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Seismic analysis of base-isolated liquid storage tanks

M.K. Shrimali, R.S. Jangid*

Department of Civil Engineering, Indian Institute of Technology Bombay, Powai, Mumbai 400 076, India

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Abstract

Three analytical studies for the seismic response of base-isolated ground supported cylindrical liquid storage tanks under recorded earthquake ground motion are presented. The continuous liquid mass of the tank is modelled as lumped masses referred as sloshing mass, impulsive mass and rigid mass. Firstly, the seismic response of isolated tanks is obtained using the modal superposition technique and compared with the exact response to study the effects of non-classical damping. The comparison of results with different tank aspect ratios and stiffness and damping of the bearing indicate that the effects of non-classical damping are insignificant implying that the response of isolated liquid storage tanks can be accurately obtained by the modal analysis with classical damping approximation.

The second investigation involves the analysis of base-isolated liquid storage tanks using the response spectrum method in which the peak response of tank in different modes is obtained for the specified response spectrum of earthquake motion and combined with different combination rules. The results indicate that the peak response obtained by the response spectrum method matches well with the corresponding exact response. However, specific combination rule should be used for better estimation of various response quantities of the isolated tanks. Finally, the closed-form expressions for the modal parameters of the base-isolated liquid storage tanks are derived and compared with the exact values. A simplified approximate method is also proposed to evaluate the seismic response of isolated tanks. The response obtained from the above approximate method was found to be in good agreement with the exact response.

1. Introduction

Liquid storage tanks are lifeline structures and strategically very important, since they have vital use in industries and nuclear power plants. Past earthquakes have demonstrated the seismic vulnerability of tanks and the damage occurred in the form of buckling of tank wall due to

*Corresponding author. Tel.: +91-22-2572-2545; fax: +91-22-2572-3480.
E-mail address: rsjangid@civil.iitb.ac.in (R.S. Jangid).

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excessive development of compressive stresses, failure of piping systems and uplift of anchorage system [1–4]. The seismic behavior of liquid storage tanks is highly complex due to liquid–structure interaction leading to a tedious design procedure from earthquake-resistant design point of view. Housner [5] and Rosenblueth and Newmark [6] developed a lumped mass model of rigid liquid storage tanks and investigated its seismic response. These models were modified by Haroun [7], which takes into account the flexibility of the tank wall in the seismic analysis.

The conventional technique to safeguard the tanks against damaging earthquakes is by strengthening, in which the size of different members is increased to resist more earthquake forces and dissipate the seismic energy. The alternative technique is base isolation by introducing the special devices between the base and foundation of the tank. One of the goals of base isolation is to shift the fundamental frequency of a structure away from the dominant frequencies of earthquake ground motion and fundamental frequency of the fixed base structure. The other purpose of an isolation system is to provide an additional means of energy dissipation, thereby reducing the transmitted acceleration into the structure. The base isolation techniques have been developed and successfully implemented to buildings in the past [8,9]. But there are very few studies reported to investigate the effectiveness of base isolation for aseismic design of liquid storage tanks. Chalhoub and Kelly [10] conducted shake table tests on fixed base and base-isolated tanks and demonstrated a reduction in the dynamic response but a slight increase in the sloshing displacement. Kim and Lee [11] experimentally investigated the earthquake performance of liquid storage tanks isolated by the elastomeric bearings and found that such system was quite effective in reducing the dynamic forces. Malhotra [12,13] investigated the seismic response of base-isolated tanks and found that isolation was effective in reducing the response of the tanks over traditional fixed base tank without any significant change in sloshing displacement. Wang et al. [14] investigated the response of liquid storage tanks isolated by FPS system and observed that the isolation was effective in reducing the response of the tanks. Recently, Shrimali and Jangid [15] investigated the seismic response of isolated liquid storage tanks with sliding systems considering the influence of parametric variation and found that the sliding systems were effective in reducing the response. Note that the above studies confirm the effectiveness of base isolation for liquid storage tanks. However, it will be interesting to investigate the feasibility of seismic response of base-isolated tanks by the modal analysis and response spectrum method, which is widely used for design practice.

In this study, the seismic response of liquid storage tanks isolated by the linear elastomeric bearings is investigated. The specific objectives of the study are: (i) to investigate the effects of non-classical damping by comparing the response of the tanks by modal superposition method and exact technique under different system parameters such as tank aspect ratio, bearing stiffness and damping of bearings, (ii) to evaluate the response of base-isolated liquid storage tanks using the response spectrum method and study the validity of different modal combination rules, (iii) to develop closed-form expressions for the modal parameters of base-isolated liquid storage tanks and (iv) to propose a simplified approximate method to evaluate the seismic response of the base-isolated liquid storage tanks.

2. Structural model of isolated liquid storage tank

The structural model considered for the base-isolated cylindrical liquid storage tank is shown in Fig. 1 in which the elastomeric bearings are installed between the base and foundation of the tank.

The contained liquid is considered as incompressible, inviscid and has irrotational flow. During the base excitation, the entire tank liquid mass vibrates in three distinct patterns: sloshing or convective mass (i.e., top liquid mass which changes the free liquid surface), impulsive mass (i.e., intermediate liquid mass vibrating along with tank wall) and rigid mass (i.e. the lower liquid mass which rigidly moves with the tank wall). There are various modes in which sloshing and impulsive masses vibrate but the response can be predicted by considering first sloshing mode and first impulsive mode as observed experimentally by Kim and Lee [11] and numerically by Malhotra [13]. Therefore, the continuous liquid is modelled as lumped masses with flexible tank, i.e., Haroun's model [7]. The sloshing, impulsive and rigid lumped masses are denoted by m_c, m_i and m_r respectively. The sloshing and impulsive masses are connected to the tank wall by corresponding equivalent spring having stiffness k_c $(= m_c\omega_c^2)$ and k_i $(= m_i\omega_i^2)$ respectively. The parameters ω_c and ω_i denote the sloshing and impulsive frequencies of the liquid mass respectively. Thus, the base-isolated tank system has three degrees of freedom under uni-directional earthquake excitation. These degrees of freedom are denoted by u_c, u_i and u_b, which denote the absolute displacement of sloshing, impulsive and rigid masses respectively. Further, the self-weight of the tank is neglected since it is very small (less than 5% of the effective weight of the tank). The geometrical parameters of the tanks considered are liquid height, H, radius, R and average thickness of tank wall, t_h.

