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A modified Ibrahim time domain algorithm for operational modal analysis including harmonic excitation

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Abstract

Operational modal analysis procedures are efficient techniques to identify modal properties of structures excited through unknown random noise produced during operation. In many practical cases, harmonic excitations are often present in addition to the white-noise and, if the harmonic frequency is close to structural frequencies, standard identification techniques fail. Here, a method is presented to take into account the harmonic excitations while doing modal parameter identification for operational modal analysis (OMA). The proposed technique is based on the Ibrahim Time Domain method and explicitly includes the harmonic frequencies known a priori. Therefore, the modified technique allows proper identification of eigenfrequencies and modal damping even when harmonic excitation frequencies are close to the natural frequencies of the structures. Experimental results are shown in the presence of multi-harmonic loads for a steel plate to validate the method.

1. Introduction

In operational modal analysis (OMA) structural modal parameters can be computed without knowing the input excitation to the system. It is therefore a valuable tool to analyze structures submitted to excitation generated by their own operation. Presently, operational modal analysis procedures are limited to the case when excitation to the system is white stationary noise. There are different ways to identify modal parameters in that case.

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One approach called the natural excitation technique (NExT) [1] consists in computing correlations between the response signals and observe that they can be compared to impulse responses of the system. Hence, the output correlation functions can be processed as the impulse responses function of the system in order to extract modal parameters. Standard time-domain identification techniques such as the least squares complex exponential method (LSCE) [2], the eigenvalue realization algorithm (ERA) [3] and the Ibrahim Time Domain method (ITD) [4–7] can be applied as identification techniques. Further details on the implementation of time-domain identification techniques can be found, e.g., in Ref. [8].

In many applications such as wind turbines, cars and ships harmonic excitation is present in addition to random loads due to unbalanced masses in rotating components or due to aerodynamic and electrical forces. A straightforward way to deal with harmonic excitations consists of considering the harmonic response as a virtual non-damped eigenmode of the system. So while doing modal identification, those virtual, non-damped, modes can be observed and identified as arising from harmonic excitation. In practice however if the excitation frequencies are close to natural frequencies of the system, identified natural frequencies and associated damping can no longer be measured accurately by the algorithms. Indeed, in that case, the harmonic response masks the actual eigenresponse and classical identification methods are not well suited to separate the harmonic and eigencomponents.

Another way of dealing with harmonic excitations is to filter out the harmonic part of the signals from the actual response of the systems. This can only be done while harmonic frequencies are well separated from the natural frequencies of the system, so that the harmonic part of the response can be filtered out efficiently without affecting the response part necessary for identification. Therefore, this approach cannot be applied when the harmonic responses are predominant and have a frequency close to natural frequencies.

In an earlier paper [9], a method to take into account harmonic excitations in OMA was proposed. That method was based on the modification of the standard LSCE method.

In this paper, a similar method is developed, but based on the single station time domain (SSTD) algorithm [10], itself a variant of the ITD approach. In a way similar to that proposed in Ref. [9], it will be assumed that the harmonic frequencies are known a priori. Note that harmonic excitation frequencies can be found easily in practice given the nature of the machine or using pre-processing of the measured data.

Experimental results obtained with the modified ITD will be compared to results obtained with standard method in the presence of harmonics. In addition to the modified ITD method results, previous modified LSCE method results will also be given to compare the effectiveness of both approaches.

2. Theoretical background

In this section the principle of the NExT as described in Ref. [1] is first outlined. Then, combining the standard ITD method [6] and the SSTD [10] algorithm, a new SIMO version of the SSTD is formulated. In the last part of this section, a modification to the SIMO–SSTD is proposed in order to account explicitly for harmonic responses.

2.1. Natural excitation technique

When a system is excited by pure stationary white noise, the correlation between dynamic responses can be exploited to identify modal parameters. One procedure to analyze structures under random excitation is based on the fact that the correlation function $x_{ij}(t)$ between the response signals i and j at a time interval of t is similar to the response of the structure at i due to an impulse on j. The theoretical basis for this results has been outlined in the NExT [1]: it is shown that correlation functions between responses to stationary white noise are expressed by

$$x_{ij}(t) = \sum_{r=1}^{N} \frac{\phi_{ri} A_{rj}}{m_r \omega_r^d} \mathrm{e}^{(-\zeta_r \omega_r^n t)} \sin(\omega_r^d t + \theta_r), \tag{1}$$

where ϕ_{ri} is the ith component of the eigenmode number r of the conservative system, A_{rj} is a constant associated to the jth response signal taken as reference, m_r is the rth modal mass, ζ_r and ω_r^n are respectively the rth modal damping ratio and no-damped eigenfrequency, $\omega_r^d = \omega_r^n \sqrt{1-\zeta_r^2}$ and θ_r is the phase angle associated with the rth modal response. Hence the correlation between signals is a superposition of decaying sinusoids having damping and frequencies equal to the damping and frequencies of the structural mode. In terms of the complex modes of the structure, the correlation function (1) can be written as (see for instance [11])

$$x_{ij}(k\Delta t) = \sum_{r=1}^{N} \psi_{ri} \mathrm{e}^{s_r k \Delta t} C_{rj} + \sum_{r=1}^{N} \psi_{ri}^{*} \mathrm{e}^{s_r^{*} k \Delta t} C_{rj}^{*}, \tag{2}$$

where $s_r = \omega_r \zeta_r + \mathrm{i}\omega_r \sqrt{(1-\zeta_r^2)}$ and where C_{rj} is a constant associated with the rth mode for the jth response signal, which is the reference signal. Δt is the sampling time step and the superscript * denotes the complex conjugate. Note that in conventional modal analysis, these constant multipliers are modal participation factors. Numbering all complex modes and eigenvalues in sequence, Eq. (2) can be written as

$$x_k = x(k\Delta t) = \sum_{r=1}^{2N} a_r \mathrm{e}^{s_r k \Delta t}, \tag{3}$$

where x_k is the correlation between i and j for a time $k\Delta t$.

