Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Journal of Sound and Vibration 275 (2004) 757–768
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Jet noise prediction using the Lighthill acoustic analogy

R.H. Self*

Institute of Sound and Vibration Research, University of Southampton, Highfield, Southampton SO17 1BJ, UK

Received 3 March 2003; accepted 30 June 2003

Abstract

A jet noise source model based on the Lighthill acoustic analogy is presented. Although much of the theory used is well known, a new feature of the model is the inclusion of frequency dependence for the time and length scales used in the turbulence two-point correlation function. It is found that allowing for this experimentally observed dependence markedly improves the agreement of the model's prediction with experimental far-field data. To illustrate this agreement the case of a single turbulent jet is considered. Using well-respected scaling laws for the mean and turbulent properties of such jets a prediction for a single jet noise spectrum is obtained which shows very good agreement with the prediction using the empirically based ESDU database. The effect of altering the frequency dependence of the moving axis timescale is briefly discussed and it is indicated how the source model can be generalized to use RANS and other CFD data to predict jet noise, for single and coaxial jets and also for more novel nozzle geometries.

1. Introduction

Jet noise is a major factor in the overall noise produced by modern aero-engines and it is important to develop ways of reducing it further than has already been achieved. Towards this goal, it is equally important that reliable prediction schemes are developed, and this paper describes the basis for such a prediction scheme.

The first theoretical model applied to the prediction of jet noise was the acoustic analogy given by Lighthill [1,2] who rearranged the full equations of motion in the form of a linear wave equation, with equivalent acoustic sources that depend on the mean and turbulent flow fields. Lighthill was able to use the fact that simple turbulent jets at high Reynolds number obey well-known similarity laws to predict that, at low Mach numbers, the overall mean square pressure radiated from a jet should scale as the eighth power of the jet velocity. Since then knowledge of the

*Corresponding author. Tel.: +44-2380-597040; fax: +44-2380-593190.
E-mail address: rhs@isvr.soton.ac.uk (R.H. Self).

0022-460X/$ - see front matter
doi:10.1016/j.jsv.2003.06.020

scaling laws together with detailed measurements on datum jets has allowed accurate predictions of the radiated noise from other similar jets to be made without the need for a detailed knowledge of the equivalent acoustic sources. For single jets this has led to several semi-empirical prediction schemes such as that based on the ESDU database [3], while for coaxial jets, an extension of this methodology has been proposed by Fisher et al. [4,5] in their Four Source Model. The SAE Aerospace Recommended Practice ARP867D [6] gives a prediction methodology for both single and coaxial jets which is also reliant on databases. All such prediction schemes depend critically on the extent to which the mean flows and turbulent properties of different jets obey the same scaling laws, regardless of nozzle geometry.

Recently there has been considerable interest in the use of complex nozzle geometries specifically designed to alter the turbulent properties of the flow and thereby obtain acoustic benefits. In such cases simple scaling laws are unlikely to hold and the traditional semi-empirical prediction schemes will be ineffective without a large (and impractical) amount of data being collected. In any case, a purely empirical scheme of this type offers no insight to the nozzle designer who wishes to reduce jet noise levels. To move forward it is necessary to measure, or model in a rational manner, both the mean and turbulent properties of the flow so that a realistic estimate of the equivalent acoustic sources can be made, as well as the influence of the mean flow on the propagation of the resulting acoustic waves through and out of the jet.

In principle the entire jet noise prediction could be accomplished by a full unsteady CFD/CAA calculation, but the complexities associated with such a task for high Reynolds number jet flows mean that this is unlikely to be practical for several years to come. One possible resolution of these difficulties is to use a relatively fast-running CFD code, such as a Reynolds-averaged Navier–Stokes (RANS) scheme, to generate input data for acoustic source and propagation models.

Coupling an acoustic source model to a steady flow prediction is not new and was considered as long ago as 1977 by Balsa and Gliebe [7] and Mani et al. [8]. Their scheme is generally referred to as the MGB method and has been extended by Khavaran [9,10] to use a RANS solution based on a k–ε turbulence model (the MGBK method). Recently Tam and Auriault [11] have also used a k–ε turbulence model with a RANS solver to provide the inputs for an acoustic source model; the subsequent propagation of sound was described by solving the linearized Euler equations. The resulting predictions are claimed to be in good agreement with measured data. However, while the MGB, the MGBK, and similar approaches all either use Lighthill's acoustic analogy, or variants such as the Lilley [12] formulation of the acoustic analogy, Tam and Auriault use an apparently novel source model which they develop in analogy to the kinetic theory of gases.