The various masses and associated natural frequencies of the tank liquid are expressed as [7]

$$m_c = m\gamma_c, \tag{1}$$

$$m_i = m\gamma_i, \tag{2}$$

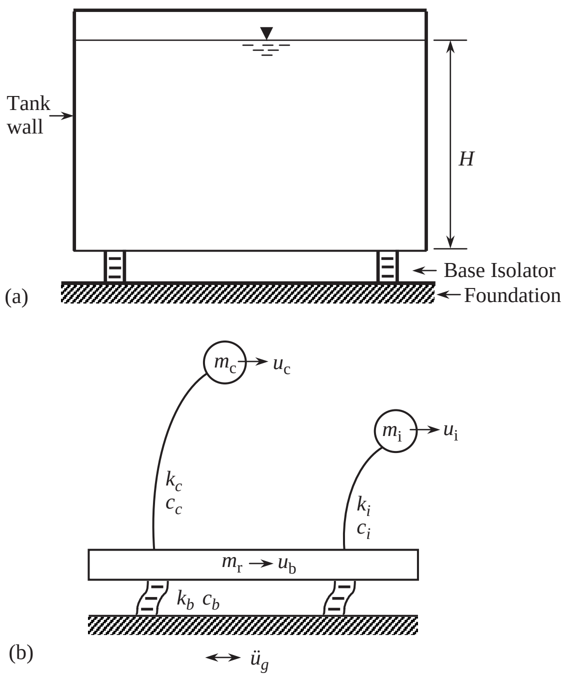

Fig. 1. (a) Isolated model of liquid storage tank and (b) mathematical model of the tank.

$$m_r = m\gamma_r, \tag{3}$$

$$m = \pi R^2 H \rho_w, \tag{4}$$

$$\omega_i = \frac{P}{H}\sqrt{\frac{E}{\rho_s}}, \tag{5}$$

$$\omega_c = \sqrt{1.84\left(\frac{g}{R}\right)\tanh(1.84S)}, \tag{6}$$

where γ_c, γ_i and γ_r are the mass ratios associated with sloshing, impulsive and rigid mass of the tank liquid, respectively; ρ_w is the mass density of the tank liquid; E and ρ_s are the modulus of elasticity and density of the tank wall, respectively; $S = H/R$ is the aspect ratio (i.e., ratio of the liquid height to radius of the tank) of the tank; g is the acceleration due to gravity and P is a dimensionless parameter. The parameters γ_c, γ_i, γ_r and P are functions of the aspect ratio of the tank, S and t_h/R. For $t_h/R = 0.004$, the above parameters are expressed by [7]

$$\begin{Bmatrix} \gamma_c \\ \gamma_i \\ \gamma_r \\ P \end{Bmatrix} = \begin{bmatrix} 1.01327 & -0.8757 & 0.35708 & 0.06692 & 0.00439 \\ -0.15467 & 1.21716 & -0.62839 & 0.14434 & -0.0125 \\ -0.01599 & 0.86356 & -0.30941 & 0.04083 & 0 \\ 0.037085 & 0.084302 & -0.05088 & 0.012523 & -0.0012 \end{bmatrix} \begin{Bmatrix} 1 \\ S \\ S^2 \\ S^3 \\ S^4 \end{Bmatrix}. \tag{7}$$

The base isolation system considered is elastomeric bearings consisting of alternate layers of rubber and steel plates. These bearings are vertically stiff and isolation is achieved through horizontal flexibility and damping. The bearings are modelled with linear force–deformation behavior having horizontal stiffness, k_b, and viscous damping, c_b. The required stiffness and damping of the bearings are designed to provide specific values of the two parameters isolation period (T_b) and damping ratio (ξ_b) which are defined by

$$T_b = 2\pi\sqrt{\frac{M}{k_b}}, \tag{8}$$

$$\xi_b = \frac{c_b}{2M\omega_b}, \tag{9}$$

where $\omega_b = 2\pi/T_b$ is the isolation frequency; $M = m_c + m_i + m_r$ is the total effective mass of the isolated liquid storage tank.

3. Governing equations of motion

The basic equations of motion of three-degrees-of-freedom model of the isolated tank subjected to earthquake excitation are expressed in the matrix form as

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = -[m]\{r\}\ddot{u}_g, \tag{10}$$

where $\{x\} = \{x_c, x_i, x_b\}^T$ is the displacement vector; $x_c = u_c - u_b$ is the relative displacement of the sloshing mass; $x_i = u_i - u_b$ is the relative displacement of the impulsive mass; $x_b = u_b - u_g$ is the displacement of bearings relative to the ground; $[m]$, $[c]$ and $[k]$ are the mass, damping and stiffness matrices, respectively; $\{r\} = \{0, 0, 1\}^T$ is the influence coefficient vector; $\ddot{u}_g$ is the earthquake acceleration and T denotes the transpose.

The matrices $[m]$, $[c]$ and $[k]$ are expressed as

$$[m] = \begin{bmatrix} m_c & 0 & m_c \\ 0 & m_i & m_i \\ m_c & m_i & M \end{bmatrix}, \tag{11}$$

$$[c] = \begin{bmatrix} c_c & 0 & 0 \\ 0 & c_i & 0 \\ 0 & 0 & c_b \end{bmatrix}, \tag{12}$$

$$[k] = \begin{bmatrix} k_c & 0 & 0 \\ 0 & k_i & 0 \\ 0 & 0 & k_b \end{bmatrix}, \tag{13}$$

where c_c and c_i are the damping of the sloshing and impulsive masses respectively.

4. Analysis by modal superposition method

The modal superposition method is a widely used method to compute the earthquake response of structures. This method directly gives the physical interpretation to the response of the structure. Seismic analysis of base-isolated liquid storage tanks by modal analysis involves the non-classical damping because of different damping associated in the vibrating fluid masses and isolation system. Therefore, the classical modal superposition method provides the seismic response of tanks by neglecting the effects of off-diagonal terms of the generalized damping matrix.

The displacement vector, $\{x\}$, can be approximated by a linear combination of three undamped modes as

$$\{x\} = [\Phi]\{q\}, \tag{14}$$

where $[\Phi] = [\phi_1, \phi_2, \phi_3]$ is the undamped modal matrix, ϕ_n is the nth modal vector; $\{q\} = \{q_1, q_2, q_3\}^T$ is the modal displacement vector and q_n is the nth modal displacement.

The solution of the following eigenvalue problem provides the nth modal vector, i.e.,

$$\omega_n^2[m]\phi_n = [k]\phi_n, \tag{15}$$

where ω_n is the nth natural frequency of the isolated tank.

