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Letter to the Editor

Response of open-plane frames on isolated footings to an excitation characterized by a white noise

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1. Introduction

There are occasions, such as multi-storey buildings founded on soft soil, when it becomes necessary to consider the effects of deformability of the foundation. These effects are generally referred to as soil–structure interaction.

While the dynamic analysis of framed structures can be made on the basis that the superstructure possesses continuously distributed properties, the analysis is complex and practicable only in the case of very simple structures. More conveniently, a finite element approach can be adopted in which framed structures are discretized into segments and the displacements of the interconnecting nodes constitute the generalized co-ordinates (or dynamic degrees of freedom) of the structure. Kinematic constraints are commonly adopted to reduce the degrees of freedom in order to save computational efforts without significant loss of accuracy.

As an extreme, for framed structures it is usually assumed that the floor slabs have considerable in-plane rigidity and that the columns are in-extensible. The mass is assumed to be concentrated at the floors and to possess only translatory degrees of freedom. An n storey frame has only n degrees of freedom along its plane. When the foundation is shallow and flexible, translation and rotation of the footing are included. Thus, an n storey structure yields an $n+2$ degree of freedom system with soil–structure interaction. This discretization may be called as the Parmelee model. The adoption of the Parmelee model for the analysis of tall framed structures with soil–structure interaction and subjected to horizontal seismic excitation is very common in practice and in the literature [1–10]. On the other hand, if the physical arrangement of the superstructure can be retained in the model adopted for analysis under dynamic loads, it will be more consistent with the model used for analyzing the structure under static loads.

The flexibility of the shallow foundation may best be represented by frequency-dependent soil impedance [1–3]. However, some studies have shown that frequency-independent impedance is

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adequate to simulate the soil–structure interaction phenomenon [4,5,11,12]. Thus, the foundation reaction can be represented by the lumped-parameter frequency-independent system [6,7,13,14]. Although the interactive system does not possess classical normal modes, satisfactory results can be obtained by assuming that the damping matrix satisfies modal orthogonality conditions [2,5,7].

Results based on the Parmelee model show that a reduction in the natural frequencies of the structure occurs due to the introduction of foundation flexibility in the non-interactive system [1,4,8]. The reduction is greater at low shear wave velocities of the supporting medium [2,6]. However, a similar trend has been reported when the superstructure is modelled as a frame [15].

It has been reported [16] that the fundamental period of an open-plane frame obtained using a shear building model is always less than that obtained for a frame type idealization. The peak response to horizontal seismic excitation, being governed by the frequency content of the excitation in case of low damping, is greatly affected by the fundamental period as well as by the mode shape. Therefore, the response of an open-plane frame to a horizontal seismic excitation is influenced by the model chosen to represent it in the analysis.

When soil–structure interaction effects are considered, it has been reported [15] that a Parmelee model idealization always has a fundamental period larger than a frame-type idealization. Increasing the ratio of stiffness of floor to column only enhances the difference in fundamental periods between the two models over a wide range of shear modulus of the soil, G_s. The difference in mode shapes and periods of vibration ensure that the response to seismic excitation of an open-plane frame with soil–structure interaction is dependent on the model adopted to idealize the framed structure. The studies reported in the literature are based on the Parmelee model and have used both artificial earthquakes [7,11] and white noise [6] to model the horizontal ground acceleration. These studies have covered a wide range of fundamental frequencies of the non-interactive structure, range of height of the top mass to foundation radius ratios, foundation eccentricity and a practical range of shear modulus G_s values [6,7,11]. The results have established that soil–structure interaction may be beneficial for certain combinations of these parameters. The findings may not be applicable when the interactive system is represented by the frame type of idealization.

The frequency content of real and artificial earthquakes can inadvertently result in resonance in the soil–structure interactive system so that conclusions drawn from a parametric study become specific to the excitation chosen. On the other hand, adoption of a white noise, which specifies equal power in all frequencies, results in each interactive system responding as a narrow band filter so that more meaningful conclusions can be drawn on the effect of soil–structure interaction on the dynamic response. Further, closed-form solutions are possible with white noise excitation whereas, when an actual earthquake is used, the deterministic approach call for adoption of numerical methods.

In this letter, the role of soil–structure interaction on the response of an open-plane frame to a ground acceleration idealized by a white noise is examined in terms of shear forces and relative displacements in the storey columns. A comparision is also made with the response obtained using a Parmelee idealization.

2. Correlation functions for the response in terms of column shear and storey sway

The member end actions f_i of a typical member m (consisting of two end moments f_1 and f_2 and axial force f_3) of the frame can be related to the joint displacements of the end nodes of this member $\{v_i\}_m$ (indicated in Fig. 1(a)) by