2.2. A variant of the ITD method

The ITD algorithm [6] uses the time response of several outputs in order to find modal parameters. In the standard ITD method, at least $2N$ response location need to be measured to identify a model of order N. The SSTD method proposed in Ref. [10] uses an approach similar to the ITD, but the identification is based on the response at a unique location, considered for different time intervals.

The method presented next is in fact a new blend of the ITD and SSTD methods to get an SIMO algorithm like the standard ITD, but where several time intervals per response are used as in the SSTD.

Writing Eq. (3) shifted for different starting time samples, gives

$$\underbrace{\begin{bmatrix} x_1 & x_2 & \cdots & \cdots & x_L \\ x_2 & x_3 & \cdots & \cdots & x_{L+1} \\ \vdots & \vdots & \cdots & \cdots & \vdots \\ x_{2N} & x_{2N+1} & \cdots & \cdots & x_{L+2N-1} \end{bmatrix}}_{\mathbf{X}^{(2N\times L)}}$$

$$= \underbrace{\begin{bmatrix} a_1 & a_2 & \cdots & a_{2N} \\ a_1 e^{s_1 \Delta t} & a_2 e^{s_2 \Delta t} & \cdots & a_{2N} e^{s_{2N}\Delta t} \\ \vdots & \vdots & \vdots\vdots\vdots & \vdots \\ a_1 e^{s_1(2N-1)\Delta t} & a_2 e^{s_2(2N-1)\Delta t} & \cdots & a_{2N} e^{s_{2N}(2N-1)\Delta t} \end{bmatrix}}_{\mathbf{A}^{(2N\times 2N)}} \underbrace{\begin{bmatrix} e^{s_1 t_1} & e^{s_1 t_2} & \cdots & \cdots & e^{s_1 t_L} \\ e^{s_2 t_1} & e^{s_2 t_2} & \cdots & \cdots & e^{s_2 t_L} \\ \vdots & \vdots & \cdots & \cdots & \vdots \\ e^{s_{2N} t_1} & e^{s_{2N} t_2} & \cdots & \cdots & e^{s_{2N} t_L} \end{bmatrix}}_{\mathbf{\Lambda}^{(2N\times L)}}, \quad (4)$$

where $t_k = k\Delta t$, N is the total number of modes considered for the identification and L is the number of correlation values per row. The identification order N is usually not known *a priori* but can be increased until the identified parameters converge.

In symbolic form, (4) can be written as

$$\underset{(2N\times L)}{\mathbf{X}} = \underset{(2N\times 2N)}{\mathbf{A}} \underset{(2N\times L)}{\mathbf{\Lambda}}. \quad (5)$$

A similar equation can be written by shifting all the discrete response values by Δt as follows:

$$\underbrace{\begin{bmatrix} x_2 & x_3 & \cdots & \cdots & x_{L+1} \\ x_3 & x_4 & \cdots & \cdots & x_{L+2} \\ \vdots & \vdots & \cdots & \cdots & \vdots \\ x_{2N+1} & x_{2N+2} & \cdots & \cdots & x_{L+2N} \end{bmatrix}}_{\hat{\mathbf{X}}^{(2N\times L)}}$$

$$= \underbrace{\begin{bmatrix} a_1 e^{s_1 \Delta t} & a_2 e^{s_2 \Delta t} & \cdots & a_{2N} e^{s_{2N}\Delta t} \\ a_1 e^{s_1 2\Delta t} & a_2 e^{s_2 2\Delta t} & \cdots & a_{2N} e^{s_{2N} 2\Delta t} \\ \cdots & \cdots & \vdots\vdots\vdots & \vdots \\ a_1 e^{s_1 2N\Delta t} & a_2 e^{s_2 2N\Delta t} & \cdots & a_{2N} e^{s_{2N} 2N\Delta t} \end{bmatrix}}_{\hat{\mathbf{A}}^{(2N\times 2N)}} \underbrace{\begin{bmatrix} e^{s_1 t_1} & e^{s_1 t_2} & \cdots & \cdots & e^{s_1 t_L} \\ e^{s_2 t_1} & e^{s_2 t_2} & \cdots & \cdots & e^{s_2 t_L} \\ \vdots & \vdots & \cdots & \cdots & \vdots \\ e^{s_{2N} t_1} & e^{s_{2N} t_2} & \cdots & \cdots & e^{s_{2N} t_L} \end{bmatrix}}_{\mathbf{\Lambda}^{(2N\times L)}}, \quad (6)$$

$$\underset{(2N\times L)}{\hat{\mathbf{X}}} = \underset{(2N\times 2N)}{\hat{\mathbf{A}}} \underset{(2N\times L)}{\mathbf{\Lambda}}. \quad (7)$$

Now let us define a system matrix $\mathbf{S}$ so that

$$\underset{(2N\times 2N)}{\mathbf{S}} \underset{(2N\times 2N)}{\mathbf{A}} = \underset{(2N\times 2N)}{\hat{\mathbf{A}}}. \quad (8)$$

Pre-multiplying Eq. (5) by **S**, gives

$$\underset{(2N\times 2N)}{\mathbf{S}}\,\underset{(2N\times L)}{\mathbf{X}} = \underset{(2N\times 2N)}{\mathbf{S}}\,\underset{(2N\times 2N)}{\mathbf{A}}\,\underset{(2N\times L)}{\mathbf{\Lambda}} \tag{9}$$

and inserting definition (8),

$$\underset{(2N\times 2N)}{\mathbf{S}}\,\underset{(2N\times L)}{\mathbf{X}} = \underset{(2N\times 2N)}{\hat{\mathbf{A}}}\,\underset{(2N\times L)}{\mathbf{\Lambda}}. \tag{10}$$