After carrying out the decomposition and neglecting the off-diagonal terms of generalized damping matrix (i.e., $[\Phi]^T[c][\Phi]$) the final transformed modal equations are expressed by

$$\ddot{q}_n + 2\xi_n\omega_n\dot{q}_n + \omega_n^2 q_n = -\Gamma_n\ddot{u}_g \quad (n = 1{-}3), \tag{16}$$

where ξ_n is the modal damping ratio and Γ_n is the modal participation factor in the nth mode of the system. The parameters ξ_n and Γ_n are defined by

$$\xi_n = \frac{C_n}{2M_n\omega_n}, \tag{17}$$

$$\Gamma_n = \frac{L_n}{M_n}, \tag{18}$$

where $M_n = \phi_n^{\mathrm{T}}[m]\phi_n$, $C_n = \phi_n^{\mathrm{T}}[c]\phi_n$ and $L_n = \phi_n^{\mathrm{T}}[m]\{r\}$.

The modal displacement of the system to specified earthquake acceleration is expressed by

$$q_n = -\Gamma_n \int_0^t \ddot{u}_g(\tau)\left(\frac{\mathrm{e}^{-\xi_n\omega_n(t-\tau)}}{\omega_{dn}}\right)\sin\{\omega_{dn}(t-\tau)\}\,\mathrm{d}\tau, \tag{19}$$

where $\omega_{dn} = \omega_n\sqrt{1-\xi_n^2}$ is the damped natural frequency of nth mode of the system.

For a base-isolated liquid storage tank, the response quantities of interest are: the base shear (F_b), the sloshing displacement (x_c) and bearing displacement (x_b). The base shear is directly proportional to earthquake force exerted in the tank. On the other hand, the sloshing and bearing displacements are important from the design point of view.

The F_b, x_c and x_b resulting from all modes are expressed by

$$F_b = \sum_{n=1}^{3} (c_b\dot{q}_n + k_b q_n)\phi_n^3, \tag{20}$$

$$x_c = \sum_{n=1}^{3} \phi_n^1 q_n, \tag{21}$$

$$x_b = \sum_{n=1}^{3} \phi_n^3 q_n, \tag{22}$$

where ϕ_n^j is the jth element of the nth modal vector.

The effects of non-classical damping on the response of base-isolated tank model as given in Fig. 1(b) are investigated by comparing the modal response with the corresponding response obtained by complex mode analysis which is referred as "exact" response. For the present study, the components S00E, N00E and N00E of the Imperial Valley (1940), Loma Prieta (1989) and Kobe (1995) earthquake ground motions recorded at El-Centro, Los Gatos Presentation Centre and JMA, respectively, are used to investigate the response of the tank. The peak ground acceleration of the Imperial Valley, Loma Prieta and Kobe earthquake records are 0.348g, 0.570g and 0.834g respectively. Two different types of tanks namely the broad ($S = 0.6$, $H = 14.6$ m, $\omega_c = 0.123$ Hz and $\omega_i = 3.944$ Hz) and slender ($S = 1.85$, $H = 11.3$ m, $\omega_c = 0.273$ Hz and $\omega_i = 5.963$ Hz) tanks filled with water are considered from Ref. [13]. The damping ratios for the sloshing mass (ξ_c) and the impulsive mass (ξ_i) are taken as 0.005 and 0.02, respectively, from Refs. [11,13]. The tank wall is considered of steel with modulus of elasticity, $E = 200$ MPa and mass density, $\rho_s = 7900$ kg/m^3. Further, the response of isolated liquid storage tank is studied with fixed value of isolation period, i.e., $T_b = 2$ s as recommended in Ref. [8] for base-isolated structures. This period of the isolated structure is significantly higher than the predominate periods of earthquake motions implying that less earthquake forces will be transmitted to the structure. The responses of the isolated tank with other isolation periods (i.e., in the range of 1.5–4 s) can be obtained from Ref. [16].

Figs. 2 and 3 show the time variation of the earthquake response for slender and broad tanks, respectively, under Imperial Valley earthquake motion considering $\xi_b = 0.1$ from Ref. [8]. The total response from modal superposition method is compared with the contribution arising from various modes and also with the exact response. It is observed that the base shear, F_b (normalized with the effective weight of the tank, $W = Mg$) for both the tanks is mainly governed by the second mode (i.e., isolation mode) while the first mode contribution is comparatively small and the third mode contribution is negligible. Further, the modal response closely matches with the exact response, implying that the effects of non-classical damping are not significant. Similar effects of non-classical damping are depicted on the sloshing displacement of fluid and the bearing displacement. The bearing displacement is mainly dominated by the second mode of vibration. On the other hand, the sloshing displacement in slender tank is governed by first two modes while for broad tank the contribution of second mode is insignificant. This is due to the fact that for a slender tank the sloshing and isolation frequency are relatively close to each other and, therefore, contribution from both the modes become dominant due to mode coupling.

The peak response of the tank obtained by the exact and the modal analysis is compared in Table 1 under different earthquake motions. The results show that the peak response quantities such as base shear (F_b/W), displacements, x_c and x_b, obtained by the exact method are close to

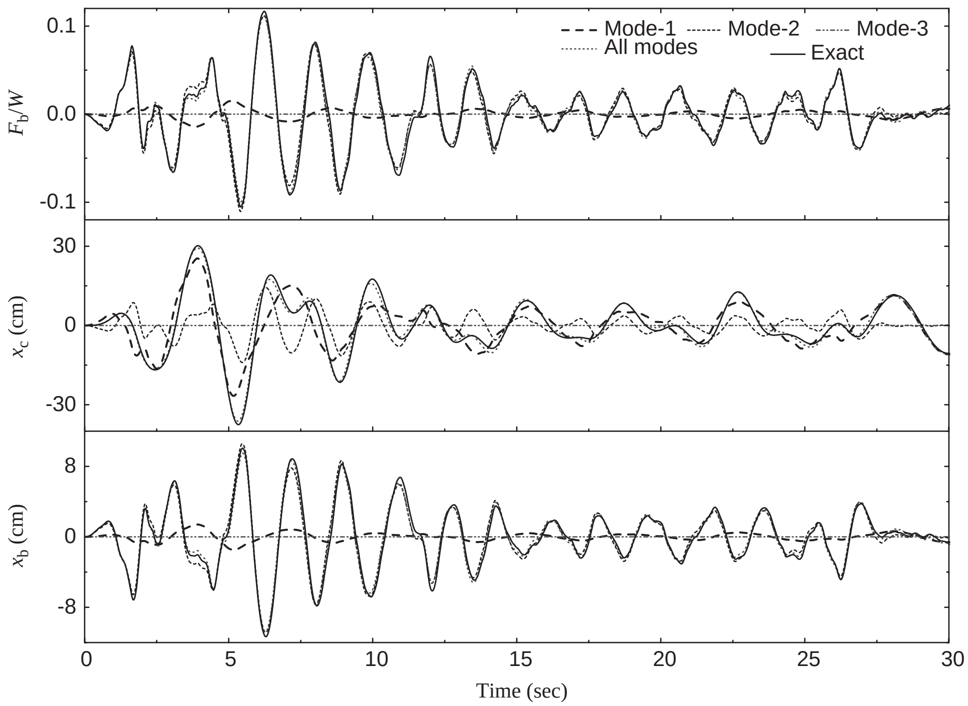

Fig. 2. Time variation of slender tank response in different modes under Imperial Valley, 1940 earthquake motion ($\xi_b = 0.1$).