The fact that Tam and Auriault's prediction scheme achieves a higher degree of agreement with measured data than some others might suggest that the novel source model they employ has advantages over the acoustic analogy models. However, a critical comparison between the model of Tam and Auriault and models based on the acoustic analogy has been made by Morris and Farassat [13]. These authors argue that the differences in prediction do not arise because of any fundamental flaw in the acoustic analogy, but because of the different models used for statistical description of the turbulent noise sources. They show that, at 90° to the jet axis, the model of Tam and Auriault gives an identical noise prediction to a model based on the acoustic analogy, provided a consistent statistical description of the turbulence is used. In both models, the far-field radiation depends on the two-point covariance of the equivalent source term which must be modelled separately. As pointed out both by Morris and Farassat, and other authors (see for example, Woodruff et al. [14,15]), the form of the cross-correlation function is central to obtaining an accurate prediction of the radiated noise spectrum. In this paper a new model for the cross-correlation of the unsteady Reynolds stresses is considered. It is shown that the resulting noise model gives improved agreement with measured data.

2. Expression for the acoustic intensity spectrum

The jet noise model below is based on the Lighthill acoustic analogy [1,2], using the self-noise source terms and disregarding the shear-noise source terms. This circumvents the problem of flow–acoustic interaction but means an accurate prediction of the far-field pressure distribution is limited to an angle of 90° to the jet axis, where such effects are unimportant. The basic theory is outlined briefly in this section. Next the frequency dependence of various parameters describing the turbulence is discussed in Section 3, and expressions are given that are consistent with the known measurements. Since the main aim of this preliminary paper is to indicate the importance of including such frequency dependence, the application in Section 4 is restricted to simple turbulent single jets, and rather than using CFD data, well-respected models for the mean velocity, overall turbulence levels, etc. for such jets are used. This is very much in keeping with the spirit of engineering modelling and the resulting far-field prediction is in good agreement with measurement. Final results are presented in Section 5 together with a brief discussion.

Following Goldstein [16] an expression for the far-field acoustic intensity spectrum can be obtained from Lighthill's equation [1,2] as follows (see Fig. 1):

$$I_\omega(\mathbf{x}) = \frac{1}{32\pi^3\rho_0 c_0^5}\frac{\omega^4}{x^2}\int\int \mathscr{S}(\mathbf{y},\boldsymbol{\eta},\omega)\mathrm{e}^{-\mathrm{i}\omega\frac{\hat{\mathbf{x}}\cdot\boldsymbol{\eta}}{c_0}}\,\mathrm{d}^3\boldsymbol{\eta}\mathrm{d}^3\mathbf{y}. \tag{1}$$

Here ρ_0 and c_0 are the ambient pressure and speed of sound respectively and, using overbars to denote time averages,

$$\mathscr{S}(\mathbf{y},\boldsymbol{\eta},\omega) = \int_{-\infty}^{+\infty}\overline{T_{xx}(\mathbf{y},t)T_{xx}(\mathbf{y}+\boldsymbol{\eta},t+\tau)}\mathrm{e}^{-\mathrm{i}\omega\tau}\,\mathrm{d}\tau = \int_{-\infty}^{+\infty}R(\mathbf{y},\boldsymbol{\eta},\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\,\mathrm{d}\tau, \tag{2}$$

is the Fourier transform of the two-point correlation of the Lighthill stress tensor components in the direction of the far-field observer [17]:

$$R(\mathbf{y},\boldsymbol{\eta},\tau) = \overline{T_{xx}(\mathbf{y},t)T_{xx}(\mathbf{y}+\boldsymbol{\eta},t+\tau)}. \tag{3}$$

For an isothermal jet, as discussed by Morris and Farassat [13], it is reasonable to approximate the Lighthill stress tensor components in Eq. (3) by the Reynolds stress

$$T_{xx} = \rho_s u_x u_x, \tag{4}$$

where ρ_s is the mean density in the source region (which can be taken as equal to the ambient density ρ_0) and u_x is the turbulent velocity fluctuation in the direction of the far-field observer. It is usual to assume that the two-point space–time correlation function takes the form

$$R(\mathbf{y},\boldsymbol{\eta},\tau) = u^4\hat{R}(\mathbf{y},\boldsymbol{\eta},\tau), \tag{5}$$

where u is a velocity characteristic of the turbulence and $\hat{R}(\mathbf{y},\boldsymbol{\eta},\tau)$ is a normalized "shape function". The latter is often taken to have a form similar to that of the normalized velocity