Finally, taking account of Eq. (7) gives

$$\underset{(2N\times 2N)}{\mathbf{S}}\,\underset{(2N\times L)}{\mathbf{X}} = \underset{(2N\times L)}{\hat{\mathbf{X}}}. \tag{11}$$

Eq. (11) uses a single-output signal due to a single input (correlation between i and j) and corresponds to the SSTD method, itself similar to the ITD [10]. In order to find a SIMO variant of the SSTD, note that the system matrix **S** is independent of the location of measurements and thus that Eq. (11) remains valid for any SISO combination. Hence the values of **S** can also be obtained by considering several responses to an input and by satisfying Eq. (11) in a least-squares sense. So let Eq. (11) for n responses to a single impulse (SIMO) be written as

$$\mathbf{S}_{(2N\times 2N)} \underset{(2N\times Ln)}{\begin{bmatrix} \mathbf{X}^1 & \mathbf{X}^2 & \cdots & \mathbf{X}^n \\ (2N\times L) & (2N\times L) & & (2N\times L) \end{bmatrix}} = \underset{(2N\times Ln)}{\begin{bmatrix} \hat{\mathbf{X}}^1 & \hat{\mathbf{X}}^2 & \cdots & \hat{\mathbf{X}}^n \\ (2N\times L) & (2N\times L) & & (2N\times L) \end{bmatrix}}, \tag{12}$$

where $\mathbf{X}^j$ is the matrix of correlation functions defined in Eq. (4) expressed between the reference i and an output j. In more compact form

$$\underset{(2N\times 2N)}{\mathbf{S}}\,\underset{(2N\times Ln)}{\mathbf{Y}} = \underset{(2N\times Ln)}{\hat{\mathbf{Y}}}. \tag{13}$$

A least-squares solution **S** of Eq. (13) can be computed (using a singular value decomposition of **S** to compute a pseudo-inverse).

Finally the eigenvalues are deduced from **S** be observing that Eq. (8) can be decomposed as

$$\mathbf{S}_{(2N\times 2N)} \underset{(2N\times 1)}{\begin{Bmatrix} a_r e^{s_r 0\Delta t} \\ \vdots \\ a_r e^{s_r(2N-1)\Delta t} \end{Bmatrix}} = \underset{(2N\times 1)}{\begin{Bmatrix} a_r e^{s_r 0\Delta t} \\ \vdots \\ a_r e^{s_r(2N-1)\Delta t} \end{Bmatrix}} e^{s_r\Delta t}, \quad r = 1, 2\ldots 2N \tag{14}$$

and therefore

$$[\mathbf{S} - e^{s_r\Delta t}\mathbf{I}] \begin{bmatrix} a_r e^{s_r 0\Delta t} \\ \vdots \\ a_r e^{s_r(2N-1)\Delta t} \end{bmatrix} = \mathbf{0}. \tag{15}$$

Eq. (15) is a standard eigenvalue problem, from which s_r can be found. Modal frequencies and damping can be computed from the values of s_r.

2.3. Modification of SIMO–SSTD in the presence of harmonic excitation

When in addition to the stationary white noise the structure is excited by harmonic excitations, the dynamic response will include the associated forced harmonic components which can be seen as virtual non-damped modes in the correlation functions. These virtual modes can in theory be identified as before by an SIMO–SSTD method, but, as illustrated in Ref. [9], identification of those non-damped virtual modes can be very difficult in practice.

Assume that the harmonic excitation has a frequency ω. The modal superposition (3) describing the impulse response now includes a forced harmonic part equivalent to a virtual modal response with frequency $s = \pm i\omega$ so that $e^{s_r \Delta t} = e^{\pm i\omega\Delta t} = \cos(\omega_r \Delta t) \pm i \sin(\omega_r \Delta t)$. This solution being known a priori the system matrix **S** will be forced to have $e^{\pm i\omega\Delta t}$ as an eigensolution: writing Eq. (14) for $s = \pm i\omega$ and rearranging, **S** must satisfy

$$\mathbf{S}_{(2N\times 2N)} \underset{(2N\times 2)}{\begin{bmatrix} 0 & 1 \\ \sin(\omega_1\Delta t) & \cos(\omega_1\Delta t) \\ . & . \\ . & . \\ \sin((2N-2)\omega_1\Delta t) & \cos((2N-2)\omega_1\Delta t) \\ \sin((2N-1)\omega_1\Delta t) & \cos((2N-1)\omega_1\Delta t) \end{bmatrix}} = \underset{(2N\times 2)}{\begin{bmatrix} \sin(\omega_1\Delta t) & \cos(\omega_1\Delta t) \\ \sin(2\omega_1\Delta t) & \cos(2\omega_1\Delta t) \\ . & . \\ . & . \\ \sin((2N-1)\omega_1\Delta t) & \sin((2N-1)\omega_1\Delta t) \\ \sin(2N\omega_1\Delta t) & \cos(2N\omega_1\Delta t) \end{bmatrix}} \tag{16}$$

which in symbolic form becomes

$$\underset{(2N\times 2N)}{\mathbf{S}}\ \underset{(2N\times 2)}{\mathbf{H}^1} = \underset{(2N\times 2)}{\hat{\mathbf{H}}^1} \tag{17}$$