$$\{f_i\}_m = [S][A]^T\{v\}_m, \tag{1}$$

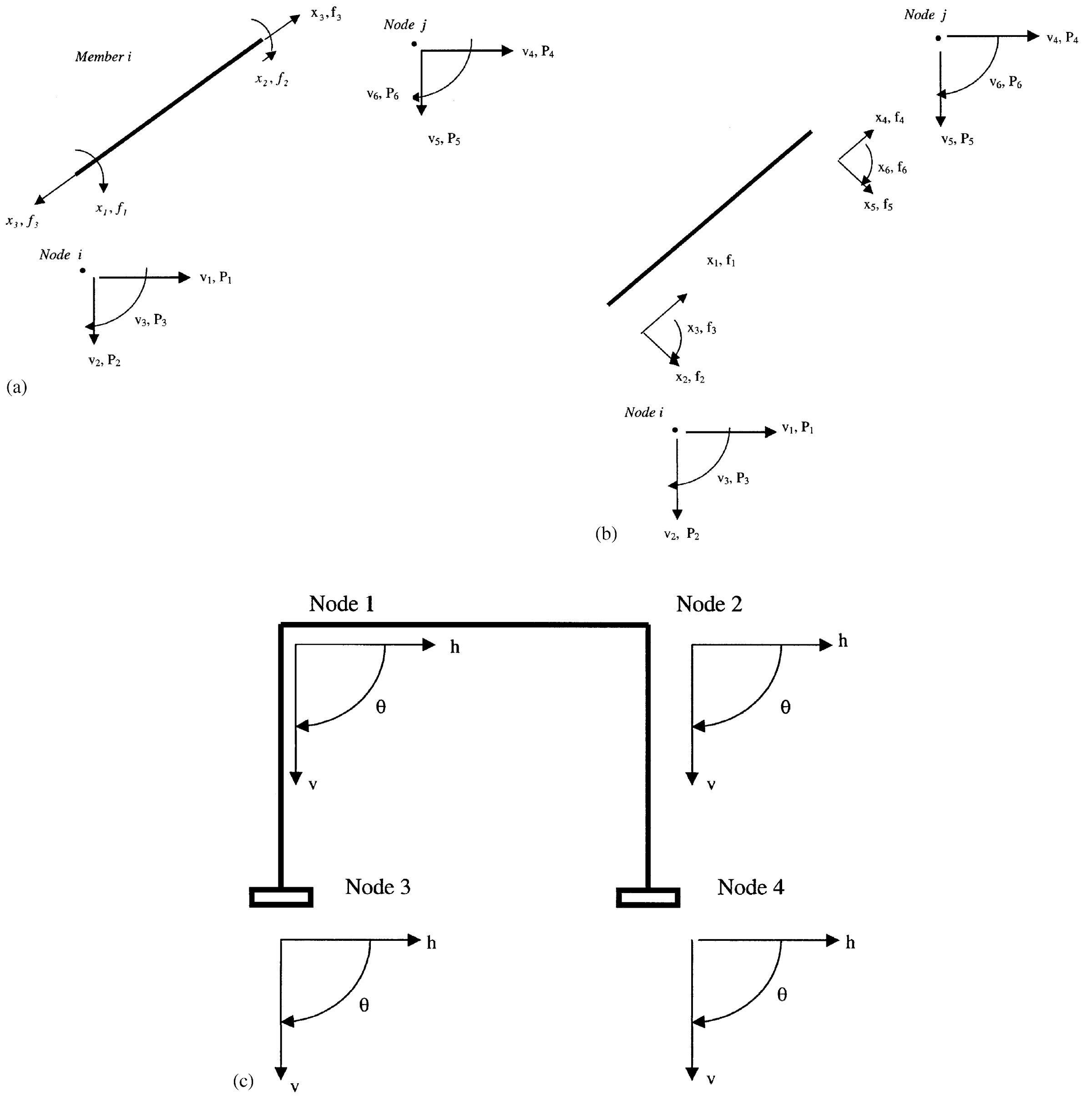

Fig. 1. (a) Plane frame element with three member end forces. (b) Plane frame element with maximum of six member end forces. (c) Nodal degrees of freedom in a typical frame with soil–structure interaction.

where $[S]$ is a 3×3 member stiffness matrix relating the member end actions to the member end displacements (end rotations x_1, x_2 and member elongation x_3). $[A]$ is a matrix that relates member end actions to nodal forces and its transpose relates member end displacements $\{x_i\}$ to nodal displacements $\{v\}_m$. For a column member which is the object of interest of this study, the column end moments

$$f_{1,m} = \sum_k SA^{\mathrm{T}}(1,k)v_{k,m}, \tag{2a}$$

$$f_{2,m} = \sum_k SA^{\mathrm{T}}(2,k)v_{k,m}. \tag{2b}$$

The subscript m identifies the member and the nodal displacements $\{v\}$ associated with it.

The column shear at time t is

$$shear_m(t) = (f_1(t) + f_2(t))/l_m, \tag{3a}$$

$$= \sum_k SA^{\mathrm{T}}(1+2,k)v_{k,m}(t)/l_m = \sum_k CSA^{\mathrm{T}}(k)v_{k,m}(t)/l_m = [CSA]^{\mathrm{T}}\{v(t)\}_m/l_m, \tag{3b}$$

where l_m is the length of the member.

The auto-correlation of the column shear is

$$R_{shear,m}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-(T/2)}^{T/2} shear(t)\, shear(t+\tau)\,\mathrm{d}t. \tag{4}$$

For the open-plane frame shown in Fig. 1(c), the equations of motion for the n-degree of freedom system under horizontal excitation $\ddot{u}_g(t)$, permitting three degrees of freedom at each footing base, are

$$\begin{bmatrix} [m_s] & [0] \\ [0] & [m_b] \end{bmatrix}\begin{Bmatrix} \ddot{v} \\ \ddot{v}_b \end{Bmatrix} + [C]\begin{Bmatrix} \dot{v} \\ \dot{v}_b \end{Bmatrix} + \begin{bmatrix} [k_s] & [k_{sf}] \\ [k_{fs}] & [k_f + k_{ss}] \end{bmatrix}\begin{Bmatrix} v \\ v_b \end{Bmatrix} = -\begin{bmatrix} [m_s] & [0] \\ [0] & [m_b] \end{bmatrix}\{r\}\ddot{v}_g(t), \tag{5}$$

where $\{r\}$ is a vector of 1 and 0's to account for the degrees of freedom influenced by the horizontal ground acceleration $\ddot{u}_g(t)$ which may be a stationary process. The diagonal matrix $[m]$ contains the mass and mass moment of inertia elements associated with the superstructure degrees of freedom $\{v\}$, the diagonal matrix $[m_b]$ refers to the footing degrees of freedom. The matrix $[k_s]$ contains the stiffness elements associated with the superstructure and the matrix $[k_{fs}]$ refers to the degrees of freedom shared by the foundation and the superstructure. The matrix $[k_f + k_{ss}]$ which pertains to the foundation degrees of freedom incorporates the effect of soil–structure interaction.

If $[\phi]$ is the matrix of eigenvectors associated with the undamped form of the equations of motion (Eq. (5)) during free vibrations and $[\omega]$ the corresponding diagonal matrix of natural frequencies, the nodal displacements at time t and $t+\tau$ are

$$\{v(t)\} = [\phi]\{q(t)\} \text{ and } \{v(t+\tau)\} = [\phi]\{q(t+\tau)\}. \tag{6a}$$

While the nodal displacements of interest (with respect to member m) can be obtained from the frame nodal displacements $\{v(t)\}$ using a selection matrix $[y]_m$:

$$\{v(t)\}_m = [y]_m\{v(t)\} = [y]_m[\phi]\{q(t)\}. \tag{6b}$$

The equations of motion after uncoupling are, using Eq. (6a)

$$\ddot{q}_r(t) + 2\xi_r\omega_r\ddot{q}_r(t) + \omega_r^2 q_r(t) = p_r(t), \tag{7a}$$

where ξ_r is the damping ratio associated with the rth mode of vibration ($r = 1, 2, \ldots, n$). And

$$p_r(t) = -\{\phi_r\}^{\mathrm{T}}[m]\{r\}\ddot{v}_g(t). \tag{7b}$$

Then the generalized response in the rth mode of vibration is

$$q_r(t) = \int g_r(\lambda_r)p_r(t - \lambda_r)\,\mathrm{d}\lambda_r, \tag{8}$$

where $g_r(t)$ is the impulse response and $p_r(t)$ is the excitation.