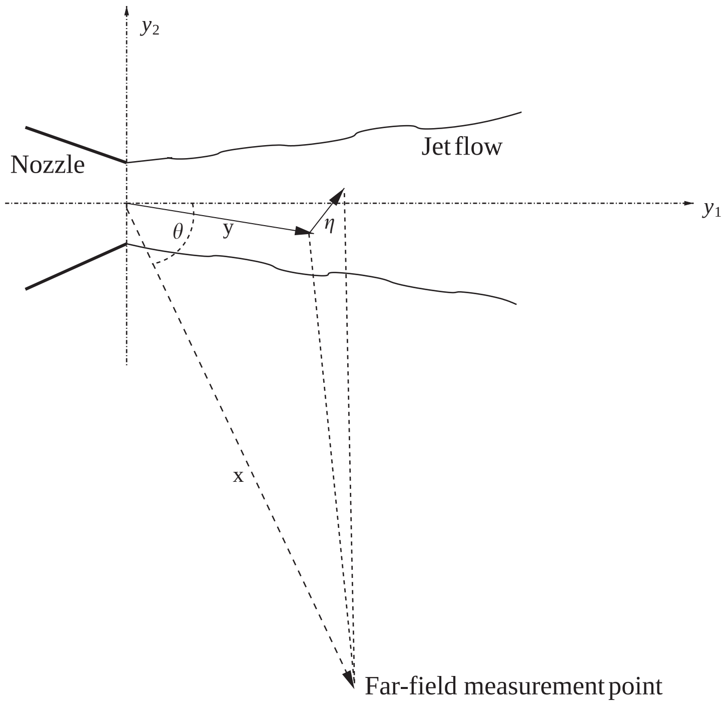

Fig. 1. Co-ordinate system used in the calculations.

correlation as measured, for instance, by Fisher and Davies [18]. This is not as gross an assumption as it might first appear so long as all parameters defining the correlation are appropriately scaled. For instance, if turbulent velocities are correlated over a time τ_s then one may well expect turbulent shear stresses to be correlated over a time of approximately $\frac{1}{2}\tau_s$.

The exact form of the function chosen to model the two-point correlation has not been considered crucial so long as it reflects the overall features of the actual correlation function. In this case several different forms can be chosen. If the turbulence is assumed to be Gaussian when measured in a reference frame moving with the convection velocity defined by the local mean velocity of the jet, then, transforming to a fixed frame, it would be expected that $\hat{R}(\mathbf{y}, \boldsymbol{\eta}, \tau)$ has the form

$$\hat{R}(\mathbf{y}, \boldsymbol{\eta}, \tau) = \exp\left\{ -\frac{\tau^2}{\tau_s^2} - \left(\frac{\eta_1 - \tau U_c}{l_1}\right)^2 - \left(\frac{\eta_2^2}{l_2^2} + \frac{\eta_3^2}{l_3^2}\right) \right\}. \tag{6}$$

Here τ_s defines the moving-axis timescale and is the characteristic time over which a turbulent eddy remains correlated; l_1, l_2 and l_3 are three lengthscales characterizing the size of the eddies (i.e., they are characteristic correlation lengths in the three axial directions); and $L_s = U_c\tau_s$ defines the moving-axis lengthscale. This latter parameter characterizes the distance an eddy travels in the time τ_s due to its convection with velocity U_c.

Choosing such a functional form for the two-point space–time correlation function is not new, and generally the predicted noise does not agree well with the measured data (see for example Ref. [13]). It is demonstrated below that this poor agreement is due to an erroneous assumption that the characteristic time and lengthscales are independent of frequency at a fixed point in the jet. However, in order to include this dependence it is somewhat easier to model the cross-spectral density function directly, rather than the space–time correlation function. The model functional form taken in the present study is

$$\mathcal{S}(\mathbf{y},\boldsymbol{\eta},\omega) = u^4 \hat{\mathcal{S}}(\mathbf{y},\boldsymbol{\eta},\omega), \tag{7}$$

where

$$\hat{\mathcal{S}}(\mathbf{y},\boldsymbol{\eta},\omega) = 2\sqrt{\pi}\,\tau_s \exp\left\{-\frac{\omega^2(1-M_c\cos\theta)^2\tau_s^2}{4}\right\}\exp-\left\{\frac{\eta_1^2}{l_1^2}+\frac{\eta_2^2}{l_2^2}+\frac{\eta_3^2}{l_3^2}\right\}, \tag{8}$$

and where the moving-axis timescale is now to be considered a function of frequency, $\tau_s = \tau_s(\omega)$. (The exact dependence is discussed in the next section.)