If m harmonic excitation frequencies exist, Eq. (17) can be generalized to

$$\mathbf{S}_{(2N\times 2N)} \underset{(2N\times 2m)}{\begin{bmatrix} \underset{(2N\times 2)}{\mathbf{H}^1} & \underset{(2N\times 2)}{\mathbf{H}^2} & \cdots & \underset{(2N\times 2)}{\mathbf{H}^m} \end{bmatrix}} = \underset{(2N\times 2m)}{\begin{bmatrix} \underset{(2N\times 2)}{\hat{\mathbf{H}}^1} & \underset{(2N\times 2)}{\hat{\mathbf{H}}^2} & \cdots & \underset{(2N\times 2)}{\hat{\mathbf{H}}^m} \end{bmatrix}} \tag{18}$$

symbolically written as

$$\underset{(2N\times 2N)}{\mathbf{S}}\ \underset{(2N\times 2m)}{\mathbf{H}} = \underset{(2N\times 2m)}{\hat{\mathbf{H}}}. \tag{19}$$

The system matrix **S** must satisfy the dynamic equation (13) and the harmonic relation (19). So **S** is solution of

$$\mathbf{S}_{(2N\times 2N)} \underset{(2N\times(2m+Ln))}{\begin{bmatrix} \underset{(2N\times 2m)}{\mathbf{H}} & \underset{(2N\times Ln)}{\mathbf{Y}} \end{bmatrix}} = \underset{(2N\times(2m+Ln))}{\begin{bmatrix} \underset{(2N\times 2m)}{\hat{\mathbf{H}}} & \underset{(2N\times Ln)}{\hat{\mathbf{Y}}} \end{bmatrix}}. \tag{20}$$

The objective is to solve Eq. (20) exactly for the harmonic part and therefore the matrices in Eq. (20) are partitioned as

$$\underset{(2N\times 2N)}{\begin{bmatrix} \underset{(2m\times 2m)}{\mathbf{S}_1} & \underset{(2m\times(2N-2m))}{\mathbf{S}_2} \\ \underset{((2N-2m)\times 2m)}{\mathbf{S}_3} & \underset{((2N-2m)\times(2N-2m))}{\mathbf{S}_4} \end{bmatrix}} \underset{(2N\times(2m+Ln))}{\begin{bmatrix} \underset{(2m\times 2m)}{\mathbf{H}_1} & \underset{(2m\times Ln)}{\mathbf{Y}_1} \\ \underset{((2N-2m)\times 2m)}{\mathbf{H}_2} & \underset{((2n-2m)\times Ln)}{\mathbf{Y}_2} \end{bmatrix}} = \underset{(2N\times(2m+Ln))}{\begin{bmatrix} \underset{(2m\times 2m)}{\hat{\mathbf{H}}_1} & \underset{(2m\times Ln)}{\hat{\mathbf{Y}}_1} \\ \underset{((2N-2m)\times 2m)}{\hat{\mathbf{H}}_2} & \underset{((2n-2m)\times Ln)}{\hat{\mathbf{Y}}_2} \end{bmatrix}}. \tag{21}$$

Eq. (21) is first solved for $\mathbf{S}_1$ and $\mathbf{S}_3$ so that the harmonic part (19) is satisfied exactly, namely

$$\mathbf{S}_1 = (\hat{\mathbf{H}}_1 - \mathbf{S}_2\mathbf{H}_2)\mathbf{H}_1^{-1}, \tag{22}$$

$$\mathbf{S}_3 = (\hat{\mathbf{H}}_2 - \mathbf{S}_4\mathbf{H}_2)\mathbf{H}_1^{-1}. \tag{23}$$

The remainder of $\mathbf{S}$ is then computed from the response equations in Eq. (12) (corresponding to Eq. (13))

$$\mathbf{S}_2(\mathbf{H}_2\mathbf{H}_1^{-1}\mathbf{Y}_1 - \mathbf{Y}_2) = (\hat{\mathbf{H}}_1\mathbf{H}_1^{-1}\mathbf{Y}_1 - \hat{\mathbf{Y}}_1), \tag{24}$$

$$\mathbf{S}_4(\mathbf{H}_2\mathbf{H}_1^{-1}\mathbf{Y}_1 - \mathbf{Y}_2) = (\hat{\mathbf{H}}_2\mathbf{H}_1^{-1}\mathbf{Y}_1 - \hat{\mathbf{Y}}_2). \tag{25}$$

These last relations are solved in a least-squares sense for $\mathbf{S}_2$ and $\mathbf{S}_4$. Once $\mathbf{S}$ is known, the system eigenvalues are obtained by solving the eigenvalue problem (15). Note that $s = \pm i\omega$ will obviously be one of the solutions by construction.

3. Experimental examples

The technique discussed above was applied to the analysis of a steel, rectangular plate in the laboratory. As shown in Fig. 1 the plate was suspended using soft strings. A shaker was applied on the structure to provide stationary noise excitation as well as harmonic loads. A total of 32,768 discrete time samples per response channel have been obtained for each experiment. Sampling was done at 2560 Hz. Six locations are chosen as shown in the diagram, where response signals were taken.

Due to the limited number of acquisition channels (four), two sets of measurements have been done for each test described hereafter. Accelerometer one has been kept at the same location in each measurement set and is used as reference signal (for discussion of practicalities, see Ref. [12]). The level of vibration has been maintained constant throughout each test.

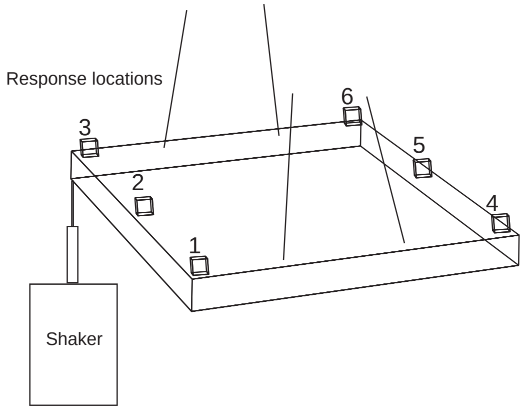

Fig. 1. Experimental set-up for a plate.