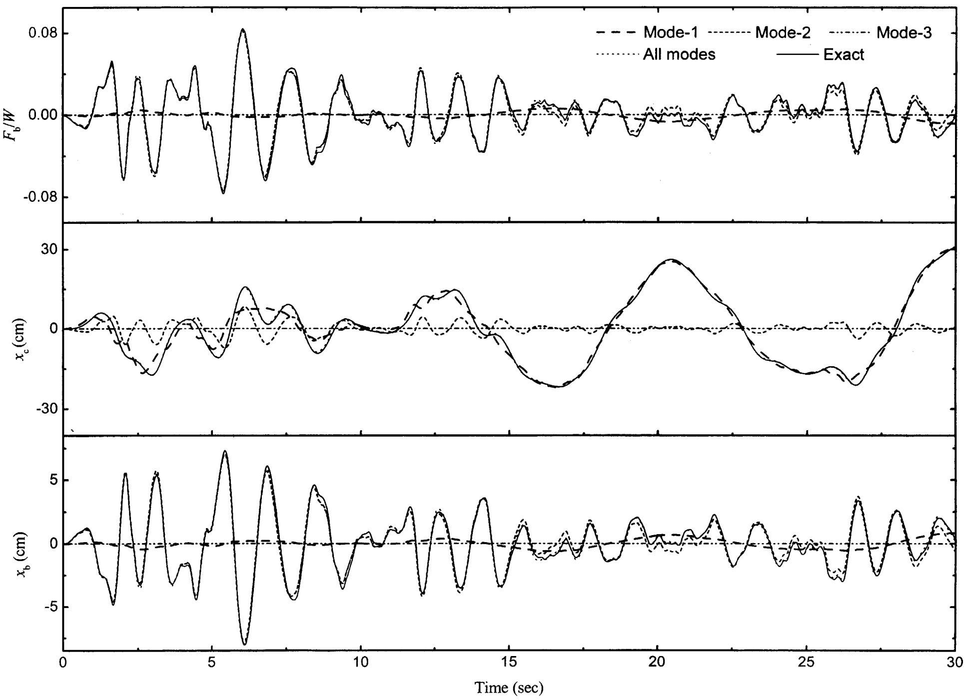

Fig. 3. Time variation of broad tank response in different modes under Imperial Valley, 1940 earthquake motion ($\xi_b = 0.1$).

Table 1
Peak response of base-isolated slender and broad tanks ($T_b = 2$ s and $\xi_b = 0.1$)

Earthquake	Type of tank	Exact response			Modal response		
		x_c (cm)	x_b (cm)	F_b/W	x_c (cm)	x_b (cm)	F_b/W
Imperial Valley, 1940	Slender	37.52	11.34	0.116	36.20	10.80	0.111
	Broad	45.03	8.05	0.084	45.20	7.76	0.082
Loma Prieta, 1989	Slender	212.6	40.19	0.419	210.0	38.10	0.398
	Broad	68.01	28.73	0.299	67.30	27.40	0.285
Kobe, 1995	Slender	65.01	31.19	0.328	64.30	30.70	0.323
	Broad	37.65	27.07	0.288	37.70	26.40	0.282

the corresponding responses obtained by the modal analysis. This indicates that the effects of non-classical damping are insignificant and the peak seismic response of isolated tanks can be accurately estimated by the modal analysis. Further, the response of isolated tanks is also compared with the corresponding response of non-isolated tank in Table 1 in order to investigate the effectiveness of base isolation. It is observed that there is significant reduction (of the order of 60–75 per cent) in the base shear of isolated tank in comparison to non-isolated tank, implying that the base isolation is effective in reducing the earthquake response of liquid storage tank.

Further, the sloshing displacement of broad tank due to isolation is not much influenced but it is slightly increased for the slender tank.

Fig. 4 shows the comparison of the peak response of the slender and broad tanks by the modal analysis and exact method against the bearing damping ratio, ξ_b. The figure indicates that the tank response by modal analysis closely matches with the exact response even for high bearing damping. Further, it is also observed from Fig. 4 that the peak base shear, sloshing and bearing displacements decrease with increase of the bearing damping. The effects of bearing damping are found to be more pronounced on the sloshing and base displacements in comparison to the base shear.

5. Analysis by response spectrum method

The response spectrum method is widely employed in seismic design of structures as it provides simple and rational basis for specifying the seismic loading. The computational efforts required

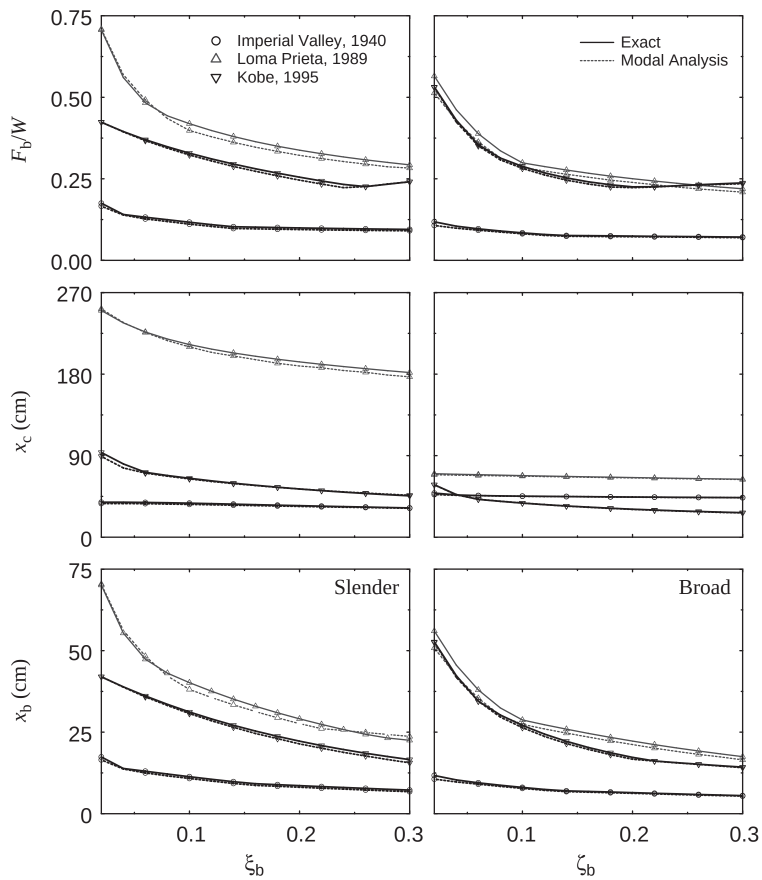

Fig. 4. Variation of peak seismic response of slender and broad tanks obtained by exact method and modal analysis against bearing damping ($T_b = 2\,\mathrm{s}$).