Or in the frequency domain

$$Q_r(\omega) = G_r(\omega)P_r(\omega), \tag{9}$$

where $G_r(\omega) = 1/(\omega_r^2 - \omega^2 + 2\mathrm{i}\xi_r\omega_r\omega)$.

Hence, Eq. (4) takes the form, using Eqs. (6a) and (8),

$$R_{shear,m}(\tau) = \lim_{T\to\infty}\frac{1}{T}\frac{1}{l_m^2}\int_{-t/2}^{t/2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[CSA]^{\mathrm{T}}[y]_m[\phi]\{g_r(\lambda_r)p_r(t-\lambda_r)\}$$
$$\times\,\{p_r(t+\tau-\lambda_s)g_s(\lambda_s)\}^{\mathrm{T}}[\phi]^{\mathrm{T}}[y]_m^{\mathrm{T}}[CSA]\,\mathrm{d}\lambda_r\,\mathrm{d}\lambda_s\,\mathrm{d}t. \tag{10}$$

Since the excitation is a stationary process,

$$R_{p_{rs}}(\tau - \lambda_s + \lambda_r) = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} p_r(t-\lambda_r)p_s(t+\tau-\lambda_s)\,\mathrm{d}t. \tag{11}$$

Eq. (10) reduces to, using Eq. (7b),

$$R_{shear,m}(\tau) = \frac{1}{l^2}\{B\}_m^{\mathrm{T}}[A_rA_s\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_r(\lambda_r)g_s(\lambda_s)R_{\ddot{v}_g}(\tau-\lambda_s+\lambda_r)\,\mathrm{d}\lambda_r\,\mathrm{d}\lambda_s]\{B\}_m, \tag{12}$$

where $A_r = \{\phi_r\}^{\mathrm{T}}[m]\{r\}$ and $A_s = \{\phi_s\}^{\mathrm{T}}[m]\{r\}$. And

$$\{B\}_m^{\mathrm{T}} = [CSA^{\mathrm{T}}][y]_m[\phi].$$

Since

$$R_{\ddot{v}_g}(\tau - \lambda_s + \lambda_r) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{\ddot{v}_g}(\omega)\,\mathrm{e}^{\mathrm{i}\omega(\tau-\lambda_s+\lambda_r)}\,\mathrm{d}\omega,$$

$$R_{shear,m}(\tau) = \frac{1}{2\pi l^2}\{B\}_m^{\mathrm{T}}[A_rA_s\int_{-\infty}^{\infty} G_r^*(\omega)G_s(\omega)S_{\ddot{v}_g}(\omega)\,\mathrm{e}^{-\omega\tau}\,\mathrm{d}\omega]\{B\}_m. \tag{13}$$

For input ground acceleration represented by a white noise S_0 and to obtain the root mean square value of the shear in the column member m, Eq. (13) takes the form

$$R_{shear,m}(0) = \frac{1}{2\pi l^2}\{B\}_m^{\mathrm{T}}S_0[A_rA_s\int_{-\infty}^{\infty} G_r^*(\omega)G_s(\omega)\,\mathrm{d}\omega]\{B\}_m. \tag{14}$$

The integrand in Eq. (14) has four simple poles $\pm\omega_s\sqrt{1-\xi_s^2} + \mathrm{i}\xi_s\omega_s$ and $\pm\omega_r\sqrt{1-\xi_r^2} - \mathrm{i}\xi_r\omega_r$, two of which lie within the contour of integration shown in Fig. 2.

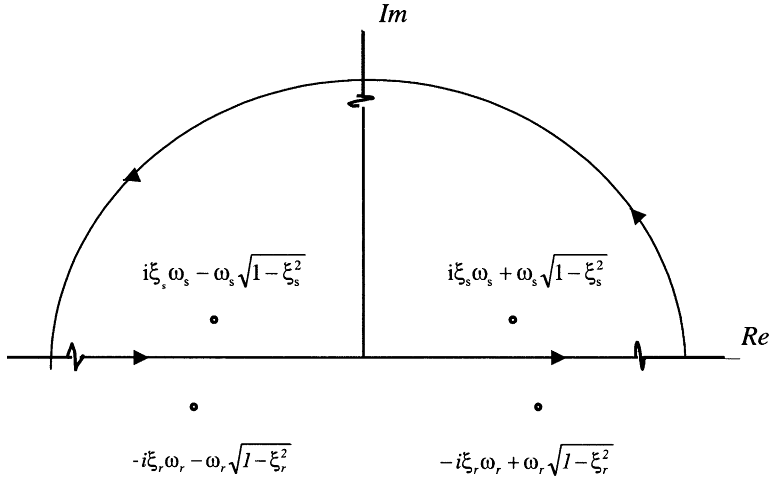

Fig. 2. Contour of integration for Eqs. (14) and (16).

When the relative transverse displacements of the column member are desired, Eq. (1) can be more conveniently reformulated so that the element has six member end actions $\{f\}$ and displacements $\{x\}$ (Fig. 1(b)). Since the response to a stationary excitation by a linear system is also a stationary process, if $x_i(t)$ and $x_j(t)$ are the transverse member end displacements,

$$R_{x_i-x_j}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} (x_i(t) - x_j(t))(x_i(t+\tau) - x_j(t+\tau))\,\mathrm{d}t$$
$$= R_{x_i}(\tau) + R_{x_j}(\tau) - R_{x_{ij}}(\tau) - R_{x_{ji}}(\tau). \quad (15)$$

The maximum relative transverse displacement for a column member (storey sway) is then

$$R_{x_i-x_j}(0) = R_{x_i}(0) + R_{x_j}(0) - R_{x_{ij}}(0) - R_{x_{ji}}(0).$$

Eq. (15) can be written to obtain the cross-correlation coefficient matrix as

$$[R_{x_{ij}}(0)] = [y][\phi]\left[\frac{S_0}{2\pi}\int_{-\infty}^{\infty} G_r^*(\omega)G_s(\omega)\,\mathrm{d}\omega\right][\phi]^{\mathrm{T}}[y]^{\mathrm{T}}. \quad (16)$$

From which the mean square value of the relative displacement (storey sway) of the column member can be found.