There are two reasons for choosing this particular functional form. Firstly, in the case where τ_s is taken to be independent of frequency, the final result for the far-field noise spectrum will reduce to that which is obtained by taking the Fourier transform of Eq. (6). Secondly, Eq. (8) predicts a Gaussian decay with spatial separation similar to that measured by Fisher and Davies [18].

After substituting Eqs. (7) and (8) into Eq. (1), it is straightforward to evaluate the inner integral, and $I_\omega(\mathbf{x})$ can be written as an integral of a source distribution over the jet:

$$I_\omega(\mathbf{x}) = \frac{1}{32\pi^3\rho_0 c_0^5}\frac{\omega^4}{x^2}\int q(\mathbf{y},\omega)\,\mathrm{d}^3\mathbf{y}, \tag{9}$$

where the source distribution, $q(\mathbf{y},\omega)$, is given by

$$q(\mathbf{y},\omega) = 2\pi l_{\|} l_{\perp}^2 \tau_s u^4 \exp\left\{-\frac{\omega^2(l_{\|}^2\cos^2\theta + l_{\perp}^2\sin^2\theta)}{4c_0^2}\right\}\exp\left\{-\frac{\omega^2(1-M_c\cos\theta)^2}{4U_c^2}\right\} \tag{10}$$

and where one has written $l_{\|} = l_1$ and assumed that $l_2 = l_3 = l_{\perp}$.

3. Modelling the moving-axis timescale

A crucial factor which governs turbulent noise generation is the rate at which the turbulence loses coherence as it is convected downstream by the mean flow of the jet [18]. Other authors have generally assumed that the turbulent eddy lifetime, or moving-axis timescale, τ_s, is inversely proportional to the local mean shear and independent of frequency. Such an assumption is at odds with the measurements by Fisher and Davies [18] for turbulent velocity fluctuations, and the measurements by Harper-Bourne [19] for Reynolds stresses. Both these studies indicate that the assumption of a constant moving-axis timescale is invalid. (They also indicate that other properties of the turbulence which are generally considered constant, such as the eddy convection velocity U_c, also vary with frequency, but to a smaller extent.)

The moving-axis timescale is related to the moving-axis lengthscale by $\tau_s = L_s/U_c$, and measurements of L_s have been made recently by Harper-Bourne [19] for a single isothermal jet at a fixed axial station a distance of four diameters downstream of the nozzle and in the centre of the shear layer. His results indicate that for low frequencies the assumption of a constant moving-axis lengthscale is reasonable, but that for higher frequencies a nearly inverse dependence on Strouhal number is obtained, as shown in Fig. 2. These results are fitted well by the analytic function

$$L_s/D = 1/(1 + 0.5St), \tag{11}$$

where D is the nozzle diameter and $St = fD/U_j$ with U_j being the jet exit velocity. Clearly, however, Eq. (11) can only apply at the particular location of Harper–Bourne's measurements. Consequently, if a relationship of this type is to be incorporated into the acoustic model, it is necessary to decide on a rational generalization of it which will apply at all other positions in the jet.

In jet-noise modelling it is often permissible to consider the jet as a line distribution of acoustic sources. Strictly the density of this line source at any axial position is obtained by integrating over the cross-section of the jet, but it is common to assume an adequate measure of density is given by considering the maximum turbulence level within the shear layer, weighted by the cross-sectional area of that shear layer. Since relationship (11) applies at the centre of the shear layer, which is the point of maximum turbulence, such an approximation is reasonable and will be adequate for the needs of this paper. Adopting this approach means that radial variations of the moving-axis lengthscale need not be considered further and Eq. (11) need only be generalized to other axial positions where it can be taken as characteristic of the shear layer as a whole. A suitable generalization is

$$L_s/W = c_1/(1 + \omega/\omega_c); \tag{12}$$

here

$$\omega_c = 2\pi c_2 U_1/W, \tag{13}$$

where W is the local shear layer width, and U_1 is the centreline jet velocity, Fig. 3. Effectively this amounts to assuming a dependence on a local Strouhal number evaluated at the current axial position and defined in terms of U_1 and W, as opposed to exit velocity U_j and nozzle diameter D. The values of the constants c_1 and c_2 depend on the ratio of W to D, and U_1 to U_j, at the axial station where Harper–Bourne's measurements were made. Assuming this was at the end of the potential core, and using the jet model discussed below, one would expect that $c_1 = 1$ and $c_2 = 2$ if