Correlation functions in the tests presented here have been computed with reference to the response of the first accelerometer. Since the first accelerometer is at the corner of the plate, it is assumed that this location does not correspond to a vibration node for the modes of interest. For the same reason, the shaker excitation is applied at location No. 3, another corner of the plate (see Fig. 1). It will be assumed that the shaker excites all fundamental modes of the plate.

In the modal identification technique, correlation between signals are treated like impulse response functions. Since a single reference signal has been used, all identification methods will work on the principle of a SIMO identification.

In the examples, modal parameters will first be computed only with stationary white noise input to get the modal parameters as would be done in a normal OMA setting. Then, the cases when harmonic excitations are present along with the random loads will be considered. For all identification methods, the number of correlation data per row and per signal (L in Eq. (4)) is taken as $L = 200$. The total number of discrete correlation values used in the identification is $L + 2N$ for each correlation function, N being the order of the identified model.

3.1. Pure stationary white noise excitation

Here, the experiment was done with stationary white noise introduced between 0 and 1000 Hz, to excite all the modes properly. Eigenfrequencies and damping values were identified with the SIMO–SSTD version of the ITD method described by Eq. (12) and the LSCE method. In Fig. 2 and in Table 1 natural frequencies and associated damping are reported. Note that the parameters identified by the SIMO–SSTD are nearly equal to the ones computed by the LSCE approach (Fig. 3).

It can be seen from the figure that the modes corresponding to peaks in the PSD plot are found in a stable manner by both identification methods. It is also observed that there is an additional stability line at 59 Hz, which is not listed in Table 1. It was found that this excitation frequency was due to an amplifier problem. Now concentrate on the mode at 271.047 Hz. In the subsequent examples, in addition to the random excitations, harmonic excitations will be added near to that 271.047 Hz mode and modal parameters of this particular mode will be identified. There is

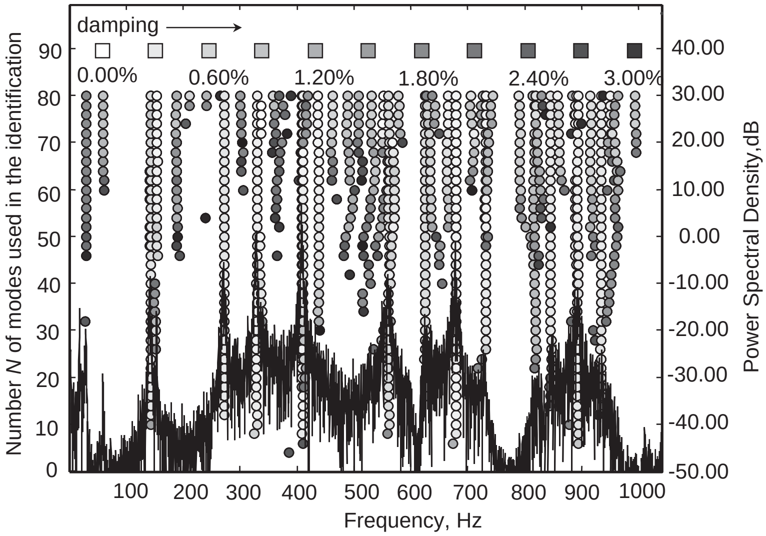

Fig. 2. Stability diagram for white noise excitation only computed by SIMO–SSTD method.

Table 1
Frequencies and associated dampings (white noise only)

SIMO–SSTD		LSCE	
Freq. (Hz)	Damp. (%)	Freq. (Hz)	Damp. (%)
28.754	1.726	28.665	1.794
141.731	0.326	141.754	0.327
271.047	0.300	271.093	0.297
328.105	0.226	328.150	0.225
408.485	0.132	408.546	0.132
560.472	0.486	560.563	0.483
624.050	0.470	624.130	0.481
678.931	0.164	679.059	0.180
731.271	0.162	731.642	0.153
844.701	0.282	844.826	0.267
892.166	0.280	892.337	0.275

particular interest in investigating the accuracy of the identification when harmonic excitations with frequencies close to the 271.047 Hz eigenfrequency are present.

3.2. Two harmonics at 274 and 276 Hz

In this experiment, two harmonic frequencies additional to the white noise are present at 274 and 276 Hz, which are close to the 271.047 Hz modal frequency of the system. In

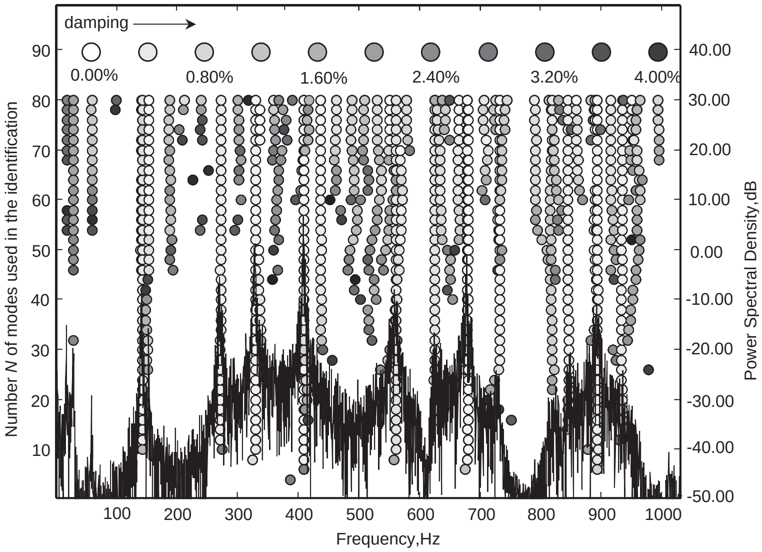

Fig. 3. Stability diagram for white noise excitation only computed by LSCE method.