for this method are also less in comparison to other methods. Since the effects of non-classical damping on the response of base-isolated liquid storage tanks are not significant, the response spectrum method can be used for the isolated tanks. The peak response quantities of isolated tanks for a specified response spectrum of the earthquake ground motion in the nth mode are expressed by

$$F_b^n = c_b \phi_n^3 \Gamma_n S_v(\xi_n, \omega_n) + k_b \phi_n^3 \Gamma_n S_d(\xi_n, \omega_n), \tag{23}$$

$$x_c^n = \phi_n^1 \Gamma_n S_d(\xi_n, \omega_n), \tag{24}$$

$$x_b^n = \phi_n^3 \Gamma_n S_d(\xi_n, \omega_n), \tag{25}$$

where $S_d(\xi_n, \omega_n)$ is the displacement spectrum of the earthquake motion for the damping ratio, ξ_n, and frequency, ω_n; $S_v(\xi_n, \omega_n)$ is the velocity spectra and the superscript to the response quantities indicates the particular mode. It is to be noted that the $S_d(\xi_n, \omega_n)$ is taken as the maximum relative displacement of a single-degree-of-freedom system with damping ratio, ξ_n, and natural frequency, ω_n, to a specified earthquake motion. The response spectra of the three earthquake motions for 5 per cent damping are reported in Ref. [15].

Note that the base shear of the isolated liquid storage tanks consists of two components arising from the peak displacement and velocity (refer Eq. (23)). Since the maximum values of these components do not occur at the same time, a rational basis to compute the peak base shear is square root of sum of squares (SRSS). Further, it is also reasonable to express the velocity spectra

Table 2
Peak seismic response of the tank obtained by response spectrum analysis

Earthquake	ξ_b	Response	Slender tank			Broad tank		
			ABS	CQC	SRSS	ABS	CQC	SRSS
Imperial Valley, 1940	0.1	x_c (cm)	40.41	29.52	29.56	50.53	42.96	42.96
		x_b (cm)	12.99	11.82	11.82	9.53	8.04	8.04
		F_b/W	0.134	0.122	0.122	0.095	0.087	0.087
	0.3	x_c (cm)	36.37	27.64	28.06	48.23	43.01	43.01
		x_b (cm)	9.18	7.94	7.87	6.55	5.51	5.5
		F_b/W	0.109	0.095	0.095	0.082	0.071	0.071
Loma Prieta, 1989	0.1	x_c (cm)	219.1	170.6	170.8	67.30	61.43	61.43
		x_b (cm)	54.14	47.10	47.08	30.78	29.19	29.19
		F_b/W	0.557	0.485	0.485	0.321	0.304	0.304
	0.3	x_c (cm)	193.69	162.55	163.98	73.06	58.04	58.08
		x_b (cm)	33.86	26.88	26.52	18.66	17.21	17.21
		F_b/W	0.396	0.32	0.316	0.238	0.222	0.222
Kobe, 1995	0.1	x_c (cm)	88.19	62.24	62.36	50.87	36.03	36.03
		x_b (cm)	36.29	34.18	34.18	28.01	27.29	27.28
		F_b/W	0.374	0.352	0.352	0.293	0.284	0.284
	0.3	x_c (cm)	68.34	50.30	51.18	37.93	27.68	27.71
		x_b (cm)	19.35	17.09	17.09	14.20	13.56	13.56
		F_b/W	0.23	0.206	0.206	0.182	0.175	0.175

by pseudo-velocity spectra (i.e., $S_v(\xi_n, \omega_n) = \omega_n S_d(\xi_n, \omega_n)$). Thus, the peak base shear in the nth mode is evaluated by

$$F_b^n = \sqrt{(c_b\omega_n)^2 + k_b^2}\phi_3^n \Gamma_n S_d(\xi_n, \omega_n). \tag{26}$$

The response of the tank obtained in each mode is combined by different combination rules such as absolute summation (ABS), SRSS rule and the complete quadratic combination (CQC). The peak tank response obtained by response spectrum method for slender and broad tanks under different earthquake motions is shown in Table 2. The results indicate that the peak base shear and bearing displacement are overpredicted by the ABS combination but the SRSS and CQC rules predict the exact response (refer to Table 1). The sloshing displacement is better predicted by the ABS rule for both tanks. Thus, it is concluded that the response spectrum method with appropriate combination rules can be used to evaluate the seismic response of the base-isolated liquid storage tanks.

The comparison of peak response obtained by the response spectrum and the exact methods for two values of bearing damping indicates that at moderate damping, both the CQC and SRSS rules predict the same response but the difference increases for higher bearing damping.

6. Modal parameters of base-isolated tanks

In this section, the closed-form expressions of the modal parameters of the isolated tanks are derived and they can be used to estimate the seismic response of tanks using modal analysis and response spectrum method. The solution of the determinant (i.e., $|\omega_n^2[m] - [k]|$) gives the characteristic equation for ω_n which is expressed as

$$\omega_n^6(\gamma_{mc} + \gamma_{mi} - 1) + \omega_n^4 \begin{Bmatrix} \omega_c^2 \\ \omega_i^2 \\ \omega_c^2 + \omega_i^2 + \omega_b^2 \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} -\gamma_{mc} \\ -\gamma_{mi} \\ 1 \end{Bmatrix} - \omega_n^2 \begin{Bmatrix} \omega_c^2 \\ \omega_i^2 \\ \omega_b^2 \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} \omega_i^2 \\ \omega_b^2 \\ \omega_c^2 \end{Bmatrix} + \omega_c^2\omega_i^2\omega_b^2 = 0, \tag{27}$$