3. Frames adopted and range of soil properties

A 1-bay single-storey and a 1-bay four-storey open-plane frame of flat slab construction were adopted for the study. Fig. 3(a) and (b) show the plan and elevation of the 1-bay four-storey frame. For both frames, a bay span of 6 m and a uniform storey height of 3 m are considered. The slab is 0.3 m thick (Fig. 3(b)) and the column dimensions are 0.2 m × 0.5 m (Fig. 3(a)). The interframe spacing is 4 m. In the transverse section, the slabs with columns constitute a flexible frame as shown shaded in Fig. 3(a).

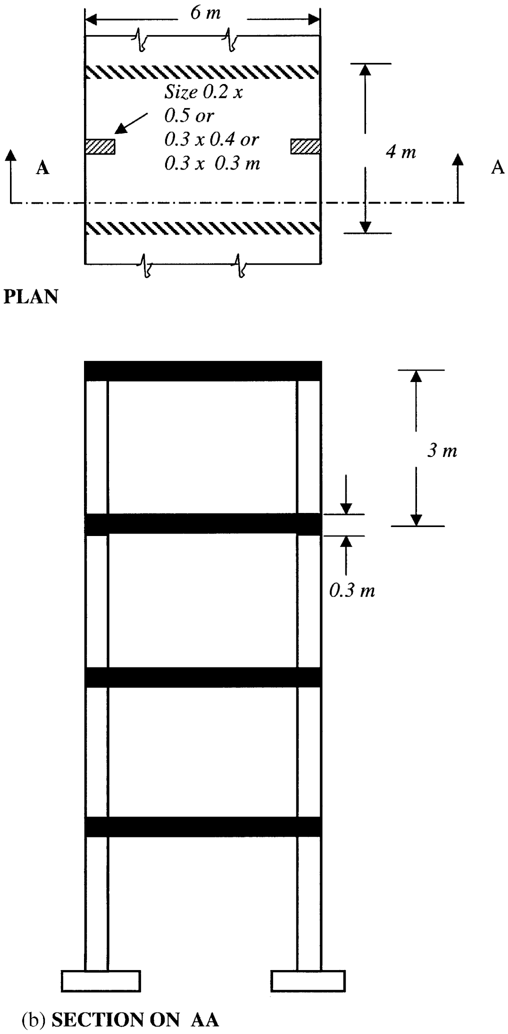

Fig. 3. Typical open-plane frame on isolated footings.

The material properties of structural members used for the linear analysis of these frames are modulus of elasticity of concrete, $E_c = 22\,\text{GPa}$ and mass density of concrete, $\rho = 2400\,\text{kg/m}^3$.

To permit soil–structure interaction in the single- and four-storey frames adopted in the study, rigid bases of concrete of size $1.0\,\text{m} \times 0.5\,\text{m}$ and 0.3 m thick were selected as footings. While this size may be adequate in medium to hard soils for a single-storey frame, and in firm to hard soils for a four-storey frame, two more footing sizes, namely, $2.0\,\text{m} \times 1.0\,\text{m}$ and $2.0\,\text{m} \times 2.0\,\text{m}$ with thickness of 1.0 m were tried to examine whether increasing the base mass can qualitatively influence the results. For the same reason, two additional column dimensions, namely, $0.3\,\text{m} \times 0.4\,\text{m}$ and $0.3\,\text{m} \times 0.3\,\text{m}$, with a storey height of 4.0 m have been also attempted so that the study covers a wide range of fundamental frequency in the non-interactive frame. Thus, for discussion of the results, the frames (both single- and four storey) are classified as Type 1 for the cases of column sections of 0.2×0.5 m with storey height of 3.0 m, Type 2 and Type 3 for column sections of 0.3×0.4 and 0.3×0.3 m, respectively, with storey height for these two types being 4.0 m. As a result, the fundamental frequency for the single-storey non-interactive structure varies from 42 to 17 rad/s. As the column section varies from 0.2×0.5 to $0.3 \times 0.3\,m^2$ when a frame model discretization is used. When a shear building idealization is used, the fundamental frequency ranges between 48 and 17 rad/s. For the four-storey structure, the fundamental frequency varies between 12 and 5.5 rad/s. Depending on the column section and inter-storey height for the frame model. In the case of the shear building discretization, the fundamental frequency varies between 16 and 6 rad/s.

The footing sizes are classified as Footing A (1.0 m length in the plane of the frame, 0.5 m width and 0.3 m thickness), Footings B and C which have a length of 2.0 m in the plane of the frame and thickness of 1.0 m but widths of 1.0 and 2.0 m, respectively.

For the non-interactive system, a constant modal damping ratio of 5% was adopted. While the soil stiffness coefficients in Eq. (5) are actually frequency dependent, to permit decoupling of the equations they have been approximated by frequency-independent ones. In the current study, those used by Pais and Kausal [17] have been used. These are suitable for circular and rectangular foundations. The damping contribution of the soil has been treated in earlier studies [2,5–7] as frequency independent and it was concluded that the assuming that the damping matrix satisfies orthogonality conditions yields satisfactory results. In the current study for the interactive structure, a constant modal damping of 5% in all the modes was assumed.

To render the results of the interactive study realistic, the shear modulus of soil G_s was varied from 10 to 500 MPa so that the results are representative of medium to hard soils where isolated footings are used to support light to medium weight structures. A value of 0.3 was adopted for the Poissons' ratio of soil, μ_s.

The eigenvalues and eigenvectors of the undamped systems necessary for computing the cross-correlation functions in Eqs. (14) and (16) were extracted using the Jacobi method.

4. Results and discussion

4.1. Eigenvalues and eigenvectors

In the absence of soil–structure interaction, the fundamental frequencies of the single-storey structure, Frames 1, 2 and 3 are 41.98, 25.02 and 17.02 rad/s, respectively, when represented by the frame model. The corresponding values for the shear building model are 47.57, 26.45 and 17.52 rad/s.