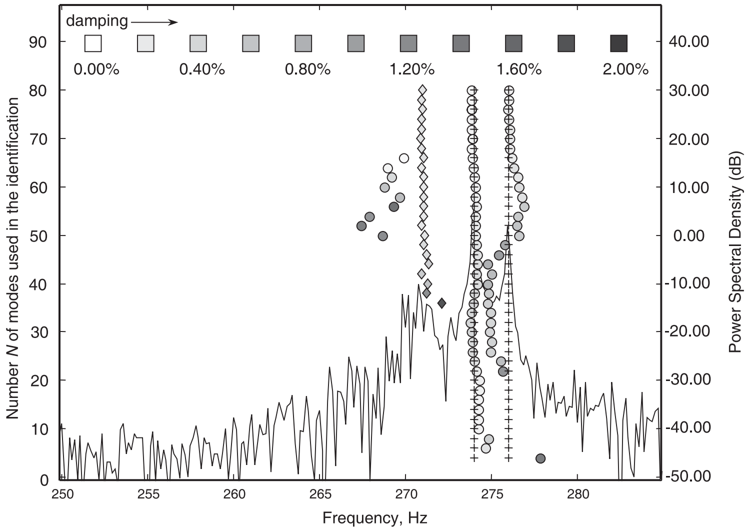

Fig. 4. PSD of the acceleration at position 1 for random loads and two additional harmonic excitations, and stabilization diagram for the SIMO–SSTD (13) and for the SIMO–SSTD modified to explicitly include harmonic components (21). $\diamond$, Eigenparameters from modified SIMO–SSTD; $+$, harmonics in modified SIMO–SSTD; $\bigcirc$, SIMO–SSTD.

Fig. 4 two peaks at 274 and 276 Hz are clearly observed from the power spectral density (PSD) plot. One secondary peak is present slightly below 274 Hz which might be representing the 271.047 Hz natural frequencies of the system. As described in the theory, four additional rows of sin and cos terms are introduced to explicitly account for the two harmonic frequencies

Table 2
Frequencies and associated damping identified by the SIMO–SSTD and the modified SIMO–SSTD

Modes	SIMO–SSTD						Modified SIMO–SSTD	
	Freq. 1 (Hz)	Damp. 1 (%)	Freq. 2 (Hz)	Damp. 2 (%)	Freq. 3 (Hz)	Damp. 3 (%)	Freq. (Hz)	Damp. (%)
80	—	—	273.849	0.069	276.031	0.136	270.031	0.341
76	—	—	273.813	0.080	275.965	0.168	271.009	0.247
72	—	—	273.856	0.099	276.042	0.202	270.945	0.246
68	—	—	273.860	0.075	276.065	0.140	270.975	0.256
64	268.936	0.078	273.948	0.121	276.295	0.248	270.970	0.249
60	268.749	0.543	274.039	0.135	276.611	0.100	271.037	0.200
56	269.325	1.358	274.110	0.160	276.916	0.468	271.095	0.257
52	267.396	1.223	274.068	0.177	276.459	0.546	270.964	0.190
48	—	—	274.123	0.223	275.75	0.814	271.072	0.316
44	—	—	274.184	0.279	274.800	0.729	271.357	0.425
40	—	—	274.176	0.295	274.764	0.664	271.301	0.545
36	—	—	273.947	0.378	274.783	0.419	272.125	1.638
32	—	—	273.773	0.356	274.938	0.357	—	—
28	—	—	273.875	0.330	275.020	0.411	—	—
24	—	—	273.999	0.236	275.532	0.570	—	—

in the response matrices (see Eqs. (17) and (21)). Fig. 4 shows the stability diagram superimposed on the auto-spectrum diagram. In Table 2 frequencies and dampings for modes in the neighborhood of 271.047 Hz are listed. These modal parameters were found with the SIMO–SSTD and the SIMO–SSTD variant including harmonics as presented in this paper.

From Table 2 and Fig. 4 it can be concluded that, when using the normal SIMO–SSTD, the identified frequencies are associated with the two fundamental harmonic frequencies. Note that the identified damping corresponding to the harmonic frequencies are not small, while they should be null in theory. Hence, in practice, it would be difficult at this point to assimilate the identified modes to harmonic responses and the analyst would probably conclude that the identified parameters correspond to true eigenmodes.

When applying the new method, two harmonic components of frequencies corresponding to the periodic excitations, namely 274 and 276 Hz are introduced in the Hankel matrix. The stabilization diagram is shown in Fig. 4. In Table 2 the identified modal frequency corresponding to the mode in the neighbourhood of 271.047 Hz and its associated damping are reported. Note that harmonic frequencies are identified exactly by construction in the modified SIMO–SSTD method and are therefore not listed in the table. It is seen that the identified eigenfrequency and the associated damping are very close to the expected values.

Modal parameters were also computed using the previously published modified LSCE method [9] which also explicitly takes account of the harmonic components, but which is based on the LSCE algorithm. The modified LSCE yields 270.911 Hz and the associated damping is 0.169%. Therefore, whereas modal parameters could not be computed extracted properly from the standard SIMO–SSTD method, the modified SIMO–SSTD and LSCE methods yield accurate eigenfrequencies even for low identification orders. Note that the damping identified by the modified SIMO–SSTD is slightly different from the one identified in the absence of harmonic excitation (namely 0.300%, see Table 1). However, it seems that the modified SIMO–SSTD yields more accurate damping ratios than the modified LSCE.

3.3. Three additional harmonics at 277, 282 and 287 Hz

Now three sine harmonic frequencies at 277, 282 and 287 Hz are given as input to the shaker along with the stationary white noise in order to investigate the effectiveness of the proposed method in the presence of harmonics components of high amplitude. As mentioned in the theory, all three harmonics have been introduced in the Hankel matrix of the proposed method to take into account the harmonic frequencies.