where $\gamma_{mc} = m_c/M$, and $\gamma_{mi} = m_i/M$. The closed-form approximate expressions for the natural frequencies of base-isolated tanks are

$$\omega_1^2 = \omega_c^2[1 - \gamma_{mc}\varepsilon_c], \tag{28}$$

$$\omega_2^2 = \frac{\omega_i^2}{1 - \gamma_{mc}}[1 + \gamma_{mc}\varepsilon_c][1 - \bar{\gamma}_i\bar{\varepsilon}_i], \tag{29}$$

$$\omega_3^2 = \frac{\omega_b^2}{(1 - \bar{\gamma}_i)}[1 + \bar{\gamma}_i\bar{\varepsilon}_i] \tag{30}$$

where $\varepsilon_c = (\omega_c/\omega_b)^2$; $\varepsilon_i = (\omega_b/\omega_i)^2$; $\bar{\varepsilon}_i = \varepsilon_i[(1 + \gamma_{mc}\varepsilon_c)/(1 - \gamma_{mc})]$ and $\bar{\gamma}_i = \gamma_{mi}[(1 + \gamma_{mc}\varepsilon_c)/(1 - \gamma_{mc})]$.

The above expressions for the natural frequency are based on the assumption that $\varepsilon_c \ll 1$ and $\varepsilon_i \ll 1$. The terms containing the second order of ε_c and ε_i are neglected. This approach was proposed by Kelly [17] to obtain the closed-form expressions for modal frequencies of single story base-isolated building. Substituting for above frequencies in Eq. (15), the corresponding modal matrix, $[\Phi]$, is given by

$$[\Phi] = \begin{bmatrix} 1 & 1 & 1 \\ \dfrac{\gamma_{mc}\varepsilon_c}{(1/\varepsilon_c\varepsilon_i) - \gamma_{mc}\varepsilon_c} & \dfrac{\varepsilon_i(\varepsilon_c - 1 - 3\gamma_{mc}\varepsilon_c)}{1 - \gamma_{mc}} & \dfrac{1 + \bar{\gamma}_i\bar{\varepsilon}_i}{\bar{\gamma}_i(1 + \bar{\varepsilon}_i)} \\ \dfrac{\gamma_{mc}\varepsilon_c}{1 - 3\gamma_{mc}\varepsilon_c} & \dfrac{\varepsilon_i(\varepsilon_c - 1 - 3\gamma_{mc}\varepsilon_c)}{1 + \gamma_{mc}\varepsilon_C} & -1 \end{bmatrix}. \tag{31}$$

The parameters L_n, M_n and C_n appearing in Eqs. (17) and (18) for modal damping and modal participation factor in each mode are explicitly given by the following formulae:

$$L_1 = M\gamma_{mc}\left(\frac{1 - \varepsilon_c\gamma_{mc} + \varepsilon_c}{1 - 3\varepsilon_c\gamma_{mc}}\right), \tag{32}$$

$$L_2 = m_c + (\varepsilon_c - 1 - 2\varepsilon_c\gamma_{mc})\left(\frac{m_i\varepsilon_i}{1 - \gamma_{mc}} + \frac{M}{1 + \gamma_{mc}\varepsilon_c}\right), \tag{33}$$

$$L_3 = \frac{m_i(1 - \bar{\gamma}_i)}{\bar{\gamma}_i(1 + \bar{\varepsilon}_i)} - m_r, \tag{34}$$

$$M_1 = \frac{m_c}{1 - 3\gamma_{mc}\varepsilon_c} + (m_i + m_r)\left(\frac{\varepsilon_c\gamma_{mc}}{1 - 3\varepsilon_c\gamma_{mc}}\right)^2, \tag{35}$$

$$\begin{aligned} M_2 = {} & m_c\left(\frac{\varepsilon_c(1 - \gamma_{mc})}{1 + \gamma_{mc}\varepsilon_c}\right)^2 + m_i(\varepsilon_c - 1 - \gamma_{mc}\varepsilon_c)^2\left(\frac{\varepsilon_i}{1 - \gamma_{mc}} + \frac{1}{1 + \gamma_{mc}\varepsilon_c}\right)^2 \\ & + m_r\left(\frac{\varepsilon_c - 1 - 2\gamma_{mc}\varepsilon_c}{1 + \gamma_{mc}\varepsilon_c}\right)^2, \end{aligned} \tag{36}$$

$$M_3 = \left(\frac{1 + 3\bar{\gamma}_i}{\bar{\gamma}_i(1 - 3\bar{\varepsilon}_i)}\right)^2 + m_r, \tag{37}$$

$$C_1 = c_c + c_i\left(\frac{\gamma_{mc}\varepsilon_c}{(1/\varepsilon_c\varepsilon_i) + \gamma_{mc}\varepsilon_c}\right)^2 + c_b\left(\frac{\gamma_{mc}\varepsilon_c}{1 + 3\gamma_{mc}\varepsilon_c}\right)^2, \tag{38}$$

$$C_2 = c_c + c_i\left(\frac{c_c - 1 - 3\gamma_{mc}\varepsilon_c}{1 - \gamma_{mc}}\right)^2 + c_b\left(\frac{c_c - 1 - 3\gamma_{mc}\varepsilon_c}{1 + \gamma_{mc}\varepsilon_c}\right)^2, \tag{39}$$

$$C_3 = c_c + c_i\left(\frac{1 + \bar{\gamma}_i\bar{\varepsilon}_i}{\bar{\gamma}_i(1 + \bar{\varepsilon}_i)}\right)^2 + c_b. \tag{40}$$

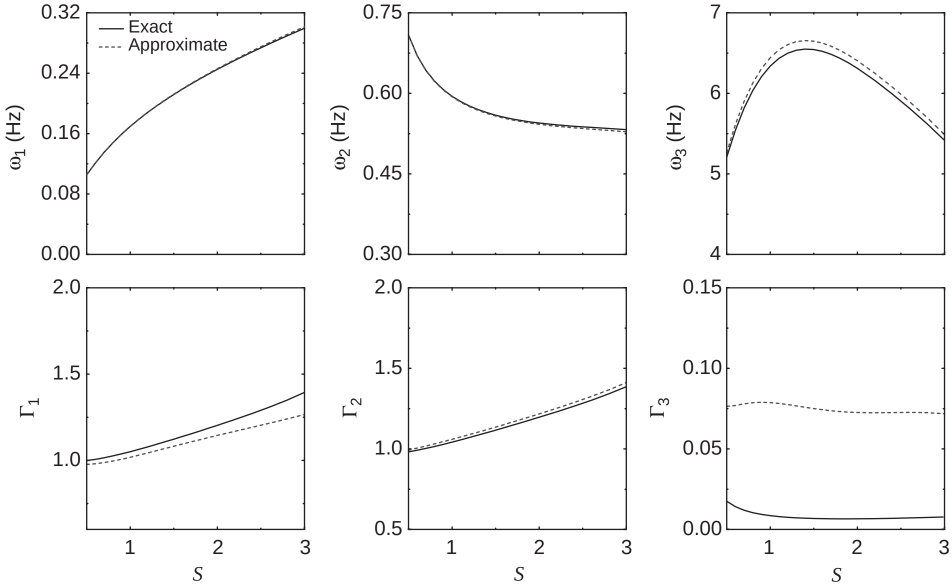

Fig. 5. Comparison of exact and approximate modal frequencies and participation factors of isolated tank against different tank aspect ratio, S.