In Table 1(a) are indicated the six lowest frequencies of the single-storey frame (Frame 1 Footing A) represented by the frame model over a range of soil shear modulus G_s values. Also included are the six natural frequencies when soil–structure interaction is not permitted. The natural frequencies of the interactive frame decrease as the shear modulus of the soil G_s decreases, and the effect in more pronounced for the fundamental frequency. The ratio of the fundamental frequency of the frame with soil–structure interaction to that of the frame without soil–structure interaction (fundamental frequency ratio) is shown in Fig. 4(a). It is seen from Fig. 4(a) that the

Table 1
Undamped natural frequencies of single-storey frame (rad/s)

G_s (MPa)	10	50	90	150	300	500	Rigid foundation
Mode							
(a) *Frame model*							
1	19.31	27.26	30.59	33.42	36.66	38.44	41.98
2	47.49	65.31	66.22	66.97	67.88	68.40	69.54
3	49.34	82.50	90.45	94.22	97.08	98.34	100.60
4	62.02	104.62	133.71	161.65	199.58	224.19	285.50
5	106.55	126.11	147.14	171.42	206.82	230.43	290.13
6	178.95	362.45	475.98	596.01	779.81	949.74	989.97
(b) *Parmelee model*							
1	6.36	13.84	18.08	22.46	29.08	33.88	47.58
2	65.19	90.88	97.51	103.65	115.23	128.56	—
3	245.31	404.34	519.20	655.26	910.77	1167.60	—

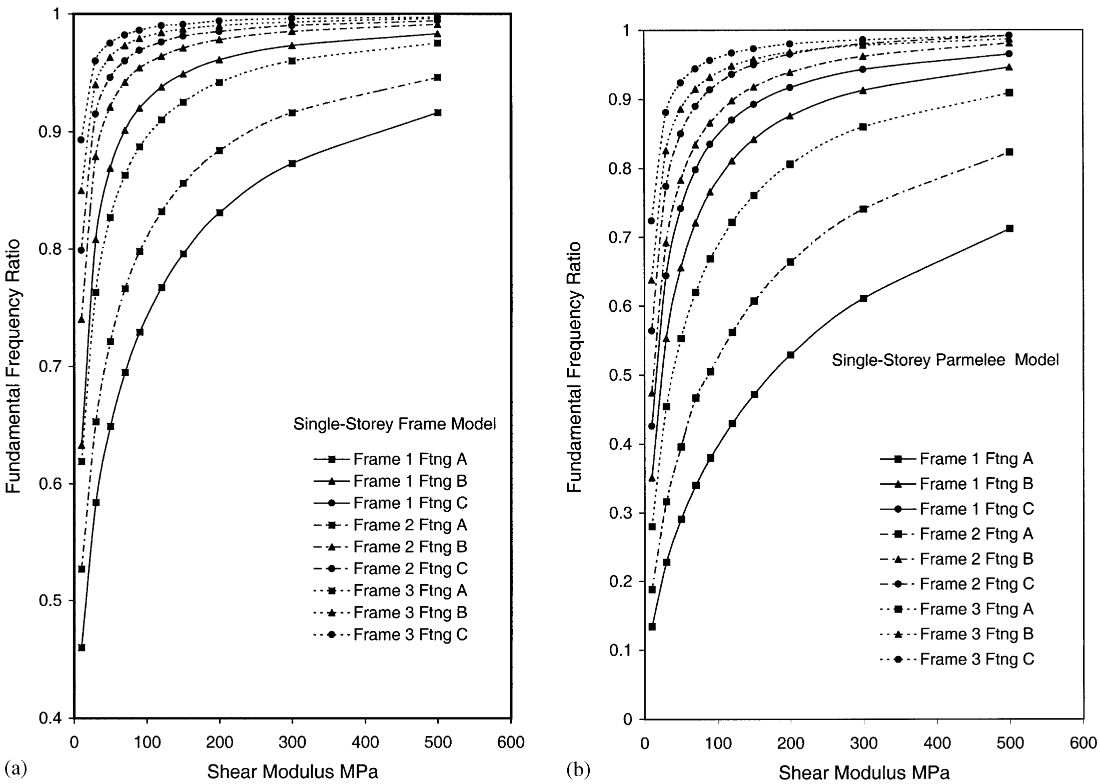

Fig. 4. Influence of soil–structure interaction on the fundamental frequency of the single-storey frame: (a) frame model and (b) Parmelee model.

fundamental frequency of the frame is affected by soil–structure interaction more severely for stiffer frames. For any frame, smaller footings with consequent smaller masses and soil impedance bring greater reduction in the magnitude of the fundamental frequency.

All the natural frequencies obtained for the alternate method of representing the frame, namely, the Parmelee model are also indicated in Table 1(b) which includes the fundamental frequency of the single-storey shear building. As reported in the literature [15,16], the shear building yields a stiffer system than the frame model. When soil–structure interaction is considered, the frame model is a stiffer system than the Parmelee model at any value of G_s in terms of the fundamental frequency. The frequencies for the higher modes obtained with the Parmelee model are much larger than those yielded by the frame model.

The effect of soil–structure interaction on the fundamental frequency of the single-storey Parmelee model is seen in Fig. 4(b). It is observed that for any G_s. as in the case of the frame model, the frequency reduction is more severe for the stiffer superstructure with a smaller footing.

Some results on the effect of type of model and G_s on the natural frequencies of the four-storey frame (Frame 1 Footing A) are available in the literature [15]. Effect of superstructure stiffness and footing size on the fundamental frequency follow the trends reported for the single-storey frame. Results regarding this observation are therefore not presented.

4.2. Superstructure response to white noise

4.2.1. Displacement response

The root mean square value (r.m.s.) of the lowest storey sway of the single- and four-storey structures subjected to a horizontal ground acceleration in the nature of a white noise was determined by evaluating the cross-correlation matrix of the nodal degrees of freedom defined in Eq. (16).

In the absence of soil–structure interaction, the r.m.s. value of the storey sway of the single-storey structure Frames 1, 2 and 3 is $1.37 \times 10^{-2}\sqrt{S_0}$ m, $3.13 \times 10^{-2}\sqrt{S_0}$ m and $5.63 \times 10^{-2}\sqrt{S_0}$ m, respectively, when represented by the frame model. The corresponding values obtained for the shear building model are $4.82 \times 10^{-3}\sqrt{S_0}$ m, $1.16 \times 10^{-2}\sqrt{S_0}$ m and $2.16 \times 10^{-2}\sqrt{S_0}$ m, respectively. That is, the shear building model always yields lesser displacement than the frame model. This is also true for the lowest storey sway in the four-storey structure. When soil–structure interaction is considered, the Parmelee model yields, through out the range of G_s considered, lesser r.m.s. values of lowest storey sway than the frame model both for single- and four-storey frames.