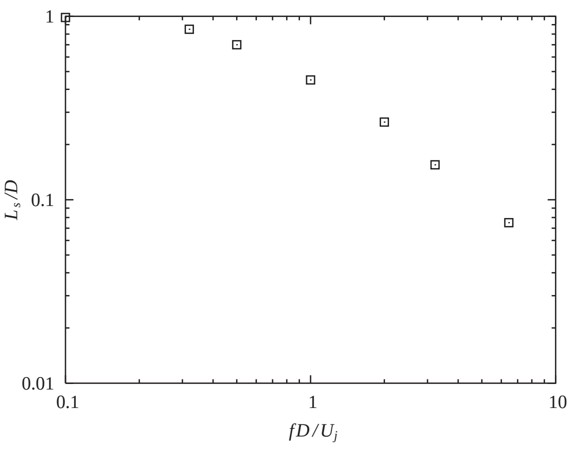

Fig. 2. Moving axis length scale as measured by Harper–Bourne [19].

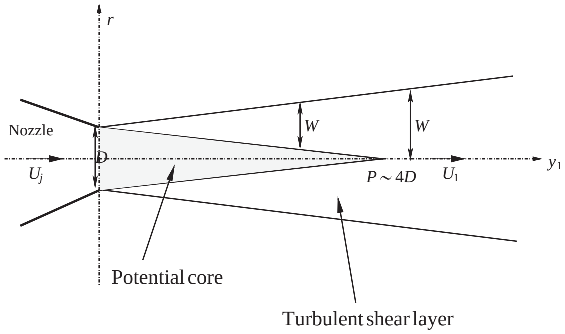

Fig. 3. Schematic of the structure of a simple, single nozzle turbulent jet.

Eqs. (11) and (13) are to be consistent. The actual values used in the calculations below are $c_1 = 1.1$ and $c_2 = 2.15$.

4. Application to a model single jet

The eventual aim is to compare the predictions of the noise source model with spectra of far-field intensity measured at 90° to the jet axis. For this purpose the model will be applied to a single-stream isothermal jet. The jet will be considered as a line source, with an axially varying source density proportional to a (characteristic) maximum turbulence level within the shear layer weighted by the cross-sectional area of that shear layer. Thus the three-dimensional integral of Eq. (9) will be approximated as a single integral over the jet axis by

$$I_\omega(\mathbf{x}) = \frac{1}{32\pi^3 \rho_0 c_0^5} \frac{\omega^4}{x^2} \int A(y_1) q(y_1, \omega)\, \mathrm{d}y_1, \tag{14}$$

where $A(y_1)$ is the local cross-sectional area of the shear layer and $q(y_1, \omega)$ is an equivalent axial source distribution obtained from a one-dimensional equivalent of Eq. (10). In this expression the moving-axis timescale will be obtained as in Section 3, but the remaining parameters, as well as the area $A(y_1)$, are yet to be modelled explicitly.

Although all these remaining source parameters could be estimated using RANS derived data, the fact that an axial source model is being used lends itself to a somewhat simpler method of estimation based on some of the well-known relationships for single jets as given, for example, by Abramovich [20] and Townsend [21].

Simple turbulent jets in which a single uniform stream with speed U_j leaves a circular nozzle and spreads into a stationary region of the same fluid are reasonably well understood. There are three distinct regions to the jet. Immediately downstream of the nozzle there is an annular shear layer surrounding a potential core within which the flow is assumed to be laminar. In this region the jet is self-similar and the shear layer grows linearly in width. At the end of the potential core there is a transition region where the centreline velocity begins to decrease and flow adapts and eventually changes to define the third region some distance downstream. In this final, fully mixed, region the flow is again self-similar. Because similarity solutions exist, the initial and fully mixed regions can both be modelled analytically. For the transition region it is necessary to smoothly match the two similarity solutions in as physically reasonable a way as possible. Based on results by Abramovich [20] the remaining properties of the jet which need to be established will be taken as follows.