The modal parameters obtained from the above closed-form expressions (referred as approximate) are compared with the exact values in order to verify their accuracy for their wide range of practical liquid storage tanks. The variation of modal frequencies and modal participation factor is shown in Fig. 5. These parameters are shown for $T_b = 2$ s. It is observed that these parameters predicted by the closed-form expressions have similar variation and closely matches the corresponding exact values. The error is found to be more in the third natural frequency and at higher value of the aspect ratio.

Fig. 6 shows the variation of modal damping ratios against the bearing damping ratio, ξ_b. The modal damping ratio linearly increases with the increase of the bearing damping in all modes. Further, the modal damping ratios predicted by the closed-form expressions are quite close to the exact values. The difference in the damping ratio by the closed-form expression and exact value is relatively more for slender tank in comparison to broad tanks due to coupling between sloshing time period and isolation period.

7. Simplified response for isolated tanks

The study of base-isolated liquid storage tanks using modal superposition method indicated that the earthquake response of the isolated tank is mainly influenced by a particular mode. Therefore, taking the influencing modes into consideration, the simplified approximate expressions to evaluate various response quantities of isolated tanks are given by

$$F_b = \sqrt{(c_b\omega_2)^2 + (k_b^2)}\left(\frac{\varepsilon_i(\varepsilon_c - 1 - 3\varepsilon_c\gamma_{mc})}{1 + \varepsilon_c\gamma_{mc}}\right)\Gamma_2 I_2, \tag{41}$$

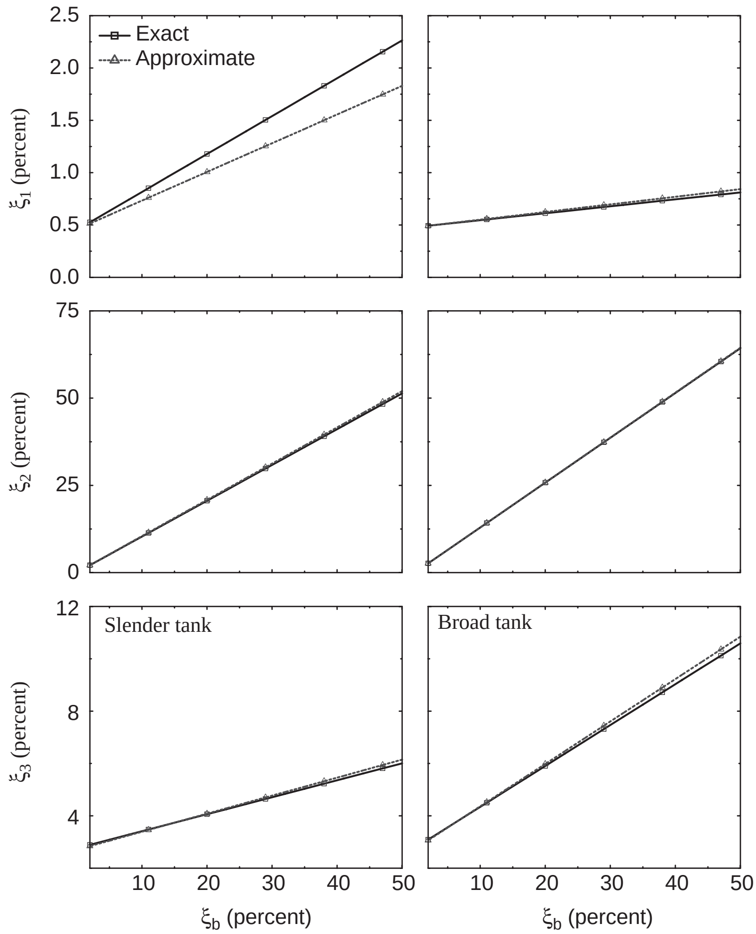

Fig. 6. Comparison of exact and approximate modal damping ratio of isolated tank against different bearing damping ratio, ξ_b.

$$x_c = \Gamma_1 I_1 + \Gamma_2 I_2, \tag{42}$$

$$x_b = \left(\frac{\varepsilon_i(\varepsilon_c - 1 - 3\varepsilon_c\gamma_{mc})}{1+\varepsilon_c\gamma_{mc}}\right)\Gamma_2 I_2, \tag{43}$$

where

$$I_n = -\int_0^t \ddot{u}_g(\tau)\left(\frac{e^{-\xi_n\omega_n(t-\tau)}}{\omega_{dn}}\right)\sin\{\omega_{dn}(t-\tau)\}\,d\tau. \tag{44}$$

For the analysis by response spectrum method, I_n shall be replaced by $S_d(\xi_n, \omega_n)$ and the ABS rule for combining the two components of the sloshing displacement is used.

A comparison of time variation of approximate base shear, sloshing displacement and bearing displacement obtained from Eqs. (41)–(44) with the exact response is carried out in Figs. 7 and 8 for slender and broad tanks respectively. The response is plotted for the Imperial Valley

