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Author's reply ☆

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It is not correct that we assumed the term a^4 in Eq. (3) to be positive for the Timoshenko beam. This term was considered as a mathematical parameter in order to obtain a unified mathematical description for the modes. This is shown in Eqs. (17) and (18) through the specific example of a fixed-supported beam. The use of the dynamical basis could allow to write characteristic frequency equations as well as Green's functions in a shorter way.

In our paper, we do not discuss the characteristic frequency equation that arises from Eq. (6) for determining the frequencies. It is formulated in such a way that any arbitrary basis of the differential equation of the modes can be employed. We used to employ the Euler basis of exponential solutions. However, we considered the use of the dynamical basis whose generator $h(x)$ can be written in terms of the usual exponential Euler basis. As we know, this later can be constructed by using the roots of the characteristic polynomial and can lead to trigonometric or hyperbolic functions.

The characteristic polynomial (7) has the roots algebraically given in terms of Eq. (8). This is mathematically consistent even in the case of a^4 being a complex number. For our purposes, there was no need to consider the nature of the roots or the parameter a^4. It is quite clear that when we vary parameters, a qualitative change on the behaviour of the system dynamics might occur. Anderson [1] deals with the characteristic polynomial (7) when inverting a Laplace transform of Timoshenko beam dynamics and considers the analyticity of its inverse which turns out to be our spatial fundamental response $h(x)$.

Since the work of Traill-Nash and Collar [2], the theoretical treatment of frequency equations like (17) and modal forms (18) have incited controversy with the so-called second spectrum of the Timoshenko beam model (TBM). Levinson and Cooke [3] discuss characteristic equations on which a relation of trigonometric and hyperbolic functions is considered for frequencies such that one root of the characteristic polynomial crosses the real axis. Recently, Chan et al. [4] discuss a superposition standing wave approach for the TBM and physical interpretation of the hyperbolic functions in the classical solution of beam vibration problems.

It should be clear that our work is a methodology that can be applied with time-invariant linear differential equations of arbitrary order. The dynamical basis being generated by a single solution

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characterized by impulsive initial conditions. Since $h(x)$ is an entire function, Taylor approximations can provide good results in local regions of the beam, even near the boundary. Modes can be theoretically predicted by analytical or numerical means [5], however the existence of the actual ones depends upon practical observation and experimentation, particularly, when approximated models are considered [6].

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