Referring to Fig. 3, the potential core length, P, and shear layer width, W, are taken as

$$P = 4D, \quad W = y_1 D/P, \tag{15, 16}$$

giving the area of the shear layer as

$$A = \begin{cases} \pi D W, & y_1 < P, \\ \pi W^2, & y_1 > P. \end{cases} \tag{17, 18}$$

In the initial region the mean centreline velocity, U_1, is constant and equal to the exit velocity U_j, while far downstream in the fully mixed region it decreases as

$$U_1/U_j = 2P/(P + y_1), \quad y_1 > P. \tag{19}$$

which has the correct $1/y_1$ behaviour as $y_1 \to \infty$. In the transition region U_1 was obtained by numerical matching these two solutions as shown in Fig. 4. The eddy convection velocity is given by $U_c = 0.6U_1$, which is consistent with experimental findings [18,19].

The overall turbulence level used in calculating $q(y_1, \omega)$ is proportional to the mean centreline velocity and a fixed percentage of 15% was chosen for this intensity (so $u = 0.15U_1$). However, since the overall level of the noise was later adjusted with an arbitrary constant the assumed value of 15% is unimportant; the main point is that in this model it is assumed constant over the noise producing regions of the jet.

Lastly, the eddy lengthscales $l_{\parallel}$ and $l_{\perp}$ were taken as $l_{\parallel} = 0.3W$ and $l_{\perp} = \beta l_{\parallel}$. The "aspect ratio" β allows for stretching of the eddies by the mean shear and was taken as 0.3. Like the turbulence intensity this is a somewhat nominal value since the spectrum is computed only for 90° to the axis.

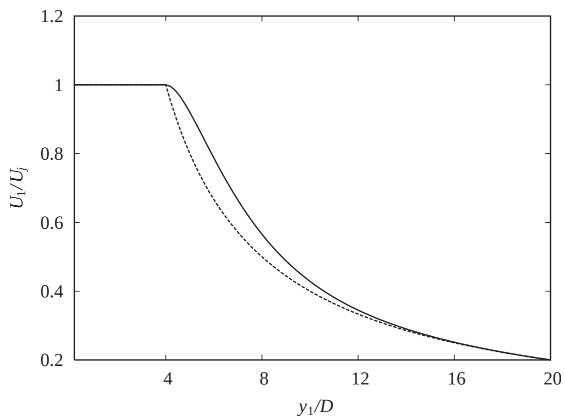

Fig. 4. Mean velocity profile used in calculation compared with the similarity solution $U_1 \sim U_j/y_1$: —, mean velocity profile used in calculations; - - -, $U_1 \sim U_j/y_1$.

5. The predicted acoustic spectrum

The model developed above does not take flow acoustic interaction into account and will therefore be unable to predict the spectrum at any angles apart from 90° to the jet axis. Even so a comparison with a known spectrum at this one angle will be a useful test as to the importance of including the frequency dependence of the moving axis timescale in prediction models.

An acoustic intensity spectrum to compare with the model's prediction was obtained using the ESDU prediction scheme which uses a known database and scales on Strouhal number. Thus so long as the model described above scales with Strouhal number, agreement with the ESDU prediction at any one set of jet conditions will necessarily imply agreement at all other conditions. It was first confirmed that the model spectra for several different jet conditions did indeed correctly collapse on Strouhal number and a comparison was then made with the ESDU derived spectrum.

The jet model and parameter values discussed in the previous sections resulted in a predicted spectrum in very good agreement with that obtained from the ESDU program. The calculation was made at a nominal distance from the jet nozzle and an additive constant was used to adjust the overall level of the model prediction. This result is shown in Fig. 5.

Next the effect of altering the frequency dependence of the moving-axis timescale was considered. This was investigated by changing the value of the parameter c_2 in the generalized Harper–Bourne model of Eqs. (12) and (13). As c_2 increases, the moving-axis timescale τ_s remains approximately constant up to higher frequencies and the model more closely predicts that which would be obtained using the overall two-point correlation model of Eq. (6). This results in a more nearly frozen pattern of turbulence and the overall level is reduced accordingly, as Fig. 6 shows. However, one also notes that the predicted peak occurs at a lower frequency and that the high-frequency fall-off is considerably enhanced. If a readjustment of parameters is made to force agreement of the peak frequency with that of the ESDU spectrum then far too narrow a spectrum results.

