changing γ_s for obtaining minimum peak value of the moment transmissibility for the considered mode, the frequency parameter λ also changes a little. As it was explained before, λ changes between λ obtained for $\gamma_s = 0$ and λ obtained for $\gamma_s = \infty$.

4. Conclusions

By using the Lagrange equation, the natural frequencies for the *AS* modes of elastically point-supported specially orthotropic square plates and the steady state response of a viscoelastically point-supported specially orthotropic square plates to a sinusoidally varying moment has been studied and compared with the existing results. Using the Lagrange equation is a good way for studying the structural behaviour of the viscoelastically point-supported plates. For the same accuracy level, it needs considerably fewer degrees-of-freedom than the finite element method and energy-based finite difference method.

By the application of the above-mentioned solution technique, for the *AS* vibration mode family, the first three values of the natural frequencies are determined, and the convergence characteristics of the frequency parameters are investigated numerically for orthotropic square plates elastically supported at four points at the corners. It is seen that the rate of convergence is very high. The effect of the orthotropy and stiffness of the supports on the frequency parameters is investigated and shown in the tables.

The response curves to a sinusoidally varying moment acting at the centre are determined numerically for orthotropic square plates viscoelastically supported at four points at the corners. The effect of the orthotropy, viscosity and stiffness of the supports on the frequency parameters and response curves is investigated and shown in the figures and table for the considered frequency ranges. All of the obtained results are very accurate and may be useful for designing mechanical systems under external dynamic moments.

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