A displacement ratio may be defined as the ratio of the r.m.s. value of the relative translation of the lowest storey column of the structure with soil–structure interaction to its value in the absence of soil–structure interaction.

The variation of the displacement ratio with G_s is indicated for nine combinations of frames and footing sizes in Fig. 5(a) for the frame model of the single-storey structure and in Fig. 5(b) for the Parmelee model. For the frame model, the displacement ratio is always greater than unity and it decreases with increase in the G_s value and progressively tends to unity as G_s tend to infinity (non-interactive case). For any superstructure stiffness (defined as the fundamental frequency of the non-interactive structure), the displacement ratio decreases as the size of the footing increases at any given G_s value. Also for a common footing size, the displacement ratio at any G_s increases

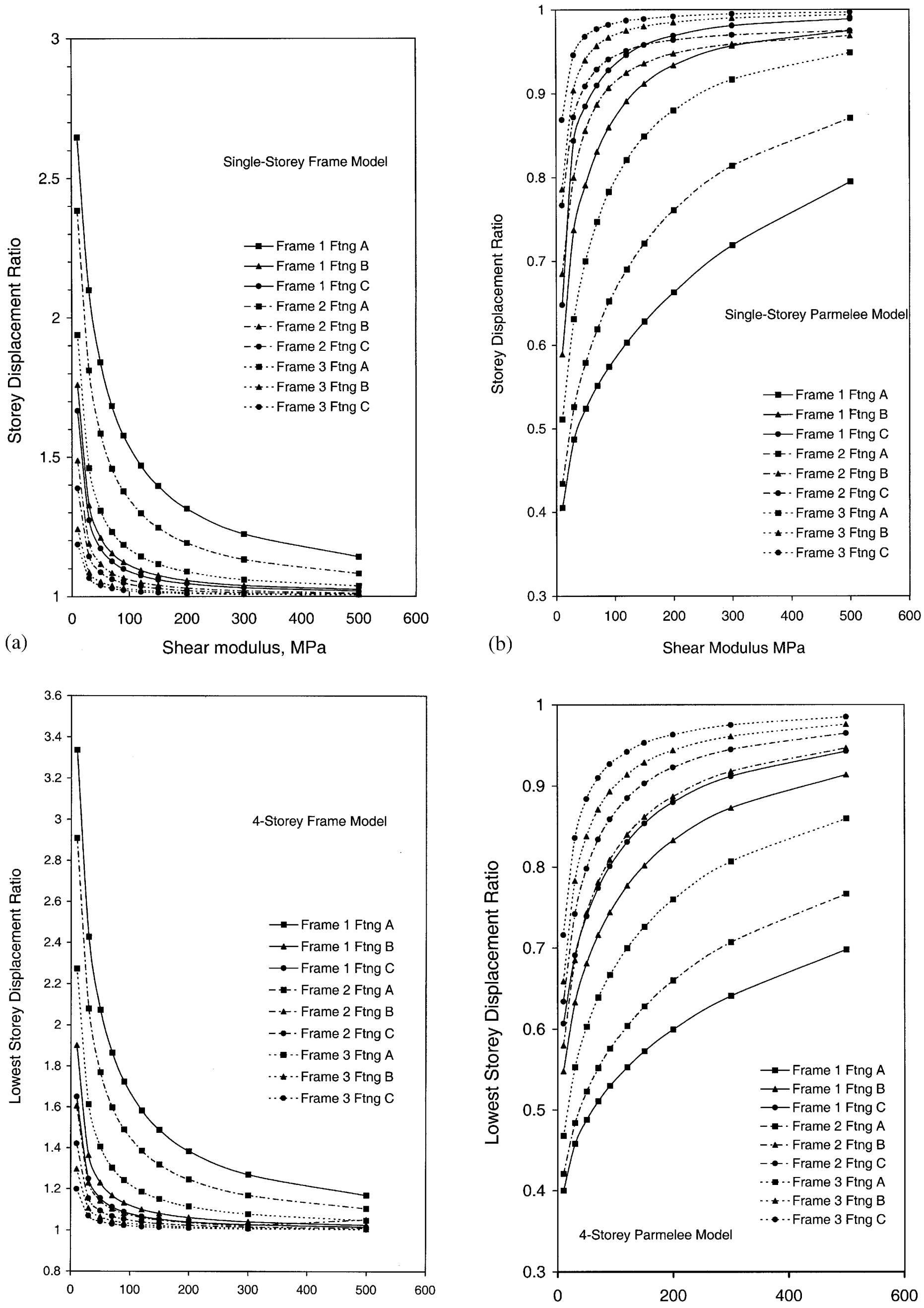

Fig. 5. (a) Influence of soil–structure interaction on storey sway for single-storey frame—frame model. (b) Influence of soil–structure interaction on storey sway for single-storey frame—Parmelee model. (c) Influence of soil–structure interaction on lowest storey sway for four-storey frame—frame model. (d) Influence of soil–structure interaction on lowest storey sway for four-storey frame—frame model.

as the superstructure stiffness increases. As G_s increases, the displacement ratio more rapidly approaches unity for the more flexible superstructure. These observations are repeated for the four-storey structure (Fig. 5(c)) and clearly indicate that the presence of soil–structure interaction always results in larger magnitudes of sway.

For the Parmelee model, the displacement ratio is found to be less than unity. The displacement ratio is found to increase with increase in G_s value unlike the trend seen for the frame model. The ratio progressively tends to unity as G_s tend to infinity (non-interactive case). The Figs. 5(b) and (d) show the variation of this ratio with G_s value, superstructure stiffness and footing size for the single- and four-storey structure, respectively. At any G_s value, the displacement ratio is greater for the more flexible superstructure mounted on the same size of footing. The ratio increases with footing size for a given superstructure. The same observations can be made in the case of the four-storey structure also. Thus, the displacement response of an open-plane frame on isolated footings to a white noise depends on the model used to idealize the structure.

The role of the number of modes in determining the r.m.s. value of storey sway was checked for the single-storey structure (Frame 1 Footing A). It was found that the fundamental mode of vibration contributes over 99.8% of the displacement response and 99.5% of the response in terms of column shear for the frame model both for the non-interactive structure and when soil–structure interaction is permitted. For the Shear building model of the four-storey structure, the fundamental mode of vibration contributed 98.5% of the r.m.s. value of the lowest storey sway. For the Parmelee model, it was found that the fundamental mode of vibration contributed about 69.5% of the sway (and also the column shear) for the single-storey structure and 62.5% of the sway of the lowest storey (and also column shear) in the four-storey structure, at low values of G_s. For very high values of G_s, this contribution rose to 97.8% and 92.3% for the single- and four-storey structures, respectively.