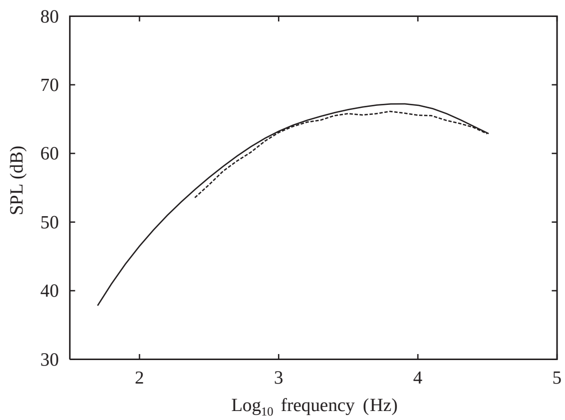

Fig. 5. Predicted far-field acoustic spectrum at to the jet axis obtained using the model (solid line —) compared with that obtained using the ESDU scheme (dashed line - - -). Jet nozzle diameter $D = 0.0254$ m, jet exit Mach number is $M_j = 0.67$.

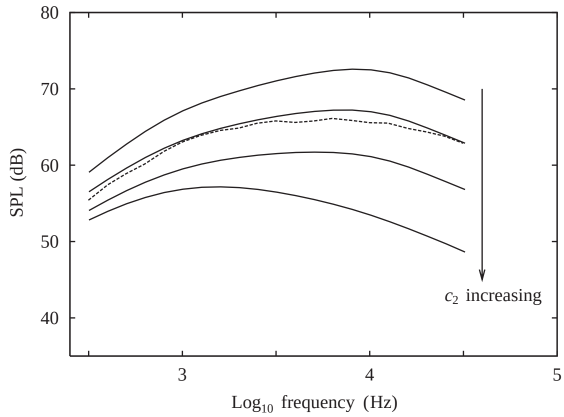

Fig. 6. Effect of altering the parameter c_2. As c_2 increases the range of frequencies over which the moving axis timescale is approximately constant increases and the turbulence becomes increasingly "frozen". (The ESDU predicted spectrum is shown as the dashed curve.)

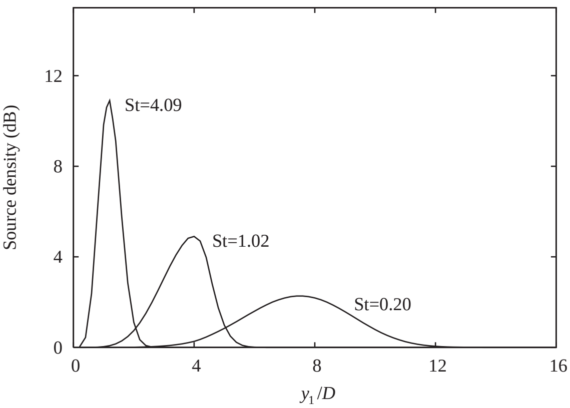

Fig. 7. Normalized source distributions obtained from the model calculations.

Next, the corresponding source distributions were examined by calculating $q(y_1, \omega)$ at fixed frequencies as a function of distance downstream of the nozzle y_1. Fig. 7 shows the results obtained at three different Strouhal numbers. The positions of the source centroids compare well with those obtained experimentally (with the centroid of the $St = 1.0$ source distribution being roughly at the end of the potential core). The shapes of the source distributions do not compare so well with as the overall spectrum, and are more symmetrical than may be expected, but this is probably a reflection of the simplicity of the model used for the mean jet velocity profile. It has been found that the mean velocity field just downstream of the potential core has a significant effect on the noise produced.

6. Discussion

As long ago as 1963 the work of Fisher and Davies [18] indicated that the moving-axis timescale as measured using velocity correlations varied with frequency. This has now been confirmed for stress correlations by the work of Harper–Bourne [19] whose experimentally observed relationship, as modelled by Eq. (11) and its generalization, can be given a straightforward (if somewhat simplistic) physical explanation. Turbulent eddies which are small compared to the shear layer width are unaware of the boundaries of that layer, and see a uniform shear. Consequently they decay on a timescale proportional to that shear. Large eddies, on the other hand, are constrained by the boundaries of the shear layer and decay more rapidly.

In this paper a model for acoustic sources for a simple turbulent jet has been obtained which includes the frequency dependence of the moving-axis timescale. Choosing a particular functional form for the turbulence spectrum and using a generalization of the results obtained by Harper–Bourne together with a simplified model jet, predictions have been made and compared with those obtained from the ESDU database resulting in good agreement. This good agreement demonstrates the crucial importance of incorporating the frequency dependence of the moving-axis timescale in any source model of this kind, and this is especially true if high frequencies are to be accurately predicted.

Equally, the simplicity of the model used for the jet aerodynamics indicates that the prediction scheme described is extremely robust and will serve as a useful starting point for more comprehensive engineering models using RANS or other CFD data as input, and for more practical jet nozzle geometries. As described above the jet is modelled as a line source so any extension will require suitable generalisation. For example, Eqs. (12) and (13) need to be written using parameters which are characteristic of each point in the shear layer rather than characteristic of the shear layer as a whole.