In Fig. 5 three harmonic frequencies can be clearly identified at 277, 282 and 287 Hz. Observe also that there is a peak, just before the first harmonic frequency, which corresponds to the natural frequency.

As shown in Table 3 and Fig. 5, the basic SIMO–SSTD method exhibits two stabilization lines. Those lines correspond to 277 and 287 Hz, which are related to two of the harmonic frequencies introduced in the signal. There is no indication of the presence of the second harmonic frequency in the graph. Although in theory those frequencies should be associated with zero damping, the results in Table 3 indicate that the damping of the identified harmonic parts are quite significant. It can thus be concluded that, when using standard identification procedure such as the SIMO–SSTD version of the ITD, the identification

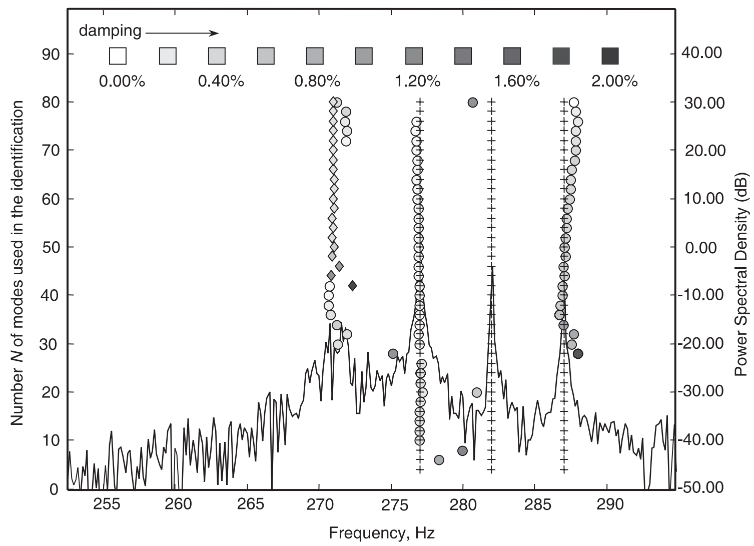

Fig. 5. PSD of the acceleration at position 1 for random loads and three additional harmonic excitations, and stabilization diagram obtained from the SIMO–SSTD and the modified SIMO–SSTD. ◇, Eigenparameters from modified SIMO–SSTD; +, harmonics in modified SIMO–SSTD; ○, SIMO–SSTD.

Table 3
Frequencies and associated damping identified by the SIMO–SSTD and the modified SIMO–SSTD

Modes	Standard ITD						Modified ITD	
	Freq. 1 (Hz)	Damp. 1 (%)	Freq. 2 (Hz)	Damp. 2 (%)	Freq. 3 (Hz)	Damp. 3 (%)	Freq. (Hz)	Damp. (%)
80	271.807	0.354	280.257	1.028	287.659	0.007	271.002	0.236
76	271.769	0.180	276.723	0.024	287.972	0.192	270.992	0.233
72	271.835	0.079	276.725	0.048	287.794	0.258	270.979	0.229
68	—	—	276.805	0.060	287.763	0.396	270.992	0.232
64	—	—	276.773	0.080	287.472	0.314	271.058	0.232
60	—	—	276.831	0.077	287.344	0.373	270.990	0.252
56	—	—	276.866	0.080	287.172	0.386	270.948	0.284
52	—	—	276.882	0.075	287.099	0.411	270.891	0.278
48	—	—	276.878	0.075	287.105	0.418	270.945	0.503
44	—	—	276.891	0.072	287.021	0.440	270.877	1.010
40	270.921	0.066	276.921	0.072	286.854	0.442	272.311	1.727
36	270.762	0.248	276.949	0.058	286.703	0.554	—	—
32	271.987	0.286	276.863	0.030	287.677	0.807	—	—
28	275.099	0.998	—	—	287.942	1.625	—	—
24	—	—	277.041	0.230	277.123	0.265	—	—

procedure breaks down in the presence of harmonic excitations with frequencies close to eigenfrequencies.

Fig. 5 and Table 3 also show the stability diagram for the modified SIMO–SSTD method proposed in this paper. The last three frequency lines belong to the three harmonics, and their frequencies have been set to exactly match the harmonic frequencies, the associated damping being zero (those frequencies and damping values are not given in Table 3). The other identified frequency and damping is listed in Table 3 and can be seen to be very close to the frequency and damping of the searched mode.

The modal parameters have also been computed by the modified LSCE method. The frequency found is 271.175 Hz and the associated damping is 0.22%.

3.4. Robustness of the modified SIMO–SSTD with respect to assumed harmonic frequencies

From all the examples, it is observed that classical identification algorithms applied to the correlation functions obtained from OMA do not work well in the presence of harmonic excitations with frequencies close to the natural frequencies of the system. The new method proposed here is able to provide reliable modal parameters in those cases.