4.2.2. Lowest storey column shear

The r.m.s. value of the column shear in the lowest storey was determined using Eq. (14) for the structure in the non-interactive condition and when soil–structure interaction is permitted. The shear building model was found to yield a higher r.m.s. value of column shear than the frame model idealization for the single- and four-storey frame. When soil–structure interaction was accounted for, the Parmelee model yielded a higher r.m.s. value of the column shear than the frame model. To study the effect of soil–structure interaction on the column shear, a column shear ratio may be defined as the ratio of the r.m.s. value of lowest storey column shear obtained at any G_s to that obtained in the non-interactive condition.

Figs. 6(a) and (c) present the variation of the column shear ratio with G_s for the frame model of the single- and four-storey structure, respectively, for nine combinations of frame geometries and footing dimensions. Figs. 6(b) and (d) correspond to the Parmelee model of the single- and four-storey structures, respectively. The column shear ratio is less than unity through out the range of G_s values adopted for the four-storey frames regardless of whether the frame model or the Parmelee model is used to idealize the interactive system. For both models, the ratio is lowest for stiffer superstructures mounted on smaller footings at any G_s value. The ratio progressively tends to unity as G_s tend to infinity (non-interactive case). For the single-storey structure idealized by the Parmelee model, the column shear ratio is always less then unity and tends to unity as G_s increases. A similar tend is observed when the frame model is used except that it is seen that the

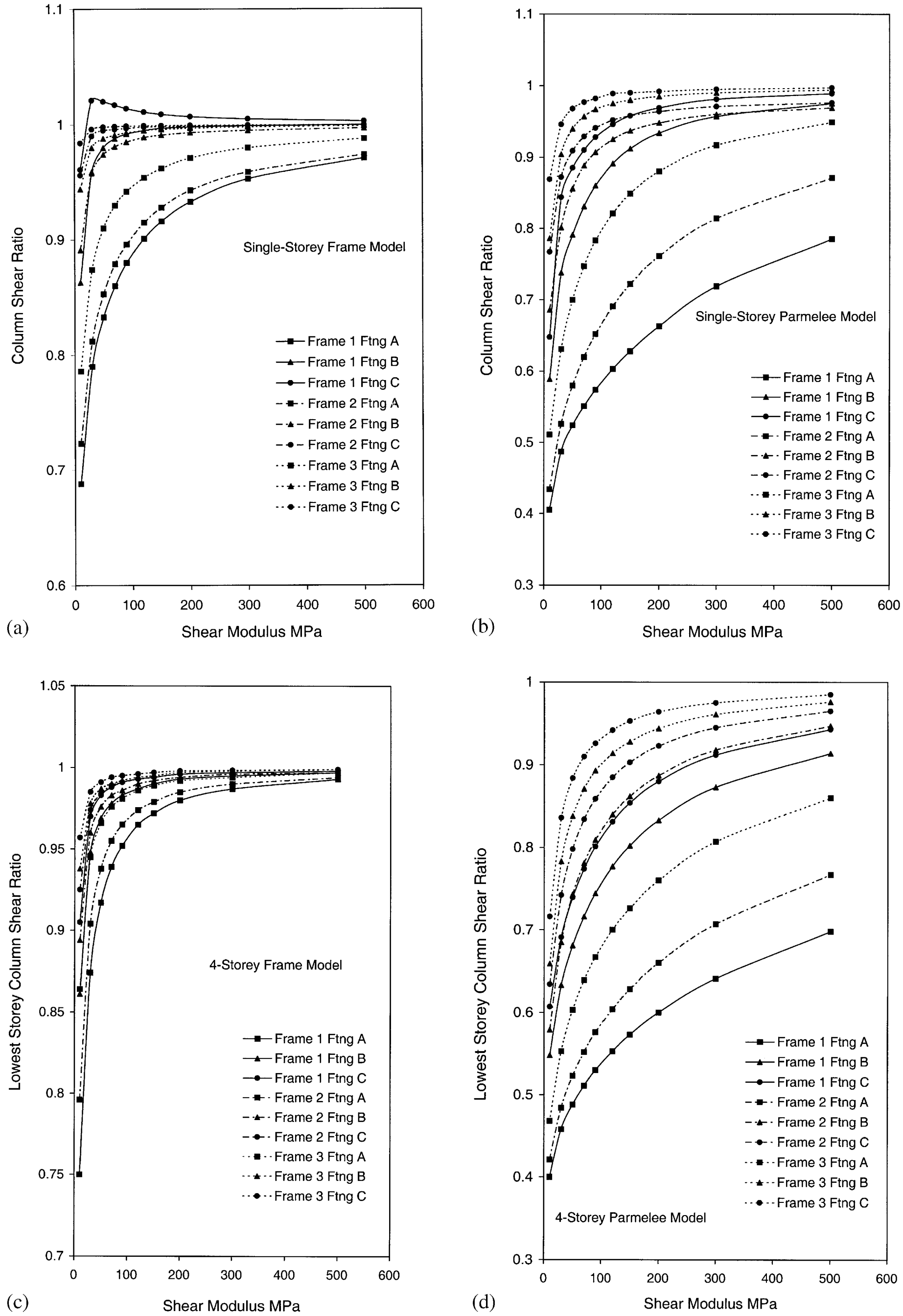

Fig. 6. (a) Influence of soil–structure interaction on the column shear for single-storey frame—frame model. (b) Influence of soil–structure interaction on the column shear for single-storey frame—Parmelee model. (c) Influence of soil–structure interaction on the lowest storey column shear for four-storey frame—frame model. (d) Influence of soil–structure interaction on the lowest storey column shear for four-storey frame—Parmelee model.

ratio marginally exceeds unity for a combination of a stiff superstructure and massive foundation (Frame 1 Footing C) for $G_s > 30$ MPa.