The model advanced in this paper is only valid at 90° to the jet axis and it is clearly necessary to extend it to arbitrary angles by including flow-acoustic interaction terms. This could be achieved by adapting the theory for use with the Lilley analogy [12]. It is also noted that, as a by-product, this scheme also yields important information about the spatial and spectral distribution of the acoustic sources, thus making geometric near-field predictions an attainable possibility. Finally the predicted source distributions obtained from the model are physically reasonable.

References

[1] M.J. Lighthill, On sound generated aerodynamically: Part 1. General theory, *Proceedings of the Royal Society of London A* 211 (1952) 564–587.

[2] M.J. Lighthill, On sound generated aerodynamically: Part 2. Turbulence as a source of sound, *Proceedings of the Royal Society of London A* 214 (1954) 1–32.

[3] ESDU International plc, ESDU 98019 and software B9819, Computer-based estimation procedure for single-stream jet noise, 1998.

[4] M.J. Fisher, G.A. Preston, W.D. Bryce, A modelling of the noise from coaxial jets, Part 1: with unheated primary flow, *Journal of Sound and Vibration* 209 (1998) 385–403.

[5] M.J. Fisher, G.A. Preston, C.J. Mead, A modelling of the noise from coaxial jets, Part 2: with heated primary flow, *Journal of Sound and Vibration* 209 (1998) 405–417.

[6] Anon, SAE aerospace recommended practice ARP876D, Gas turbine exhaust noise prediction, Society of Automotive Engineers Inc., Warrendale, PA, 1994.

[7] T.F. Balsa, P.R. Gliebe, Aerodynamics and noise of coaxial jets, *American Institute of Aeronautics and Astronautics Journal* 15 (1977) 1550–1558.

[8] R. Mani, T.F. Balsa, P.R. Gliebe, High-velocity jet noise source location and reduction, Task 2, Federal Aviation Administration Report, FAA-RD-76-II, 1978.
[9] A. Khavaran, Computation of supersonic jet mixing noise for an axisymmetric convergent-divergent nozzle, *American Institute of Aeronautics and Astronautics Journal of Aircraft* 31 (1993) 603–612.
[10] A. Khavaran, Role of anisotropy in turbulent mixing noise, *American Institute of Aeronautics and Astronautics Journal* 37 (1999) 832–841.
[11] C.K.W. Tam, L. Auriault, Jet mixing noise from fine-scale turbulence, *American Institute of Aeronautics and Astronautics Journal* 37 (1999) 145–153.
[12] G.M. Lilley, The generation and radiation of supersonic jet noise, Vol. IV – theory of turbulence generated noise, noise radiation from upstream sources, and combustion noise. Part II: generation of sound in a mixing region, Air Force Aero Propulsion Laboratory Technical Report 72–53, 1972.
[13] P.J. Morris, F. Farassat, Acoustic analogy and alternative theories for jet noise prediction, *American Institute of Aeronautics and Astronautics Journal* 40 (2002) 671–680.
[14] S.L. Woodruff, J.M. Seiner, M.Y. Hussaini, G. Erlebacher, Evaluation of turbulence-model performance in jet flows, *American Institute of Aeronautics and Astronautics Journal* 39 (2001) 2402–2404.
[15] S.L. Woodruff, J.M. Seiner, M.Y. Hussaini, G. Erlebacher, Implementation of new turbulence spectra in the Lighthill analogy source terms, *Journal of Sound and Vibration* 242 (2001) 197–214.
[16] M.E. Goldstein, *Aeroacoustics*, McGraw-Hill, New York, 1976.
[17] I. Proudman, The generation of noise by isotropic turbulence, *Proceedings of the Royal Society of London A* 214 (1952) 119–132.
[18] M.J. Fisher, P.O.A.L. Davies, Correlation measurements in a non-frozen pattern of turbulence, *Journal of Fluid Mechanics* 18 (1963) 97–116.
[19] M. Harper-Bourne, Jet near field noise prediction, AIAA Paper No. 99-1838, 1999.
[20] G.N. Abramovich, *Theory of Turbulent Jets*, MIT Press, Cambridge, MA, 1963.
[21] A.A. Townsend, *The Structure of Turbulent Shear Flow*, 2nd Edition, Cambridge University Press, Cambridge, 1976.