Obviously, it is important to introduce the exact harmonic frequencies in the identification a priori (see Eq. (21)). Consider again the experimental example where two harmonic excitations are acting at 274 and 276 Hz (as in Section 2.2). In order to investigate the sensitivity of the identification to the accuracy of the harmonic frequency specified in the modified SIMO–SSTD, assumed harmonic frequencies different from the actual ones will be introduced in the algorithm. In Figs. 6 and 7, the eigenfrequency identified by the modified SIMO–SSTD as a function of the

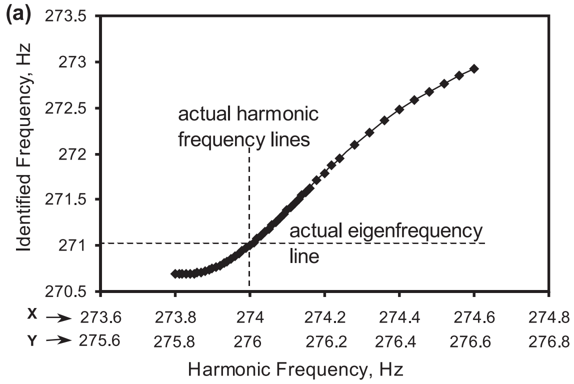

Fig. 6. Eigenfrequency versus assumed harmonic frequency in the modified SIMO–SSTD (harmonic excitations at 274 and 276 Hz). X, assumed first harmonic; Y, assumed second harmonic.

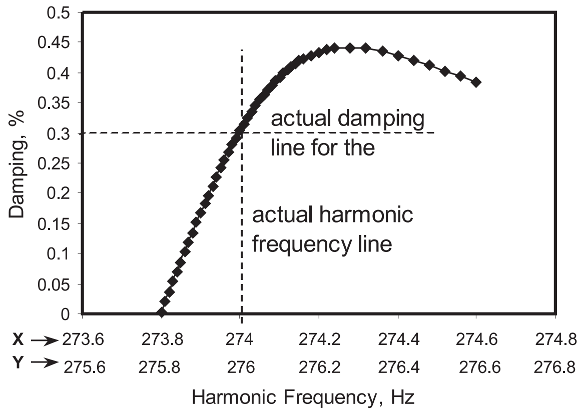

Fig. 7. Damping versus assumed harmonic frequencies in the modified SIMO–SSTD (harmonic excitations at 274 and 276 Hz). X, assumed first harmonic; Y, assumed second harmonic.

assumed harmonic frequencies is plotted for an identification order of 46. The actual harmonic frequencies are 274 and 276 Hz while the assumed ones introduced in the identification are varied simultaneously.

It is observed that if the assumed harmonic frequencies are different from the actual ones, the identified eigenfrequency varies linearly with respect to the harmonic frequency input in the vicinity of the exact harmonic frequency. Also the identified damping ratio is very sensitive to the accuracy of the harmonic frequencies specified in the identification procedure. Hence, it is essential to introduce the assumed harmonic frequencies into the new algorithm accurately. Otherwise the harmonic components explicitly included in the identification are not capable of representing the harmonic part of the signal properly and the new method gives no better results than standard procedures.

4. Conclusions

In this paper, a SIMO–SSTD variant of the Ibrahim Time Domain algorithm has been introduced to identify modal parameters from measurements in operational modal analysis. In particular, a modification is proposed in the identification procedure to account explicitly for harmonic components that might be present in addition to the response to stationary white noise.

The proposed modified SIMO–SSTD method allows eigenfrequencies of the system to be identified accurately even when harmonic components with frequencies close to an eigenfrequency are present. Damping parameters are however difficult to identify accurately if harmonic excitations hide the actual eigenresponse of the system. Standard identification methods fail to identify the system parameters properly in that case. Results of the method described here are slightly better than identification results obtained with a previously proposed modified LSCE approach [9] (typically dampings are better identified).

In these tests, it was found that in order to identify eigenparameters accurately, the harmonic frequencies introduced *a priori* into the identification should be known precisely. If the assumed harmonic frequencies are different from the actual ones, the identification with the modified algorithm yields results similar to those obtained from standard methods.

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References

[1] G.H. James, T.G. Carne, J.P. Lauffer, Sandia National Laboratories, The Natural Excitation Technique (NExT) for modal parameter extraction from operating structures, *Journal of Analytical and Experimental Modal Analysis* 10 (4) (1995) 260–277.

[2] D.L. Brown, R.J. Allemang, R. Zimmerman, M. Mergeay, Parameter estimation techniques for modal analysis, *Seventh International Seminar on Modal Analysis*, Katholieke Universiteit Leuven, Leuven, 1985.

[3] J.N. Juang, R.S. Pappa, An eigensystem realization algorithm for modal parameter identification and model reduction, *Journal of Guidance, Control, and Dynamics* 8 (5) (1985) 620–627.

[4] S.R. Ibrahim, E.C. Mikulcik, A time domain vibration test technique, *The Shock and Vibration Bulletin* 43 (4) (1973) 21–37.

[5] S.R. Ibrahim, E.C. Mikulcik, The experimental determination of vibration parameters from time responses, *The Shock and Vibration Bulletin* 46 (5) (1976) 187–196.

[6] S.R. Ibrahim, E.C. Mikulcik, A method for direct identification of vibration parameters from the free response, *The Shock and Vibration Bulletin* 47 (4) (1977) 183–198.

[7] R.S. Pappa, S.R. Ibrahim, A parametric study of the Ibrahim time domain modal identification algorithm, *The Shock and Vibration Bulletin* 51 (3) (1981) 43–57.

[8] M. Silva, H. Lieven, L. Skingle, T. Urgueira, *Theoretical and Experimental Modal Analysis*, Research Studies Press Ltd., Somerset, 1997.

[9] P. Mohanty, D.J. Rixen, Operational modal analysis in the presence of harmonic excitation, *Journal of Sound and Vibration*, in press.
[10] S.A. Zaghlool, Single station time-domain (SSTD) vibration testing technique: theory and application, *Journal of Sound and Vibration* 72 (2) (1980) 205–234.
[11] M.Gėradin, D. Rixen, *Mechanical Vibrations, Theory and Application to Structural Dynamics*, 2nd Edition, Wiley, Chichester, 1997.
[12] L. Hermans, H. van der Auweraer, Modal testing and analysis of structures under operational conditions: industrial applications, *Mechanical System and Signal Processing* 13 (2) (1999) 193–216.