In this frame the footing mass (Footing C) is comparable to (6% larger) to the superstructure mass concentrated at the column slab junction (Fig. 1(c)). This is an unlikely combination because in foundation engineering practice, the weight of the isolated footing rarely exceeds 15% of the column load. For soft soils, wherever more massive footings are anticipated, a raft foundation is provided to control differential settlements and the resulting secondary forces in the superstructure. The column shear ratio was also computed for Frame 1 Footing C in the G_s range of 0.1–10 MPa. It was found that the column shear ratio increases monotonically from 0.292 at $G_s = 0.1$ MPa to 0.961 at $G_s = 10$ MPa. However, when the footing thickness was reduced to 0.5 m, it was found that the column shear ratio monotonically increased from 0.909 at $G_s = 10$ MPa to 0.999 at $G_s = 500$ MPa and did not exceed unity even for very large G_s.

As mentioned earlier, the fundamental mode governs the r.m.s. value of the lowest storey sway and the column shear induced in the frame model by the white noise excitation. The coefficients of the fundamental eigenvector for a stiff superstructure frame and three footing sizes (Frame 1 combined with footings A–C) are listed in Table 2 for different G_s values for the single-storey frame. Nodes 1 and 2 are located at the floor slab-column junction (Fig. 1(c)) while nodes 3 and 4

Table 2
Coefficients of the fundamental eigenvector for the single-storey frame model

G_s (MPa)	Node 1			Node 2			Node 3			Node 4			$\theta_1+\theta_3$	h_1-h_3
	h_1	v_1	θ_1	h_2	v_2	θ_2	h_3	v_3	θ_3	h_4	v_4	θ_4		
Frame 1 Footing A														
10	72.2[a]	−11.6	7.4	72.2	11.6	7.4	12.8	−11.2	24.3	12.8	11.2	24.3	31.7	59.4
50	73.4	−3.8	6.9	73.4	3.8	6.9	5.1	−3.3	22.5	5.1	3.3	22.5	29.4	68.3
120	73.3	−2.3	7.7	73.3	2.3	7.7	3	−1.6	16.7	3	1.6	16.7	24.4	70.3
300	72.9	−1.6	8.9	72.9	1.6	8.9	1.5	−0.8	9.8	1.5	0.8	9.8	18.7	71.4
500	72.8	−1.3	9.4	72.8	1.3	9.4	1	−0.5	6.7	1	0.5	6.7	16.1	71.8
Fixed	72	−1	11	72	1	11	—	—	—	—	—	—	11.0	72.0
Frame 1 Footing B														
10	72.4	−8.8	7.8	72.4	8.8	7.8	13.1	−8.3	16.2	13.1	8.3	16.2	24.0	59.3
50	72.6	−3.2	9.2	72.6	3.2	9.2	4.7	−2.4	7.6	4.7	2.4	7.6	16.8	67.9
120	72.5	−2.0	9.9	72.5	2.0	9.9	2.2	−1.1	3.8	2.2	1.1	3.8	13.7	70.3
300	72.4	−1.4	10.3	72.4	1.4	10.3	1	0.5	1.7	1	0.5	1.7	12.0	71.4
500	72.4	−1.2	10.4	72.4	1.2	10.4	0.6	0.3	1	0.6	0.3	1	11.4	71.8
Frame 1 Footing C														
0.1	54.4	−27.5	9.3	54.4	27.5	9.3	25.2	−27.5	9.9	25.2	27.5	9.9	19.2	29.2
1	60.2	−22.1	8.6	60.2	22.1	8.6	23.5	−22	13	23.5	22	13	21.6	36.7
10	70.8	−7.7	8	70.8	7.7	8	11.9	−7.1	13.6	11.9	7.1	13.6	21.6	58.9
50	72.4	−2.7	9.6	72.4	2.7	9.6	3.7	−1.9	5.3	3.7	1.9	5.3	14.9	68.7
120	72.4	−1.7	10.1	72.4	1.7	10.1	1.7	−0.8	2.5	1.7	0.8	2.5	12.6	70.7
300	72.4	−1.3	10.4	72.4	1.3	10.4	0.7	−0.3	1.1	0.7	0.3	1.1	11.5	71.7
500	72.3	−1.1	10.5	72.3	1.1	10.5	0.4	−0.2	0.6	0.4	0.2	0.6	11.1	71.9

[a] All coefficient values $\times 10^{-4}$.

are at the column-footing junction. It is seen that the mode shape is not qualitatively affected by footing size and mass.

The difference of the 1st and the 7th coefficients (or 4th and 10th) represents the net horizontal translation of the frame. This is found to increase with G_s increase and it determines the magnitude of the storey sway. It may be observed from the table that the super-storey nodal rotation (3rd and 6th coefficients) increases with G_s while the footing nodal rotation (9th and 12th coefficients) decreases. The algebraic sum of the 3rd and 9th coefficients (or 6th and 12th), which pertain to node rotations, is found to monotonically decrease as G_s increases. These net rotations, as reported earlier [15], tend to attenuate the forces generated by storey sway in the fundamental mode of vibration.

5. Conclusions

The response in terms of r.m.s. value of lowest storey displacements of an open-plane frame on isolated footings to a ground acceleration idealized by a white noise is found qualitatively affected by the model used to idealize the interactive system. The frame model yields a larger r.m.s. value of the lowest storey sway displacement than the shear building model in the non-interactive case. With soil–structure interaction, the frame model yields larger displacements than the Parmelee model.

For the frame model, the displacement ratio (defined as the ratio of the r.m.s. value of the lowest storey sway in the structure with soil–structure interaction to that for the fixed base structure) is found to be greater that unity. This ratio decreases with increase in G_s. It also decreases with increase in footing size for a given G_s and superstructure stiffness. The ratio also increases with increase in superstructure stiffness for any G_s and foundation size.

When the Parmelee model is used to represent the interactive system, the displacement ratio is always less than unity and tends to increase with G_s. The ratio is found to increase with increase in footing size for a given superstructure stiffness and G_s. Unlike the frame model, the displacement ratio decreases with increase in superstructure stiffness for any G_s and footing size.

The shear building model yields a larger value of the r.m.s. value of the lowest storey column shear than the frame model in the non-interactive case.

The lowest storey column shear ratio is found to less than unity for both the Parmelee model and the frame model (particularly when the latter represents realistically proportioned frames and footings). This ratio increases with G_s. For both models, the ratio is lower for stiffer superstructures on smaller footings.

Thus, while the responses of the two idealizations used to represent open-plane frames on isolated footings differ in terms of displacements, in terms of lowest storey column shear the two idealization yield qualitatively similar responses. It may be concluded, in terms of lowest storey column shear, disregard of soil–structure interaction in the analysis will be conservative.

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