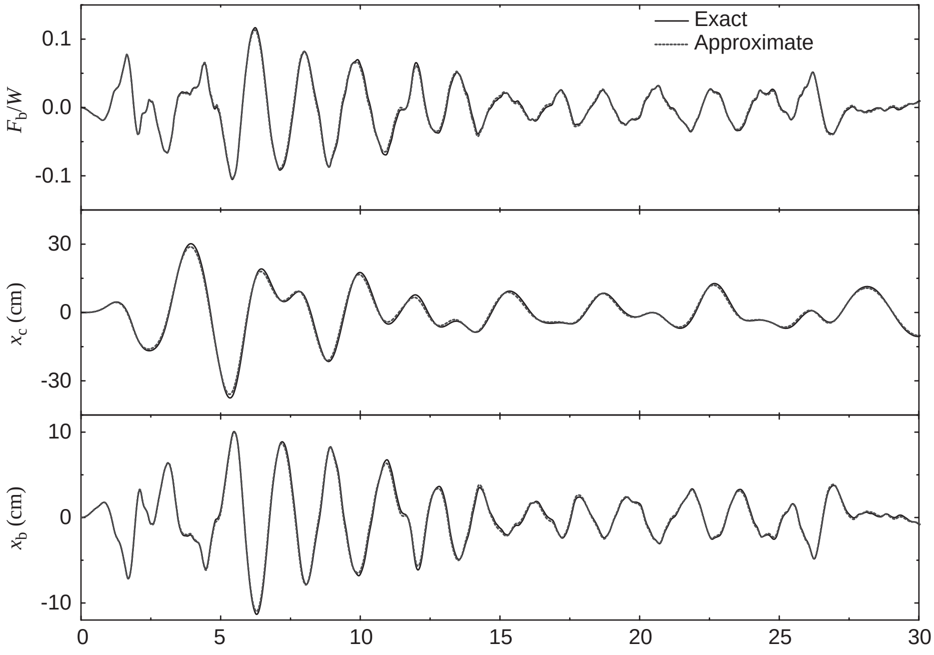

Fig. 7. Comparison of approximate and exact seismic response of slender tank under Imperial Valley, 1940 earthquake motion ($T_b = 2$ s and $\xi_b = 0.1$).

Table 3
Comparison of exact and approximate response of the isolated tank ($T_b = 2$ s and $\xi_b = 0.1$)

Earthquake	Response	Slender tank		Broad tank	
		Exact	Approximate	Exact	Approximate
Imperial Valley, 1940	x_c (cm)	37.52	40.39	45.30	50.48
	x_b (cm)	11.34	11.75	8.05	8.28
	F_b/W	0.116	0.121	0.084	0.086
Loma Prieta, 1989	x_c (cm)	212.6	219.04	68.01	83.34
	x_b (cm)	40.19	46.45	28.73	29.16
	F_b/W	0.419	0.478	0.299	0.304
Kobe, 1995	x_c (cm)	65.01	88.16	37.65	50.79
	x_b (cm)	31.19	34.11	27.07	27.27
	F_b/W	0.328	0.351	0.288	0.284

earthquake, 1940 earthquake motion with $\xi_b = 0.1$. These figures indicate that the approximate response closely matches the corresponding exact response. Thus, the proposed approximate expressions for the isolated liquid storage tanks accurately predict the earthquake response of the system.

Comparison of the peak responses of the isolated slender and broad tanks by approximate method (using the response spectrum) and exact method is shown in Table 3 for different

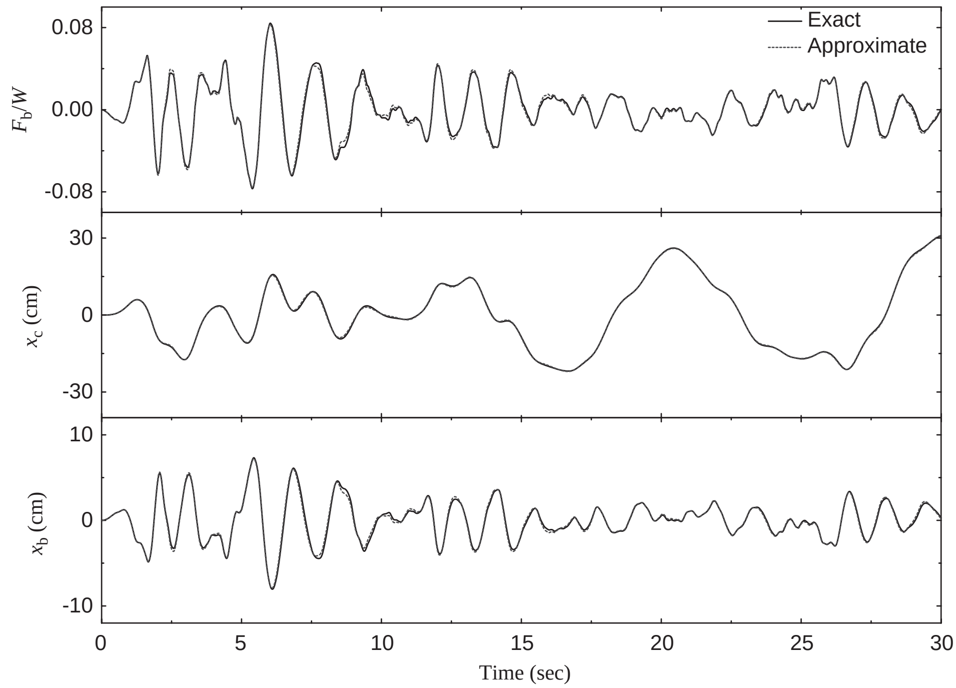

Fig. 8. Comparison of approximate and exact seismic response of broad tank under Imperial Valley, 1940 earthquake motion ($T_b = 2$ s and $\xi_b = 0.1$).

earthquake motions. The table indicates that the base shear predicted by the simplified method closely matches the corresponding exact response. The sloshing displacements of slender and broad tanks obtained by the approximate method are higher in comparison to the exact method because the ABS rule is used to combine the peak response of first and second modes. The peak base displacement predicted by the proposed expression closely matches the exact value for both slender and broad tanks. Thus, the proposed closed-form expressions accurately estimate the peak response of base-isolated liquid storage tanks.

8. Conclusions

The following conclusions are drawn from the trends of the results of the present study:

1. For a base-isolated liquid storage tank the effects of non-classical damping are found to be insignificant and the classical modal superposition method can be used for the earthquake response of the system.
2. The base shear and bearing displacement of the isolated tanks are mainly contributed by the second mode, i.e., isolation mode.
3. The sloshing displacement of broad tank is mainly governed by the first mode (i.e., sloshing mode). However, for slender tank both first and second modes contribute significantly.

4. The CQC and SRSS combination rules generally predict the identical seismic response of the isolated liquid storage tanks. On the other hand, the ABS rule provides conservative estimate of the peak base shear and base displacement but predicts better estimate for the sloshing displacement.
5. At moderate isolation damping the CQC and SRSS rules provide better estimate of peak base shear and base displacement. However, at higher isolation damping the SRSS rule gives better estimate of the response for slender tank.
6. The modal parameters predicted by the closed-form expressions for the base-isolated tanks matches the corresponding exact values with marginal errors in the third mode. However, this does not influence the response, since its contribution to total response is negligible.
7. The seismic response of isolated tanks predicted by the proposed simplified method closely matches the corresponding exact response.

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